An ACI Manual

ACI Reinforced Concrete Design Handbook

A Companion to ACI 318-19

Volume 1: Member Design

MNL-17(21)

American Concrete Institute
Always advancing

ACI MNL-17(21)

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DEDICATION

This edition of *The ACI Reinforced Concrete Design Handbook*, MNL-17(21), is dedicated to the memory of Daniel W. Falconer and his many contributions to the concrete industry. He was Managing Director of Engineering for the American Concrete Institute from 1998 until his death in July 2015.

Dan was instrumental in the reorganization of "Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)" as he served as ACI staff liaison to ACI Committee 318, Structural Concrete Building Code; and ACI Subcommittee 318-SC, Steering Committee. His vision was to simplify the use of the Code for practitioners and to illustrate the benefits of the reorganization with MNL-17. His oversight and review comments were instrumental in the development of the ninth edition of the Handbook.

An ACI member since 1982, Dan served on ACI Committees 344, Circular Prestressed Concrete Structures, and 373, Circular Concrete Structures Prestressed with Circumferential Tendons. He was also a member of the American Society of Civil Engineers. Prior to joining ACI, Dan held several engineering and marketing positions with VSL Corp. Before that, he was Project Engineer for Skidmore, Owings, and Merrill in Washington, DC. He received his BS in civil engineering from the University at Buffalo, Buffalo, NY and his MS in civil and structural engineering from Lehigh University, Bethlehem, PA. He was a licensed professional engineer in several states.

In his personal life, Dan was an avid golfer, enjoying outings with his three brothers whenever possible. He was also an active member of Our Savior Lutheran Church in Hartland, MI, and a dedicated supporter and follower of the Michigan State Spartans basketball and football programs. Above all, Dan was known as a devoted family man dedicated to his wife of 33 years, Barbara; his children Mark, Elizabeth, Kathryn, and Jonathan; and two grandsons, Samuel and Jacob.

In his memory, the ACI Foundation has established an educational memorial. For more information visit http://www.scholarshipcouncil.org/Student-Awards. Dan will be sorely missed for many years to come.

FOREWORD

The ACI Reinforced Concrete Design Handbook provides assistance to professionals engaged in the design of reinforced concrete buildings and related structures. This edition is a major revision that brings it up-to-date with the approach and provisions of "Building Code Requirements for Structural Concrete" (ACI 318-19).

The ACI Reinforced Concrete Design Handbook provides dozens of design examples of various reinforced concrete members, such as one- and two-way slabs, beams, columns, walls, diaphragms, footings, and retaining walls. For consistency, many of the numerical examples are based on a fictitious seven-story reinforced concrete building. There are also many additional design examples not related to the design of the members in the seven-story building that illustrate various ACI 318-19 requirements.

Each example starts with a problem statement, then provides a design solution in a three-column format—Code provision reference, short discussion, and design calculations—followed by a drawing of reinforcing details, and finally a conclusion elaborating on a certain condition or comparing results of similar problem solutions.

In addition to examples, almost all chapters in *The ACI Reinforced Concrete Design Handbook* contain a general discussion of the related ACI 318-19 chapter.

This edition of *The ACI Reinforced Concrete Design Handbook* was updated and enhanced by ACI staff engineers under the auspices of the ACI Technical Activities Committee (TAC). Each chapter was reviewed by at least two reviewers, who provided valuable comments, suggestions, and insights. The following reviewers are gratefully acknowledged and thanked:

Michael E. Ahern	Christopher C. Ferraro	Ian S. McFarlane	Brandon Ross
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Special thanks are due to a number of outside contributors to this Manual. Dirk Bondy and Kenneth Bondy provided software used to analyze and design the post-tensioned beam example, in addition to their valuable comments and suggestions. StructurePoint and Computers and Structures, Inc. (SAP 2000 and Etabs) provided use of their software to perform analyses of structure and members. The Bridge Software Institute (BSI) provided use of their software and their expertise in the development of the design examples on deep foundations.

The ACI Reinforced Concrete Design Handbook is published in two volumes: Chapters 1 through 11 are published in Volume 1 and Chapters 12 through 15 are published in Volume 2. Design aids and a moment interaction diagram Excel spreadsheet are available for free download from the following ACI webpage links:
https://www.concrete.org/MNL1721Download1
https://www.concrete.org/MNL1721Download2

Keywords: anchoring to concrete; beams; columns; cracking; deflection; diaphragm; durability; flexural strength; footings; frames; pile caps; piles; post-tensioning; punching shear; retaining wall; shear strength; seismic; slabs; splicing; stiffness; structural analysis; structural systems; strut-and-tie; walls.

Trey Hamilton
Managing Editor

VOLUME 1: CONTENTS

CHAPTER 1—INTRODUCTION

1.1—Introduction

This Manual is intended to assist with the design of reinforced concrete structures using ACI 318-19 (hereinafter referred to as the Code). The focus is on the application of the Code requirements to the individual members with respect to both structural design requirements and detailing provisions. As with the Code, the design procedures and detailing practices illustrated in this Manual do not replace sound professional judgment or the licensed design professional's (LDP's) knowledge of the specific factors surrounding a project.

To illustrate the procedures and details, it is necessary to generate the member actions for which the design will be conducted. Although this Manual provides background and context regarding the analysis of structural concrete systems, it is assumed that the user of this Manual has a basic understanding of structural analysis and the development of the member design actions from such an analysis.

This chapter describes the overall organization of this Manual and additionally describes the loads, geometry, and other details of the example building used to generate actions for member or component design illustrated in subsequent chapters.

1.2—Organization and use

A structural system consists of members, joints, and connections, each performing a specific role or function. Structural systems and their component members must provide sufficient stability, strength, and stiffness so that overall structural integrity is maintained, design loads are resisted, and serviceability limits are met.

This Manual is organized into chapters listed below that follow the general progression of the structural design of a building. The early chapters describe the overall building configuration, loads, and development of actions from structural analysis followed by chapters devoted to the design of the individual members within the example structure.

(a) Horizontal floor and roof members (one-way and two-way slabs, Chapters 7 and 8)

(b) Horizontal support members (beams and joists, Chapter 9)

(c) Vertical members (columns and structural walls, Chapters 10 and 11)

(d) Diaphragms and collectors (Chapter 12)

(e) Foundations—isolated footings, mats, pile caps, and piles (Chapter 13)

(f) Plain concrete—unreinforced foundations, walls, and piers (Chapter 14)

(g) Joints and connections (Chapters 15 and 16)

In Table 1.2, Code chapters are correlated with the chapters in Volumes 1 and 2 of this Manual.

The fictitious example building depicted in Fig. 1.2a through 1.2d was created to demonstrate how, by various examples in this Manual, to design and detail a typical structural concrete building according to the Code. This example building is seven stories above ground and has a one-story basement. The building has evenly spaced columns along the grid lines in both directions. One column has been removed along Grid C on the second level to provide open space for the lobby. The building dimensions are:

- Width (north/south) = 72 ft (5 bays @ 14 ft)
- Length (east/west) = 218 ft (6 bays @ 36 ft)
- Height (above ground) = 92 ft
- Basement height = 10 ft

The basement is used for storage, building services, and mechanical equipment. It is 10 ft high and has an extra column in every bay along Grids A through F to support a two-way slab at the second level. There are basement walls at the perimeter.

A single specific gravity load system is not specified herein but rather is left unknown to enable demonstration of the design of several structural systems including nonprestressed and prestressed one-way beam and slab systems; nonprestressed and prestressed two-way slab systems; nonprestressed and prestressed transfer girder to accommodate column removal; and nonprestressed and prestressed beams of various types and sizes. Lateral loads are resisted by concrete shear walls in the north/south direction and concrete moment frames in the east/west direction; both systems are designated as *ordinary* for the purposes of seismic design and detailing. In some cases, member examples are expanded to demonstrate the change in design and detailing procedures when elements or systems are designated as *intermediate* or *special*, but using the results from the original structural

Table 1.2—Member chapters

Volume No. ACI MNL-17(21)	Chapter name ACI MNL-17(21)	Chapter No. ACI 318-19	Chapter No. ACI MNL-17(21)
I	Building system	—	1
	Structural systems	4 and 5	2
	Structural analysis	6	3
	Durability	19	4
	One-way slab	7	5
	Two-way slab	8	6
	Beams	9	7
	Diaphragm	12	8
	Columns	10	9
	Walls	11	10
	Foundations	13	11
II	Retaining walls	7 and 13	12
	Serviceability	24	13
	Strut and tie	23	14
	Anchoring to concrete	17	15

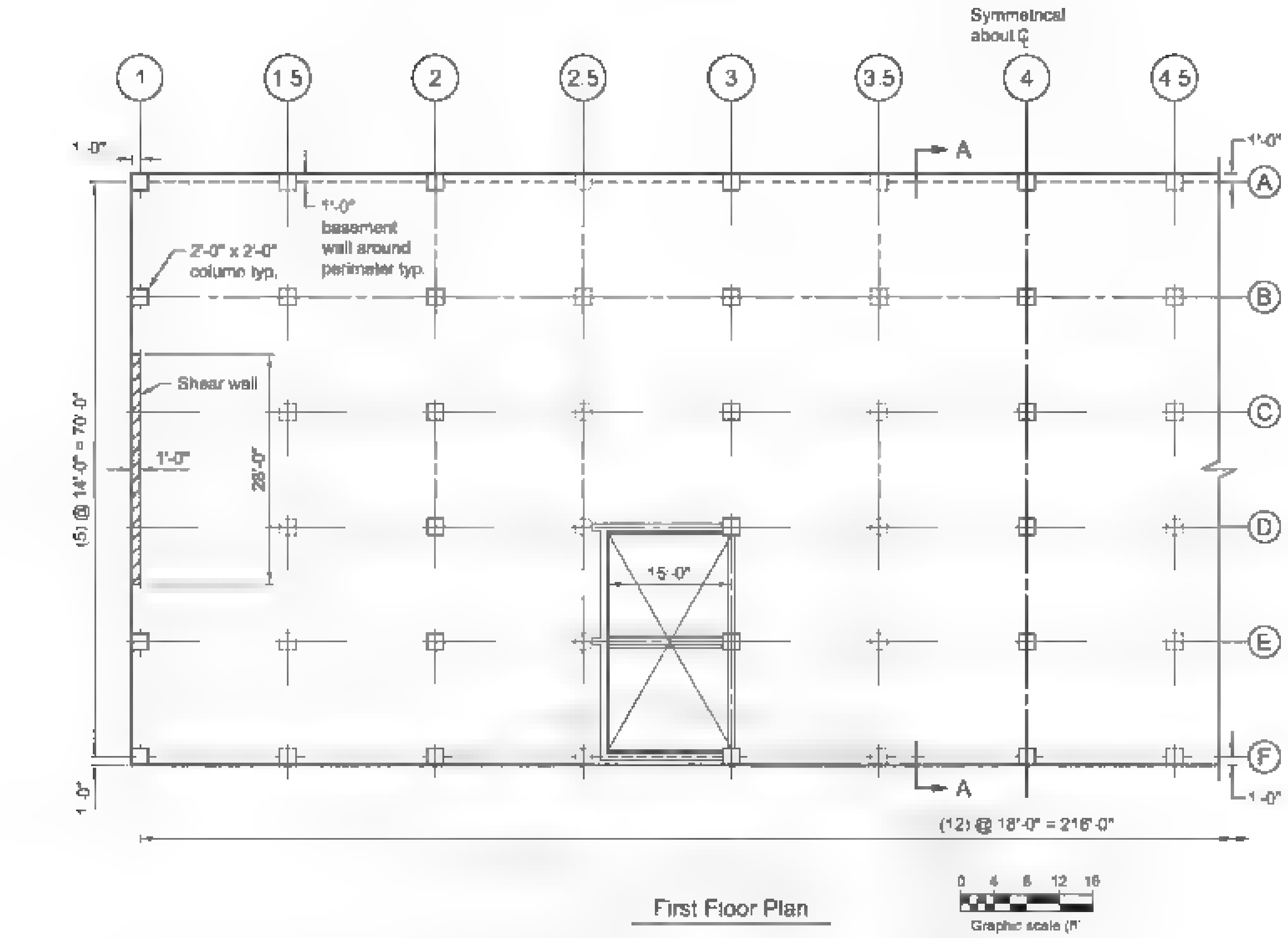


Fig. 1.2a—Example building first floor plan

analysis. Those examples may modify this initial data to demonstrate some specific code requirement.

This building example was created for the purpose of illustrating design of structural concrete members and systems. Other aspects of building design that may affect the structural layout such as occupancy, egress, fire protection, or other architectural constraints have not been addressed.

1.3—Building plans and elevation

The following building plans and elevation illustrate the structural layout and some details of the example building.

1.4—Loads

The following loads for the example building are generated in accordance with ASCE/SEI 7. Risk Category II is assumed.

Gravity Loads

Dead Load, D:
- Self weight
- Additional D = 15 lb/ft²
- Perimeter walls = 15 lb/ft²

Live Load
- First and Second Floors: Lobbies, public rooms, and corridors serving them = 100 lb/ft²
- Typical Floor: Private rooms and corridors serving them 65 lb/ft²

Roof Live Load
- Unoccupied = 20 lb/ft²

Snow Load
- Ground load, P_g = 20 lb/ft²
- Thermal, C_t = 1.0
- Exposure, C_e = 1.0
- Importance, I_s = 1.0
- Flat roof load, P_f = 20 lb/ft²

Lateral Loads

Wind Load
- Basic (ultimate) wind speed = 115 mph
- Exposure category = C
- Wind directionality factor, K_d = 0.85
- Topographic factor, K_{zt} = 1.0
- Gust effect factor, G_f = 0.85 (rigid)
- Internal pressure coefficient, GC_{pi} = ±0.18

Directional Procedure Seismic Load
- Importance, I_e = 1.0
- Site class: D
- S_S = 0.15, S_{DS} = 0.16
- S_1 = 0.08, S_{D1} = 0.13
- Seismic design category: B
- Equivalent lateral force procedure

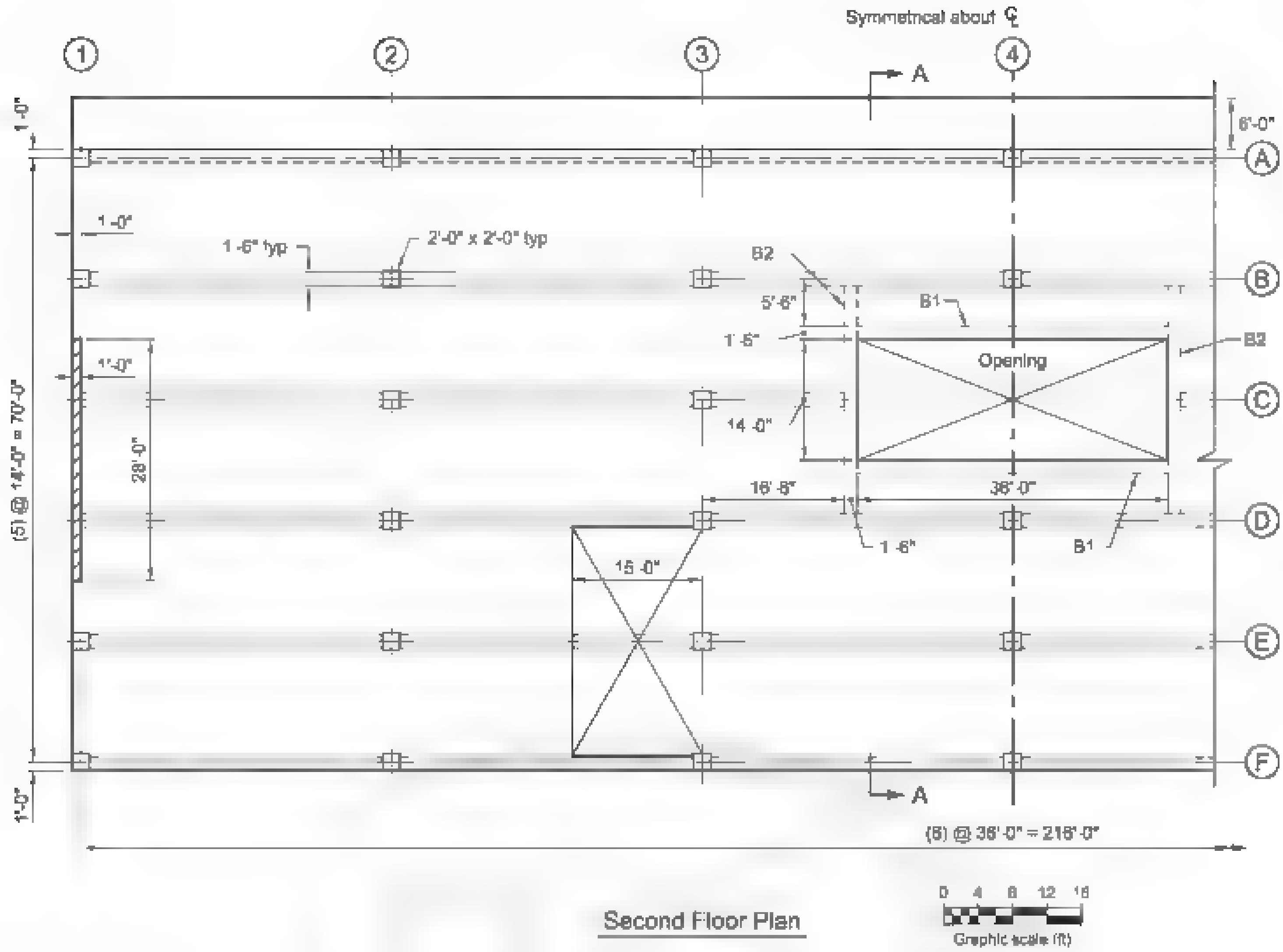


Fig 1 2b—Example building second floor plan

- Ordinary reinforced concrete shear walls in the north-south direction
  - $R = 5$
  - $C_s = 0.046$
- Ordinary reinforced concrete moment frame in the east-west direction
  - $R = 3$
  - $C_s = 0.032$

1.5—Material properties

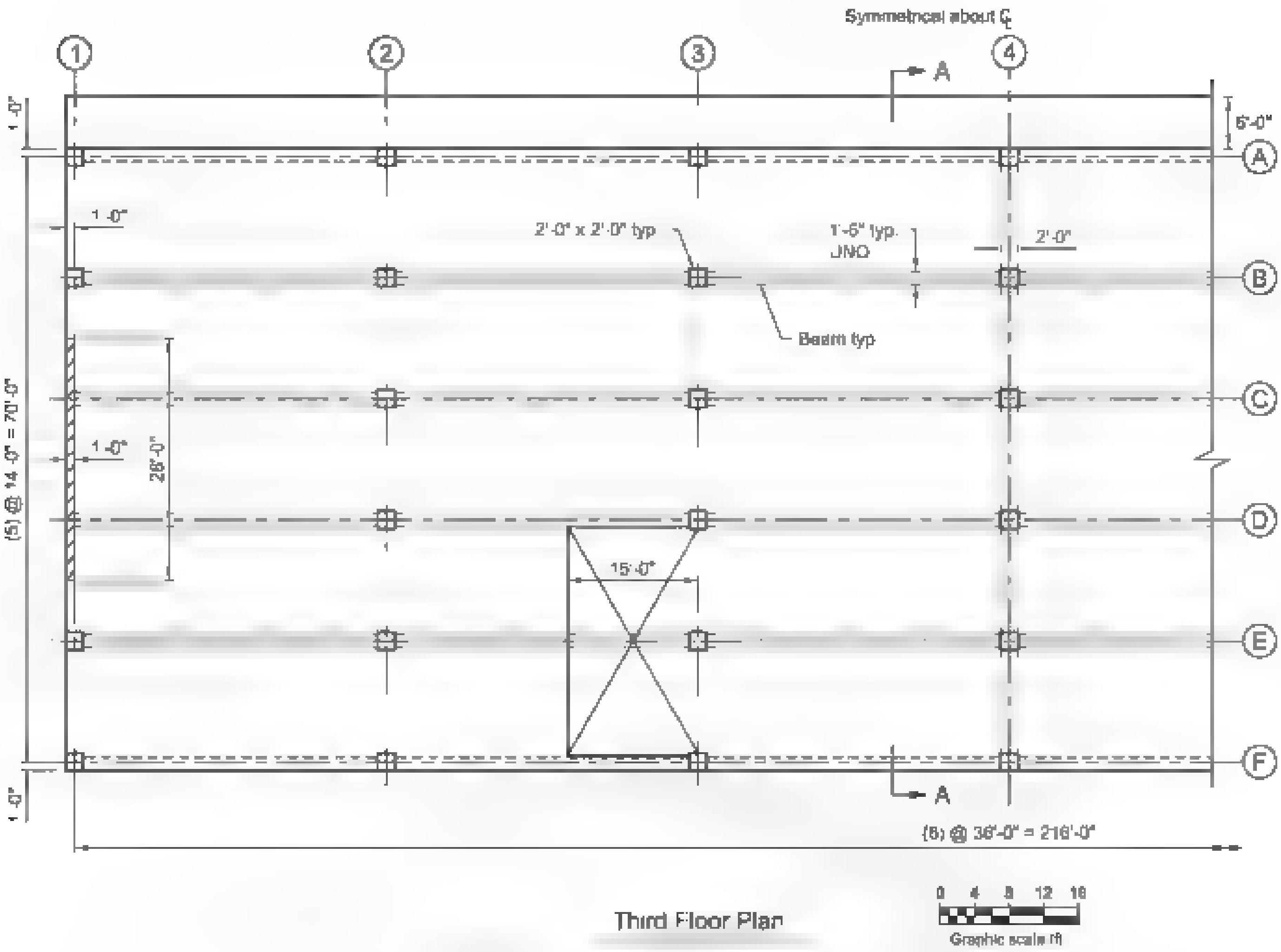
The designer should investigate and acquire a reasonable knowledge of locally available concrete and steel materials. Concrete properties are typically selected based on both mechanical properties and durability. Code Chapter 19 provides limitations, requirements, and guidance on the selection of f_c'. The chapter also provides requirements for durability of concrete, which will be discussed further in Chapter 4 of this Manual.

Code Table 19.2.1.1 provides minimum f_c' for use with a variety of structural systems and seismic design categories (SDCs). Minimum required strength for general use in SDC A, B, or C is 2500 psi. For typical floor spans and loads, however, an f_c' of 4000 psi is usually sufficient to satisfy strength requirements. For the example used in this Manual, the building height is moderate and the loads are typical. Assume that the locally available aggregate is a durable dolomitic limestone. Thus, the concrete can readily have a higher f_c' than the initial assumption of 4000 psi. A check of the durability requirements of Code Table 19.3.2.1 shows that 5000 psi will satisfy minimum f_c' for all exposure classes. The following concrete material properties are chosen

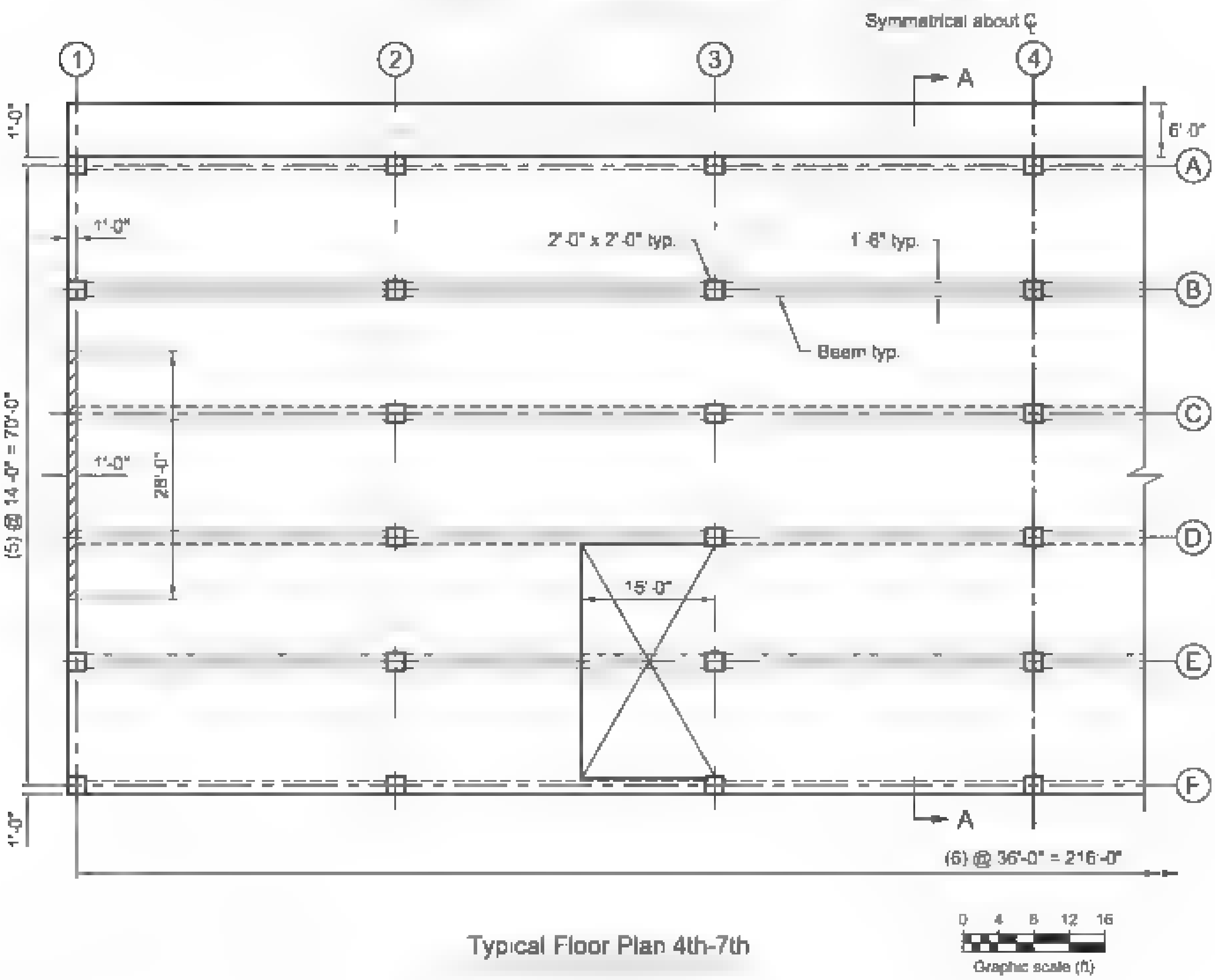
- $f_c' = 5000$ psi
- Normalweight $w_c = 150$ lb/ft^3
- $E_c = 4{,}030{,}000$ psi (Code Eq. (19.2.2.1b))
- $\nu = 0.2$
- $e_{th} = 5.5 \times 10^{-6}$/F (ACI 209R)

To minimize space occupied by heavily loaded columns and walls in multi-story buildings, the designer may choose to use a larger f_c' for columns than is used for the floor system. Concrete placement usually proceeds in two stages for each story; first, the vertical members, such as columns, and second, the floor members, such as beams and slabs. This results in column loads being transferred through the lower-strength concrete of the floor system. It may be desirable to specify f_c' of the floor system to be greater than $0.7 \times f_c'$ of the vertical members to avoid having to place higher-strength concrete in the floor system in the area of contact between floor and column (Code Sections 15.5.1 and 15.5.1a). Usually this situation only becomes an issue for taller buildings.

The most common and most available nonprestressed reinforcement is ASTM A615 Grade 60. Higher grades are

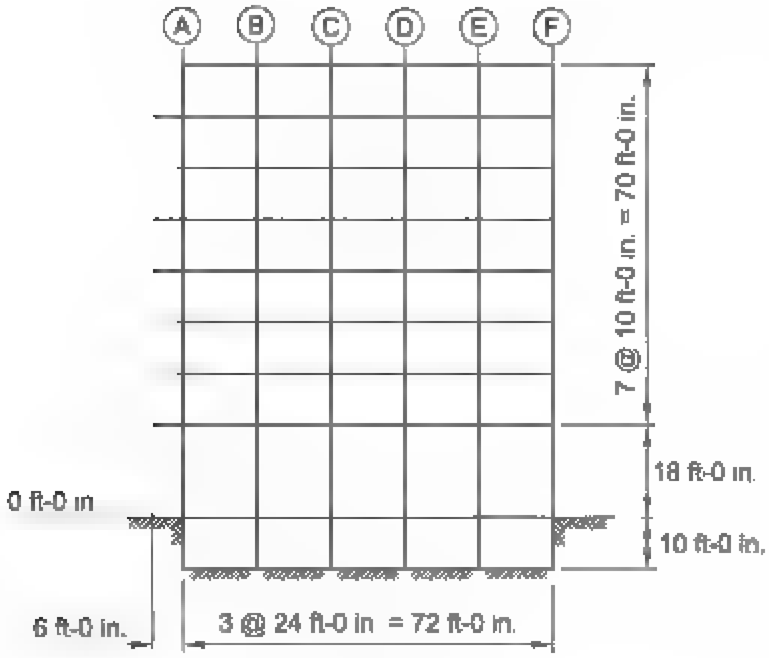


Third Floor Plan



Typical Floor Plan 4th-7th

Fig. 1.2c—Example building floor plans for third through seventh floor.



Section A-A

Fig. 1.2d—Example building section.

available but are subject to some limitations by the Code. The modulus of elasticity for reinforcement, E_p, is given in Code Section 20.2.2.2. The Code allows the use of wire, strand, or bar as prestressed reinforcement. The most commonly used prestressed reinforcement, however, is ASTM A416 Grade 270 seven-wire prestressing strand.

Assumed reinforcement material properties

- f_y = 60,000 psi
- f_{yt} = 60,000 psi
- E_s = 29,000,000 psi
- f_{pu} = 270,000 psi
- E_p = 28,500,000 psi

The use of lightweight concrete can reduce seismic forces, column loads, and foundation loads, which will allow savings in both concrete and reinforcement in taller or heavier buildings. Because this building is of moderate height, carries modest loading, and is designed for low seismic risk, the use of lightweight concrete is not likely to provide an economic advantage over normalweight concrete.

REFERENCES

American Concrete Institute

ACI 209R-92(08)—Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures

American Society of Civil Engineers

ASCE/SEI 7-16—Minimum Design Loads for Buildings and Other Structures

ASTM International

A416/A416M-18—Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete

A615/A615M-18ε1—Standard Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement

CHAPTER 2—STRUCTURAL SYSTEMS

2.1—Introduction

Structural concrete design has evolved from emphasizing the design of individual members to designing the structure as an entire system. Ultimately, however, the individual members must be designed and detailed according to their distinctive role in the system. Chapter 4 of the Code addresses the role and use of an individual member in the context of the entire structural concrete system. This chapter of the Manual provides an introduction and overview of the design lateral loads that might be encountered in design. In addition, common lateral load and gravity load systems used in structural concrete buildings are described.

Prior to the 1970s, reinforced concrete buildings that were of moderate height (less than 20 stories), not in seismically active areas, or constructed with nonstructural masonry walls and partitions, were seldom explicitly designed for lateral forces (ACI Committee 442 1971). Continuing research, advancement in materials science, and improvements in analysis tools have allowed structural engineers to develop economical building designs with more reliable structural performance.

For a typical structural concrete building, the structural system can be subdivided into the gravity load-resisting system and the lateral-force resisting system. Elements such as the floor system, walls, columns, and foundation contribute to one or both of the load-resisting systems. A structural engineer's primary concern is to design these systems to harmoniously support the anticipated design loads in a safe and serviceable manner.

2.2—Design lateral loads

The Code provisions are intended to address dead, live, earthquake, and wind loads such as those recommended in ASCE/SEI 7. These loads are applied to the structural system or directly to individual members, as applicable. Gravity loads are typically assumed to be applied vertically. Earthquake and wind loads are typically applied horizontally and are assumed to act in orthogonal directions. This section covers the following loads as they relate to Code provisions:

1. Wind loading (Code Chapter 6, elastic analysis)
2. Earthquake loading (Code Chapter 18)

Wind and earthquake loads are dynamic in nature, however, they differ in the manner in which these loads are induced on a structure. Wind loads are externally applied loads and, hence, are related to the structure's exposed surface. Earthquake loads are inertial forces related to the magnitude and distribution of the mass in the structure.

2.2.1 *Wind loading*—Wind kinetic energy is transformed into potential energy when it is resisted by an obstruction. Wind pressure is related to the wind velocity, building height, building surface, the surrounding terrain, and the location and size of other local structures. The structural response to a turbulent wind environment is predominantly in the first mode of vibration.

The quasi-static approach to wind load design has generally proved sufficient. Aeroelastic effects in tall buildings, however, may cause vibrations that cause occupant discomfort or damage partitions or glass. Therefore, to determine design wind loads for very tall buildings, wind tunnel testing may be required.

2.2.2 *Earthquake loading*—The main objective of structural design is life safety—that is, preserving the lives of occupants and passersby. Serviceability and minimizing economical loss, however, are also important objectives. By studying the results of previous earthquakes on various structural systems, improvements to code provisions and design practices have been achieved. These improvements have led to a reduction in damage of reinforced concrete structures that experience an earthquake. Some code improvements for members that resist the load effects of significant seismic accelerations include design and detailing requirements that:

1. Ensure that columns in a frame are flexurally stronger than beams—the so-called "strong column-weak beam" concept
2. Increase ductility and improve energy dissipation (with less deterioration in stiffness and strength)
3. Ensure members yield in flexure before reaching nominal shear strength, thus protecting the energy dissipation capability of the plastic hinge.
4. Ensure connections are stronger than the members framing into them
5. Limit structural system irregularities.

For most structures, the equivalent lateral force procedure given in ASCE/SEI 7 is used.

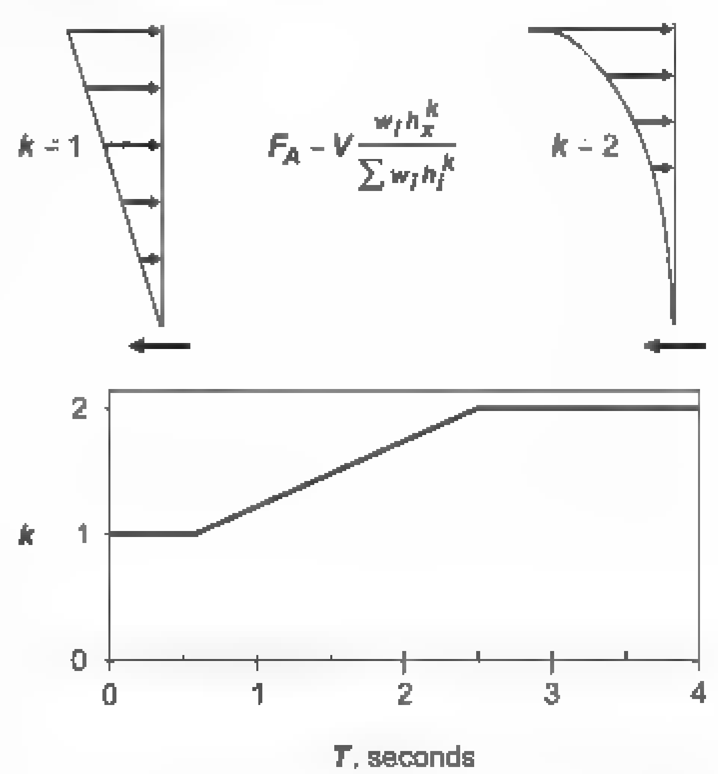


Fig. 2.2.2—Typical distribution of equivalent static lateral forces representing seismic forces (adapted from ASCE/SEI 7).

Based on this procedure, the distribution of design forces along the height of a building roughly approximates the building's fundamental mode of vibration (Fig. 2.2.2).

Applying recorded earthquake motions to a structure through elastic dynamic analyses usually result in greater force demands than from the earthquake design forces specified by most codes. This is because codes generally account for force reductions due to inelastic response. For example, ASCE/SEI 7 applies an R factor (response modification factor), which accounts for the ductility of a building, system overstrength, and energy dissipation through the soil foundation system (ASCE/SEI 7). It simplifies the seismic design process such that linear static elastic analysis can be used for building designs. The R factor reduces the calculated lateral loads and assumes a building may be damaged during an earthquake event, but will not collapse. The higher the R-value, the lower the lateral design load on a structure. R-values range from 1-1/2 for structures with stiff systems having low ductility to 8 for ductile systems having significant ductility. In a design-level earthquake, it is expected that some building members will yield. To promote appropriate inelastic behavior, the Code contains provisions meant to ensure inelastic deformation capacity in regions where yielding is likely, which then protects the overall integrity and stability of the building.

Dynamic (modal) analysis is commonly used for larger structures, important structures, or for structures with an irregular vertical or horizontal distribution of stiffness or mass. For very important and potentially critical structures—for example, nuclear power plants—inelastic dynamic analysis may be used (ACI Committee 442 1988).

2.3—Structural systems

All structures must have a continuous load path that can be traced from all load sources or load application to the foundation. The joints between the vertical members (columns and walls) and the horizontal members (beams, slabs, diaphragms, and foundations) are crucial to this concept. Properly detailed cast-in-place (CIP) reinforced concrete joints transfer moments and shears from the floor into columns and walls, thus creating a continuous load path. Joint design strength is covered in Chapter 15 of the Code. Further design and detailing information can be found in ACI 352R.

Engineers commonly refer to a structure's gravity-load-resisting system (GLRS) and lateral-force-resisting system (LFRS). All members of a CIP reinforced concrete structure contribute to the GLRS and most contribute to both systems. For low-rise structures, the inherent lateral stiffness of the GLRS is often sufficient to resist the design lateral forces without changes to the design or detailing of the GLRS members. As the building increases in height, however, the importance of designing and detailing the LRFS to resist lateral loads increases. When sufficient building height is reached, stiffness rather than strength will govern design of the LFRS. In the design process, the type of LFRS is usually influenced by architectural considerations and construction requirements.

There are several types of structural systems or a combination thereof to resist gravity, lateral, and other loads, with deformation behavior as follows:

1. **Frames**—Lateral deformations are primarily due to story shear. The relative story deflections therefore depend on the horizontal shear applied at each story level.
2. **Walls**—Lateral deformations are due to both shear and bending. The predominate behavior mode depends on the wall's height-to-width aspect ratio.
3. **Dual systems**—Dual systems are a combination of moment-resisting frames and structural walls. The moment resisting frames support gravity loads, and up to 25 percent of the lateral load. The structural walls resist the majority of the lateral loading.
4. **Frames with closely spaced columns, known as cantilevered column system or a tube system**—Lateral deformations are due to both shear and bending, similar to a wall. Wider openings in a tube, however, can produce a behavior intermediate between that of a frame and a wall.

Regardless of the system, a height is reached at which the resistance to lateral sway will govern the design of the structural system. At such a height, stiffness, not strength, controls the building design.

ASCE/SEI 7 provides provisions for assigning a structure to a Seismic Design Category (SDC A through F). As a building's Seismic Design Category increases, from A through F, ASCE/SEI 7 requires a progressively more rigorous seismic design and a more ductile system to maintain an acceptable level of seismic performance.

The Code provides three categories of earthquake detailing: ordinary, intermediate, and special. These categories provide an increasing level of system ductility and toughness.

Building height limits in ASCE/SEI 7 are related to the LFRS.

For buildings in SDC A and B, wind load will usually control the design of the LFRS.

For buildings in SDC C, seismic loads are likely to control design forces, and seismic detailing is required. LFRSs are not limited in height for most systems for this SDC, but interstory drift limits from ASCE/SEI 7 must be met. Again, stiffness, not strength, will likely control the LFRS design.

For buildings in SDC D, E, and F, seismic loads almost always control design forces, and increased seismic detailing is required. LFRSs often have height limitations based on assumed structural performance. Figure 2.3 shows approximate height limits for different structural systems.

Table 2.3 provides ASCE/SEI 7 limits for choosing a structural system for a particular building. The ranges of applicability shown are influenced by occupancy requirements, architectural considerations, internal traffic flow (particularly in the lower floors), the structure's height and aspect ratio, and load intensity and types (live, wind, and earthquake).

2.3.1 *Gravity-load-resisting systems*—A GLRS is composed of horizontal floor members and vertical members that support the horizontal members.

Gravity loads are resisted by reinforced concrete members through axial, flexural, shear, and torsional stiffness and

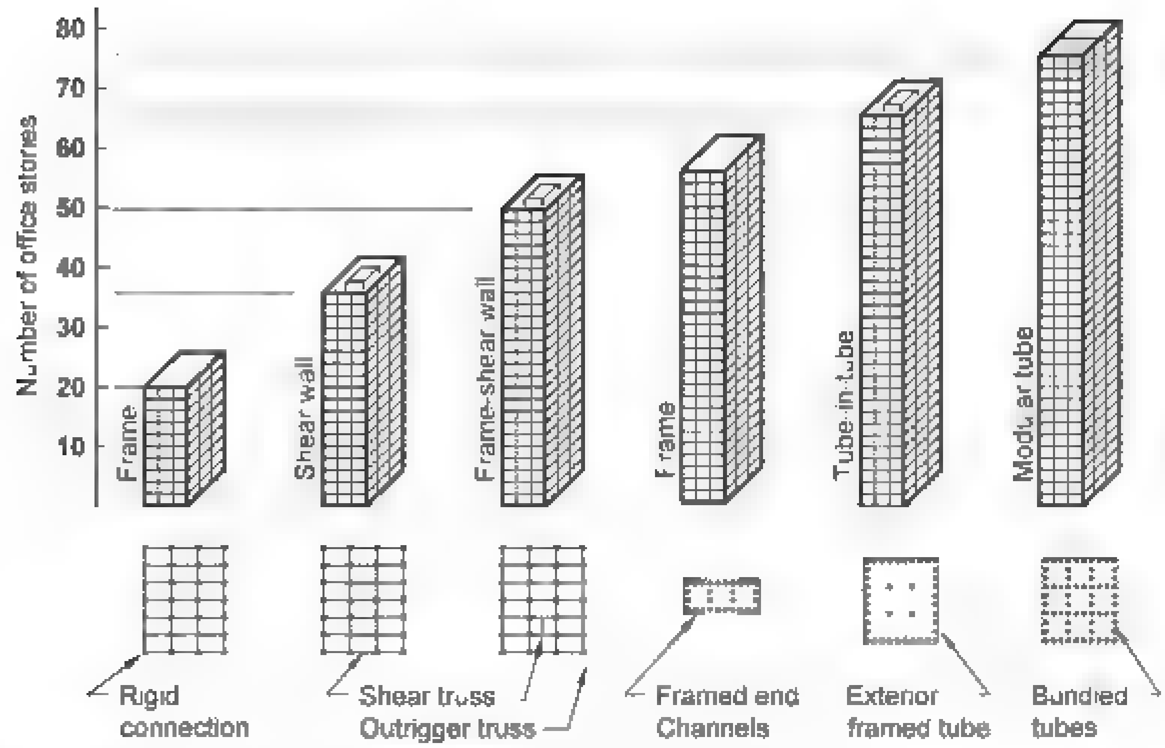


Fig. 2.3—Structural systems and optimum height limitations (Ali and Moon 2007).

Table 2.3—Approximate building height limits for various LFRS

Type of LFRS	Practical limit of system (ASCE/SEI 7 limit according to SDC)				
	SDC				
	A and B	C	D	E	F
Moment-resisting frames (only):					
Ordinary moment frame (OMF)	NL*	NP*	NP	NP	NP
Intermediate moment frame (IMF)	NL	NL	NP	NP	NP
Special moment frame (SMF)	NL	NL	NL	NL	NL
Structural walls (only):					
Building frame systems (structural walls are the primary LFRS and frames are the primary GLRS)					
Ordinary structural wall (OSW)	NL	NL	NP	NP	NP
Special structural wall (SSW)†	NL	NL	160 ft	160 ft	00 ft
Bearing wall systems (structural walls are the primary lateral- and gravity-load-resisting system):					
OSW	NL	NL	NP	NP	NP
SSW†	NL	NL	60 ft	160 ft	00 ft
Dual systems (structural walls are the primary LRFS, and the moment-resisting frames carry at least 25% of the lateral load).					
OSW with OMF	NL	NP	NP	NP	NP
OSW with IMF	NL	NL	NP	NP	NP
OSW with SMF	NL	NL	NP	NP	NP
SSW with OMF	NP	NP	NP	NP	NP
SSW with IMF	NL	NL	160 ft	100 ft	100 ft
SSW with SMF	NL	NL	NL	NL	NL

*NL is no limit. NP is not permitted.

†Height limits can be increased per ASCE/SEI 7 Section 12.2.5.4.

strength. The related deformations are exaggerated and shown in Fig. 2.3.1.

2.3.2 *Lateral load resisting system*—An LFRS must have an adequate toughness to maintain integrity during high wind loading and design earthquake accelerations. Buildings are basically cantilevered members designed for strength (axial, shear, torsion, and moment) and serviceability (deflection and creep must be considered for tall buildings).

Code Section 18.2.1 lists the design and detailing requirements for each SDC as it applies to a specific seismic-force-resisting system. The following LFRSs are addressed as follows.

2.3.2.1 *Moment resisting frames*—Cast-in-place moment resisting frames derive their load resistance from member

strengths and connection rigidity. In a moment-resisting frame structure, the lateral displacement (drift) is the sum of three parts: 1) deformation due to bending in columns, beams, slabs, and joint deformations; 2) deformation due to shear in columns and joints; and 3) deformations due to axial force in columns.

Yielding and plastic hinge formation in frame members can significantly increase the lateral displacement. The effect of secondary moments caused by column axial forces multiplied by lateral displacement (*P*-Δ effect) will further increase the lateral displacement.

In buildings, moment-resisting frames are usually arranged parallel to the principal orthogonal axes of the structure and the frames are interconnected by floor diaphragms (Code Chapter 12). Moment-resisting frames usually allow the maximum flexibility in space planning and are an economical solution up to a certain height.

2.3.2.2 *Shear walls*—Reinforced concrete shear walls are often introduced into multi-story buildings because of their high in-plane stiffness and strength to resist lateral forces or when the building program is conducive to layout of structural walls. For buildings without a significant moment frame, shear walls behave as vertical cantilevers. Walls can be designed with or without openings (Fig. 2.3.2.2a). Separate walls can be coupled to act together by beams/slabs or deep beams, depending on design forces and architectural

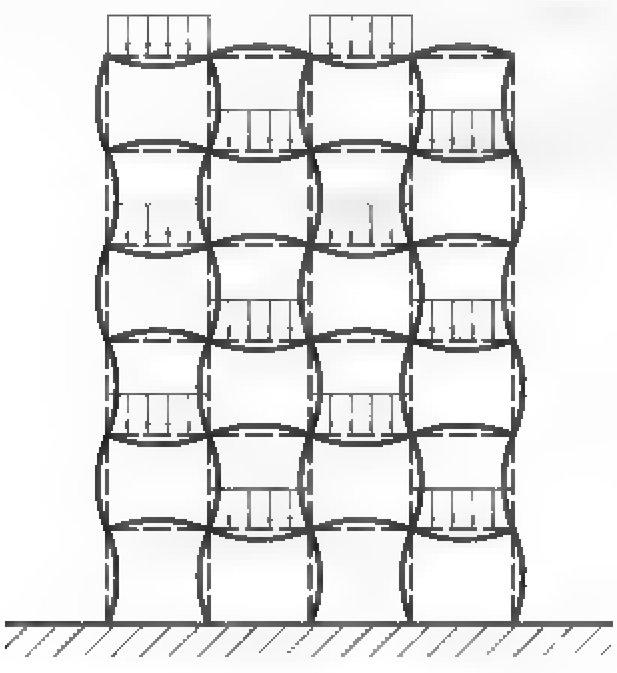


Fig. 2.3.1—Deflections due to gravity load.

requirements. Coupling of shear walls introduces frame action to the LFRS and thus reduces lateral deflection of the system. Reinforced concrete walls are often used around elevator and stair shafts to achieve the required fire rating. For shear wall types and functions, refer to Table 2.3.2.2.

A shear wall building usually consists of a series of parallel shear walls in orthogonal directions that resists lateral loads and supports vertical loads.

In multi-story bearing wall buildings, significant discontinuities in mass, stiffness, and geometry should be avoided. Bearing walls should be located close to the plan perimeter if possible and should preferably be symmetric in plan to reduce torsional effects from lateral loading (refer to Fig. 2.3.2.2b).

2.3.2.3 *Staggered wall-beam system*—This system uses story-high solid or pierced walls extending across the entire width of the building and supported on two lines of columns placed along exterior faces (Fig. 2.3.2.3). By staggering the locations of these wall beams on alternate floors, large clear areas are created on each floor.

The staggered wall-beam building is suitable for multistory construction having permanent interior partitions such as apartments, hotels, and student residences.

An advantage of the wall-beam building is the large open area that can be created in the lower floors when needed for parking, commercial use, or even to allow a highway to pass under the building. This system should be considered in low seismic areas because of the stiffness discontinuity at each floor.

2.3.2.4 *Tubes*—A tube structure consists of closely spaced columns in a moment frame, generally located around the perimeter of the building (Fig. 2.3.2.4(a)).

Because tube structures generally consist of girders and columns with low span-to-depth ratios (in the range of 2 to 4), shearing deformations often contribute to lateral drift and should be included in analytical models. Tubes are often thought of as behaving like a perforated diaphragm.

Frames parallel to direction of force act like webs to carry the shear from lateral loads, while frames perpendicular to the direction of force act as flanges to carry the moment from lateral loads. Gravity loads are resisted by the exterior frames and interior columns.

A reinforced concrete braced tube is a system in which a tube is stiffened and strengthened by infilling in a diagonal pattern over the faces of the building (Fig. 2.3.2.4(b)). This

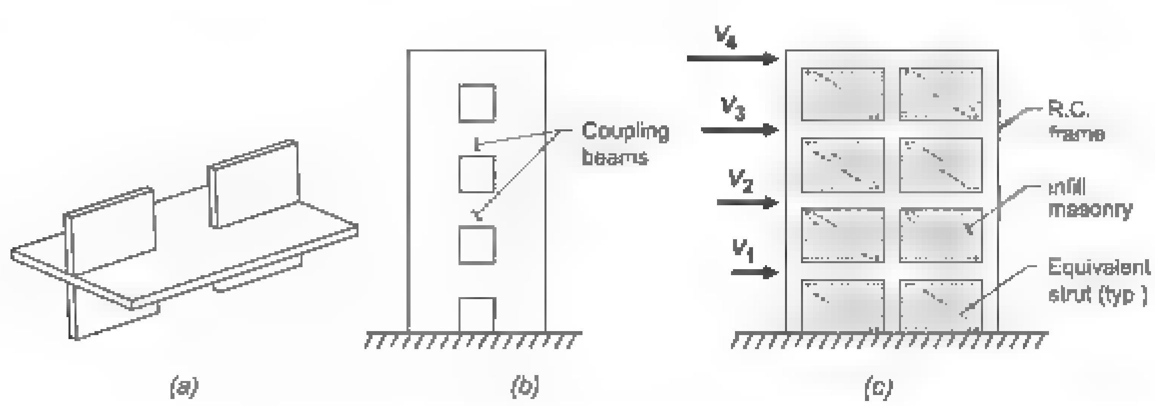


Fig. 2.3.2.2a—Coupled and infill walls: (a) shallow coupling beams or slabs; (b) coupling beams; and (c) infill walls.

Table 2.3.2.2—Shear wall types and functions

Wall type	Behavior	Reinforcement	Remarks
Short—height-to-length ratio does not exceed 2	Lateral design is usually concerned only with shear strength	Bars evenly distributed horizontally and vertically	Wall foundation must be capable of resisting the actions generated in the wall. Consider sliding resistance provided by foundation.
Height-to-length ratio is greater than 2	Lateral design must consider both the wall's shear and moment strength	Evenly distributed vertical and horizontal reinforcement. Part of the vertical reinforcement may be concentrated at wall ends—boundary elements. Vertical reinforcement in the web contributes to the flexural strength of the wall.	Wall foundation must be capable of resisting the actions generated in the wall. Consider overturning resistance provided by foundation.
Ductile structural wall	Lateral design is heavily influenced by flexure stiffness and strength	Flexural bar spacing and size should be small enough so that flexural cracking is limited if yielding occurs. Over-reinforcing for flexure is discouraged because flexural yielding is preferred over shear failure	Acceptable ductility can be obtained with proper attention to axial load level, confinement of concrete, splicing of reinforcement, treatment of construction joints, and prevention of out-of-plane buckling.
Coupled walls with shallow coupling beams or slabs (Fig. 2.3.2.2a(a))	Link slab flexural stiffness deteriorates quickly during inelastic reversed loading	Place coupling slab bars to limit slab cracking at the stress concentrations at the wall ends	Punching shear stress around the wall ends in the slab needs to be checked.
Coupled walls with coupling beams (Fig. 2.3.2.2a(b))	Depending on span-to-depth ratio, link beams may be designed as deep beams	Main reinforcement placed horizontally or diagonally. Diagonal reinforcement is placed corner to corner of the beam and may be confined by spirals or closed ties and designed to resist flexure and shear directly	Properly detailed coupling beams can achieve ductility. Coupling beams should maintain their load-carrying capacity under reverse inelastic deformation
Infilled frames (structural or nonstructural) (Fig. 2.3.2.2a(c))	Frames behave as braced frames, increasing the lateral strength and stiffness. The infilling acts as a strut between diagonally opposite frame corners, and creates high shear forces in the columns.	Infill walls should either be sufficiently separated from the moment frame (making them nonstructural), or detailed to be connected structurally with the moment frame	Uneven infilling can cause irregularities of the moment frame. If there are no infills at a given story level, that story acts as a weak or soft story that is vulnerable to concentrated damage and instability

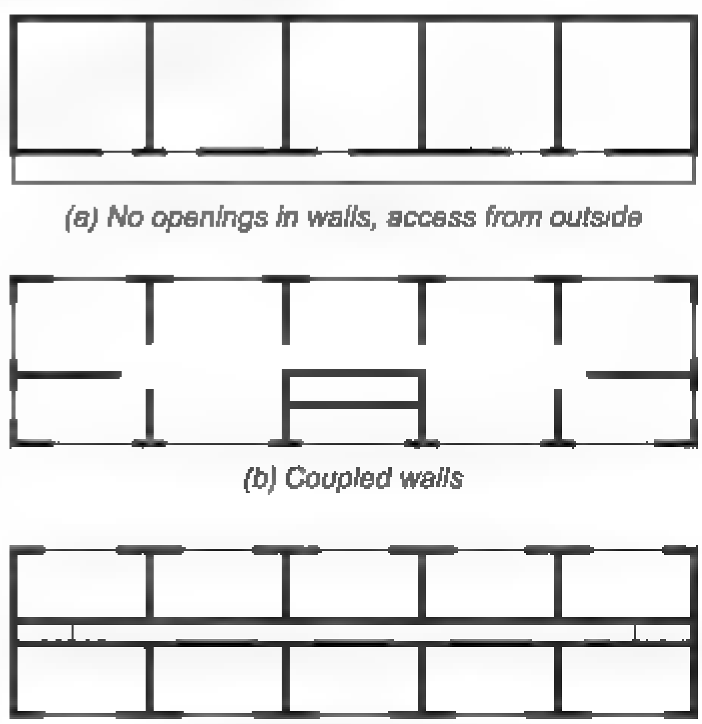
bracing increases the structure's lateral stiffness, reduces the moments in the columns and girders, and reduces the effects of shear lag.

2.3.3 *Dual systems*—Dual systems consist of combined shear walls and moment-resisting frames. They are used to achieve specific response characteristics, particularly with respect to seismic behavior. Some of the more common dual systems are discussed in Sections 2.3.3.1 through 2.3.3.6.

2.3.3.1 *Wall-frame systems*—Rigid-jointed frames and isolated or coupled structural walls can be combined to produce an efficient LFRS. Because of the different shear and flexural lateral deflection characteristics of moment frames and structural walls, careful attention to the interaction between the two systems can improve the structure's lateral response to loads by reducing lateral deflections (Fig. 2.3.3.1).

The wall's overturning moment is greatly reduced by interaction with the frame. Because drift compatibility is forced on both the frame and the wall, and the frame-alone and wall-alone drift modes are different, the building's overall lateral stiffness is increased. Design of the frame columns for gravity loads is also simplified in such cases, as the frame columns are assumed to be braced against sidesway by the walls.

The wall-frame dual system permits the structure to be designed for a desired yielding sequence under strong ground motion. Beams can be designed to experience significantly yielding before inelastic action occurs at the bases of the walls. By creating a hinge sequence, and considering the relative economy with which yielded beams can be repaired, wall-frame structures are appropriate for use in higher seismic zones. However, note that the variation



(a) No openings in walls, access from outside

(b) Coupled walls

(c) Longitudinal corridor walls

Fig. 2.3.2.2b—Example of shear wall layouts

of shears and overturning moments over the height of the wall and frame is very different under inelastic versus elastic response conditions.

2.3.3.2 *Outrigger systems*—An outrigger system uses orthogonal walls, girders, or trusses, one or two stories in height, to connect the perimeter columns to central core walls, thus enhancing the structure lateral stiffness (Fig. 2.3.3.2).

In addition to the outrigger girders that extend out from the core, girders or trusses are placed around the perimeter of the structure at the outrigger levels to help distribute lateral forces between the perimeter columns and the core walls. These perimeter girders or trusses are called "hat" or "top-hat" bracing if located at the top, and "belt" bracing if located at intermediate levels. Some further reductions in total drift and core bending moments can be achieved by increasing the cross section of the columns and, therefore, the axial stiffness, and by adding outriggers at more levels. Outriggers are effective in increasing overall building stiffness and, thus, resist wind loads with less drift. Design of outrigger type systems for SDC D through F must consider the effect of the high local stiffness of the outriggers on the inelastic response of the entire system. Members framing into the outriggers should be detailed for ductile response.

2.3.3.3 *Tube-in-tube*—For tall buildings with a reasonably large service core, it is generally advantageous to use shear walls enclosing the entire service core (inner tube) as part of the LFRS. The outer tube is formed by the closely spaced column-spandrel beam frame. A bundled tube system consists of several framed tubes bundled into one larger structure that behaves as a multicell perforated box (Fig. 2.3.3.3).

The tube-in-tube system combines the advantages of both the perimeter-framed tube and the inner shear walls. The inner shear walls enhance the structural characteristics of the perimeter-framed tube by reducing the shear deformation of the columns in the framed tube. The tube-in-tube system can be considered a refined version of the shear wall-frame interaction type structure.

2.3.3.4 *Bundled tubes*—A bundled tube system consists of several framed tubes bundled into one larger structure that behaves as a multicell perforated box. Individual tubes can be terminated at different heights. The bundled tube system offers considerable flexibility in layout and possesses large torsional and flexural stiffness.

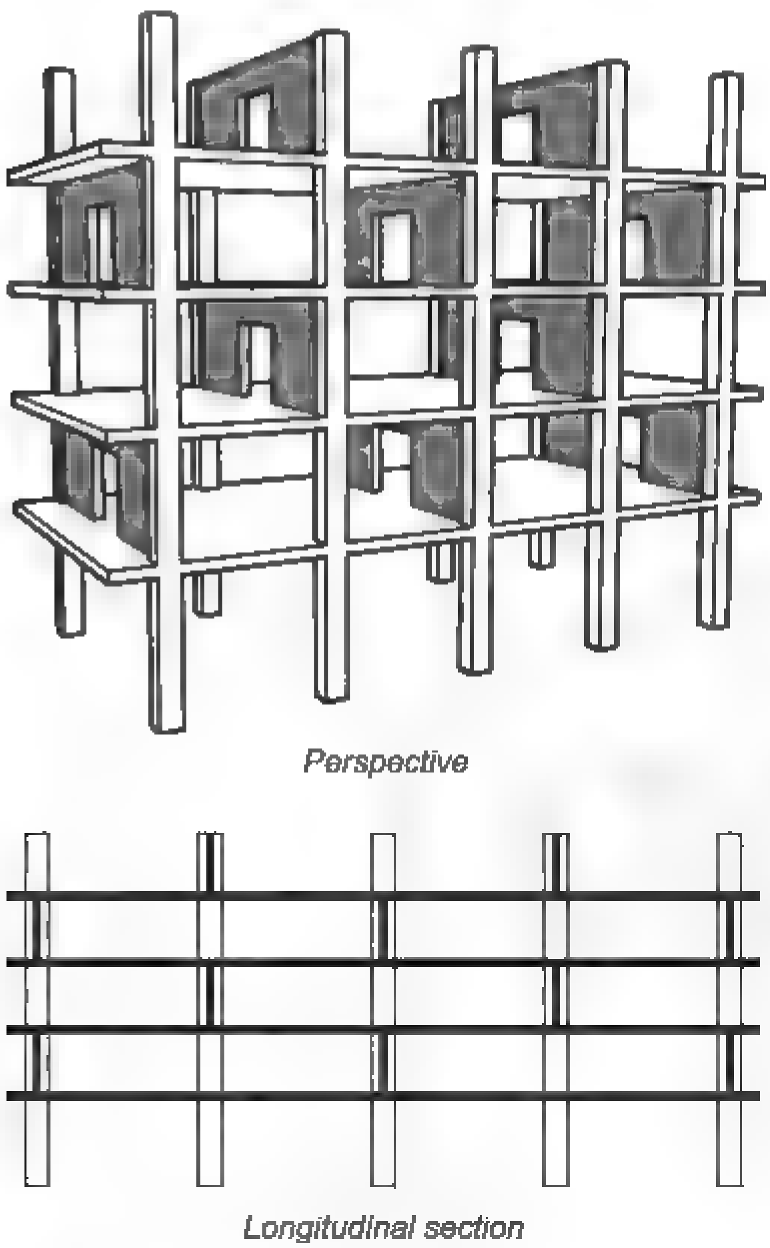
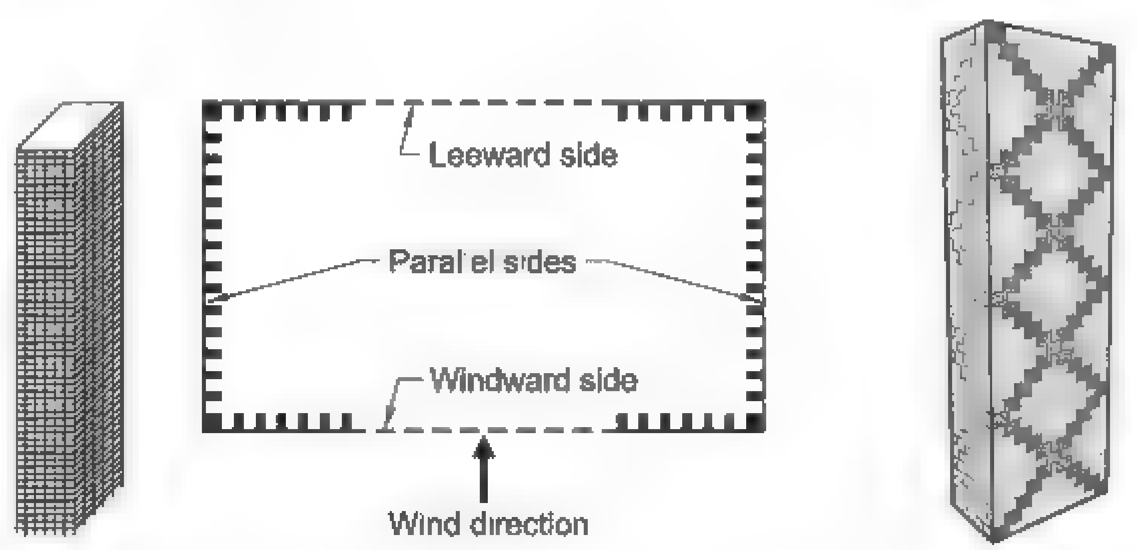


Fig. 2.3.2.3—Staggered wall-beam system.



(a)—Typical tube system

(b)—Braced tube system

Fig. 2.3.2.4—Tube systems.

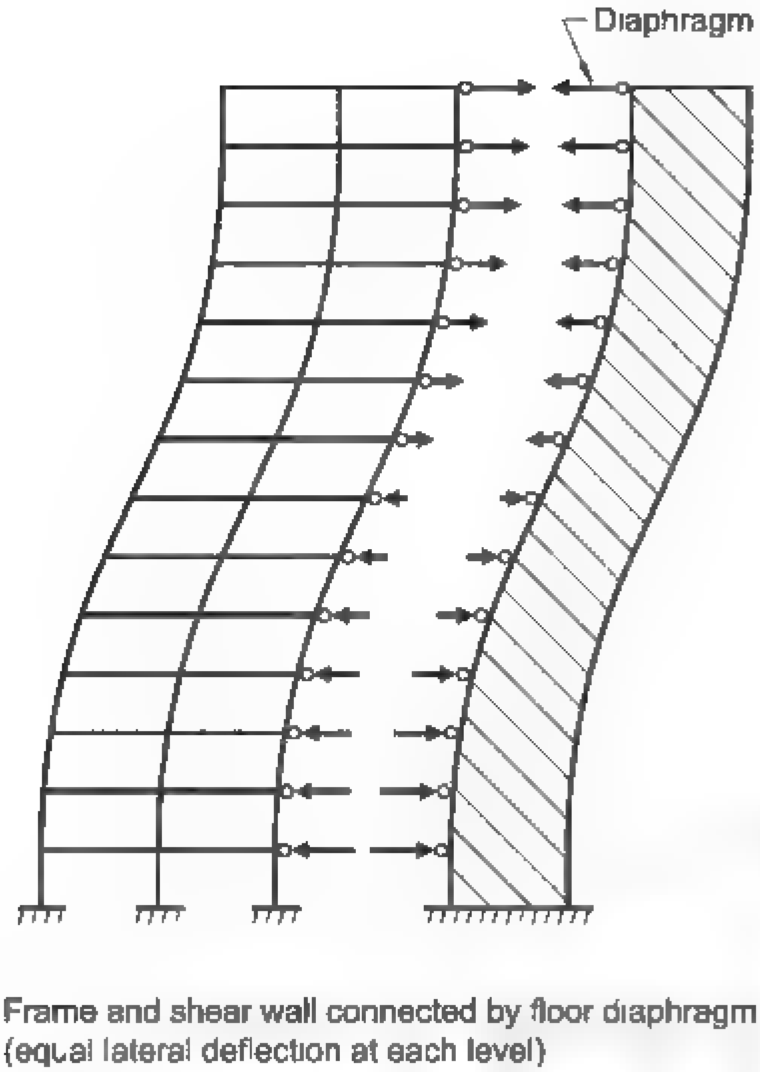


Fig. 2.3.3.1—Shear wall and moment frame system.

2.3.3.5 *Mixed concrete-steel structures*—Mixed concrete-steel systems consist of interacting concrete and steel assemblies. The resulting composite structure displays most or all of the advantages of steel structures (large spans and lightweight construction) as well as the favorable characteristics of concrete structures (high lateral stiffness of shear walls and cores, and high damping). Engineers must address the differential vertical creep and shrinkage between steel and concrete to prevent uneven displacement. Because the erection of steel and concrete structures involves different building trades and equipment, engineers who design mixed construction should consider scheduling issues.

2.3.3.6 *Precast structures*—Precast concrete members are widely used as components in frame, wall, and wall-frame systems. Mixed construction, consisting of precast concrete assemblies connected to a cast-in-place concrete core, is also used. The efficiency of such systems depends on the extent of standardization, the ease of manufacture, the simplicity of assembly, and the speed of erection.

Precast floor systems include large standardized reinforced (and usually prestressed) concrete slabs, with or without interior cylindrical voids (hollow core), as well as prefabricated rib slabs. Rigid-jointed frames are usually assembled from H- or T-units, and shear walls and cores are assembled from prefabricated single-story panels. Planning and designing appropriate connection details for panels, frame members, and floor assemblies is the single most important operation related to precast systems.

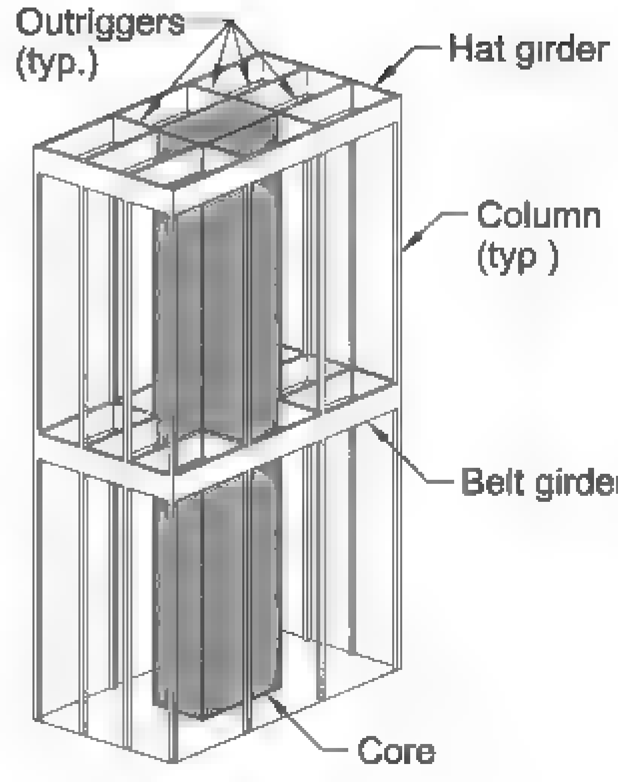


Fig. 2.3.3.2—Outrigger system.

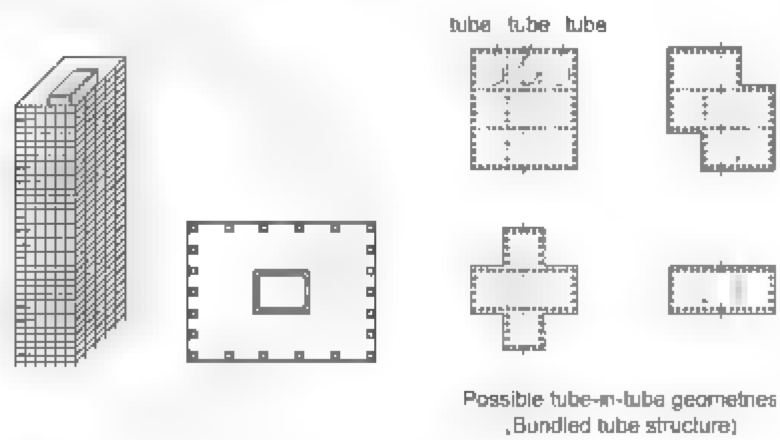


Fig. 2.3.3.3—Tube-in-tube and bundled tube systems.

Three main types of connections are described as follows.

1. Steel reinforcing bars protruding from adjacent precast members are made continuous by mechanical connectors, welding, or lap splices, and the joint between the members is filled with cast-in-place concrete. If welding is used, the engineer should specify weldable reinforcement and appropriate reinforcement material and welding procedures to ensure connections with suitable ductility.
2. Steel inserts (plates and angles) cast into the precast members are bolted or welded together and the gaps are grouted.
3. The individual precast units are post-tensioned together across the joint, with or without a mortar bed.

The behavior of a precast system subjected to seismic loading depends to a considerable degree on the characteristics of the connections. Connection details can be developed that ensure satisfactory performance under seismic loadings, provided that the engineer pays particular attention to steel ductility and positive confinement of concrete in the joint area.

Structural Systems

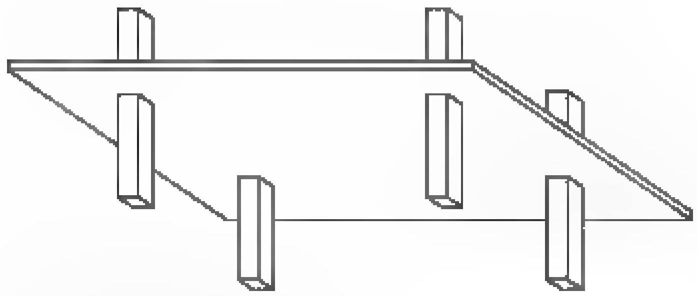


Fig. 2.4.1—Two-way flat plate system

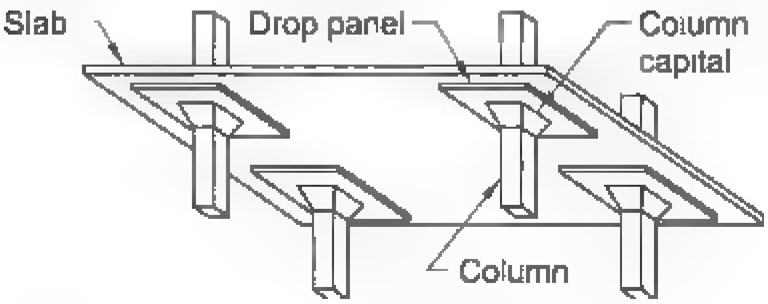


Fig. 2.4.2—Flat slab with drop panels and capitals

2.4—Floor framing systems

Selection of the floor system significantly affects a structure's cost as well as the performance of its lateral-force-resisting system. The primary function of a floor system is to resist gravity load. Additional important functions in most buildings are:

(a) Diaphragm action. The slab's in-plane stiffness maintains the plan shape of the structure and distributes horizontal forces to the LFRS

(b) Moment resistance: The flexural stiffness of the floors may be an integral and necessary part of the LFRS.

Concrete structures are commonly analyzed for lateral loads assuming the floor system acts as a diaphragm, infinitely stiff in its plane. This assumption is not valid for all configurations and geometries of floor systems. Factors affecting diaphragm stiffness are span-to-depth ratio of the slab's plan dimensions relative to the location of the lateral load-resisting members, slab thickness, locations of slab openings and discontinuities, and type of floor system used. The floor system flexural stiffness can add to the lateral stiffness of the structure. If the slab is assumed to act as part of a frame to resist lateral moments, engineers usually limit the effective slab width (acting as a beam within the frame) to between 25 and 50 percent of the bay width.

2.4.1 *Flat plates*—A flat plate is a two-way slab supported by columns, without column capitals or drop panels (Fig. 2.4.1).

The flat-plate system is a very cost-effective floor for commercial and residential buildings. Simple formwork and reinforcing patterns, as well as lower overall building height, are advantages of this system. In designing and detailing plate-column connections, particular attention must be paid to the transfer of shear and unbalanced moment between the slab and the columns (Chapters 8 and 15 of the Code). This is achieved by using a sufficient slab thickness or shear reinforcement (stirrups or headed shear studs) at the slab-column joint, and by concentrating slab flexural reinforcement over the column area.

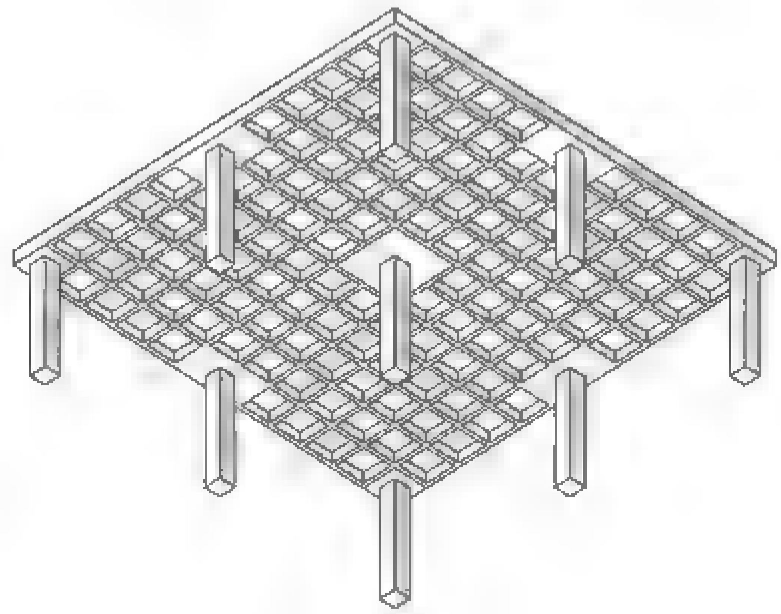


Fig. 2.4.3—Two-way grid (waffle) slab

2.4.2 *Flat slabs with drop panels, column capitals, or both*—The shear strength of flat slabs can be improved by thickening the slab around columns with drop panels, column capitals (either constant thickness or tapered), shear caps, or a combination (Fig. 2.4.2). Like flat plates, flat slab systems normally act as diaphragms transmitting lateral forces to columns and walls.

Drop panels increase a slab's flexural and shear strength at the column, and thus improve the ability of the flat slab to participate in the LFRS. Shear caps and column capitals improve the slab shear strength by increasing the slab thickness around the column. To improve the slab shear strength without increasing the slab thickness, engineers can provide closely spaced stirrups or shear studs radiating out from the column.

LFRSs consisting only of flat slab or flat plate frames, without ductile frames, structural walls, or other bracing members, are unsuitable in high seismic areas (SDC D through F).

2.4.3 *Two-way grid (waffle) slabs*—For longer spans, a slab system consisting of a grid of ribs intersecting at a constant spacing can be used to achieve an appropriate slab depth for the longer span with much less dead load than a solid slab (Fig. 2.4.3).

The ribs are formed by standardized dome or pan forms that are closely spaced. The slab thickness between the ribs is thin and normally governed by fire rating requirements. Some pans adjacent to the columns are omitted to form a solid concrete drop panel, to satisfy requirements for transfer of shear and unbalanced moment between the slab and columns.

A waffle slab provides an adequate shear diaphragm. The solid slab adjacent to the column provides significant two-way shear strength. Slab flexural and punching shear strength can be increased by the addition of closely spaced stirrups radiating out from the column face in two directions. Stirrups may also be used in the ribs.

Because a waffle slab behaves similarly to a flat slab, LRFSs consisting only of waffle slab frames are unsuitable in high seismic design areas (SDC D through F).

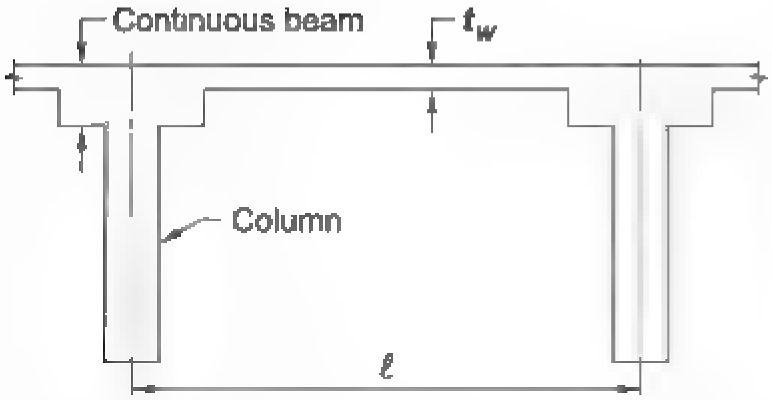


Fig. 2.4.6—One-way banded one-way slab

2.4.4 *One-way slabs on beams and girders*—One-way slabs on beams and girders consist of girders that span between columns and beams that span between the girders. One-way slabs span between the beams. This system provides a satisfactory diaphragm, and uses the girder-column frames and beam-column frames to resist lateral loads. Adequate flexural ductility can be obtained by proper detailing of the beam and girder reinforcement.

The beams and slabs can be placed in a composite fashion (with precast elements). If composite, shear connectors are placed at the beam-slab interface to ensure composite action. This system can provide good lateral force resistance, provided that the shear connectors are detailed with sufficient strength and ductility. Some examples of this type of slab system include:

(a) Precast concrete joists with steel shear connectors between the top of the beam and a cast-in-place concrete slab. The concrete joists are usually fabricated to readily support the formwork for the cast-in-place slab. In this system, the joists are supported on walls or cast-in-place concrete beams framing directly into columns

(b) Steel joists with the top chord embedded in a cast-in-place concrete slab. The slab formwork is supported from the joists, which supports the fresh slab concrete

(c) Steel beams supporting a noncomposite steel deck with a cast-in-place concrete slab. Note that the Code does not govern the structural design of concrete slabs for composite steel decks

2.4.5 *One-way ribbed slabs (joists)*—One-way ribbed slab (joist) systems consist of concrete ribs in one direction, spanning between beams, which span between columns. The size of pan forms available usually determines rib depth and spacing. As with a two-way ribbed system, the thickness of the thin slab between ribs is often determined by the building's fire rating requirements.

This system provides an adequate shear diaphragm and is used in a structure whose lateral resistance comes from a moment-resisting frame or shear walls. One row of pans can be eliminated at column lines, giving a wide, flat beam that may be used as part of the LFRS. Even if the slab system does not form part of the designated LRFS, the engineer should investigate the actions induced in the ribs by building drift.

2.4.6 *One-way banded slabs*—A one-way banded slab is a continuous drop panel (shallow beam) spanning between columns, usually in the long-span direction, and a one-way slab spanning in the perpendicular direction (Fig. 2.4.6). The

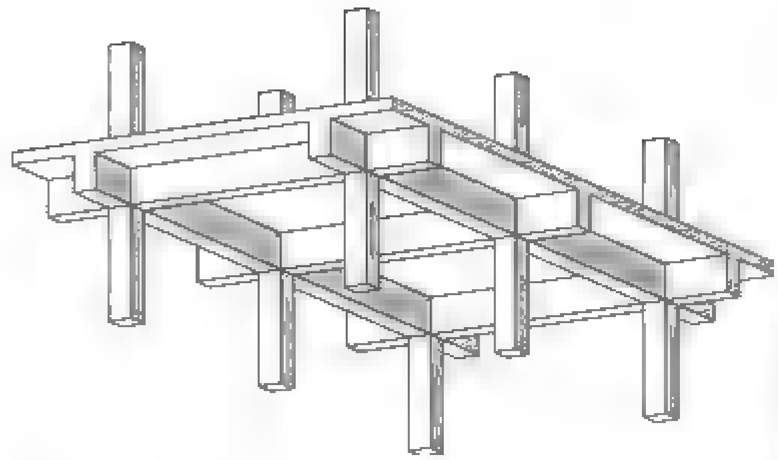


Fig. 2.4.7—Two-way slab with beams

shallow beam can be reinforced with closely spaced stirrups near the support to increase the slab's shear strength. This system is also sometimes referred to as wide-shallow beams with one-way slabs.

A structure using this type of floor system is less stiff laterally than a structure using a ductile moment frame with beams of normal depth. LFRSs consisting only of flat slab or flat plate frames, without ductile frames, structural walls, or other bracing members, are not suitable in SDC D through F.

2.4.7 *Two-way slabs with beams*—As shown in Fig. 2.4.7, the slab is supported by beams in two directions on the column lines. This system is useful where a beam-column frame is required as part of the LFRS. The slab provides high diaphragm stiffness, and the perimeter beams can provide sufficient lateral stiffness and strength though frame action for use in SDC D through F.

For longer spans, a two-way grid (Section 2.4.3) slab with beams may be used rather than a slab with beams.

2.4.8 *Precast slabs*—Precast, one-way slabs are usually supported by bearing walls, precast beams, or cast-in-place beams. Precast slabs may be solid, hollow-core slabs, or single- or double-T sections. They are sometimes topped by a thin cast-in-place concrete layer, referred to as a "topping slab."

Welded connections are normally used to transfer in-plane shear forces between precast slabs and their supports. Because precast slabs are individual units interconnected mechanically, the ability of the assembled floor system to act as a shear diaphragm must be examined by the engineer. Boundary reinforcement may be required, particularly where the lateral-force-resisting members are far apart. In areas of high seismicity, the connections between the precast slab system and the LFRS must be carefully detailed. A concrete topping bonded to the precast slab improves the ability of the slab system to act as a shear diaphragm, and can be used in SDC D through F.

2.5—Foundations

Foundation design must consider the weight of the building, live loads, and the transmission of lateral forces to the ground. A distinction should be drawn between external forces, such as wind, and inertia forces that result from the building's response to ground motions during an earthquake

External lateral forces can include static pressures due to water, earth or fill, and equivalent static forces representing the effects of wind pressures, where a gust factor or impact factor is included to account for their dynamic nature.

The soil type and strata usually dictate whether a deep or shallow foundation is required. A soils report from a licensed geotechnical engineer provides the detailed information and foundation recommendations that the licensed design professional (LDP) needs to design the foundation. For shallow footings, the geotechnical engineer provides an allowable soil bearing pressure for the soil at the foundation elevation. That pressure limit targets a certain amount of soil deflection and includes consideration of the anticipated use of the building. If allowable soil pressure is less than 2500 lb/ft^2, the soil is very soft and deep foundation options are usually considered. Other soil situations, such as expansive clay or nonstructural fill, may preclude the use of shallow foundations. If the building is below grade, concrete earth retaining walls can be part of the foundation system.

The two types of deep foundations are caissons (also known as piers) and piles. If hard rock is not far below existing grade, caissons can transfer a column load directly to the bedrock. Bearing values for solid rock can be more than 10 kip/ft^2. Caissons are large in diameter, usually starting at approximately 30 in. Piles are generally smaller in diameter, starting at approximately 12 in., and can be cast-in-place in drilled holes or precast piles that are driven into place. Piles are usually designed for lighter loads than caissons. Groups of piles may be used where bedrock is too deep for a caisson. Tops of piles or caissons are bridged by pile caps and grade beams to distribute column loads as needed.

Shallow foundations are referred to as footings. Types of footings are isolated, combined, and mat. Isolated rectangular or square footings are the most common types. Combined footings are often needed if columns are too close together for two isolated footings, if an exterior column is too close to the boundary line, or if columns are transmitting moments to the footing, such as if the column is part of an LFRS. If the column loads are uniformly large, such as in multi-story buildings, or if column spacing is small, mat foundations are considered.

2.5.1 *Resistance to lateral loads*—The vertical foundation pressures resulting from lateral loads are usually of short duration and constitute a small percentage of the total vertical load effects that govern long-term soil settlements. Allowing a temporary peak in vertical bearing pressures under the influence of short-term lateral loads is usually preferred to making the footing areas larger.

The geotechnical engineer should report the likelihood of liquefaction of sands or granular soils in areas with a high groundwater table, or the possibility of sudden consolidation of loose soils when subjected to jarring. The capacity of friction piles founded in soils susceptible to liquefaction or consolidation should be checked.

2.5.2 *Resistance to overturning*—The engineer should investigate the safety factor of the foundation against overturning and ensure it is within the limits of the local building code. Overturning calculations should be made with removable soil fill or live load completely removed and should be based on a safe (low) estimate of the building's actual dead load.

2.6—Structural analysis

The analysis of concrete structures "shall satisfy compatibility of deformations and equilibrium of forces," as stated in Section 4.5.1 of the Code. The LDP may choose any method of analysis as long as these conditions are met. This discussion is intended to be a brief overview of the analysis process as it relates to structural concrete design. For more detailed information on structural analysis, refer to Chapter 3 of this Manual.

2.7—Durability

Reinforced concrete structures are expected to be durable. The design of the concrete mixture proportions should consider exposure to temperature extremes, snow and ice, and ice-removing chemicals. Chapter 19 of the Code provides mixture requirements to protect concrete and reinforcement against various exposures and deterioration. Chapter 20 of the Code provides concrete cover requirements to protect reinforcement against steel corrosion. For more information, refer to Chapter 4 of this Manual.

2.8—Sustainability

The Code provisions for strength, serviceability, and durability are minimum requirements to achieve a safe and durable concrete structure. To improve sustainability of the structure, however, the LDP or owner are permitted to specify requirements that are more stringent that those mandated by the Code. For example, the LDP may choose to specify a higher concrete strength or design with a more restrictive deflection limit to improve service life, which is one aspect of sustainability. The strength, serviceability, and durability requirements specified in the Code, however, are required to take precedence over sustainability considerations.

For more information, the reader is directed to ACI 130R, which provides in-depth discussion on the connection between materials selection and their impact on sustainable development. Topics include efficient use of materials, life-cycle assessment, energy, and replacement materials, among others.

2.9—Structural integrity

Code provisions for *structural integrity* are intended to "improve the redundancy and ductility in structures so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be confined to a relatively small area and the structure will have a better chance to maintain overall stability" (ACI Committee 442 1971). It is currently defined in the Code as the "ability of a structure through strength, redundancy, ductility, and detailing of reinforcement to redistribute stresses and maintain overall stability if localized damage or significant overstress occurs." The means by which the Code ensures structural integrity varies depending on the member or system type, load characteristics, and details. Table 2.9.1

shows the different member types that have specific structural integrity requirements.

Many of the Code sections quoted in the table are focused on reinforcement detailing to maintain overall stability in the event of an overload or the loss of a supporting element. These detailing provisions are relatively inexpensive and unobtrusive methods that are intended to restrict the extent of damage and provide overall stability. The fundamental mechanism is illustrated in Fig. 2.9a. In the event a supporting element is lost, whether through a shear failure or outright loss of a column, the flexural strength of the horizontal element will likely be exceeded by doubling of the span length. The failed element will fall on the floor below and precipitate failure of that element, resulting in a progressive collapse. Reinforcing bars that are properly anchored and spliced, however, can behave as a cable and through catenary action support the element, thus stabilizing the structure and preventing further collapse. Mitchell and Cook (1984) cite experimental evidence that this mechanism can stabilize two-way slab systems by developing a "net" of reinforcing bars that can prevent progressive collapse.

Rather than attempting to compute the loads associated with a collapse scenario, most of the Code provisions in the table call for a specific number of bars that are already in place for positive reinforcement to be continuous. The use of bottom bars is critical to the success of these provisions. For example, Code Section 9.7.7 requires that one-quarter of the positive reinforcement of interior beams be continuous, with

Table 2.9.1—Structural integrity Code provisions for specific member types

Member type	Section
Nonprestressed one-way cast-in-place slabs	7.7.7
Nonprestressed two-way slabs	8.7.4.2
Prestressed two-way slabs	8.7.5.6
Nonprestressed two-way joist slabs	8.8.1.6
Cast-in-place beam	9.7.7
Nonprestressed one-way joist system	9.8.1.6
Precast joints and connections	16.2.[illegible].8

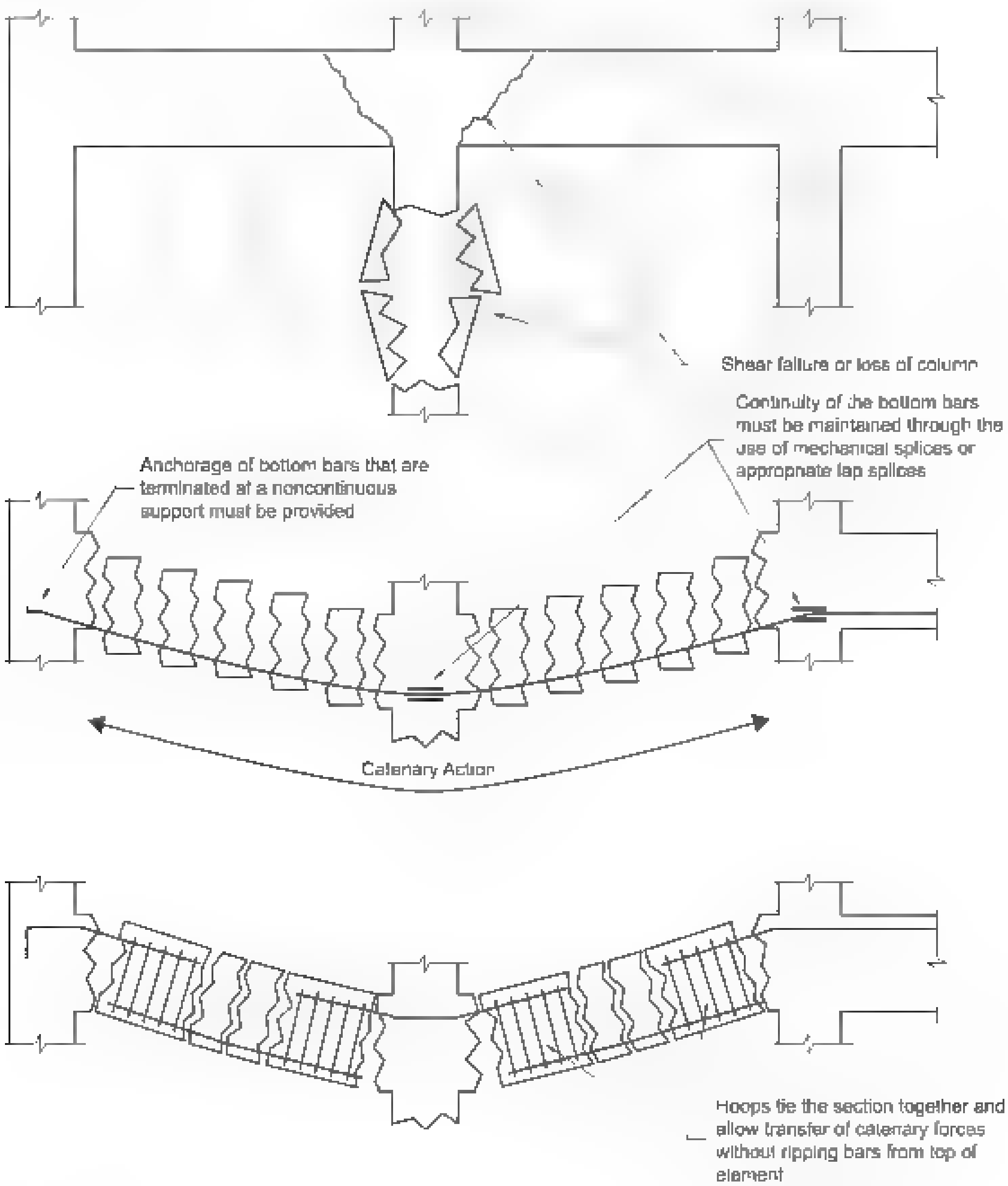


Fig. 2.9a—Role of integrity reinforcement in maintaining overall structural stability following loss of support.

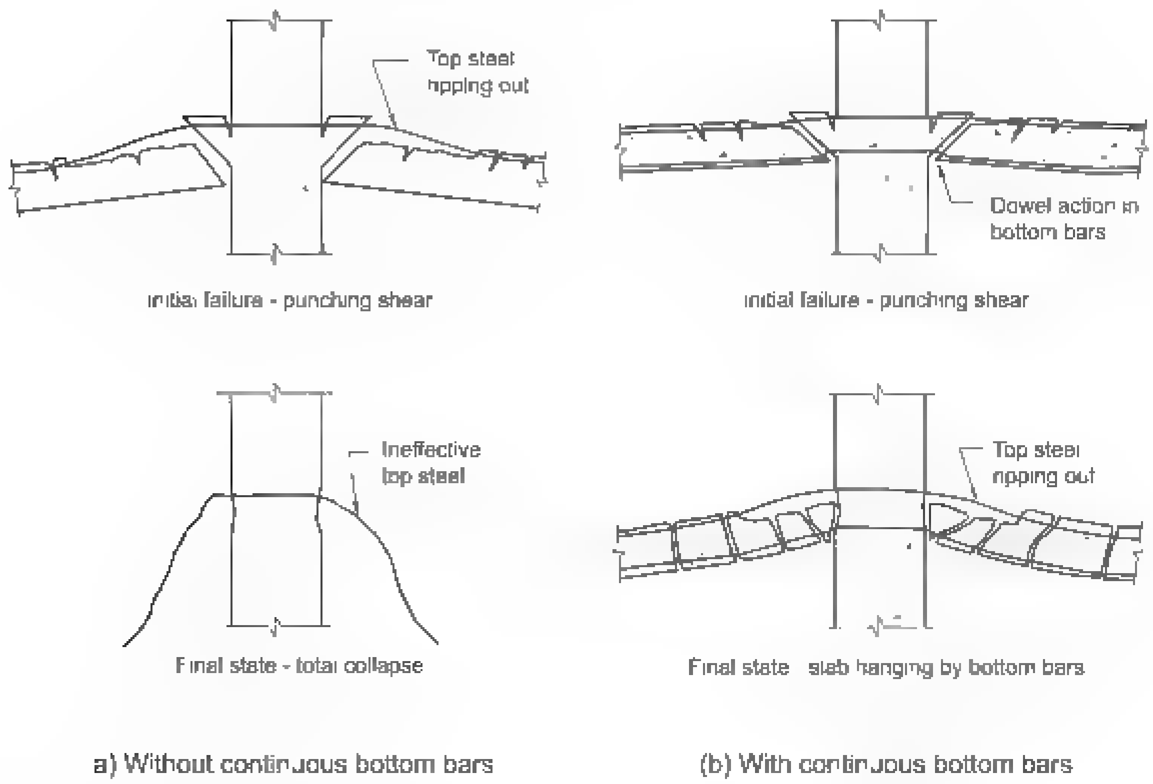


Fig. 2.9b—Catenary action of bottom flexural reinforcement after punching failure.

a minimum of two bars or strands. Alternatively, top reinforcement can be engaged by enclosing both top and bottom steel with closed stirrups in accordance with 25.7.1.6.

Figure 2.9b illustrates the effect of reliance on top bars to behave as catenary support for the horizontal element. Following a shear failure or other loss of support, the slab or beam will hang from the top bars. This will result in the top bars ripping out of the top of the slab or beam and allowing collapse to continue. As illustrated in Fig. 2.9a, using bottom bars that are continuous from end to end of the structure ensures that a continuous catenary element is available at any location in the structure. Furthermore, if the continuous bars are properly anchored in the exterior supports and mechanically spliced or appropriately lap spliced in other locations, then the approach will be successful.

2.10—Fire resistance

Minimum cover specified in Chapter 20 of the Code is intended to protect reinforcement against fire; however, the Code does not provide a method to determine the fire rating of a member. The International Building Code (IBC) 2015 Section 722 permits calculations that determine fire ratings to be performed in accordance with ACI 216.1 for concrete, concrete masonry, and clay masonry members.

2.11—Post-tensioned/prestressed construction

The introduction of post-tensioning/prestressing to concrete floor, beams, and wall elements imparts an active, permanent force within the structural system. Because cast-in-place structural systems are monolithic, this force affects the behavior of the entire system. The engineer should consider how elastic and plastic deformations, deflections, changes in length, and rotations due to post-tensioning/prestressing affect the entire system. Special attention must be given to the connection of post-tensioned/prestressed members to other members to ensure the proper transfer of forces between, and maintain a continuous load path. Because the post-tensioning/prestressing force is permanent, the system creep and shrinkage effects require attention.

2.12—Quality assurance, construction, and inspection

Code Section 4.13 requires that the specifications used for the execution of construction must be prepared in accordance with the provisions in Chapter 26. In addition, inspection must be conducted in accordance with the provisions in Chapter 26 and the general building code. These two general provisions, along with the more specific provisions provided in Chapter 26, establish the minimum level of quality. In the context of the Code, quality means that mechanical and durability properties of the construction materials meet or exceed either values explicitly assumed by the designer or values that are implicit in the Code provisions. Quality also denotes the level of workmanship used in assembling the materials into their final form. Chapter 26 also requires verification, typically through inspection, that the Work was constructed in accordance with the construction documents and thus meets the intent of the structural design.

At first glance, Chapter 26 may appear to apply directly to contractors. As indicated in the scope of the chapter, however, it is "directed to the LDP responsible for incorporating project requirements into the construction documents." Construction documents, which contain written and graphic instructions for the construction of the Work, are prepared by the LDP to ensure that the structural system meets the structural system requirements of Code Chapter 4. As such, the LDP should neither expect nor require the Contractor to read and interpret the Code. Construction documents should be free of general or specific references to the Code. Such provisions as are necessary to ensure compli-

ance with the Code should be interpreted by the designer and included in the construction documents. For instance, use of material standards published by ASTM International should be included in the construction documents to ensure that materials used by the contractor are manufactured correctly. Other standards, such as ACI 301, which was written to be consistent with the requirements of the Code, can be used by the designer to aid in preparation of their construction specifications. ACI 301 is a reference specification that the designer can apply by citing it in the construction documents. A mandatory requirements checklist and an optional requirements checklist are provided to assist the designer in implementing this Specification.

Chapter 26 also provides the LDP with minimum information that should be included in the construction documents such as information developed in the structural design that must be conveyed to the contractor, provisions directing the contractor on required quality, and inspection requirements to verify compliance with the construction documents.

REFERENCES

American Concrete Institute

ACI 130R-19—Report on the Role of Materials in Sustainable Concrete Construction

ACI 216.1-14—Code Requirements for Determining Fire Resistance of Concrete and Masonry Construction Assemblies

ACI 301-16—Specifications for Structural Concrete

ACI 352R-02—Recommendations for Design of Beam-Column Connections in Monolithic Reinforced Concrete Structures

ACI 442R-71—Response of Buildings to Lateral Forces

ACI 442R-88—Response of Concrete Buildings to Lateral Forces

American Society of Civil Engineers

ASCE/SEI 7-16—Minimum Design Loads for Buildings and Other Structures

International Code Council

IBC 2015—International Building Code

Authored documents

Ali, M. M., and Moon, K. S., 2007, "Structural Development in Tall Buildings: Current Trends and Future Prospects," *Architectural Science Review*, V. 50, No. 3, pp. 205-223.

Mitchell, D., and Cook, W. D., 1984, "Preventing Progressive Collapse of Slab Structures," *Journal of Structural Engineering*, ASCE, V. 110, No. 7, pp. 1513-1532.

Structural Systems

CHAPTER 3—STRUCTURAL ANALYSIS

3.1—Introduction

Structural engineers mathematically model reinforced concrete structures, in part or in whole, to calculate member moments, forces, and displacements under the design loads that are specified by a standard such as ASCE/SEI 7. In all conditions, equilibrium of forces and compatibility of deformations must be maintained. The stiffness values of individual members for input into the model, under both service loads and factored loads, are discussed in detail in Code Chapter 6. The factored moments and forces resulting from the analysis are used to determine the required strengths for individual members. The calculated displacements and drift are also checked against commonly accepted serviceability limits.

3.2—Overview of structural analysis

3.2.1 *General*—The analysis of concrete structures "shall satisfy compatibility of deformations and equilibrium of forces," as stated in Section 4.5.1 of the Code. The licensed design professional (LDP) may choose any method of analysis as long as these conditions are met. The Code specifically recognizes four methods of analysis. Section 6.6 addresses the most common, which is linear elastic first-order analysis. Section 6.7 covers linear elastic second-order analysis, which is typically used to calculate slenderness effects. Section 6.8 addresses inelastic analysis, both first- and second-order. In addition, the Code permits the use of strut-and-tie modeling for the analysis of discontinuous regions.

Except as noted in Code Chapter 18, structural concrete members are typically assumed to be prismatic and behave elastically under design loads. Although these assumptions result in behavior that differs from the actual behavior of the concrete member, they provide a reasonable estimate of the distribution of the demands on the members. The Commentary suggests that both serviceability and strength requirements can be addressed by conducting separate analyses with varying stiffness assumptions to bound the actual solution, especially where stiffness can significantly affect results.

3.2.2 *Elastic analysis*—Load effects in most concrete structures are determined using elastic analysis techniques that are implemented with computer software. If a first-order analysis is used, then Code Section 6.6.1.1 indicates that slenderness effects should be considered by using the moment magnification approach covered in Section 6.6.4. In this approach, moments obtained from a first-order frame analysis are multiplied by a moment magnifier that is based on the axial load and buckling strength of the column. Frames that are braced against sway are treated differently from those that are not. Alternatively, a second-order analysis can be used to determine the slenderness effects directly by considering the loads applied to the deformed structure. This approach is typically implemented with computer software and involves iterative analyses where the stiffness matrix is reformulated as the geometry of the structure changes under load. A valid second-order analysis requires that the structure be in equilibrium in the final deformed shape.

3.2.3 *Inelastic analysis*—Inelastic analysis considers material nonlinearities such as concrete cracking and steel yielding. As with elastic analyses, a first-order inelastic analysis must satisfy equilibrium in the undeformed (original) geometry. A second-order inelastic analysis must satisfy equilibrium in the deformed configuration. Code Commentary R6.8.1.1 suggests that material nonlinearities may be affected by duration of loads, shrinkage, and creep. The Code indicates that inelastic analysis procedures should be validated by comparison of strength and deformation results with that of physical tests. Inelastic analyses are typically used for "push-over" analysis, which is used in seismic retrofit of existing buildings, design of materials and

Table 3.2.5—Common analysis types and tools

Analysis type	Applicable member or assembly	Analysis tool
First-order Linear elastic Static load Hand calculations	One-way slab	Analysis tables*
	Continuous one-way slab	Simplified method in Section 6.5 of the Code
	Two-way slab	Direct design method in Section 8.10 of ACI 318-14†
		Equivalent frame method in Section 8.11 of ACI 318-14†
	Beam	Analysis tables*
	Continuous beam	Simplified method in Section 6.5 of the Code
	Column	Interaction diagrams*
	Wall	Interaction diagrams*
		Alternative method for out-of-plane slender wall analysis in Section 11.8 of the Code
	Two-dimensional frame	Portal method in Section 3.3.3 of this chapter
First-order Linear elastic Static load Computer programs	Gravity-only systems	Spreadsheet program based on the analysis tools for hand calculations above
		Program based on matrix methods but only analyze floor assemblies for gravity loads
	Two-dimensional frames and walls	Program based on matrix methods without iterative capability
Second-order Linear elastic Static or dynamic load Computer programs	Two-dimensional frames and walls	Programs based on matrix methods with iterative capability
	Three-dimensional structure	Programs based on finite element methods with iterative capability
Second-order Inelastic	Three-dimensional structure	Beyond the scope of this Manual

*Information can be downloaded from the ACI website; refer to Foreword for links.

†Direct design method and equivalent frame method are not included in ACI 318-19 but are still permitted.

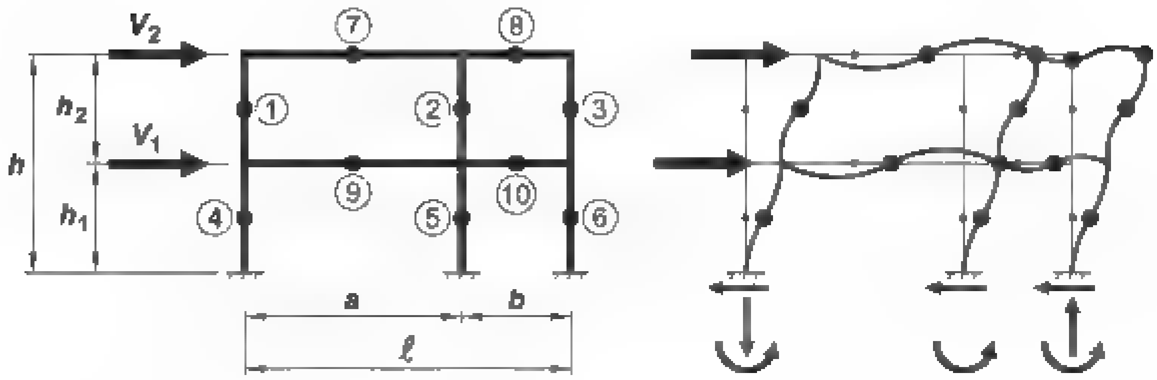


Fig. 3.3.3—Frame analyzed by portal method.

systems not covered by the Code, and evaluation of building performance above Code minimum requirements (Deierlein et al. 2010). This Manual does not include any examples of inelastic analyses.

3.2.4 *Strut-and-tie*—The strut-and-tie method in Code Chapter 23 is another analysis method that is permitted by the Code. This method does not assume that plane sections of unloaded members remain plane under loading. Because this method also provides design provisions, it is considered both an analysis and design method. This method is applicable where the sectional strength assumptions in Code Chapter 22 do not apply for a discontinuity region of a member or a local area.

3.2.5 *Analysis types and tools*—The Code identifies three general types of analysis, refer to Section 3.2.1: 1) first-order linear elastic, 2) second-order linear elastic; and 3) second-order inelastic. Table 3.2.5 shows some common analysis tools used for different analysis methods, loads, and systems.

3.3—Hand calculations

3.3.1 *General*—Before computers became widely available, designers used simplified analysis methods to calculate gravity design moments and shears (Code Section 6.5) along with a simplified frame analysis technique such as the portal method to calculate frame moments and shears due to lateral forces. In limited applications, the design of an entire building using hand calculations is still possible with today's set of building codes. For the large majority of building designs, however, a hand calculation design approach is not practical due to the large number and complexity of design load combinations necessary to fully meet ASCE/SEI 7 requirements.

3.3.2 *Code design equations for moment and shear*—The simplified Code equations are useful for purposes of preliminary estimating or member sizing, for designing isolated members or subassemblies, and to complete rough checks of computer program output. Because these equations and expressions are easy to incorporate into electronic spreadsheets and equation solvers, they continue to be helpful. In the member chapters of this Manual, examples of hand calculations are provided.

3.3.3 *Portal method*—The portal method was commonly used before computers were readily available to calculate a frame's moments, shears, and axial forces due to lateral forces (Hibbeler 2015). This method has been virtually abandoned as a design tool with the widespread use of commercial design software programs. The portal method has limitations as stated in the assumptions and considerations that follow but it is still a useful tool for the designer. With complex, three-dimensional modeling becoming commonplace, there is always a chance of modeling error. The portal method allows the designer to independently and quickly find approximate moments and shears in a frame. This can be useful for spot-checking the program results (Fig. 3.3.3).

The basic assumptions in the portal method are:

(a) Apply only the lateral load to the frame

(b) Exterior columns resist the overturning from lateral loads

(c) Shear at each column is based on plan tributary area

(d) Inflection points are assumed to be located at midheight of column and midspan of beams

(e) Shear in the beam is the difference between column axial forces at a joint

(f) Beam axial force is to be zero

These assumptions reduce a statically indeterminate problem to a statically determinate one.

The following should be considered when using this method:

(a) Discontinuity in geometry or stiffness—such as setbacks, changes in story height, and large changes in member sizes—can cause member moments to differ significantly from those calculated by a computer analysis

(b) The lateral deformation will be larger than the lateral deformation calculated by a computer analysis

(c) Axial column deformation is ignored

3.4—Computer programs

3.4.1 *General*—Computing power and structural software have advanced significantly from the time computers were introduced to the designer. Numerous complex computer programs and specialized analysis tools have been developed, taking advantage of increasing computer speeds. Currently, designers commonly use finite element analysis to design structures. A multi-story building only takes minutes of computing time on a personal computer compared to the past, when it would take several hours on a large mainframe computer. Computer software has also greatly improved, user interfaces have become more intuitive, members can be automatically meshed, and input and output data can be reviewed in graphical and in tabular form in a variety of preprogrammed or user-defined menus. Sophisticated user interfaces, however, further disconnects the designer from the analysis process, which can lead to modeling errors or incorrect interpretation of output. It is important, therefore,

for the designer to have a solid understanding of structural behavior and design and of the software being used in the analysis. This makes hand calculations discussed in the previous section an important tool that can be used to verify that the designer has used the program properly and has interpreted the output correctly

Although three-dimensional models are becoming commonplace, many engineers still analyze the building as a series of two-dimensional frames. Matrix methods mentioned in Table 3.2.5 are programs based on the direct stiffness method. Simpler programs model the structure as discrete members connected at joints. The members can be divided into multiple elements to account for changes in member properties along its length. The two-dimensional stiffness method is a key component of many commercially available software packages and is typically used to model frame and shear wall structural systems

For complex geometric shapes, boundary conditions, or loads, the finite element analysis (FEA) can be used. FEA involves the discretization of a member or system into multiple discrete solid elements that are typically connected together at nodes (similar to frame members). Each member of the structure consists of multiple elements. Subdividing members into an assembly of elements is called "meshing" and can be a time-consuming task. A large amount of data is generated from this type of analysis, which may be tedious for the designer to review and process. For straightforward designs, a designer may more efficiently analyze the structure by dividing it into parts and using less complicated programs to analyze each part

3.4.2 *Two-dimensional frame modeling*—A building can be divided into parts that are analyzed separately. For example, structures are often symmetrical with regularly spaced columns in both directions. There may be a few isolated areas of the structure where columns are irregularly spaced. These columns can be designed separately for gravity load and checked for deformation compatibility when subjected to the expected overall lateral deflection of the structure

Buildings designed as moment-resisting frames can often be effectively modeled as a series of parallel planar frames. The complete structure is modeled using orthogonal sets of crossing frames. Compatibility of vertical deflections at crossing points is not required. The geometry of beams can vary depending on the floor system. For slab-column moment frames, the direct design method or equivalent frame method may be used (Code Section 8.2.1). For beam-column moment frames, it is permitted to model T-beams, with the limits on geometry given in Section 6.3.2 of the Code; however, it is often simpler to conservatively ignore the slab and use the section properties of the stem of the T-beam in the analysis. For beams in intermediate or special moment frames, however, the assumption of a rectangular section may not be conservative; refer to Sections 18.4.2.2, 18.6.5.1, and 18.7.3.2 in the Code

The stiffness of the beam-column joint is underestimated if the beam is assumed to span between column centerlines and the beam is modeled as prismatic along the entire span. Consequently, most computer programs adjust their analysis procedures to account for the contribution of the column stiffness to the overall beam stiffness. To do this, a program may add a rigid zone that extends from the face of the column to the column centerline. If the program does not provide this option, the designer can increase the beam stiffness in the column region 10 to 20 percent to account for this change of rigidity (ACI 442-71). Walls with aspect ratios of total height-to-width greater than 2 can sometimes be modeled as column elements. A thin wall may be too slender for a conventional column analysis, and a more detailed evaluation of the boundary elements and panels may be necessary. Where a beam frames into a wall that is modeled as a column element, a rigid link should be provided between the edge of the wall and centerline of the wall

Wall elements can be modeled as either a frame element that is very stiff or, if the program has the capability, the wall can be modeled using FEA. Where a finite beam element frames into a finite wall element, rotational compatibility should be assured. Walls with openings can be more difficult to analyze with frame elements because the rigidity of the joints near the openings must be carefully considered (Fig. 3.4.2a(a)). Similar to the beam-column joint modeling discussed previously, a rigid link should be modeled from the centerline of the wall to the edge of the opening (Fig. 3.4.2a(b)). Alternatively, FEA can be used to analyze walls with openings (Fig. 3.4.2a(c)). Once the distribution of forces in the wall has been determined from FEA, the wall can be analyzed using the strut-and-tie method to check section size and reinforcement quantities and distribution

For lateral load analysis, all the parallel plane frames in a building are linked into one plane frame to enforce lateral deformation compatibility. Alternately, two identical frames can be modeled as one frame with doubled stiffness; one method to double the stiffness is to double the modulus of elasticity in the concrete material model. Structural walls, if present, should be linked to the frames at each floor level (Fig. 3.4.2b). Note that torsional effects should be considered after the lateral deformation compatibility analysis is run. For seismic loads, accidental torsion must be considered when the diaphragm is rigid (ASCE/SEI 7 Section 12.8.4.2). For wind loads, a torsional moment should be applied according to Fig. 27.3-8 in ASCE/SEI 7

3.4.3 *Three-dimensional modeling*—A three-dimensional model allows the designer to observe structural behavior that a two-dimensional model would not reveal. The effects of structural irregularities and torsional response can be directly analyzed. Current computer software that provides three-dimensional modeling are capable of running a modal response spectrum analysis, seismic response history procedures, and can perform a host of other time-consuming mathematical tasks.

To reduce computation time, concrete floors are sometimes modeled as rigid diaphragms, reducing the number of dynamic degrees of freedom to only three per floor (two horizontal translations and one rotation about a vertical axis). ASCE/SEI 7 allows for diaphragms to be modeled as rigid if following conditions are met

(a) For seismic loading, no structural irregularities and the span-to-depth ratios are 3 or less (Section 12.3.1.2 in ASCE/SEI 7)

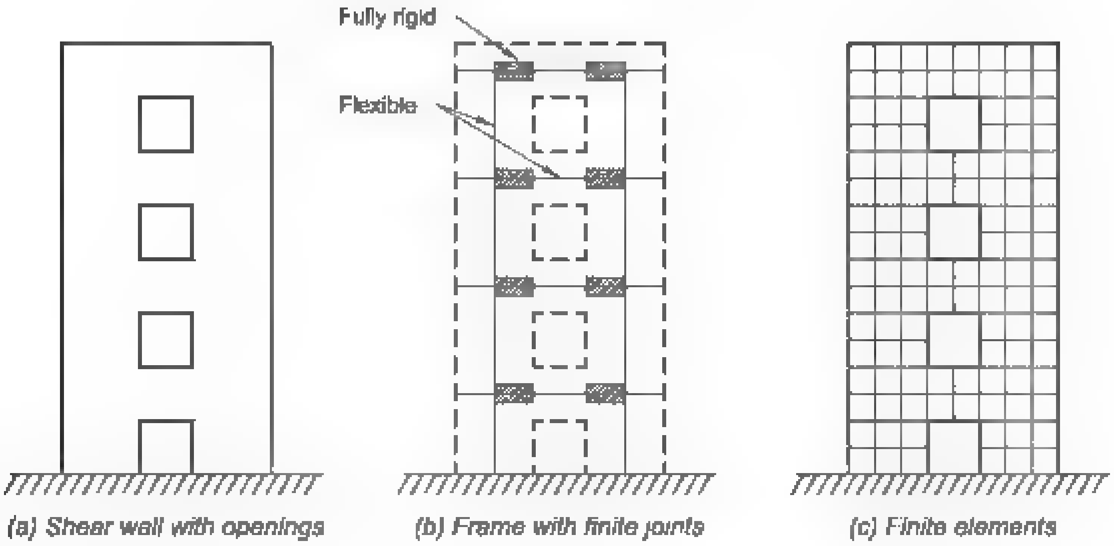


Fig. 3.4.2a—Element and frame analogies.

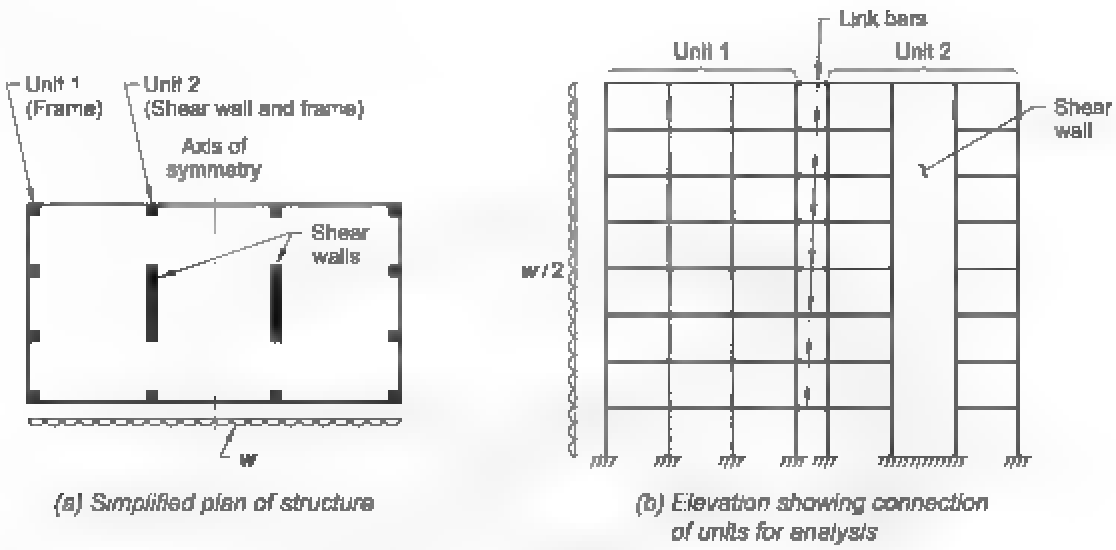


Fig. 3.4.2b—Idealization for plane frame analysis.

(b) For wind loading, the span-to-depth ratios are 2 or less (Section 27.4.5 in ASCE/SEI 7)

If a rigid diaphragm is assumed, the stresses in the diaphragm are not calculated and need to be derived from the reactions in the walls above and below the floor. A semi-rigid diaphragm requires more computation power but provides a distribution of lateral forces and calculates slab stresses. A semi-rigid diaphragm can also be helpful in analyzing torsion effects. For more information on torsion, refer to ASCE/SEI 7 Section 12.8.

3.5—Structural analysis in ACI 318

3.5.1 *Arrangement of live loads*—Section 4.3.3 in ASCE/SEI 7 states that the "full intensity of the appropriately reduced live load applied only to a portion of a structure or member shall be accounted for if it produces a more unfavorable load effect than the same intensity applied over the full structure or member." This is a general requirement that acknowledges greater moments and shears may occur with a pattern load than with a uniform load. There have been a variety of methods used to meet this requirement. Cast-in-place concrete is inherently continuous, and Section 6.4 in the Code provides acceptable arrangements of pattern live load for continuous one-way and two-way floor systems.

3.5.2 *Simplified method of analysis for nonprestressed continuous beams and one-way slabs*—Section 6.5 in the Code provides approximate equations for conservative design moments and shears, which greatly simplifies the design of continuous floor members. This method is probably used more often to estimate initial member sizes for computer input, or for initial cost estimates, than for final design.

3.5.3 *First-order analysis*—Code requirements for first-order analysis are provided in Section 6.6 in the Code.

3.5.3.1 *Section properties*—Section properties for elastic analysis are given in Table 6.6.3.1.1(a) of the Code. The moment of inertia values have a stiffness reduction ϕ_k of 0.875 already applied. These properties are acceptable for the analysis of the structure for strength design. For service level load analysis, the moment of inertia values in Table 6.6.3.1.1(a) can be multiplied by 1.4. Table 6.6.3.1.1(b) offers a more accurate estimation of stiffness by including the effects of axial load, eccentricity, reinforcement ratio, and concrete compressive strength. These equations can also be used to calculate member stiffness at factored load levels by using the factored axial load and moment, as presented, but the equations can be used to calculate member stiffness for any given axial load and moment. These moment-of-inertia equations also have the 0.875 stiffness reduction ϕ_k already applied. Section 6.6.3.1.2 of the Code also allows a simplification of using $0.5I_g$ for all members in a factored lateral load analysis. This is helpful for hand-calculation methods such as the portal method.

It is important to note that the stiffness reduction factor used for moment of inertia discussed previously is for global building behavior. The moment of inertia for second-order effects related to an individual column or wall should have a stiffness reduction ϕ_K of 0.75, as discussed in R6.6.4.5.2 of the Code.

3.5.3.2 *Slenderness effects*—A first-order analysis in the Code assumes that only primary stresses are calculated. Secondary stresses caused by the lateral deflection caused by the design loads are not calculated. First-order analysis is typical when hand-calculation methods are used or basic matrix analysis computer programs are used that are not programmed for iterative analysis.

This method ignores both P-Δ and P-δ effects, which are the second-order moments caused by axial loads acting on the deformed geometry (Fig. 3.5.3.2). P-Δ is typically considered the second-order moment due to sidesway of the structure in which Δ is the lateral relative story drift. P-δ is the second-order moment due to the flexural deformation, δ, of the column relative to its supporting ends. To approximately account for these secondary effects, a moment magnifier is applied to first-order column design moments.

The designer must account for slenderness in a first-order analysis. Figure R6.2.5.3 in the Code provides a flowchart that illustrates the options to account for slenderness. In summary, slenderness can be neglected if the column or wall meets the requirement of Section 6.2.5 in the Code. If slenderness cannot be neglected, the next step is to determine if the building story being analyzed is braced against sidesway or unbraced. If the story is braced, the column or wall end moments are magnified for moment effects along the member (P-δ). If the story is unbraced, the column or wall end moments are magnified for moment effects along the member (P-δ) and at the ends due to story drift (P-Δ).

3.5.3.3 *Superposition*—Linear analysis allows for superposition to be used when combining loads. This is helpful when performing hand calculations. The designer calculates the member moment, shear, and axial load for each load. The reactions are then superimposed according to the applicable load combination. Many hand-calculation tools, such as the moment magnification method, assume that the designer is performing a linear analysis with superposition of multiple load effects.

3.5.3.4 *Redistribution of moments*—The Code allows for the designer to adjust design slab and beam moments and shears by taking advantage of the ductility provided through Code detailing requirements. Ductile detailing is required for continuous members at supports and midspan. Moment redistribution can be helpful in creating economical designs. For example, in a final design, moment redistribution may permit the designer to specify uniform beam sizes over multiple beam spans. If column spacing is not uniform, some beam design moments may be slightly lower than the beam required moments. Once steel yielding has developed at factored loads, however, additional moments will be carried by regions that have not yet yielded, and the beam design moments will satisfy the beam required moments throughout the multiple beam spans.

3.5.4 *Second-order analysis*—Code Section 6.7 indicates that a second-order analysis must consider the effect

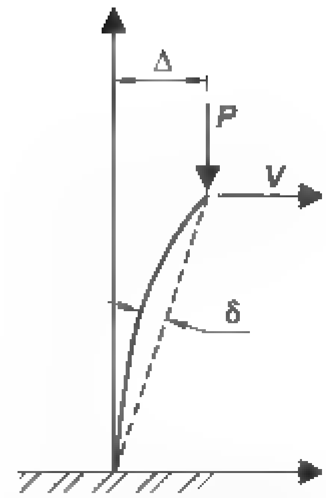


Fig. 3.5.3.2—P-Δ effects.

of loads on the laterally deformed structure. The initial P-Δ effects on the member due to story drift are computed. A computer algorithm then automatically carries out a series of iterative analyses using the new deflection values until the solution converges to the final secondary moments. Note that linear material properties are used with this method, but the results of a second-order analysis is a nonlinear solution. This is referred to as "geometric nonlinearity." This means that the load cases cannot be computed separately and then combined for the calculation of the secondary moments.

Software should be checked to determine if it accounts for both P-Δ and P-δ effects. Software can easily calculate the additional moment due to building lateral deformation but some software does not calculate the secondary moments along the member length. The designer may have to model the column as at least two segments to capture this effect. Even though a column member is modeled by two elements, the designer must account for the smaller stiffness reduction factor for the moment of inertia (refer to Section 3.5.3.1) because deflection along the member is a local effect. Because of the difficulty of appropriately capturing the secondary moment along the column length, many programs calculate the secondary moments due to P-Δ effects and then use Code Section 6.6.4.5 in a post-processing program to account for the secondary moments due to P-δ effects.

3.5.5 *Inelastic second-order analysis*—The consideration of the nonlinear behavior of structures arising from nonlinearity of the stress-strain curve for concrete and steel reinforcement, particularly under large deformations, may be important in seismic analysis. Nonlinearities in structural response, whether arising from material properties as for concrete or steel, loading conditions (for example, axial load effects on bending stiffness), or geometry (for example, second moments) are best handled by numerical iterative or step-by-step procedures. For inelastic second-order analysis, the principle of superposition should not be used. Nonlinear analysis is beyond the scope of this Manual. Several references that provide further information on nonlinear analysis are ASCE 41, FEMA 440, and Deierlein et al. (2010).

3.5.6 *Finite element analysis*—The finite element analysis of concrete structures is permitted by the Code and can be used to satisfy any of the modeling approaches in the Code as long as the element types are compatible with the response required.

Structural Analysis

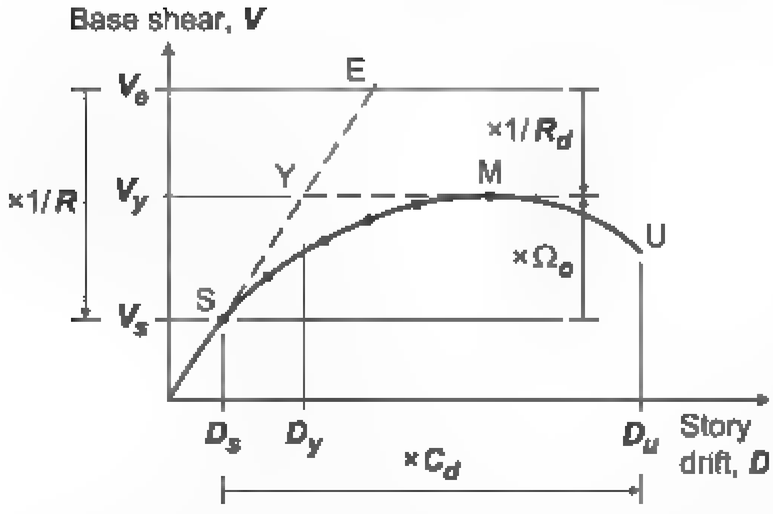


Fig. 3.6—Inelastic force-deformation curve (SEAOC Seismology Committee 2008).

Section 6.9 in the Code was added to acknowledge that finite element analysis is a widely used and acceptable tool for analysis. Many programs are based on finite element analysis and have sophisticated auto-mesh capabilities. Finite element analysis is a tool that may be used for either linear or nonlinear analyses, but care should be exercised in selecting element types, numerical solver methods, and nonlinear element properties.

3.6—Seismic analysis

When exposed to substantial ground shaking, structures can experience multiple cycles of significant inelastic deformations. Early dynamic evaluation of structures that had survived earthquakes indicated that the computed inertial forces were much higher than the resistance of the structures that survived earthquakes. In many cases, the differences were well beyond what might be accounted for by safety factors and overstrength. This ability to resist what appear to be very high loads is attributed to structural ductility and the ability to dissipate energy through post-elastic deformations. Simulation of post-elastic deformation is complex and computationally intensive. Consequently, the equivalent lateral force procedure was developed, which allows the use of the elastic analysis method using spectral response accelerations based on a single degree of freedom system in combination with reduction factors that account for the ductility of the system being analyzed.

ASCE/SEI 7 provides the equivalent lateral force (ELF) analysis procedure (Section 12.8 of ASCE/SEI 7) to allow a linear elastic analysis even though the structure will behave inelastically. ELF analysis is a commonly used analysis method and is adequate for many structures. ELF analysis assumes an approximately uniform distribution of mass and stiffness along the building height with minor torsional effects. Structures analyzed using the ELF procedure for seismic loads must comply with several limitations. Depending on the Seismic Design Category (SDC), the building height and type, and the type of structural irregularities, a Modal Response Spectrum analysis (Section 12.9 of ASCE/SEI 7) or Seismic Response History procedure, either linear or nonlinear (Chapter 16 of ASCE/SEI 7), may be required. Regardless of irregularities, the ELF procedure is acceptable for all buildings in SDC A and B up to 160 ft in height.

Section 12.3 of ASCE/SEI 7 provides limitations related to types of structural irregularities. Five horizontal irregularities are described: 1) torsion; 2) reentrant corner; 3) diaphragm discontinuity; 4) out-of-plane offset; and 5) nonparallel systems. Five vertical irregularities are also described: 1) stiffness-soft story; 2) weight; 3) vertical geometry; 4) in-plane discontinuity in lateral force-resisting systems (LFRSs); and 5) discontinuity in lateral strength.

Because the structure will likely undergo greater deflections and stresses than predicted by an ELF analysis, ASCE/SEI 7 and the Code have additional requirements to account for the anticipated behavior. Lateral-force-resisting systems for concrete structures are defined in ASCE/SEI 7 and Chapter 18 of the Code. Each LFRS has a response modification coefficient R, overstrength factor Ω_o, and deflection amplification factor C_d (Fig. 3.6) used in analysis. These factors account for the difference between the estimated elastic inertial forces and their actual effect on the structure when accounting for inelastic response. Detailed explanations of these factors and their application can be found in ASCE/SEI 7 and FEMA P-750.

For reinforced concrete structures, the Code provides structures with the ability to deform inelastically by enforcing special seismic detailing requirements. The seismic detailing requirements in Chapter 18 of the Code are additive to the detailing requirements in the member chapters, or the seismic detailing requirements supersede the member chapter requirements. The detailing requirements for a particular LFRS need to be applied even if seismic loads do not govern the required strength of the structure.

REFERENCES

American Concrete Institute

ACI 442-77—Response of Buildings to Lateral Forces

American Society of Civil Engineers

ASCE/SEI 7-16—Minimum Design Loads for Buildings and Other Structures

ASCE 41-13—Seismic Evaluation and Retrofit Rehabilitation of Existing Buildings

Federal Emergency Management Agency

FEMA 440-05—Improvement of Nonlinear Static Seismic Analysis Procedures

FEMA P-750-09—NEHRP Recommended Seismic Provisions for New Buildings and Other Structures

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Deierlein, G. G., Reinhorn, A. M., and Willford, M. R., 2010, "Nonlinear Structural Analysis for Seismic Design," NEHRP Seismic Design Technical Brief No. 4 (NIST GCR 10-917-5), National Institute of Standards and Technology, Gaithersburg, MD, 36 pp.

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CHAPTER 4—DURABILITY

4.1—Introduction

Durability of structural concrete is its ability, while in service, to resist possible deterioration due to the surrounding environment, and to maintain its engineering properties. This can be accomplished by proper proportioning and selection of materials for the concrete mixture design. Other aspects influencing durability include reinforcing bar selection, detailing, and construction practices. The Code provides minimum requirements to protect the structure against early serviceability deterioration. Depending on exposure conditions, structural concrete may be required to resist chemical or physical attack, or both. The attack mechanisms the Code covers include exposure to freezing and thawing, soil and water sulfates, wetting and drying, and reinforcement corrosion due to chlorides. All these failure mechanisms depend on transport of water through concrete. For this reason, it is essential to understand the mechanisms themselves and how different concrete-making materials, including admixtures and their proportions, influence concrete's resistance to these mechanisms.

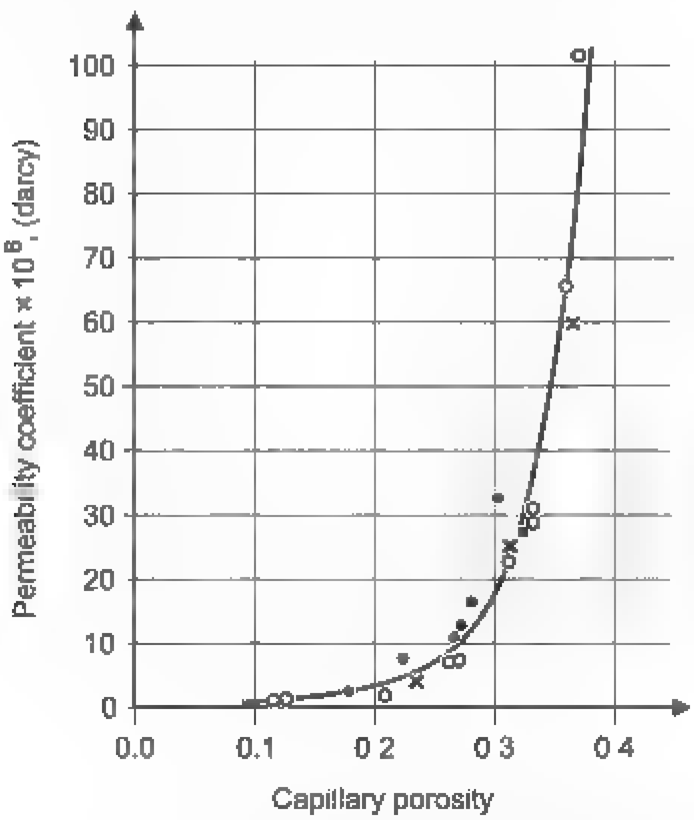

Fig. 4.1.1a—Permeability versus capillary porosity for cement paste. Different symbols designate different cement pastes (Powers 1958).

4.1.1 *Permeability*—Permeability can be defined as "the ease with which a fluid can flow through a solid" or as "the ability of concrete to resist penetration by water or other substances (liquid, gas, or ions)" (ACI 365.1R, Kosmatka and Wilson 2011). Low-permeability concretes are more resistant to resaturation, freezing and thawing, sulfate and chloride ion penetration, and other forms of chemical attack (Kosmatka and Wilson 2011). Concrete permeability is related to porosity (volume of voids/pores in concrete) and connectivity of these pores. Out of the pores present in concrete, capillary pores in cement paste are most relevant to concrete durability, as they are responsible for the transport properties of concrete (ACI 201.2R, Kosmatka and Wilson 2011). The influence of capillary porosity in cement paste on permeability was reported by Powers (1958) (Fig. 4.1.1a).

Concrete permeability, diffusivity, and electrical conductivity can be reduced with lower water-cement ratios (*w/c*), the use of supplementary cementitious materials (SCMs), and extended moist curing (Kosmatka and Wilson 2011). Effects of the *w/c* and duration of the moist curing on permeability is presented in Fig. 4.1.1b.

4.1.2 *Freezing and thawing*—When water freezes in concrete, it causes cement paste to dilate destructively by generating hydraulic and osmotic pressure. While hydraulic pressure forces water away from the freezing water filled capillary cavities, osmotic pressure is produced by water entering partly frozen capillary cavities (Powers 1958). Hydraulic pressures in cement paste are generated by the 9 percent increase in volume of water when it freezes and

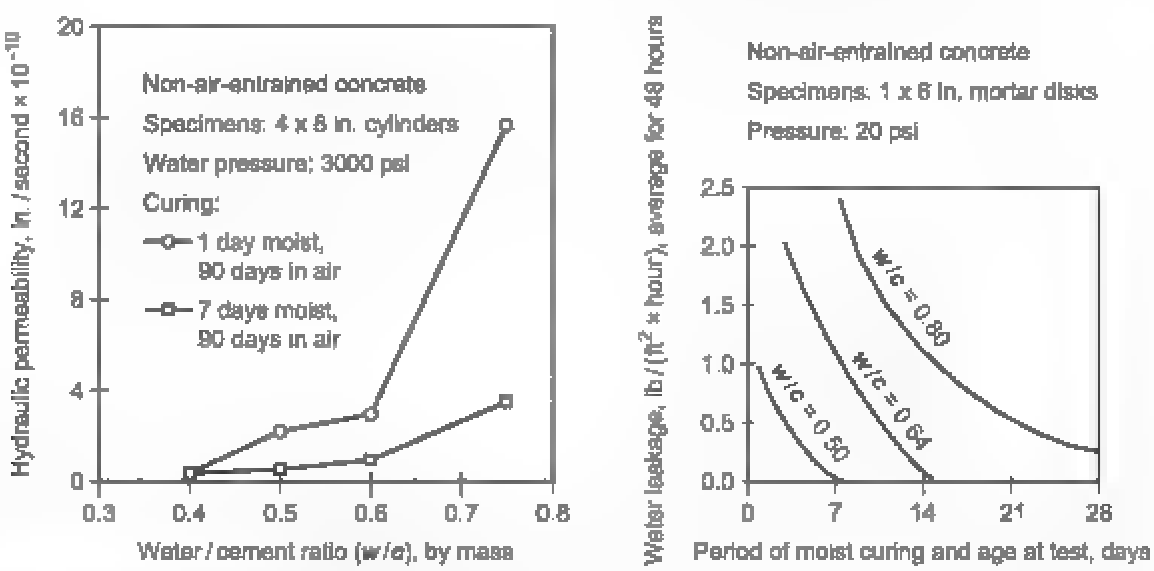

Fig. 4.1.1b—(left) Effect of w/c and initial curing on hydraulic (water) permeability, and (right) effect of w/c and curing duration on permeability (leakage) of mortar (Kosmatka and Wilson 2011).

Durability

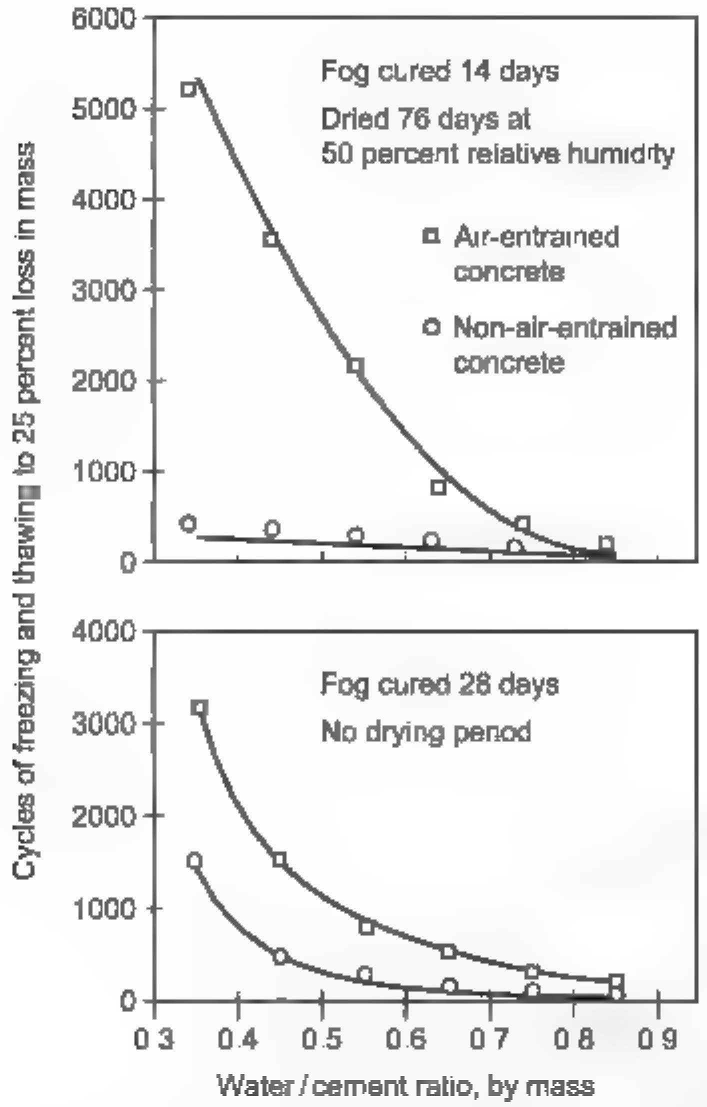

Fig. 4.1.2—Effect of w/c, air entrainment, and curing/drying on resistance to freezing and thawing of concrete (Kosmatka et al. 2008).

changes to ice. For the freezing to take place, a capillary has to reach its critical saturation: 91.7 percent filled with water (Kosmatka and Wilson 2011). Osmotic pressures develop due to differential concentrations of ionic species in the pore solution within the paste.

When pressure in concrete due to freezing exceeds the tensile strength of concrete, some damage occurs, especially if concrete is saturated with water and exposed to repeated cycles of freezing and thawing. The resulting damage is cumulative as it increases with additional repetitions of freezing and thawing cycles. Deterioration due to freezing and thawing can appear in the form of cracking, scaling, or disintegration, or all three of these (Kosmatka and Wilson 2011).

Low permeability and low absorption are main characteristics needed for concrete to be frost resistant, while air entraining admixtures are used to control the pressure generated in concrete paste during freezing-and-thawing cycles. In other words, high resistance to freezing and thawing is associated with entrained air, low *w/c*, and a drying period prior to freezing-and-thawing exposure, which is demonstrated by data presented in Fig. 4.1.2.

4.1.3 *Sulfates*—Sulfates present in soil and water can react with hydrated compounds in the hardened cement paste and induce sufficient pressure to disintegrate the concrete. However, formation of new crystalline substances due to

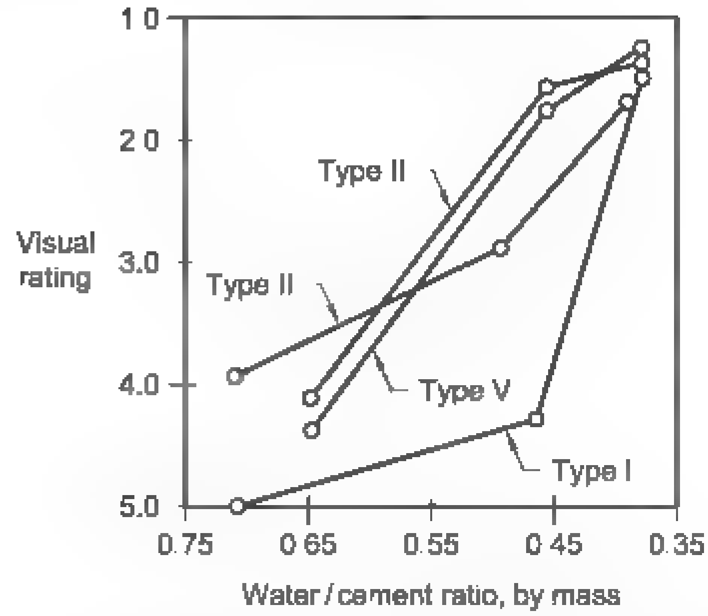

Fig. 4.1.3—Effect of w/c on sulfate resistance for different ASTM C150/C150M types of cement (lower visual rating indicates better resistance) (Stark 1989).

those reactions is partly responsible for the expansion. If water can freely diffuse out of capillaries and into the cement paste, the volume of growing crystals cannot exceed the space available to them. Additionally, the swelling pressure can arise from the diffusion of the sulfate salts into the gel pores, which disturbs the equilibrium between the gel and its surrounding liquid phase, resulting in the movement of external water into the gel pores (Hewlett 1998).

Although ordinary portland cements are most susceptible to sulfate attack, the use of sulfate-resistant cements will not necessarily prevent sulfate attack, either (Fig. 4.1.3). Resistance to sulfate attack can be greatly increased by decreasing the permeability of concrete through reduction of the water-cementitious materials ratio (*w/cm*) (Stark 1989).

4.1.4 *Corrosion*—The alkaline nature of concrete (pH greater than 13) will induce formation of a passive, noncorroding layer on reinforcing steel. If, however, chloride ions are present in concrete, they can reach and disrupt that layer, which can lead to corrosion of steel in the presence of water and oxygen. Once corrosion initiates, corrosion products form and may cause cracking, spalling, or delamination of concrete. This allows for easier access of aggressive agents to the steel surface and increases the rate of corrosion. Cross-sectional area of the corroding steel will decrease and the load-carrying capacity of the member will be reduced (Neville 2003).

Chlorides can be introduced to concrete with materials used to produce the mixture (contaminated aggregate or water, or some admixtures), with deicing chemicals, or through marine exposure (seawater or brackish water). To reduce likelihood of corrosion initiation, total chloride-ion content should not exceed a certain concentration value, referred to as the chloride threshold. A literature review of reported chloride threshold values revealed "that there is no single threshold value, but a range based on the conditions and materials in use" and were found to vary from 0.1 to 1 percent by mass of cement (Taylor et al. 1999). The Code limits water-soluble chlorides to 0.15 percent by mass of cement for nonprestressed concrete and 0.06 percent

for prestressed concrete exposed to external chlorides or seawater. Value of 0.40 percent total chloride by mass of cement is given in British and European Standards (Neville 2003, Whiting et al. 2002). Corrosion of the reinforcing steel in concrete can be reduced or prevented by minimizing the *w/cm* (permeability), ensuring maximum cover depth of concrete over steel (Stark 2001), use of corrosion inhibitors, corrosion-resistant reinforcement, and membranes and sealers, which are popular in Europe.

4.2—Background

To produce durable structural concrete, concrete materials and mixture proportions are selected based on design strength requirements, anticipated exposure conditions, and required service life of the structure. The selection of materials and mixture proportions has to be accompanied by appropriate field practices, such as quality control, testing, inspection, and proper placement, finishing, and curing practices.

As stated in Section 1.3.1 of the Code, "The purpose of this Code is to provide for public health and safety by establishing minimum requirements for strength, stability, serviceability, durability, and integrity of concrete structures." Section 4.8 of the Code addresses global durability requirements related to material selection for concrete mixtures and corrosion protection of reinforcement. Code Chapters 19 and 20 provide detailed durability requirements for concrete and reinforcing steel, respectively. Code Chapter 26 discusses what durability requirements must be specified in a project's construction documents.

The durability section of the Code focuses mainly on concrete resistance to fluid penetration, which is primarily affected by the *w/cm* and the composition of the constitutive materials of concrete. The use of SCMs, such as Type F and Type C fly ashes, slag cement, silica fume, calcined shale, calcined clay or metakaolin, or their combinations, can result in a significant improvement in concrete durability. The SCMs affect concrete properties in many ways, depending on their type, dosage, and other mixture proportions and composition. In general, SCMs have the following impacts on hardened concrete properties (Kosmatka and Wilson 2011):

- Increase long-term strength
- Have varied effect on early-age strength gain (Type F fly ash, calcined shales, and calcined clays lower early strength, silica fume and metakaolin increase early strength gain)
- Reduce permeability and absorption
- Increase sulfate resistance (with the exception of Type C fly ash, which may have either a positive or negative effect)
- Have no significant impact on abrasion resistance, drying creep and shrinkage, and freezing and thawing
- May reduce resistance to deicer scaling

The Code does not cover all topics related to concrete durability. It does not include recommendations for extreme exposure conditions (that is, acids, high temperature, or exposure to fire), alkali-aggregate reaction, or abrasion. The Code commentary (R4.8) identifies the importance of preventive maintenance, however, the topic is not explicitly addressed in the Code. Additionally, the Code does not cover waterproofing, routine inspections, condition assessment, or service life prediction. Information related to these topics is found in other ACI documents, including:

- ACI 201.2R—Guide to Durable Concrete
- ACI/TMS 216.1—Code Requirements for Determining Fire Resistance of Concrete and Masonry Construction Assemblies
- ACI 221.1R—Report on Alkali-Aggregate Reactivity
- ACI 222R—Protection of Metals in Concrete Against Corrosion
- ACI 222.2R—Report on Corrosion of Prestressing Steels
- ACI 222.3R—Guide to Design and Construction Practices to Mitigate Corrosion of Reinforcement in Concrete Structures
- ACI 224.1R—Causes, Evaluation, and Repair of Cracks in Concrete Structures
- ACI 311.4R—Guide for Concrete Inspection
- ACI 362.1R—Guide for the Design and Construction of Durable Concrete Parking Structures
- ACI 365.1R—Report on Service Life Prediction
- ACI 515.2R—Guide to Selecting Protective Treatments for Concrete
- ACI 562—Code Requirements forAssessment, Repair, and Rehabilitation of Existing Concrete Structures and Commentary

4.3—Requirements for concrete in various exposure categories

The Code addresses durability by requiring that four exposure categories be assigned to each concrete member. The four exposure categories are:

1. F: concrete exposed to moisture and cycles of freezing and thawing (with or without deicing chemicals);
2. S: concrete in contact with soil or water containing deleterious amounts of water-soluble sulfate ions;
3. W: concrete in contact with water but not exposed to freezing and thawing, chlorides, or sulfates;
4. C: concrete exposed to conditions that require additional protection against corrosion of reinforcement.

Each exposure category is divided into exposure classes that define severity of the exposure, starting with 0 for a negligible exposure. Once all structural members are assigned exposure classes and the concrete mixtures for these members satisfy the requirements outlined in those exposure classes, the Code's minimum durability requirements are met.

4.3.1 *Freezing and thawing (F)*—The volume of ice is 9 percent larger than water. As water freezes in saturated concrete, cement phase and aggregates are subject to internal pressure, which then causes concrete tensile stresses. If those stresses are greater than the tensile strength of concrete, cracking will occur. The cumulative expansion after many cycles of freezing and thawing may lead to significant concrete damage. One method to protect concrete from freezing-and-thawing damage is to reduce moisture penetration so it does not become critically saturated, however, this is not always possible. The other method is to generate small

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air bubbles in fresh concrete by addition of an air-entraining admixture, which creates voids for the freezing water to expand into without creating internal stress.

The Code requires the use of air entrainment in concrete in structural members exposed to cycles of freezing and thawing. Air entrainment significantly improves resistance of saturated concrete to freezing and thawing. ACI 212.3R provides an in-depth discussion on these air-entraining admixtures and their applications, dosage rates, effects on fresh and hardened concrete, and other factors they influence.

Although the specified amount of air entrainment depends primarily on the frequency of exposure to water (exposure class), but also on nominal maximum aggregate size and concrete compressive strength. To achieve similar freezing-and-thawing protection, higher air content is generally required for concrete mixtures with smaller nominal maximum aggregate size. For example, concrete with 3/8 in. aggregate requires 50 percent higher air content than concrete with 2 in. aggregate (Code Table 19.3.3.1). The Code requires that the licensed design professional (LDP) specify the nominal maximum aggregate size for each concrete mixture in the construction documents. Nominal maximum aggregate size depends on locally available aggregates, as well as construction issues such as size of formwork, member depth, and clear bar spacing. The criteria for maximum size selection are given in Code Section 26.4.2.1. Table 19.3.3.1 lists target air content for Exposure Classes F1, F2, and F3, which depends on the nominal maximum aggregate size. Code Table 19.3.3.3 also lists target air content for shotcrete.

Another factor affecting selection of target air content is compressive strength. An air content reduction of 1 percent is allowed for concrete with specified compressive strength exceeding 5000 psi (Code Section 19.3.3.6). The reason for air content reduction is that concretes with higher strengths are characterized by lower *w/cm* and reduced porosity, which improve resistance to freezing-and-thawing cycles.

For example, a structural member in Exposure Class F2 with 1/2 in. nominal maximum aggregate size requires concrete with a target air content of 7 percent (or 6 percent for concrete with compressive strength exceeding 5000 psi). Because exact air content is difficult to achieve, the Code allows tolerance for air content in as-delivered concrete of ±1.5 percentage points. This is consistent with the tolerances in ASTM C94/C94M and ASTM C685/C685M (Section R26.4.2.1(a)(5)). The required air content range, therefore, is from 5.5 to 8.5 percent (or 4.5 to 7.5 percent for concrete with compressive strength exceeding 5000 psi).

Additional requirements or limitations, such as minimum compressive strength, minimum *w/cm*, or limits on cementitious materials depend on the exposure class assigned to a particular member. Interior members, foundations below the frost line, or structures in climates where freezing temperatures are not anticipated are assigned Exposure Class F0. These conditions therefore do not require air entrainment and there is no limit on maximum *w/cm* or on the use of cementitious materials. The minimum compressive strength for concrete in Exposure Class F0 is the Code minimum 2500 psi.

Freezing-and-thawing cycles have little effect on concrete that is not critically saturated. Structural members exposed to freezing-and-thawing cycles, but with low likelihood of being saturated, are assigned Exposure Class F1. Concrete for this exposure must be air-entrained (Code Table 19.3.3.1) in case there is occasional saturation during freezing. In addition, the concrete should have a maximum *w/cm* of 0.55 and at least 3500 psi compressive strength.

Exposure Classes F2 and F3 are assigned to concrete in structural members having frequent exposure to water during freezing. The distinction between the two classes is that Class F2 anticipates no exposure to deicing chemicals or seawater, while Class F3 does. Concrete in F2 and F3 exposure classes must be air-entrained (Code Table 19.3.3.1) and have a maximum *w/cm* of 0.45 and 0.40, respectively. The minimum concrete compressive strengths for F2 and F3 classes are 4500 and 5000 psi, respectively. The most severe class of exposure, F3, also has a limit on cementitious materials in concrete mixtures, given in Code Table 26.4.2.2(b).

Requirements for concrete in Exposure Category F are listed in Table 19.3.2.1 of the Code.

4.3.2 *Sulfate (S)*—All soluble forms of sulfate, sodium, calcium, potassium, or magnesium have a detrimental effect on concrete. Depending on the sulfate form, they react with hydrated cement phases and result in formation of ettringite or gypsum. Depending on the reaction product, the concrete either expands and cracks (ettringite), or softens and loses strength (gypsum). The most effective measure to reduce the effects of sulfate reactions, apart from reducing moisture ingress, is to use cements with a low content of tricalcium aluminate (C_3A). A more detailed discussion on sulfate's effect on concrete can be found in ACI 201.2R.

Exposure Category S applies to structural members that will likely be affected by external source of sulfates, which predominantly come from exposure to soil, groundwater, or seawater. The exposure classification (class) is selected based on the concentration of sulfate ions (SO_4^{2-}), which should be determined in accordance with ASTM C1580 for soil samples and with ASTM D516 or ASTM D4130 for water samples. The Code requires the LDP to specify the exposure class by comparing field test results with concentration ranges in Table 19.3.1.1 of the Code. Note that seawater exposure is classified as S1 even though the sulfate concentration (in seawater) is usually higher than 1500 ppm. The reason for lower class for seawater is the presence of chloride ions, which inhibit expansive reaction due to sulfate attack.

Class S0 is assigned to concrete in members not exposed to sulfates and there is no restriction on *w/cm*, or type or limit on cementitious materials. The only requirement for concrete classified as S0 is the minimum compressive strength be at least 2500 psi. Greater minimum compressive strength and maximum *w/cm* limits are imposed on concrete in Exposure Classes S1 through S3. For these exposure classes, the type of cement is the major requirement.

A summary of all requirements for concrete in Exposure Category S is listed in Table 19.3.2.1 of the Code.

4.3.3 *In contact with water (W)*—The durability of structural members in direct contact with water, such as foundation walls below the groundwater table, may be affected by water penetration into or through concrete. Apart from external systems, such as drainage systems or waterproofing membranes for foundations, the most effective way to reduce concrete permeability is to keep the *w/cm* low.

Concrete for members assigned to Exposure Class W0 is required to have a minimum compressive strength of 2500 psi, but no additional requirements. Concrete in structural members assigned to Exposure Class W1 does not have specific requirements for low permeability, but because of exposure to water. No direct Code requirements are given for addressing aggregate reactivity. Instead, Code Section 26.4.2.2(d) requires that evidence be submitted documenting either that the aggregates are not alkali-silica reactive or the intended mitigation measures if they are reactive. Further documentation must be provided that the aggregates are not alkali-carbonate reactive. Such documentation is typically provided by the concrete supplier. Commentary R26.4.2.2(d) suggests consulting ASTM C1778 for methods and criteria for determining the reactivity of aggregates and guidance for reducing the risk of occurrence. Use of aggregates that are alkali-carbonate reactive is also prohibited. Concrete in structural members assigned to Exposure Class W2 requires low permeability. Table 19.3.2.1 of the Code requires *w/cm* not to exceed 0.50 and compressive strength to be at least 4000 psi. Exposure Class W2 has the same aggregate reactivity requirements as that of W1. Note that additional requirements are imposed if the member's durability is to be affected by reinforcement corrosion, sulfate exposure, or exposure to cycles of freezing and thawing. Recommendations for the design and construction of water tanks and reservoirs are provided in ACI 350.4R, ACI 334.1R, and ACI 372R.

Requirements for concrete in Exposure Class W are listed in Table 19.3.2.1 of the Code.

4.3.4 *Corrosion (C)*—Corrosion of reinforcement may significantly affect durability and structural capacity of a member. Reinforcement corrosion usually occurs as a result of the presence of chlorides or steel depassivation due to carbonation. Corrosion products (rust) are larger in volume than the original steel and therefore exert internal pressure on the surrounding concrete, causing it to crack or delaminate. A significant loss of reinforcing bar cross section leads to increased steel stresses under service load and reduced member nominal strength. Because moisture and oxygen must be present at the steel surface for corrosion to occur, the quality of concrete and the reinforcing bar cover are of great importance. Corrosion can be mitigated by proper mixture design and construction practices, application of sealers, coatings, or membranes that protect concrete from moisture and chloride penetration, use of corrosion-resistant reinforcement, or inclusion of corrosion inhibitors in the mixture to elevate the corrosion threshold concentration. Refer to ACI 222R, ACI 222.3R, and ACI 212.3R for additional information.

Each exposure class within the corrosion exposure category has a limit on water-soluble chloride-ion content in concrete. The chloride-ion content is measured in accordance with ASTM C1218/C1218M, which requires the sample be representative of the concrete constituents—that is, cementitious materials, fine and coarse aggregate, water, and admixtures. Because chloride limits are imposed on concrete in Exposure Class C0, all structural concrete must comply with the Code's maximum chloride ion limits. Chloride limits for nonprestressed concrete, expressed as percent of cementitious materials weight, are 1 percent for Class C0, 0.30 percent for Class C1, and 0.15 percent for Class C2. Chloride limit for prestressed concrete is 0.06 percent by cement weight, regardless of exposure class. Apart from chloride limits, Exposure Classes C0 and C1 have no additional requirements, as there is no limit on *w/cm* and the minimum compressive strength is 2500 psi.

Class C2 requires concrete strength of at least 5000 psi, a maximum *w/cm* of 0.40, and reinforcing steel specified cover to satisfy the Code's minimum concrete cover provisions. The minimum concrete cover depends on exposure to weather, contact with ground, type of member, type of reinforcement, diameter and arrangement (bundling) of reinforcement, method of construction (cast-in-place or precast), and if the member is prestressed. Tables 20.5.1.3.1, 20.5.1.3.2, 20.5.1.3.3, and 20.5.1.3.4 of the Code provide cover provisions for cast-in-place nonprestressed, cast-in-place prestressed, precast nonprestressed or prestressed concrete members, produced under plant conditions, and deep foundation members, respectively. If the design requires bundled bars, check Section 20.5.1.3.5 of the Code for specific requirements. Concrete cover requirements in corrosive environments or other severe exposure conditions are more stringent and are provided in Section 20.5.1.4 of the Code.

Requirements for concrete in Exposure Class C are listed in Table 19.3.2.1 of the Code.

4.4—Concrete evaluation, acceptance, and inspection

Durability requirements are met once concrete proportions and properties satisfy the minimums set by the Code. To assure that the delivered concrete achieves the desired durability, the LDP should specify concrete evaluation and acceptance criteria consistent with Code Section 26.12 and field inspection consistent with Code Section 26.13.

4.5—Examples

The following examples illustrate one approach of implementing minimum durability requirements of the Code. In some cases, durability requirements for material properties may exceed those of the structural design. This is more likely for severe exposure conditions, which require a minimum compressive strength of 5000 psi. In some cases, SCMs may be required, which may extend setting time and reduce early-age strength, and result in modifications to construction schedule. For these reasons, consultation with engineers experienced with concrete materials and mixture proportioning, and with concrete suppliers is recommended.

4.5.1 *Example 1. Interior suspended slab not exposed to moisture or freezing and thawing*—Consider the design of a cast-in-place, nonprestressed slab in a multi-story

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office building. It is located in a climate zone with frequent freezing and thawing cycles; however, the slab will be constructed during summer and the temperatures at night during construction are expected to remain above 40 to 45°F. It is desirable for the slab to quickly gain strength to meet the construction schedule. For this reason, calcium chloride was proposed as an accelerating admixture. The required minimum compressive strength, from structural analysis, is 4000 psi. The slab is 7 in. thick with top and bottom mats of No. 5 bars spaced at 8 in. What additional information should be specified for the slab concrete to meet durability requirements?

Answer: The first step is to assign exposure classes within every exposure category to each structural member or group of members. Once exposure classes are assigned, the Code guides the LDP to satisfy the durability requirements. The step-by-step instructions are as follows.

Step description, action item	Selection and discussion	Code reference
Assign exposure classes within each exposure category	**F0** (concrete not exposed to freezing-and-thawing cycles), **S0** (soil not in contact with concrete; injurious sulfate attack is not a concern); **W0** (there are no specific requirements for low permeability); **C0** (concrete dry or protected from moisture)	Table 19.3.1.1
Assign required minimum compressive strength	**2500 psi** (based on F0)	Table 19.3.2.1
Assign maximum *w/cm*	**Not limited** (based on F0)	Table 19.3.2.1
Assign minimum concrete cover	**0.75 in.** (not exposed to weather, slabs, No. 11 bars and smaller)	Table 20.5.1.3.1
Assign nominal maximum size of aggregate	**2 in.** (3/4 x 3 in. clear bar spacing of top and bottom mat or 1/3 x 7 in. of slab thickness); **use 1 in.** as readily available	26.4.2.1(a)(5)
Assign alkali-aggregate reaction documentation	**Not exposed to moisture.** No additional documentation required.	Table 19.3.2.1
Assign required air content	**Not air-entrained**	Table 19.3.2.1
Assign limits on cementitious materials	**No limits**	Table 19.3.2.1
Assign limits on calcium chloride admixture	**No restriction** (based on S0) [Note: chloride ions from the admixture will significantly affect measured chloride ion content in concrete.]	Table 19.3.2.1
Assign maximum water-soluble chloride ion (Cl^-) content in concrete, percent by weight of cement	**1.00** (based on C0; water-soluble chloride-ion content from all concrete ingredients determined by ASTM C1218/C1218M at age between 28 and 42 days)	Table 19.3.2.1

4.5.2 *Example 2: Balcony slab exposed to moisture and freezing and thawing*—An LDP designs a cast-in-place, nonprestressed balcony slab in a multi-story office building, located in a climate zone with frequent freezing-and-thawing cycles. It is anticipated that the balconies will be exposed to moisture, but not deicing salts. The required minimum compressive strength, from structural analysis, is 4000 psi. The balcony slabs are 6 in. thick with top mat of No. 4 bars spaced at 6 in. What additional information should be specified in the contract documents to ensure that the balcony concrete meets Code durability requirements?

Answer: Durability requirements are met once the most rigorous requirements of the Code are satisfied. The first step is to assign exposure classes within each exposure category to each structural member or group of members. Once exposure classes are assigned, the Code guides the LDP to set the minimum durability requirements. The step-by-step instructions are as follows.

Step description, action item	Selection and discussion	Code reference
Assign exposure classes within each exposure category	**F2** (concrete exposed to freezing-and-thawing cycles with frequent exposure to water), **S0** (soil not in contact with concrete; injurious sulfate attack is not a concern); **W1** (no specific requirements for low permeability, exposure to moisture triggers Code requirements to address alkali-aggregate reactivity), **C1** (concrete exposed to moisture but not to an external source of chlorides)	Table 19.3.1.1
Assign required minimum compressive strength	**4500 psi** (based on F2); because 4500 psi is greater than design strength of 4000 psi, the 4500 psi governs	Table 19.3.2.1
Assign maximum *w/cm*	**0.45** (based on F2)	Table 19.3.2.1
Assign minimum concrete cover	**1.5 in.** (exposed to weather, No. 5 bar and smaller)	Table 20.5.1.3.1
Assign nominal maximum size of aggregate	**2 in.** (1/3 x 6-in. – slab thickness, 3/4 x 6-in. clear bar spacing of top mat bars); **use 1 in.** as readily available	26.4.2.1(a)(5)
Assign alkali-aggregate reaction documentation	For concrete exposed to water, confirm low alkali-aggregate reactivity or provide mitigation. ASTM C1778 is referenced in R26.5.2.2(d) for this use.	Table 19.3.2.1, 26.4.2.2(d)
Assign required air content	**6% ± 1.5%** (for 1 in. aggregate and F2 class) [Note: 1 in. aggregate can be substituted for 3/4 in. aggregate with no air content change]	Table 19.3.3.1 and Section R26.4.2.1(a)(6)
Assign limits on cementitious materials	**No limits**	Table 19.3.2.1

cont. from previous page

Assign maximum water-soluble chloride ion (Cl^-) content in concrete, percent by mass of cementitious materials	**0.30**	Table 19.3.2.1 26.4.2.2(e)(1) or 26.4.2.2(e)(2)
Provide guidance on cold weather construction	Consult ASTM C94/C94M, ACI 306R, and ACI 301 for guidance on temperature limits for concrete delivered in cold weather.	Section 26.5.4.1

4.5.3 *Example 3: Wall foundation exposed to sulfate soil and deicing salts while in service*—An LDP designs a cast-in-place, nonprestressed foundation wall of a partially underground parking structure. The structure is located in a northern climate zone with frequent freezing-and-thawing cycles and high sulfate soil content (6 percent SO_4^{2-} by mass). Exposure to deicing salts is anticipated from melt-water runoff from the nearby streets and a sidewalk. It is desirable for the foundation wall to quickly gain strength to reduce possible frost damage and to meet the construction schedule. The required minimum compressive strength from structural design is 4000 psi. The foundation wall is 8 in. thick with inside face and outside face mats of No. 4 bars spaced at 12 in. What additional information should be specified in the contract documents to ensure that the foundation wall concrete meets Code durability requirements?

Answer: The first step is to assign exposure classes within every exposure category to each structural member or group of members. Once exposure classes are assigned, the Code guides the LDP to set the minimum durability requirements. The step-by-step instructions are as follows:

Step description, action item	Selection and discussion	Code reference
Assign exposure classes within each exposure category	**F3** (concrete exposed to freezing-and-thawing cycles with frequent exposure to water and exposure to deicing chemicals); **S3** (structural concrete members in direct contact with elevated soluble sulfates in soil or water); **W2** (concrete in contact with moisture and low permeability is required, exposure to moisture triggers Code requirements to address alkali-aggregate reactivity); **C2** (concrete exposed to moisture and an external source of chlorides from deicing chemicals)	Table 19.3.1.1
Assign minimum compressive strength	**5000 psi** (based on F3 and C2), because 5000 psi is greater than design strength of 4000 psi, 5000 psi governs	Table 19.3.2.1
Assign maximum *w/cm*	**0.40** (based on F3 and C2)	Table 19.3.2.1
Assign minimum concrete cover	**2.0 in.**—outside face of wall (1.5 in. cover is listed in Table 20.5.1.3.1 for exposure to weather or in contact with ground for No. 5 bar and smaller; cover increased to 2.0 in. based on 20.5.1.3.1) **3/4 in.**—inside face of wall (side of the wall not exposed to weather or in contact with ground)	Table 20.5.1.3.1 20.5.1.4.1
Assign alkali-aggregate reaction documentation	For concrete exposed to water, confirm low alkali-aggregate reactivity or provide mitigation. ASTM C1778 is referenced in R26.4.2.2(d) for this use	Table 19.3.2.1 26.4.2.2(d)
Assign nominal maximum size of aggregate	**1.5 in.** (1.5 x 8 in. = wall thickness, 3/4 x 3-1/4 in. clear bar spacing between interior and exterior mats of reinforcing steel); **use 1.5 in.**	Section 26.4.2.1(a)(5)
Assign limits on calcium chloride admixture	Not permitted (based on S2 and C2)	Table 19.3.2.1
Assign required air content	**5.5% ± 1.5%** (for 1.5 in. aggregate and F3 class) [Notes: 1. Changing to lower nominal maximum aggregate size will require higher air content; 2. Air content reduction of 1% (to 4.5% ± 1.5%) is allowable if concrete compressive strength exceeds 5000 psi; refer to 19.3.3.3]	Table 19.3.3.1
Assign limits on cementitious materials	S3 Option 2 limited to ASTM C595 HS or ASTM C1157 HS. Can use ASTM C150 Type V as sole cementitious material if it meets limitations on expansion	Table 19.3.2 Table 26.4.2.2(b) Table 26.4.2.2(c)
Assign limits on calcium chloride admixture	Not permitted (based on S3)	Table 19.3.2.1
Assign maximum water-soluble chloride-ion (Cl^-) content in concrete, percent by mass of cementitious materials	0.15	Table 19.3.2.1 26.4.2.2(e)(1) or 26.4.2.2(e)(2)

REFERENCES

American Concrete Institute

ACI 201.2R-08—Guide to Durable Concrete

ACI 212.3R-10—Report on Chemical Admixtures for Concrete

ACI 222R-01—Protection of Metals in Concrete Against Corrosion

ACI 222.3R-11—Guide to Design and Construction Practices to Mitigate Corrosion of Reinforcement in Concrete Structures

ACI 301-10—Specification for Structural Concrete

ACI 306R-10—Guide to Cold Weather Concreting

Durability

ACI 334.1R-92—Concrete Shell Structures Practice and Commentary

ACI 350.4-04—Design Considerations for Environmental Engineering Concrete Structures

ACI 372R-13—Guide to Design and Construction of Circular Wire-and-Strand-Wrapped Prestressed Concrete Structures

ASTM International

ASTM C94/C94M-15—Standard Specification for Ready-Mixed Concrete

ASTM C1012/C1012M-13—Standard Test Method for Length Change of Hydraulic-Cement Mortars Exposed to a Sulfate Solution

ASTM C1218/C1218M-99—Standard Test Method for Water-Soluble Chloride in Mortar and Concrete

ASTM C150/C150M-12—Standard Specification for Portland Cement

ASTM C685/C685M-14—Standard Specification for Concrete Made by Volumetric Batching and Continuous Mixing

ASTM C1580-09—Standard Test Method for Water-Soluble Sulfate in Soil

ASTM D516-11—Standard Test Method for Sulfate Ion in Water

ASTM D4130-15—Standard Test Method for Sulfate Ion in Brackish Water, Seawater, and Brines

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Hewlett, P. C., ed., 1998, *Lea's Chemistry of Cement and Concrete*, fourth edition, John Wiley & Sons, Inc., New York, 1057 pp.

Kosmatka, S. H., Kerkhoff, B., and Panarese, W. C., 2002, *Design and Control of Concrete Mixtures (EB001)*, 14th edition, fourth printing (rev.), Portland Cement Association, Skokie, IL, Feb., 358 pp.

Kosmatka, S. H., and Wilson, M. L., 2011, *Design and Control of Concrete Mixtures (EB001)*, 15th edition, Portland Cement Association, Skokie, IL, 444 pp.

Neville, A., ed., 2003, *Neville on Concrete: An Examination of Issues in Concrete Practice*, American Concrete Institute, Farmington Hills, MI, 510 pp.

Powers, T. C., 1958, "Structure and Physical Properties of Hardened Portland Cement Paste," *Journal of the American Ceramic Society*, V. 41, No. 1, pp. 1-6.

Stark, D., 1989, "Durability of Concrete in Sulfate-Rich Soils (RD097)," Portland Cement Association, Skokie, IL, 14 pp.

Stark, D., 2001, "Long-Term Performance of Plain and Reinforced Concrete in Seawater Environments (RD119)," Portland Cement Association, Skokie, IL, 14 pp.

Taylor, P. C.; Nagi, M. A.; and Whiting, D. A., 1999, "Threshold Chloride Content for Corrosion of Steel in Concrete: A Literature Review (RD2169)," Portland Cement Association, Skokie, IL, 32 pp.

Whiting, D. A.; Taylor, P. C.; and Nagi, M. A., 2002, "Chloride Limits in Reinforced Concrete (RD2438)," Portland Cement Association, Skokie, IL, 72 pp.

CHAPTER 5—ONE-WAY SLABS

5.1—Introduction

One-way slabs are generally used in buildings with vertical supports (columns or walls) that are unevenly spaced, creating a longer span in one direction and a shorter span in the perpendicular direction. One-way slabs typically span in the short direction and are supported by beams that span in the long direction (refer to the building example in Chapter 1 of this Manual). During preliminary design, the designer determines the loads and spans, reinforcement type (prestressed or nonprestressed), and slab thickness. The designer determines the concrete strength based on experience and the exposure and durability provisions of the Code.

One-way slabs are designed in accordance with Code Chapter 7 for strength and serviceability. Generally, one-way slabs are designed for distributed loads specified by the building code. In some cases, such as parking garages, point loads must also be considered. These result in local shear forces on the slab, requiring verification of punching shear strength of the slab. Punching shear for slabs is addressed in Chapter 6 for two-way slabs in this Manual.

For relatively small slab openings, trim bars can be used to limit crack widths caused by geometric stress concentrations. For larger openings, a local increase in slab thickness, as well as additional reinforcement, may be necessary to provide adequate serviceability and strength.

5.2—Analysis

The Code allows for the designer to use any analysis procedure that satisfies equilibrium and geometric compatibility, as long as design strength and serviceability requirements are met. The Code includes a simplified method of analysis for one-way slabs that relies on coefficients to calculate moments and shears.

5.3—Service limits

5.3.1 *Nonprestressed slab – Minimum thickness*—For nonprestressed slabs, the Code allows the designer for slabs not supporting or attached to partitions or other construction likely to be damaged by large deflection to either calculate deflections or simply satisfy a minimum slab thickness (Code Section 7.3.1). In the case where loads are heavy, nonuniform, or deflection is a concern, deflections should be calculated to verify that short- and long-term deflections are within Code limits (Section 24.2.2).

5.3.2 *Prestressed slabs – Minimum thickness*—For prestressed slabs, the Code does not provide a minimum span-to-depth ratio, but rather requires that both immediate and time-dependent deflections be calculated in accordance with Code Section 24.2 and checked against the limits in Code Section 24.2.2. Table 9.3 of the *Post-Tensioning Manual* (Post-Tensioning Institute [PTI] 2006) lists span-to-depth ratios that have been found from experience to provide satisfactory structural performance.

5.3.3 *Deflections*—For all prestressed slabs and nonprestressed slabs that have depths less than those in Code Table 9.3.1.1, deflections must be calculated. The calculated deflections must not exceed the limits given in Code Section 24.2. Deflections can be calculated by any appropriate method, such as classical equations or structural analysis software.

Note that the spacing of slab reinforcing bars, timing of form removal, concrete quality, timing of construction loads, and other construction variables all could affect the actual deflection. These variables should be considered when assessing the accuracy of deflection calculations. In addition, creep will increase the immediate deflections.

If prestressed slabs are designed to remain uncracked, then slab deflections are usually small because deflections can be calculated using gross section properties.

5.3.4 *Nonprestressed slabs – Concrete service stress*—Nonprestressed slabs are designed for strength but do not have limitations placed on concrete service flexural stress.

5.3.5 *Prestressed slabs – Concrete service stress*—For prestressed slabs, the analysis of concrete flexural tension stresses is a critical part of the design. The slab is classified according to Code 7.3.4.1 and 24.5.2 as Class U, T, or C based on the maximum net tensile stresses in the precompressed tensile zone. These classifications are used to determine the appropriate section properties for use in calculating stresses and deflections. Class U members are considered uncracked, Class C are considered cracked, and Class T is the transition between the two.

In post-tensioned (PT) slab construction, the tendon profile must be designed before the slab flexural stresses in a design strip can be calculated. Both profile and tendon force are directly related to slab forces and moments created by the prestressing force. A common approach to calculate PT slab moments is the use of the "load balancing" concept. Tendons are typically placed in a parabolic profile such that the tendon is at the minimum cover requirements at midspan and over supports; this maximizes the parabolic drape. Anchors are typically placed at middepth at the slab edge (Fig. 5.3.5).

The tendon exerts a uniform upward force along its length that counteracts a portion of the gravity loads, usually 60 to 80 percent of the slab self-weight according to Libby (1990); hence, the term "load balancing." The load effect from the prestressing force in the tendon is then combined with the load effect of the gravity loads to determine net concrete stresses.

To achieve Code stress limits, the designer can use an iterative or direct approach. In the iterative approach, the

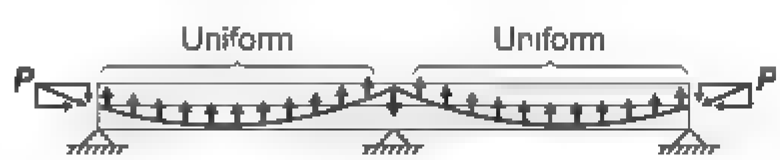


Fig. 5.3.5—Load balancing concept.

tendon profile is defined and the tendon force is assumed. The analysis is executed, flexural stresses are calculated, and the designer then adjusts the profile or force or both, depending on results and design constraints.

In the direct approach, the designer determines the highest tensile stress permitted, then rearranges equations so that the analysis calculates the tendon force required to achieve the stress limit.

The Code does not impose a minimum concrete compressive stress due to the effective prestress force, but 125 psi is commonly used as a minimum for cast-in-place PT slabs, which is the same as the required amount for prestressed two-way slabs as noted in Code Section 8.6.2.1. For slabs exposed to aggressive environments, the minimum concrete compressive stress is usually set to a higher value.

5.4—Required strength

The design of one-way slabs is typically controlled by moment strength, not concrete stress or shear strength. Assuming a uniform load, the designer calculates the unit (usually a 1 ft width) factored slab moments. The required area of flexural reinforcement over a unit slab width is calculated with the same assumptions as a beam.

5.4.1 *Calculation of required moment strength*—For nonprestressed reinforced slabs, a quick way to calculate gravity design moments (if the slab meets the specified geometric and load conditions) is by using the moment coefficients in Section 6.5 of the Code. Chapter 6 of the Code permits other, more sophisticated analysis methods.

For indeterminate PT slabs, effects of reactions induced by prestressing (secondary moments) should be added to the factored gravity moments per Section 5.3.11 of the Code to calculate M_u. Secondary moments in the slab are a result of the beam's vertical restraint of the slab against the PT "load" at each support. Because the PT force and drape are determined during the service stress checks, secondary moments can be quickly calculated by the "load-balancing" analysis concept. A simple method for calculating the secondary moment is to subtract the tendon force multiplied by the tendon eccentricity (distance from the neutral axis) from the total balance moment, expressed mathematically as $M_2 = M_{bal} - Pe$.

The critical locations to calculate M_u along the span are usually at the support and midspan. Section 7.4.2.1 of the Code allows M_u to be calculated at the face of support rather than the support centerline.

5.4.2 *Calculation of required shear strength*—Assuming a uniform load, the designer calculates the unit (usually a 1 ft width) factored slab shear force by either the coefficient method or more sophisticated calculations.

5.5—Design strength

One-way slabs must have adequate one-way shear strength and moment strength in all design strips.

5.5.1 *Calculation of design moment strength*—The required area of flexural reinforcement for a nonprestressed and PT unit slab width are calculated using the same assumptions as for a beam, Chapter 7, of this Manual.

Table 5.6.1a—Maximum spacing of bonded reinforcement in nonprestressed and Class C prestressed members (Code Table 24.3.2 excerpt for deformed bars or wires only)

Lesser of	$15\left(\frac{40,000}{f_s}\right)$ 2.5C
	2,40,000/f_s

5.5.2 *Calculation of design shear strength*—Discussion for nominal one-way shear strength is the same as for a beam, Chapter 7, of this Manual.

5.6—Detailing of flexural reinforcement

The Code requires a minimum area of flexural reinforcement be placed in tension regions to ensure that the slab deformation and crack widths are limited when the cracking strength of the slab is exceeded. If more than the minimum area is required by analysis, that reinforcement area must be provided. Reinforcement in one-way slabs is usually uniformly spaced, unless there is a large point load or opening.

5.6.1 *Nonprestressed slab – Flexural reinforcement area and placing*—For nonprestressed slabs, the minimum area of flexural reinforcement, $A_{s,min}$, is $0.0018A_g$ in Code Section 7.6.1.1.

The maximum spacing of deformed flexural bars is given in Table 24.3.2 of the Code (Table 5.6.1a of this Manual). Because the service load stress in the reinforcement (f_s) is usually taken as 40,000 psi, the maximum spacing will not exceed 12 in.

The bar termination rules in Section 7.7.3 of the Code cover general conditions that apply to beams, but because one-way slab bars are usually placed at the maximum spacing, bars generally cannot be terminated without violating the maximum spacing. This usually results in all bottom bars extending full length into the beams.

5.6.2 *Prestressed slab – Flexural tendon area and placing*—For prestressed one-way slabs, the Code does not have a limit for minimum tendon area or a minimum compressive stress due to the effective prestress force. This is consistent with the flexible approach on service stresses.

5.6.3 *Prestressed slab – Flexural reinforcing bar area and placing*—The Code requires the minimum bonded reinforcement area, $A_{s,min}$, to be placed close to the slab face at the bottom at midspan and the top at the support. For one-way prestressed slabs, $A_{s,min} = 0.004A_{ct}$. Because the one-way slab strip is rectangular, $A_{ct} = 0.5A_g$. This minimum is independent of service stress level.

The maximum spacing of reinforcing bars in Class T or C prestressed slabs containing unbonded tendons is the lesser of $3h$ and 18 in.

If the slab design moment strength is fully satisfied by the tendons alone, the termination length of $A_{s,min}$ bars for bottom bars is (a) and for top bars is (b):

(a) At least $\ell_n/3$ in positive moment areas and be centered in those areas

(b) At least $\ell_n/6$ on each side of the face of support

The termination length for bars that are required for strength are the same as for nonprestressed slabs

5.6.4 *Temperature and shrinkage reinforcement and placing*—Shrinkage and temperature slab reinforcement is required and could be either reinforcing bar or tendons placed perpendicular to flexural reinforcement

If the designer uses reinforcing bars, the minimum area of temperature and shrinkage Grade 60 bar is $0.0018A_g$.

If the designer uses tendons to resist shrinkage and temperature stresses, the minimum slab effective compression force due to temperature and shrinkage tendons is 100 psi

The purpose of this reinforcement is to restrain the size and spacing of slab cracks, which can occur due to volume variations caused by temperature changes and slab shrinkage over time In addition, if the slab is restrained against movement, the Code requires the designer to provide reinforcement that accounts for the resulting tension stress in the slab.

Authored references

Libby, J., 1990, *Modern Prestressed Concrete. Design Principles and Construction Methods*, fourth edition, Springer, 871 pp

Post-Tensioning Institute (PTI), 2006, *Post-Tensioning Manual*, sixth edition, PTI TAB 1-06, 354 pp.

One-Way Slabs

5.7—Examples

One-way Slab Example 1 *Nonprestressed one-way slab—*

Design and detail the second-story floor slab of the seven-story building. The one-way slab consists of five spans of 14 ft each. The slab is supported by 18 in. wide beams. A 6 ft cantilever extends at the left end of the slab (Fig. E1.1).

<u>**Given:**</u>

Load—

Service live load L = 100 psf

Concrete—

f_c' = 5000 psi (normalweight concrete)

f_y = 60,000 psi

Geometry—

Span length: 14 ft

Beam width: 18 in.

Column dimensions: 24 in. x 24 in.

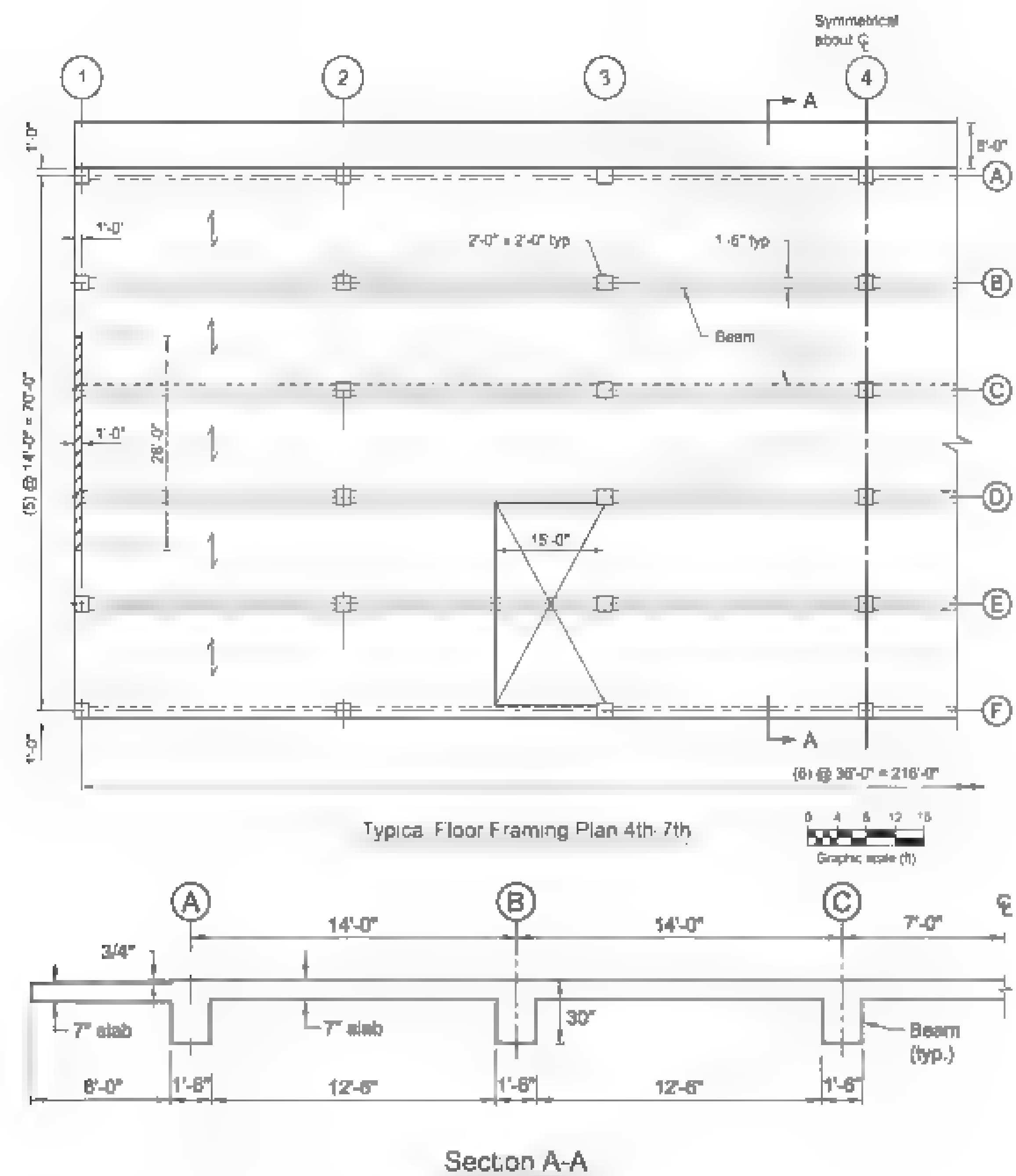


Fig. E1.1—Plan and section of five-span one-way slab.

ACI 318	Discussion	Calculation
Step 1: Geometry		
7.3.1.1	Determine slab thickness using ratios from Table 7.3.1.1.	$h \geq \frac{\ell}{24} = \frac{(14\text{ ft})(12\text{ in./ft})}{24} = 7\text{ in.}$
	Determine cantilever thickness:	$h \geq \frac{\ell}{10} = \frac{(6\text{ ft})(12\text{ in./ft})}{10} = 7.2\text{ in., say, }7\text{ in.}$ Because the slab and cantilever satisfy the Code span-to-depth ratios (Table 7.3.1.1), the designer does not need to check deflections, unless the slab is supporting or attached to partitions or other construction likely to be damaged by large deflections.
	Note: Architectural requirements specify a 3/4 in. step at the cantilever. Detail to maintain 7 in. slab thickness.	
	Self weight Slab	 $w_s = (7\text{ in./12 in./ft})(150\text{ lb/ft}^3) = 87.5\text{ psf}$
Step 2: Loads and load patterns		
5.3.1	The service live load is 100 psf in assembly areas and corridors per Table 4-1 in ASCE/SEI 7. For cantilever use 100 psf. To account for weights from ceilings, partitions, HVAC systems, etc., add 15 psf as miscellaneous dead load.	
	$U = 1.4D$ (5.3.1a) $U = 1.2D + 1.6L$ (5.3.1b)	The required strength equations to be considered are: $U = 1.4\,(87.5\text{ psf} + 15\text{ psf}) = 143.5\text{ psf}$ $U = 1.2\,(102.5\text{ psf}) + 1.6\,(100\text{ psf})$ $= 123\text{ psf} + 160\text{ psf} = 283\text{ psf}$ **Controls**
	The slab resists gravity only and is not part of a lateral force-resisting system, except to act as a diaphragm.	
	Both ASCE/SEI 7 and the Code provide guidance for addressing live load patterns. Either approach is acceptable. To simplify the analysis, the Code allows the use of the following two patterns, Fig. E1.2.	
6.4.2	Factored dead load is applied on all spans and factored live load is applied as follows: (a) Maximum positive M_u near midspan occurs with factored live load on the span and on alternate spans. (b) Maximum negative M_u at a support occurs with factored live load on adjacent spans only.	

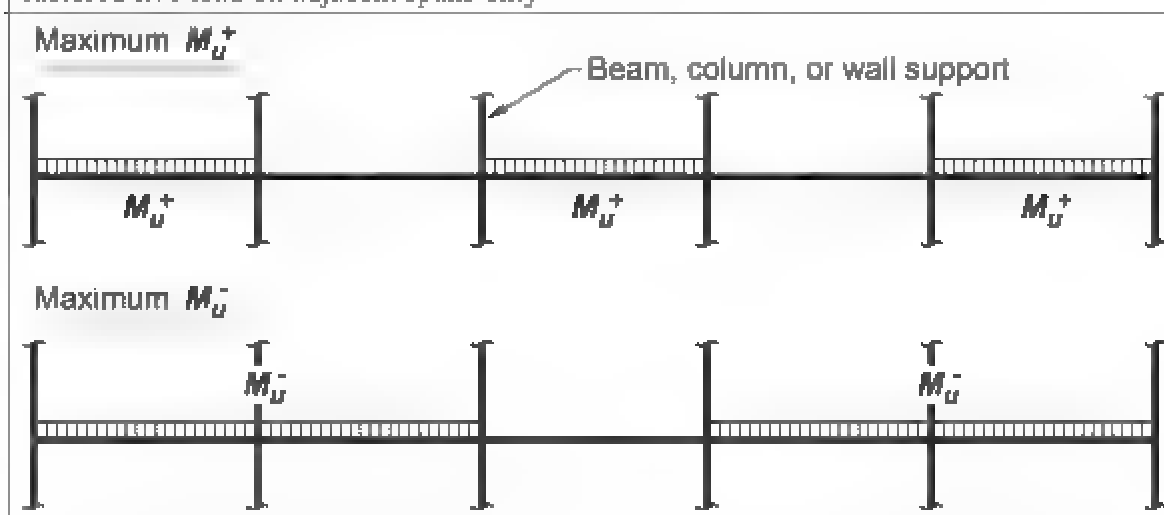


Fig. E1.2—Live load loading pattern.

Step 3: Concrete and steel material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements of the Code. The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes.	By specifying that the concrete mixture must be in accordance with ACI 301-10 and providing the exposure classes, Code Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be 5000 psi.
26.4.3.1	ACI 301 is a reference construction specification that is coordinated with the Code. The Code allows the use of ACI 301 for compliance of concrete mixture proportioning. There are several mixture options within ACI 301, such as the use of admixtures and supplementary cementitious materials, which the designer can require, permit, or review if suggested by the contractor	
7.2.2.2	The reinforcement must satisfy Chapter 20 of the Code The designer specifies the grade of bar and whether the reinforcing bar should be coated by epoxy, galvanized, or both	By specifying the reinforcing bar grade and any coatings, and that the reinforcing bar must be in accordance with ACI 301-10, Chapter 20 requirements are satisfied. Use ASTM A615 Grade 60 uncoated reinforcement
Step 4: Slab structural analysis		
6.3	System is braced by shear walls and moment-resisting frames. Assume slab is braced and that moment effects in slab caused by lateral loads may be ignored.	Modeling assumptions Assume a constant moment of inertia for the entire length of the slab. Ignore torsional stiffness of beams Only the slab at this level is considered.
6.6	Figure E1.3 shows the results of the first-order analysis of the slab for moments due to pattern gravity load. The figure shows a moment envelope that was developed by the software for various pattern loadings	Analysis approach The connection to the beams is monolithic, however, when the slab is fully loaded, flexural cracking will reduce joint stiffness.
Step 5: Required moment strength		
7.4.2	The negative design moments are taken at the face of support as is permitted by the Code (Fig. E1.3)	

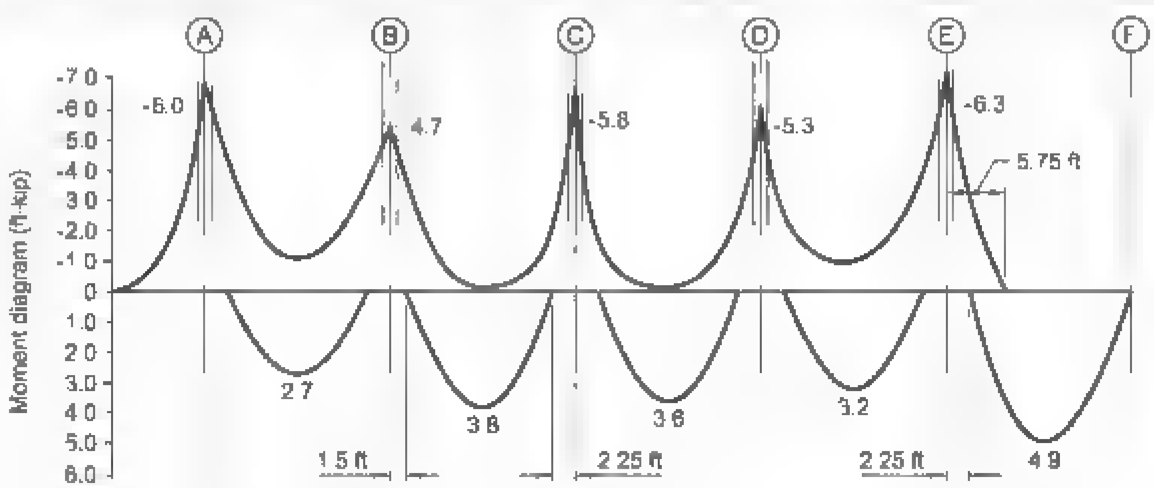


Fig. E1.3—Moment envelope

The maximum positive moment is located in the end span, EF, and is 4.9 ft-kip. The inflection points for positive moments are 0.0 ft from the exterior support centerline and 2.25 ft from the first interior support centerline, column line (CL) E.

The maximum negative moment is located at the face of the first interior support from the right end (CL E) and is 6.3 ft-kip. The negative moment inflection point is 5.75 ft from the support centerline. On the left side and for the full length of the slab, there is no inflection point. Make top bars continuous over the full length of the slab with the exception of span E-F.

The maximum positive moment in the interior span, CL BC, is 3.8 ft-kip. The inflection points for positive moments are 1.5 ft from the first interior support centerline and 2.25 ft from the second interior support centerline. Because of pattern loading, a small negative moment can exist across all spans with the exception of the last span.

The maximum negative moment at the exterior left support, CL A, is 6 ft-kip because of the cantilevered slab.

Table E.1—Maximum moments at supports and midspans

Required strength	Location from left to right along the slab					
	Support A	Midspan AB	Support B	Midspan BC	Support C	Midspan CD
M_u, ft-kip	-6.0	+2.7	-4.7	+3.8	-5.8	+3.6

Continue:

Required strength	Location from left to right along the slab				
	Support D	Midspan DE	Support E	Midspan EF	Support F
M_u, ft-kip	-5.3	+3.3	-6.3	+4.4	0

Step 6: Required shear strength

7.4.3.1 The maximum shear is taken at the support centerline for simplicity. The maximum shear under any load pattern is 2.4 kip (Fig. E1.4).

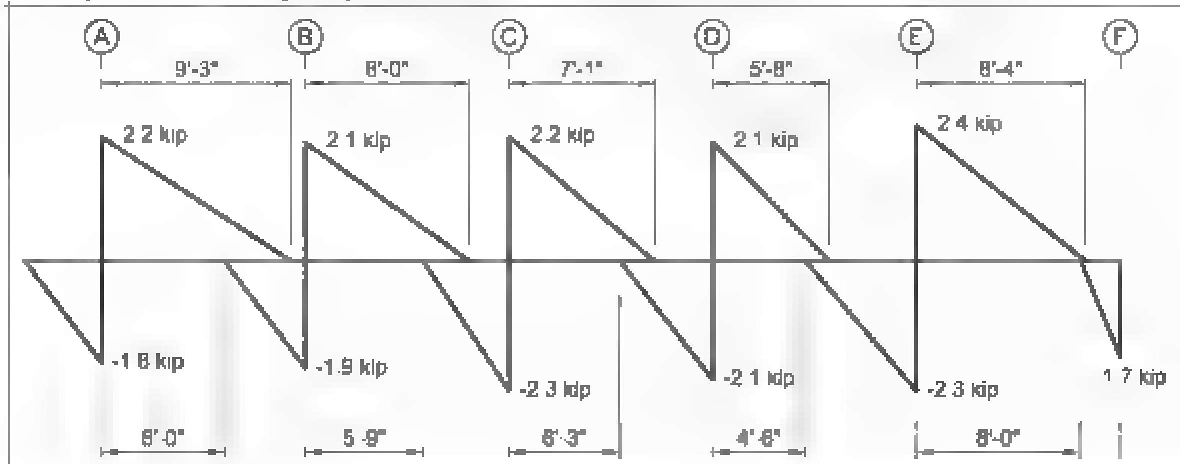
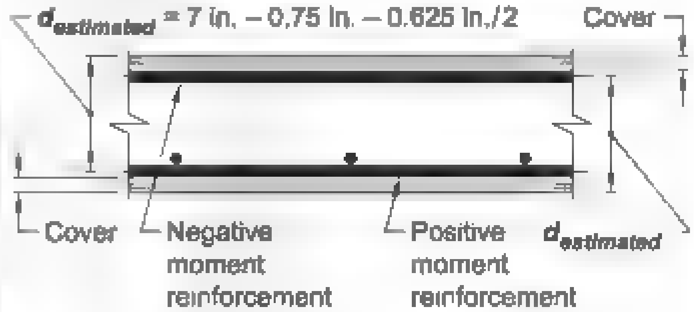


Fig. E1.4—Shear envelope.

Step 7: Design moment strength		
7.5.1	The two common strength inequalities for one-way slabs, moment and shear, are noted in Section 7.5.1.1.	
7.5.2	The one-way slab chapter refers to Section 22.3 for calculation of flexural strength	
7.3.3.1	Slab must be tension-controlled in accordance with Table 21.2.2. This provision limits the amount of reinforcement to provide warning by excessive deflection and cracking. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members. Beginning with the 2019 Code, this limit is revised to require that the section be tension-controlled.	$\varepsilon_{ty} = \dfrac{f_y}{E_s} = \dfrac{60{,}000 \text{ psi}}{29{,}000{,}000 \text{ psi}} = 0.002$ $\varepsilon_t \geq \varepsilon_{ty} + 0.003 = 0.002 + 0.003 = 0.005$
21.2.2	Because section must be tension-controlled, the strength reduction factor is 0.9	$\phi = 0.9$
20.5.1.3	Determine the effective depth assuming No. 5 bars and 0.75 in. cover (Fig. E1.5).	$d_{estimated} = t - \text{cover} - d_b/2$ $d_{estimated} = 7 \text{ in.} - 0.75 \text{ in.} - 0.625 \text{ in.}/2$  *Fig. E1.5—Effective depth for positive and negative bending*

7.7.1.1 20.5.1.3.1	One row of reinforcement $d = t - cover - d_b/2$	d = 7 in. – 0.75 in. – 0.625 in./2 = 5.93 in., say, 5.9 in.
22.2.2.1	The concrete compressive strain at nominal moment strength is assumed equal to $\varepsilon_{cu} = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and is approximately 10 to 15 percent of the concrete compressive strength. The Code neglects the concrete tensile strength to calculate nominal strength.	
22.2.2.3	Determine the equivalent concrete compressive stress at nominal strength: The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distribution of $0.85f_c'$ with a depth of:	
22.2.2.4.1	$a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3	
22.2.2.4.3	For f_c' = 5000 psi:	$\beta_1 = 0.85 - \dfrac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$
22.2.1.1	Find the depth of equivalent rectangular stress block, a, by equating the compression force to the tension force within the slab unit width: $C = T$ $0.85f_c'ba = A_sf_y$ Unit width of slab; assumed to be unit width for design convenience, 12 in.	$0.85(5000\text{ psi})(b)(a) = A_s(60{,}000\text{ psi})$ $a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(12\text{ in.})} = 1.176A_s$

	The slab is designed for the maximum flexural moments obtained from the analysis.	
7.5.1.1	Moment strength at first interior support will be designed for the larger of the two moments. The beam's design strength must be at least the required strength at each section along its length $\phi M_n \geq M_u$ $\phi V_n \geq V_u$	
	Calculate the required reinforcement area. $\phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right) \ge M_u$ A No. 5 bar has a d_b = 0.625 in. and an A_s = 0.31 in.^{2}	Maximum positive moment $4.9\text{ ft-kip} \le (0.9)(60\text{ ksi})A_s\left(6.0\text{ in.} - \frac{1.176A_s}{2}\right)$ $A^+_{s,req'd} = 0.19\text{ in.}^2/\text{ft}$ Use No. 4 at 12 in. on center or No. 5 at 18 in. center bottom. Try No. 5 at 18 in. on center $A_{s,prov'd} = (0.31\text{ in.}^2/\text{ft})(12\text{ in.}/18\text{ in.}) = 0.21\text{ in.}^2/\text{ft}$ $A_{s,prov'd} = 0.21\text{ in.}^2/\text{ft} > A^+_{s,req'd} = 0.19\text{ in.}^2/\text{ft}$ **OK** Maximum negative moment $6.3\text{ ft-kip} \le (0.9)(60\text{ ksi})A_s\left(6.0\text{ in.} - \frac{1.176A_s}{2}\right)$ $A^-_{s,req'd} = 0.25\text{ in.}^2/\text{ft}$ Use No. 5 at 12 in. on center top $A_{s,prov'd} = 0.31\text{ in.}^2/\text{ft} > A^-_{s,req'd} = 0.25\text{ in.}^2/\text{ft}$ **OK**
	Reinforcement limits of negative moment reinforcement will control. Check if the calculated strain exceeds 0.005 in./in. (tension controlled) using similar triangles in strain profile (Fig. E1.6). $a = \frac{A_s f_y}{0.85 f_c' b}$ and $c = a/\beta_1$ where β_1 = 0.8 for f_c' = 5000 psi $\varepsilon_t = \frac{\varepsilon_{cu}}{c}(d - c)$	Top reinforcement $a = 1.176A_s = (1.176)(0.31\text{ in.}^2) = 0.36\text{ in.}$ $c = 0.36/0.8 = 0.46\text{ in.}$ $\varepsilon_t = \frac{0.003}{0.46\text{ in.}}(6\text{ in.} - 0.46\text{ in.}) = 0.036 \ge 0.005$ **OK**

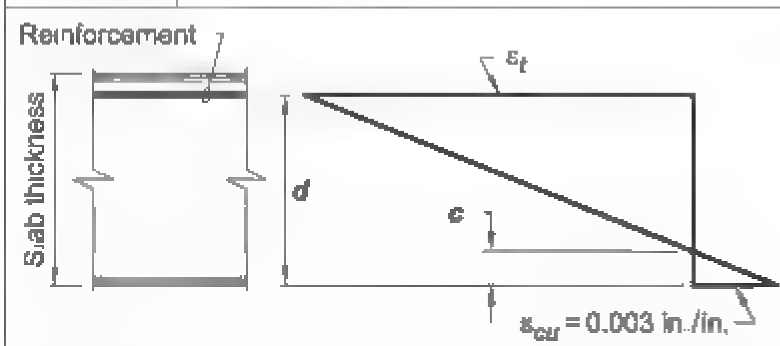


Fig. E1.6—Strain distribution in negative moment reinforcement used to check reinforcement limits.

Step 8. Minimum flexural reinforcement		
7.6.1	Check if design reinforcement exceeds the minimum required by the Code	$A_{s,min}$ = 0.0018 × 7 × 12 = 0.15 in.^{2}/ft At all critical sections, the required A_s is greater than the minimum
Step 9. Shrinkage and temperature reinforcement		
7.6.4 24.4.3.2 24.4.3.3	For nonprestressed one-way slabs, the minimum area of shrinkage and temperature (S+T) bars is 0.0018A_g. The maximum spacing of S+T reinforcing bars is the lesser of 5h and 18 in.	S+T steel area = 0.0018 × 12 × 7 = 0.15 in.^{2}/ft Based on S+T steel area, solutions are No. 4 at 16 in. or No. 5 at 18 in.; use No. 4 at 16 in. placed atop and perpendicular to the primary positive moment reinforcement
Step 10: Minimum and maximum spacing of flexural reinforcement		
7.7.2.1 25.2.1	The minimum spacing between bars must not be less than the greatest of (a) 1 in. (b) d_b (c) (4/3)d_{agg} Assume 1 in. maximum aggregate size	(a) 1 in. (b) 0.625 in. (c) (4/3)(1 in.) = 1.33 in. **Controls**
7.7.2.2 24.3.2	For reinforcement closest to the tension face, the spacing between reinforcement is the lesser of (a) and (b) (a) 12(40,000/f_s) (b) 15(40,000/f_s) – 2.5c_c	(a) 12(40,000/40,000) = 12 in. **Controls** (b) 15(40,000/40,000) – 2.5(0.75 in.) = 13.1 in.
24.3.2.1	f_s = (2/3)f_y = 40,000 psi	
7.7.2.3	The maximum spacing of deformed reinforcement is the lesser of 3h and 18 in.	3(7 in.) = 21 in. > 18 in. Therefore, Section 24.3.2 controls, 12 in.

Step 11 Design shear strength		
	Shear reinforcement is not typically used in one-way slabs so all of the shear strength is provided by the concrete contribution ($\phi V_n = \phi V_c$).	
7.6.3.1	Minimum shear reinforcement is required when $V_u > \phi V_c$.	
22.5.5.1c	Use appropriate equation from Table 22.5.5.1c $V_c = \left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \dfrac{N_u}{6A_g}\right]b_w d$	
21.2.1	Strength reduction factor for shear from Table 21.2.1b	$\phi = 0.75$
	Effective depth to centroid of reinforcement.	$d = 7\text{ in.} - 0.75\text{ in.} - 0.5 \times 0.625\text{ in.} = 5.9\text{ in.}$
	Consider No. 5 at 12 in. spacing. Use largest anticipated spacing to ensure shear strength check will cover all reinforcement conditions.	$s = 12\text{ in.}$
		$A_s = \dfrac{0.31\text{ in.}^2}{s} = \dfrac{0.31\text{ in.}^2}{1\text{ ft}} = 0.31\text{ in.}^2$ per unit width of slab
	Unit slab width of 12 in.	$b_w = 12\text{ in.}$
2.2	Reinforcement ratio of flexural reinforcement relative to the web width, which is the slab width of 12 in. $\rho_w = \dfrac{A_s}{b_w d}$	$\rho_w = \dfrac{0.31\text{ in.}^2}{12\text{ in.}(5.9\text{ in.})} = 0.00438$
	Axial load is zero.	$N_u = 0$
22.5.5.1.3	Size effect factor $\lambda_s = \sqrt{\dfrac{2}{1+0.1d}}$	$\lambda_s = \sqrt{\dfrac{2}{1+(0.1)(5.9)}} = 1.12$ use $\lambda_s = 1.0$ $\lambda = 1.0$ $V_c = \left[8(1.0)(1.0)(0.00438)^{1/3}\cdot\sqrt{5000}\right](12)(5.9)\dfrac{1}{1000}$ $V_c = 6.55$ kip/ft $\phi V_c = 0.75(6.5) = 4.9 > V_u = 2.4$ kip/ft **OK** Design shear strength from concrete contribution is nearly twice the factored shear. Slab thickness is adequate.

Step 12. Select reinforcing bar size and spacing		
	Based on the above requirement, use No. 5 bars Spacing on top and bottom bars is 12 in. Note that there is no point of zero negative moment along all spans except the last bay, so continue the top bars across all spans. Also, No. 4 bars can be used instead of No. 5. While this solution is slightly conservative (No. 5 versus No. 4 bars), the engineer may desire consistent spacing and reinforcing bar use for easier installation and inspection.	
Step 13. Top bar cutoff at exterior support of span E-F		
	The inflection point for negative moment near exterior support of span E-F is 5.0 ft from support centerline.	
7.7.3.3	Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	Bar cutoff Extend bars beyond the inflection point at least: d = 5.9 in. or (12)(0.625 in.) = 7.5 in. Therefore, use 7.5 in.
7.7.3.8.4	At least one-third of the negative moment reinforcement at a support shall have an embedment length beyond the point of inflection at least the greatest of d, $12d_b$, and $\ell_n/16$.	33 percent of the bars to extend beyond the inflection point at least (14 ft − 1.5 ft)(12)/16 = 9.4 in. > $12d_b$ = 7.5 in. > d = 5.9 in. Because the reinforcing bar is already at maximum spacing, no percentage of bars (as permitted by Section 7.7.3.3 check) can be cut off in the tension zone. Make top bars continuous over full length.
Step 14: Development and splice lengths		
7.7.1.2 25.4.2.4	ACI provides two equations for calculating development length, simplified and detailed. In this example, the detailed equation is used. $\ell_d = \left(\dfrac{3}{40}\dfrac{f_y}{\lambda\sqrt{f'_c}}\dfrac{\psi_t\psi_e\psi_s}{\left(\dfrac{c_b+K_{tr}}{d_b}\right)}\right)d_b$ where ψ_t = bar location, not more than 12 in. of fresh concrete below horizontal reinforcement ψ_e = coating factor; uncoated ψ_s = bar size factor, No. 7 and larger ψ_g = reinforcement grade; Grade 60 However, the expression: $\dfrac{c_b+K_{tr}}{d_b}$ must not be taken greater than 2.5.	The development length of a No. 5 black bar in a 7 in. slab with 0.75 in. cover is: $\ell_d = \left(\dfrac{3}{40}\dfrac{60{,}000\text{ psi}}{(1.0)\sqrt{5000\text{ psi}}}\dfrac{(1.0)(1.0)(0.8)}{1.7\text{ in.}}\right)(0.625\text{ in.})$ = 19 in. ψ_t = 1.0, because not more than 12 in. of concrete is placed below bars ψ_e = 1.0, because bars are uncoated ψ_s = 0.8, because bars are smaller than No. 7 ψ_g = 1.0, because bars are Grade 60 c_b = 0.75 in. + 0.5(0.625 in.) = 1.06 in. $\dfrac{1.06\text{ in.}+0}{0.625\text{ in.}} = 1.7$ in.

7.7.1.3 25.5 25.5.1.1	Top bar splices The maximum bar size is No. 5, therefore, splicing is permitted.	
25.5.2.1	Because 100% of the reinforcement will be spliced, tension lap splice length, ℓ_{st}, for deformed bars in tension must be the greater of:	
	$1.3\ell_d$ and 12 in.	$\ell_s = (1.3)(19\text{ in.}) = 24.7\text{ in.}$, use 26 in.
Step 15: Bottom bar cutoff in span E-F		
	The inflection points for positive moments are 0 ft from exterior support centerline at CL F and 2.25 ft from the first interior support centerline CL E	
7.7.3.3	Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers	
7.7.3.4	Continuing flexural tensile reinforcement must have an embedment length not less than ℓ_d beyond the point where bent or terminated tensile reinforcement is no longer required to resist flexure	This condition is satisfied at any section along the beam span
7.7.3.5	Flexural tensile reinforcement must not be terminated in a tensile zone unless (a), (b), or (c) is satisfied. $V_u \leq (2/3)\phi V_n$ at the cutoff point Note that (b) and (c) do not apply.	2400 lb < (2/3)(10,800 lb) = 7200 lb **OK**
7.7.3.8.2	At least one-fourth the maximum positive moment reinforcement must extend along the slab bottom into the continuous support a minimum of 6 in.	
7.7.3.8.3	At points of inflection, d_b for positive moment tensile reinforcement shall be limited such that ℓ_d for that reinforcement satisfies condition (b), because end reinforcement is not confined by a compressive reaction	
	$\ell_d < M_n/V_u + \ell_a$ ℓ_a is the greater of d and $12d_b = 7.5$ in. $M_n = A_s f_y\left(d - \dfrac{a}{2}\right)$	Check if bar size is adequate M_n for an 7 in. slab with No. 5 at 12 in., 0.75 in. cover is: $M_n = (0.31\text{ in.}^2/\text{ft})(60{,}000\text{ psi})(5.9\text{ in.} - 0.18\text{ in.}) =$ 106,392 in.-lb ≅ 106,000 in.-lb
	The elastic analysis indicates that V_u at inflection point is 1800 lb.	$\ell_d = 19\text{ in.} \leq \dfrac{108{,}000\text{ in.-lb}}{1800\text{ lb}} + 7.5\text{ in.} = 67.5\text{ in.}$ Therefore, No. 5 bar is **OK**

Step 16: Bottom bar cutoffs in other spans		
7.7.3.3	Bars close to their maximum spacing limit may not be cut off within the tensile zones because the maximum reinforcing bar spacing would then be exceeded. All bottom bars must extend at least 8 in. beyond the positive moment inflection points	Greater of d = 7.9 in. and $12d_b$ = 12(0.625 in.) = 7.5 in. use 8 in.
7.7.3.5	Because all bottom bars will be extended past the inflection points, 7.7.3.5 is not applicable	
		Because the cutoff location is close to the right support and for field placing simplicity, extend all bars 6 in. into both supports
Step 17: Slab integrity steel		
7.7.7	Provide structural integrity reinforcement in the slab to protect overall structural stability	
7.7.7.1	At least 1/4 of the bottom bars must be continuous	Extend bottom bars into support to lap with bars from adjacent spans
7.7.7.3	Provide Class B tension lap splices	
25.5.2.1	$1.3\ell_d$ and 12 in.	lap = 1.3(19 in.) = 24.7 in. Use 2 ft 2 in.
Step 18: Slab detailing		

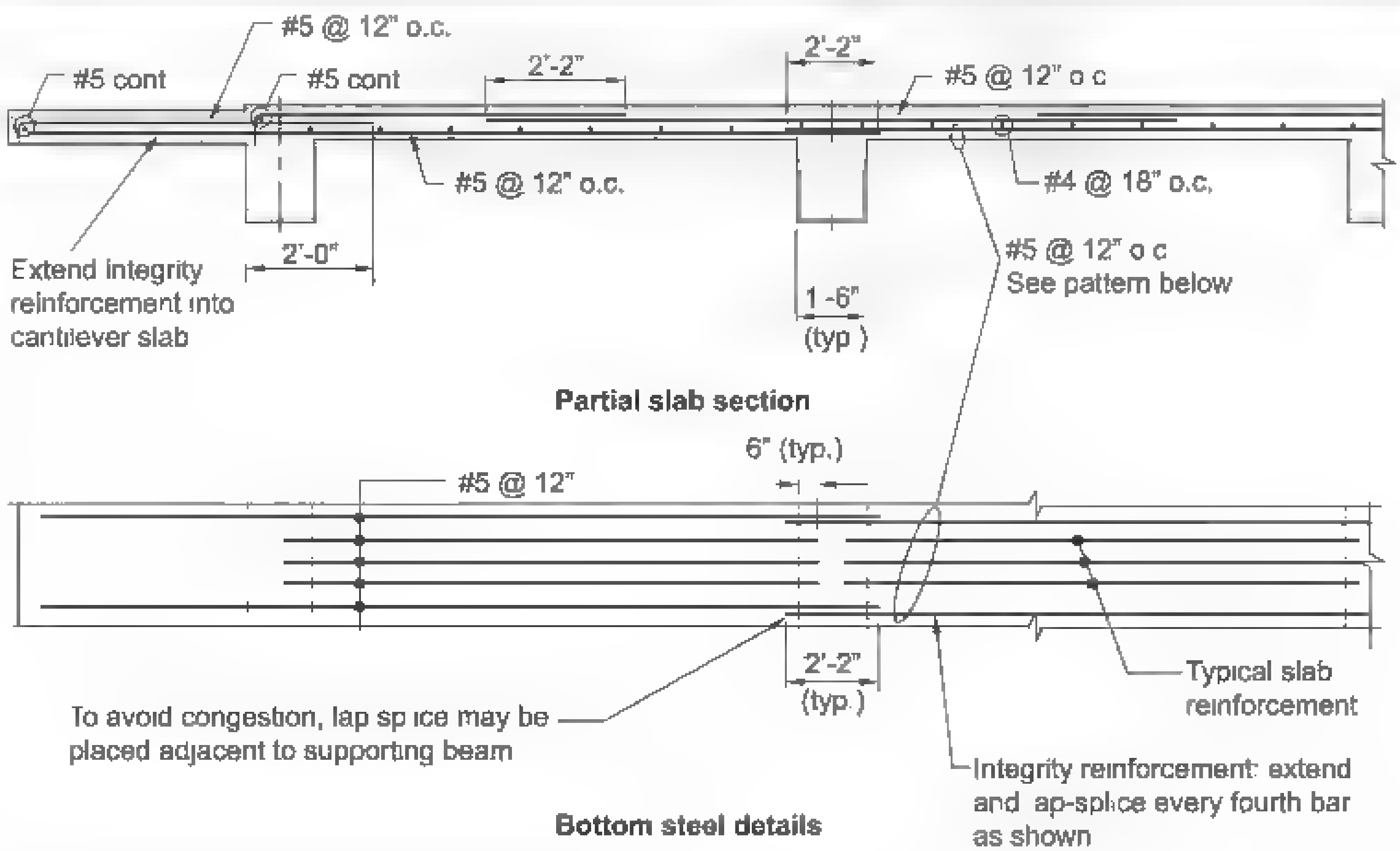


Fig. E1.7—One-way slab reinforcement details

One-way Slab Example 2. *Assembly loading*—

Design and detail a one-way nonprestressed reinforced concrete slab both for service conditions and factored loads. The one-way slab spans 20 ft-0 in. and is supported by 12 in. thick walls on the exterior, and 12 in. wide beams on the interior.

Given:

Load—

Live load L = 100 psf

Concrete unit weight γ_c = 150 lb/ft^3

Geometry—

Span = 20 ft

Slab thickness t = 9 in.

Material properties—

f_c' = 5000 psi (normalweight concrete)

f_y = 60,000 psi

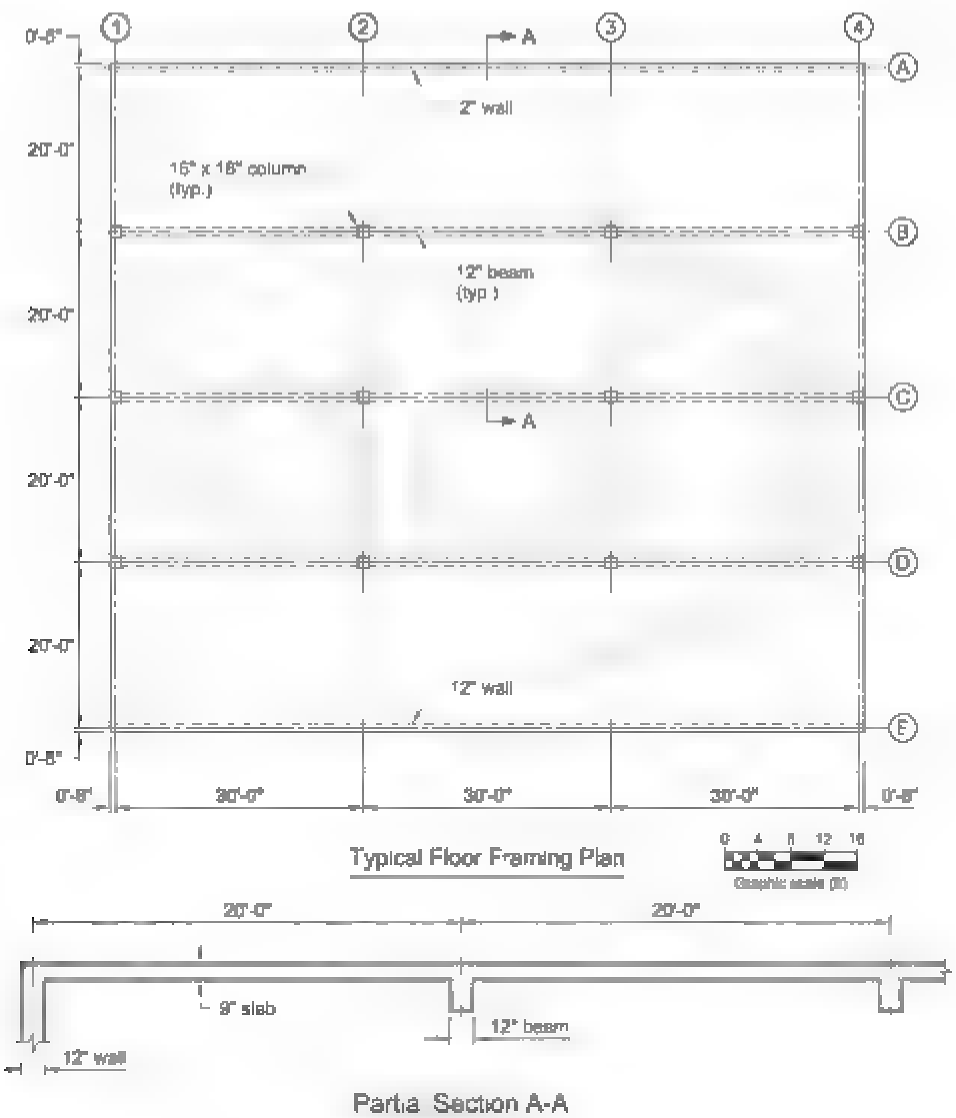


Fig. E2.1—Plan and section of four-span one-way slab.

ACI 318	Discussion	Calculation
Step 1: Geometry		
7.3.1.1	The specified slab thickness is 9 in. Since the slab satisfies the Code limit on span-to-depth ratios for fixed-fixed condition (Table 7.3.1.1), the designer does not need to check deflections unless supporting or attached to partitions or other construction likely to be damaged by large deflections.	$h \ge \frac{\ell}{27} = \frac{(20\text{ ft})(12\text{ in./ft})}{27} = 8.89$ in., say, 9 in.

Step 2. Loads and load patterns		
5.3.1	For hotel lobbies, the live load is assembly occupancy, the design live load is 100 psf per Table 4-1 in ASCE/SEI 7. A 9 in. slab is a 112 psf dead load. To account for loads due to ceilings, partitions, HVAC systems, etc., add 10 psf as miscellaneous dead load. $U = 1.4D$ (5.3.1a) $U = 1.2D + 1.6L$ (5.3.1b) The slab resists gravity only and is not part of a lateral force-resisting system, except to act as a diaphragm. Both ASCE/SEI 7 and the Code provide guidance for addressing live load patterns. Either approach is acceptable. The Code allows the use of the following two patterns, Fig. E2.2.	The required strength equations to be considered are: $U = 1.4(122) = 171$ psf $U = 1.2(122) + 1.6(100) = 146 + 160 = 306$ psf **Controls**
6.4.2	Factored dead load is applied on all spans and factored live load is applied as follows. (a) Maximum positive M_u near midspan occurs with factored live load on the span and on alternate spans (b) Maximum negative M_u at a support occurs with factored live load on adjacent spans only	

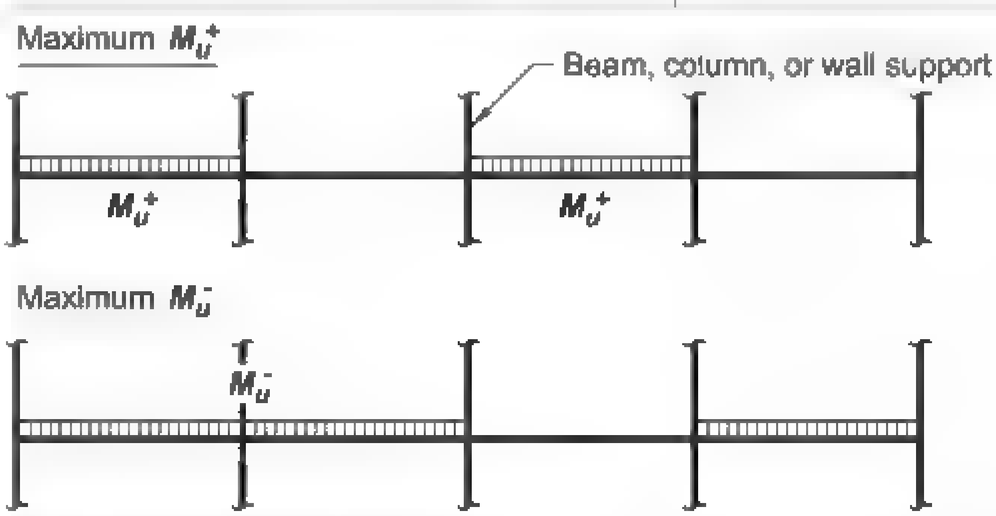


Fig. E2.2—Live load loading pattern.

Step 3. Concrete and steel material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements of the Code. The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes.	By specifying that the concrete mixture must be in accordance with ACI 301-10 and providing the exposure classes, Code Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be 5000 psi.
26.4.3.1	ACI 301 is a reference construction specification that is coordinated with the Code. The Code allows the use of ACI 301 for compliance of concrete mixture proportioning. There are several mixture options within ACI 301, such as the use of admixtures and supplementary cementitious materials, which the designer can require, permit, or review if suggested by the contractor.	
7.2.2.2	The reinforcement must satisfy Chapter 20 of the Code. The designer specifies the grade of bar and whether the reinforcing bar should be coated by epoxy, galvanized, or both.	By specifying the reinforcing bar grade and any coatings, and that the reinforcing bar must be in accordance with ACI 301-10, Chapter 20 requirements are satisfied. Use ASTM A615 Grade 60 uncoated reinforcement.
Step 4: Slab analysis		
6.3	System is braced by shear walls and moment-resisting frames. Assume slab is braced and that moment effects in slab caused by lateral loads may be ignored.	Modeling assumptions Assume constant moment of inertia for the entire length of the slab. Ignore torsional stiffness of beams. Only the slab at this level is considered.
6.6	Figure E2.3 shows the results of the first-order analysis of the slab for moments due to gravity load. The figure shows a moment envelope that was developed by the software for various pattern loadings. Analysis approach: The connection to the wall is monolithic, however, when the slab is fully loaded, flexural cracking will reduce joint fixity. Rather than attempting to estimate an appropriate level of softening, bound the problem by analyzing the slab assuming that (a) the support provides near-zero flexural restraint and then (b) assuming the support provides full flexural restraint. To simulate near-zero flexural restraint, reduce support flexural stiffness by increasing the length of support to 100 ft. To simulate full flexural restraint by the supporting member, use the gross moment of inertia (12 in. x 12 in. section) and a 10 ft length. Moments in the exterior joint from this analysis may be used to design the exterior wall.	
Step 5: Required moment strength		
7.4.2	The negative design moments are taken at the face of support, as is permitted by the Code.	

Analysis (a) reduced flexural restraint at exterior wall

Note: The moment at the exterior support is near zero (refer to Fig. E2.3)

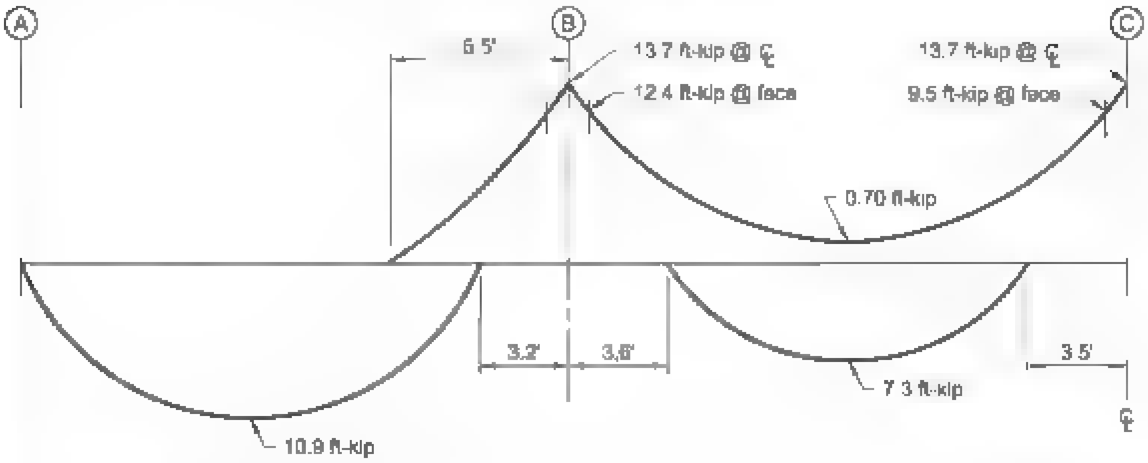


Fig. E2.3—Moment envelope.

The negative moment at the centerline of the exterior support is 0.0 ft-kip

The maximum positive moment in the end span is 10.9 ft-kip. The inflection points for positive moments are 0.0 ft from the exterior support centerline and 3.2 ft from the first interior support centerline

The maximum negative moment at the face of the first interior support is 12.4 ft-kip. The negative moment left inflection point is 6.5 ft from support B. On the right side of support B, there is no inflection point. Because of pattern loading, a small negative moment can exist across the span.
The maximum positive moment in the interior span is 7.3 ft-kip. The inflection points for positive moments are 3.6 ft from the first interior support centerline and 3.5 ft from the second interior support centerline

The maximum negative moment at the face of the second interior support is 9.5 ft-kip. On the left side, there is no inflection point.

Table E2.1—Maximum factored moments for (a) reduced flexural restraint at exterior support

Required strength	Location from left to right along the slab				
	Exterior support	Midspan AB	Support B	Midspan BC	Support C
M_u, ft-kip	0.0	10.9	12.4	+7.3	9.5

Analysis (b) full flexural restraint at exterior support

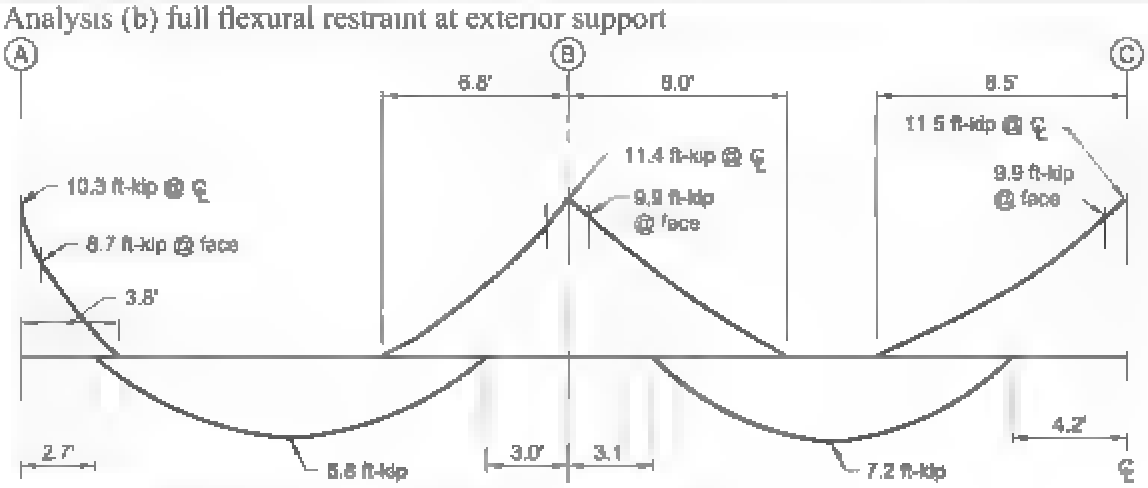


Fig. E2.4—Moment envelope with fixed end condition.

The negative moment at the face of the exterior support is 8 7 ft-kip. The negative moment inflection point is 3 8 ft from support A.

The maximum positive moment in the end span is 6.8 ft-kip. The inflection points for positive moments are 2 7 ft from the exterior support centerline and 3 0 ft from the first interior support centerline

The maximum negative moment at the face of the first interior support is 9 9 ft-kip The negative moment inflection points are 6.8 ft left and 8.0 ft right of support B

The maximum positive moment in the interior span is 7 2 ft-kip The inflection points for positive moments are 3 1 ft from the first interior support centerline and 4.2 ft from the second interior support centerline.
The maximum negative moment at the face of the second interior support is 9 9 ft-kip The negative moment left inflection points are 8 5 ft from the support centerline and the right inflection point is 8 5 ft from the support centerline.

Following are the maximum moments from a combination of Analysis (a) and (b) (conservative approach)

Table E2.2—Maximum factored moments for (b) full flexural restraint at exterior wall

Required strength	Location from left to right along the slab				
	Exterior support	Midspan AB	Support B	Midspan BC	Support C
M_u, ft-kip	–8.7	+6 8	–9.9	+7 2	–9.9

Step 6. Required shear strength

7 4.3 1 The maximum shear is taken at the support centerline for simplicity The maximum shear under any load pattern is 3 4 kip.

Step 7. Design moment strength

7 5 1 The two common strength inequalities for one-way slabs, moment and shear, are noted in Section 7.5 1 1

7 3 3 1 The Code requires slabs to be tension controlled in accordance with Code Table 21.2 2, which results in a strength reduction factor of 0 9

7 5 2 The one-way slab chapter refers to Code Section 22 3 for calculation of design moment strength.

Generate the required area of steel for the maximum factored moments from analysis

To calculate A_s in terms of the depth of the compression block, a, set the section's concrete resultant force (C) equal to steel force at yield (T)

$$T \quad C$$

22 2 2 4 1 $A_sf \quad 0.85f'(b)a$

$$A_s \quad \frac{0.85(5000\text{ psi})(12\text{ in.})a}{60{,}000\text{ psi}} \quad 0.85a$$

The effective depth, d, is the overall slab height minus the cover (3 4 in.) minus half the bar diameter (for a single layer of reinforcing bars) Assuming a No 5 bar, therefore,

$d \quad 9\text{ in.} \quad 0.75\text{ in.} \quad 0.625\text{ in.}/2$
$\quad 7.9\text{ in.}$

To calculate required A_s, set the design moment strength equal to the factored moment

$$\phi M_n \quad \phi A_sf_y(d \quad a/2) \quad M_u$$

$$0.9A_s(60{,}000\text{ psi})\left(7.9\text{ in.}-\frac{A_s}{2(0.85)}\right) \quad M_u$$

	Table E2.3 shows the required area of steel corresponding to an envelope formed from the maximum moments from Analysis (a) and (b).	

Table E2.3—Maximum moment

	Location from left to right along the slab				
	Exterior support	Midspan AB	Support B	Midspan BC	Support C
M_u envelope, ft-kip	-8.7	+10.9	12.4	+7.3	9.9
Req'd A_s, in.^{2} per foot	0.26	0.32	0.37	0.22	0.29

21.2.2	To ensure a ductile failure mode, the slab is required to be tension controlled. Table 21.2.2 defines a section as tension controlled when the steel strain is at least equal to the yield strain plus 0.003 when the section is at nominal strength.	To calculate reinforcing bar strain, begin with force equilibrium within the section $T = C$ $A_s f_y = 0.85 f_c' ba$ where b = 12 in./ft, f_y = 60,000 psi; and the maximum reinforcement is A_s = 0.37 in.^{2} From above calculations $A_s = 0.85a$ or $a = A_s/0.85$
22.2.2.1	Maximum strain at the extreme concrete compression fiber is assumed equal to: ε_{cu} = 0.003 in./in. Use strain diagram (Fig. E2.5) to determine steel strain at nominal strength.	Therefore, a = 0.44 in. where $a = \beta_1 c$ and β_1 = 0.80 for f_c' of 5,000 psi, so c = 0.55 in. From similar triangles (Fig. E2.5) $\varepsilon = \dfrac{0.003(7.9\text{ in.} - 0.55\text{ in.})}{(0.55\text{ in.})} = 0.040 \geq 0.004$ Therefore, section is tension-controlled

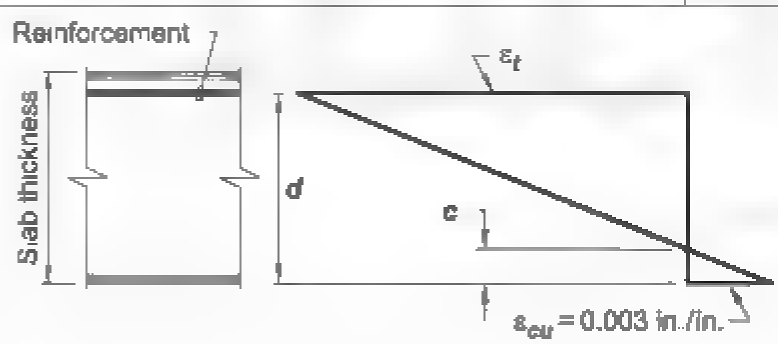


Fig. E2.5—Strain distribution in negative moment reinforcement used to check reinforcement limits.

Step 8: Minimum flexural reinforcement		
7.6.1	Check if design reinforcement exceeds the minimum required reinforcement by Code	$A_{s,min}$ = 0.0018 x 9 in. x 12 in. = 0.20 in.^{2}/ft At all critical sections, the required A_s is greater than the minimum
Step 9: Shrinkage and temperature reinforcement		
7.6.4 24.4.3.2 24.4.3.3	For nonprestressed one-way slabs, the minimum area of shrinkage and temperature (S+T) bars is $0.0018A_g$. The maximum spacing of S+T reinforcing bars is the lesser of $5h$ and 18 in.	S+T steel area = 0.0018 x 12 in. x 9 in. = 0.20 in.^{2}/ft Based on S+T steel area, solutions are No. 4 at 12 in. or No. 5 at 18 in., use No. 5 at 18 in. placed atop and perpendicular to the primary reinforcement
Step 10: Minimum and maximum spacing of flexural reinforcement		
7.7.2.1 25.2.1	The minimum spacing between bars must not be less than the greatest of: (a) 1 in. (b) d_b (c) $(4/3)d_{agg}$ Assume 1 in. maximum aggregate size	(a) 1 in. (b) 0.625 in. (c) (4/3)(1 in.) = 1.33 in. **Controls**
7.7.2.2 24.3.2	For reinforcement closest to the tension face, the spacing between reinforcement is the lesser of (a) and (b) (a) $12(40,000/f_s)$ (b) $15(40,000/f_s) - 2.5c_c$	(a) 12(40,000/40,000) = 12 in. **Controls** (b) 15(40,000/40,000) – 2.5(0.75 in.) = 13.1 in.
24.3.2.1	$f_s = (2/3)f_y$ = 40,000 psi	
7.7.2.3	The maximum spacing of deformed reinforcement is the lesser of $3h$ and 18 in.	3(9 in.) = 27 in. > 18 in. Therefore, Section 24.3.2 controls, 12 in.

Step 11: Design shear strength

ACI 318-19 Section	Discussion	Calculation
	Shear reinforcement is not typically used in one-way slabs so shear strength is provided by the concrete contribution ($\phi V_n = \phi V_c$).	
7.6.3.1	Minimum shear reinforcement is required when $V_u > \phi V_c$	
22.5.5.1c	Use appropriate equation from Table 22.5.5.1c $$V_c = \left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d$$	
21.2.1	Strength reduction factor for shear from Table 21.2.1b	$\phi = 0.75$
	Effective depth to centroid of reinforcement	$d = 9\text{ in.} - 0.75\text{ in.} - 0.5(0.625\text{ in.}) = 7.9\text{ in.}$
	Consider No. 5 at 11 in. spacing. Use largest anticipated spacing to ensure shear strength check will cover all reinforcement conditions (Table E3.2).	$s = 11\text{ in.} = 0.917\text{ ft}$
		$A_s = \frac{0.31\text{ in.}^2}{s} = \frac{0.31\text{ in.}}{0.917\text{ ft}} = 0.338\text{ in.}^2$ per unit width of slab
	Unit slab width of 12 in.	$b_w = 12\text{ in.}$
2.2	Reinforcement ratio of flexural reinforcement $$\rho_w = \frac{A_s}{b_w d}$$	$\rho_w = \frac{0.338\text{ in.}^2}{12\text{ in.}(7.93\text{ in.})} = 0.00356$
	Axial load is zero	$N_u = 0$
22.5.5.1.3	Size effect factor $$\lambda_s = \sqrt{\frac{2}{1+0.1d}}$$	$\lambda_s = \sqrt{\frac{2}{1+0.1(7.93)}} = 1.056$ use $\lambda_s = 1.0$
		$\lambda = 1.0$
		$V_c = \left[8(1.0)(1.0)(0.00356)^{1/3}\sqrt{5000}\right](12)(7.93)\frac{1}{1000}$
		$V_c = 8.20\text{ kip/ft}$
		$\phi V_c = 0.75(8.20) = 6.1 > V_u = 3.4\text{ kip/ft}$ **OK**
		Design shear strength from concrete contribution is about twice the factored shear. Slab thickness is adequate.

Step 12: Select bar size and spacing

Table E2.4—Bar spacing

Bar size	Location from left to right along the slab				
	Exterior support	Midspan AB	Support B	Midspan BC	Support C
No. 4 at spacing, in.	9	7	6	10	8
No. 5 at spacing, in.	12	11	10	12	12
No. 6 at spacing, in.	12	12	12	12	12

Refer to Fig. E2.6 for provided bar spacing

	Based on the above, use No. 5 bars. Spacing of top bars at exterior support is 12 in. and at interior supports is 10 in. Note that there is no point of zero negative moment along the second and third span, so continue the top bars across both spans. While this solution is slightly conservative (10 in. versus 12 in. spacing), the engineer may desire consistent spacing for easier installation and inspection.	No. 5 at 10 in.: $A_{prov} = \dfrac{0.31\text{ in.}^2(12\text{ in./ft})}{10\text{ in.}} = 0.38\text{ in.}^2$
7.3.3.1	Recheck reinforcement strain limits using maximum provided reinforcement of No. 5 at 10 in.	$c = A_s f_y/(0.85\beta_1) = 0.38\text{ in.}^2/(0.85)(0.80) = 0.56\text{ in.}$ $\varepsilon_t = \dfrac{0.003(7.9\text{ in.} - 0.56\text{ in.})}{(0.56\text{ in.})} = 0.039 \geq 0.005$ Section is tension-controlled considering provided reinforcement
Step 13: Top bar cutoff at exterior support of exterior span		
	The inflection point for negative moment is 3.8 ft from support centerline	
	<u>Bar cutoff</u>	
7.7.3.3	Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	Extend bars beyond the inflection point at least: d = 7.9 in. or (12)(0.625 in.) = 7.5 in., Therefore, use 8 in. ~ 7.9 in.
7.7.3.8.4	At least one-third the negative moment reinforcement at a support shall have an embedment length beyond the point of inflection at least the greatest of d, $12d_b$, and $\ell_n/16$	33 percent of the bars to extend beyond the inflection point at least (19 ft × 12 in./ft)/16 ≅ 15 in. > d = 7.9 in. > $12d_b$ = 7.5 in. Because the reinforcing bar is already at maximum spacing, no percentage of bars (as permitted by Code Section 7.7.3.3) can be cut off in the tension zone <u>Solution</u> The top bar length is 6 in. (wall beyond centerline) plus 3.8 ft (inflection) plus an extension of either 8 in. or 15 in. Because the two cut off locations are close together, use a 15 in. extension for all bars. A practical length for top bars is:
	<u>Bars at the wall connection</u> It is assumed that the wall is placed several days before the first floor slab. Because the wall and the slab will be firmly connected, the wall will tend to restrain the slab from shrinking as it cures. Many designers place extra reinforcement along the slab edge, parallel to the wall, to limit widths of possible cracks due to this restraint	3.8 ft + 0.5 ft + 1.25 ft = 5.55 ft, say, 6 ft

Step 14. Development and splice lengths		
7.7.1.2 25.4.2.4	ACI provides two equations for calculating development length, simplified and detailed. In this example, the detailed equation is used $\ell_d = \left(\dfrac{3}{40}\dfrac{f_y}{\lambda\sqrt{f_c'}}\dfrac{\psi_t\psi_e\psi_s\psi_g}{\left(\dfrac{c_b+K_{tr}}{d_b}\right)}\right)d_b$ where ψ_t = bar location, not more than 12 in. of fresh concrete below horizontal reinforcement ψ_e = coating factor; uncoated ψ_s = bar size factor; No. 7 and larger ψ_g = reinforcement grade: Grade 60 But the expression: $\dfrac{c_b+K_{tr}}{d_b}$ must not be taken greater than 2.5.	The development length of a No. 5 black bar in a 9 in. slab with 0.75 in. cover is $\ell_d = \left(\dfrac{3}{40}\dfrac{60{,}000\text{ psi}}{(1.0)\sqrt{5000\text{ psi}}}\dfrac{(1.0)(1.0)(0.8)(1.0)}{1.7\text{ in.}}\right)(0.625\text{ in.})$ $= 19$ in. ψ_t = 1.0, because not more than 12 in. of concrete is placed below bars. ψ_e = 1.0, because bars are uncoated ψ_s = 0.8, because bars are smaller than No. 7 ψ_g = 1.0, because bars are Grade 60 c_b = 0.75 in. + 0.5(0.625 in.) = 1.06 in. $\dfrac{1.06\text{ in.}+0}{0.625\text{ in.}} = 1.7$ in.
7.7.1.3 25.5 25.5.1.1 25.5.2.1	Splice The maximum bar size is No. 5, therefore, splicing is permitted Tension lap splice length, ℓ_{st}, for deformed bars in tension must be the greater of: 1.3ℓ_d and 12 in.	 ℓ_{st} = (1.3)(19 in.) = 24.7 in., use 26 in.
Step 15: Bottom bar cutoff in exterior span		
	The inflection points for positive moments are 0.0 ft from the exterior support centerline (analysis (a)) and 3.0 ft from the first interior support centerline (analysis (b))	
7.7.3.3	Bar cutoff Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply supported spans and at free ends of cantilevers	
7.7.3.4	Continuing flexural tensile reinforcement must have an embedment length not less than ℓ_d beyond the point where bent or terminated tensile reinforcement is no longer required to resist flexure	This condition is satisfied at any section along the beam span
7.7.3.5	Flexural tensile reinforcement shall not be terminated in a tensile zone unless (a), (b), or (c) is satisfied (a) $V_u < (2/3)\phi V_n$ at the cutoff point Note that (b) and (c) do not apply	
7.7.3.8.2	At least one-fourth the maximum positive moment reinforcement must extend along the slab bottom into the continuous support a minimum of 6 in.	

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7.7.3.8.3	At points of inflection, d_b for positive moment tensile reinforcement must be limited such that ℓ_d for that reinforcement satisfies condition (b), because end reinforcement is not confined by a compressive reaction	
	$\ell_d < M_n/V_u + \ell_a$	Check if bar size is adequate M_n for an 9 in. slab with No. 5 at 12 in., 0.75 in. cover is
	$M_n = A_s f_y\left(d - \frac{a}{2}\right)$	$M_n = (0.31\text{ in.}^2)(60{,}000\text{ psi})(7.9\text{ in.} - 0.4\text{ in.})$ $= 140{,}000\text{ in.-lb}$
	The elastic analysis indicates that V_u at inflection point is 2800 lb. The term ℓ_a is 8 in.	$\ell_d \leq \frac{140{,}000\text{ lb}}{2800\text{ lb}} + 8\text{ in.} = 58\text{ in.} > 19\text{ in.}$ Therefore, No. 5 bar is **OK**
Step 16: First span bottom bar length		
7.7.3.3	Bars close to their maximum spacing limit may not be cut off within the tensile zones because the maximum reinforcing bar spacing would then be exceeded. All bottom bars must extend at least 8 in. beyond the positive moment inflection points	Greater of d = 7.9 in. and $12d_b$ = 12(0.625 in.) = 7.5 in. use 8 in.
7.7.3.5	Because all bottom bars will be extended past the inflection points, 7.7.3.5 is not applicable	
		Because the cutoff location is close to the right support and for field placing simplicity, extend all bars 6 in. into both supports
Step 17: Bottom bar cutoffs in interior span		
	The inflection points for positive moments are 3.6 ft from the left support centerline and 4.2 ft from the right support centerline	Create a partial length bar that is symmetrical within the span, so assume both inflection points are 3.6 ft. The minimum length is 20 ft − 3.6 ft − 3.6 ft + (2 ft)(0.5) = 13.8 ft, say, 14 ft 0 in.
	Bar cutoffs Similar to the first interior span, all bottom bars must extend at least 8 in. past inflection points. The Code requires at least 25 percent of bottom bars be full length, extending 6 in. into the support.	In a repeating pattern, use 3 No. 5 at 14 ft long and 1 No. 5 extended into support to satisfy integrity steel requirement.
Step 18: Top bar cutoffs at middle support		
	There are no inflection points over either span that frame into the middle support.	The required reinforcing bar is No. 5 at 12 in. in the middle support. Because the top bar from the first support is No. 5 at 10 in., extend the No. 5 at 10 in. top over the middle support for simplicity
	Bar cutoffs Because the top bars will be continuous, no bars are cut off	

Step 19: Slab integrity steel		
7.7.7	Provide structural integrity reinforcement in the slab to protect overall structural stability	
7.7.7.1	At least 1/4 of the bottom bars must be continuous.	Extend bottom bars into support to lap with bars from adjacent spans. Adjust bar spacing in interior spans to match that of exterior span (11 in.)
7.7.7.3	Provide Class B tension lap splices at interior supports	
25.5.2.1	$1.3\ell_d$ and 12 in.	lap = 1.3 × 19 in. = 24.7 in. Use 2 ft 2 in.
7.7.7.2	Provide hook at exterior (noncontinuous) support to develop bar yield strength at face of support.	
25.4.3.1	$\ell_{dh} = \left(\dfrac{f_y\psi_e\psi_r\psi_o\psi_c}{55\lambda\sqrt{f_c'}}\right)d_b^{1.5}$	
	Table 25.4.3.2 Reinforcement uncoated	$\psi_e = 1.0$
	Hook confinement (or spacing)	hook spacing = 4(11 in.) = 44 in. $> 6d_b = 3.75$ in. $\psi_r = 1.0$
	Hook location and side cover	hook side cover $> 6d_b = 3.75$ in. $\psi_o = 1.0$
	Concrete strength	for $f_c' < 6000$ psi $\psi_c = (f_c'/15{,}000) + 0.6 = 5/15 + 0.6 = 0.93$
25.4.3.4	For hook development at the discontinuous ends of members, 25.4.3.4 must be checked.	Side, top, and bottom cover over hook is greater than 2.5 in. Ties are not required.
25.4.3.1a		$\ell_{dh} = \dfrac{60{,}000\text{ psi}(1.0)(1.0)(1.0)(0.93)}{55(1.0)\sqrt{5000\text{ psi}}}(0.625)^{1.5} = 7.1$ in.
25.4.3.1b		or $8d_b = 5$ in.
25.4.3.1c		or 6 in.
		Provided develop length = 12 in. – 2 in. (cover) = 10 in. > 7.1 in. **OK**

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Step 20: Detailing

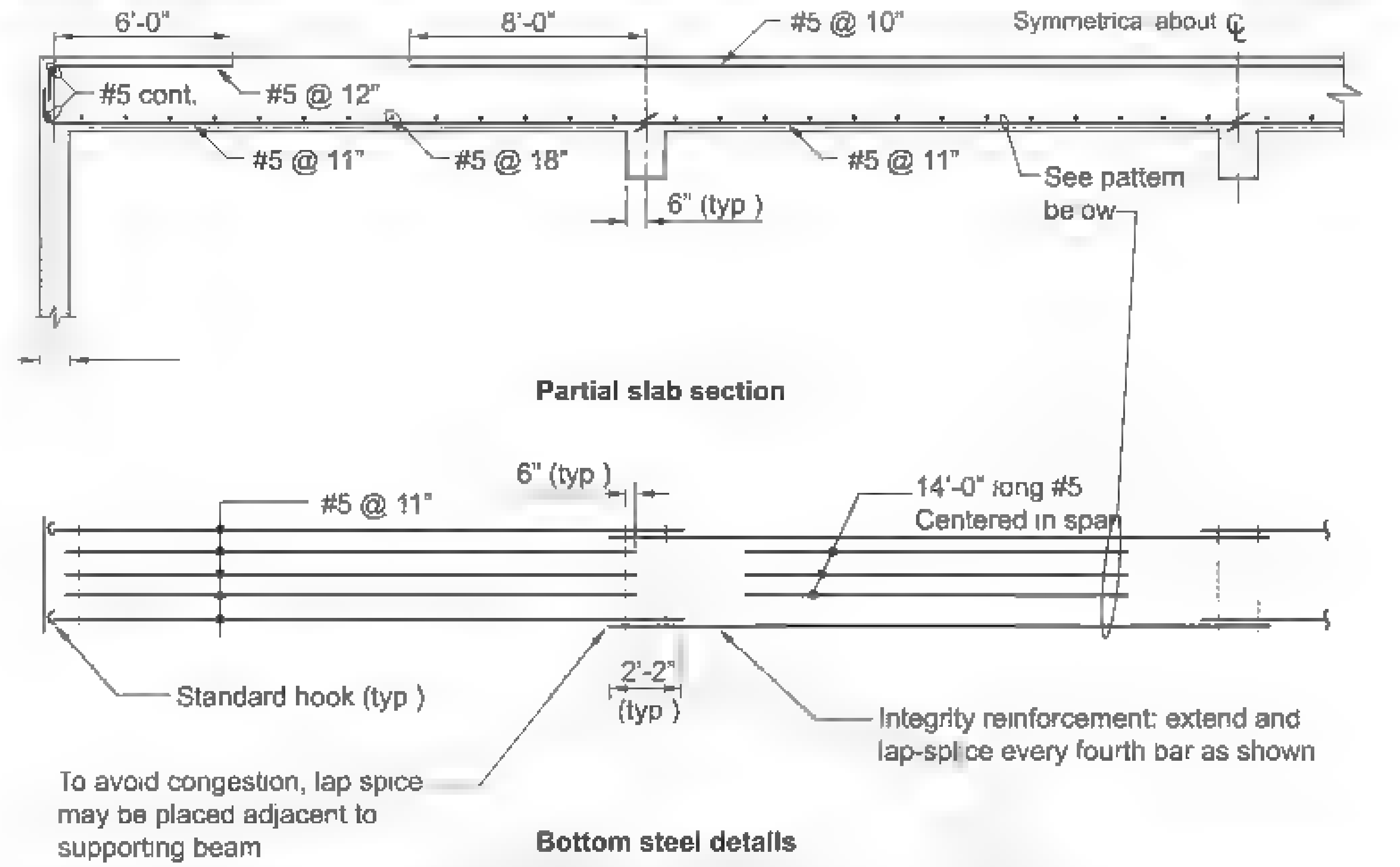

Fig. E2.6—One-way slab reinforcement details.

One-way Slab Example 3: *One-way slab post-tensioned—Hotel loading*

There are four spans of 20 ft 0 in. each, with a 3 ft 0 in. cantilever balcony at each end. The slab is supported by 12 in. walls on the exterior, and 12 in. wide beams on the interior (Fig. E3.1). This example will illustrate the design and detailing of a one-way post-tensioned (PT) slab, both for service conditions and factored loads.

Given:

Load—

Service live load $L = 40$ psf

Concrete—

$f_c' = 5000$ psi (normalweight concrete)

$f_y = 60{,}000$ psi

$f_{pu} = 270{,}000$ psi

Geometry—

Span length: 20 ft

Beam width: 12 in.

Column dimensions: 16 in. x 16 in.

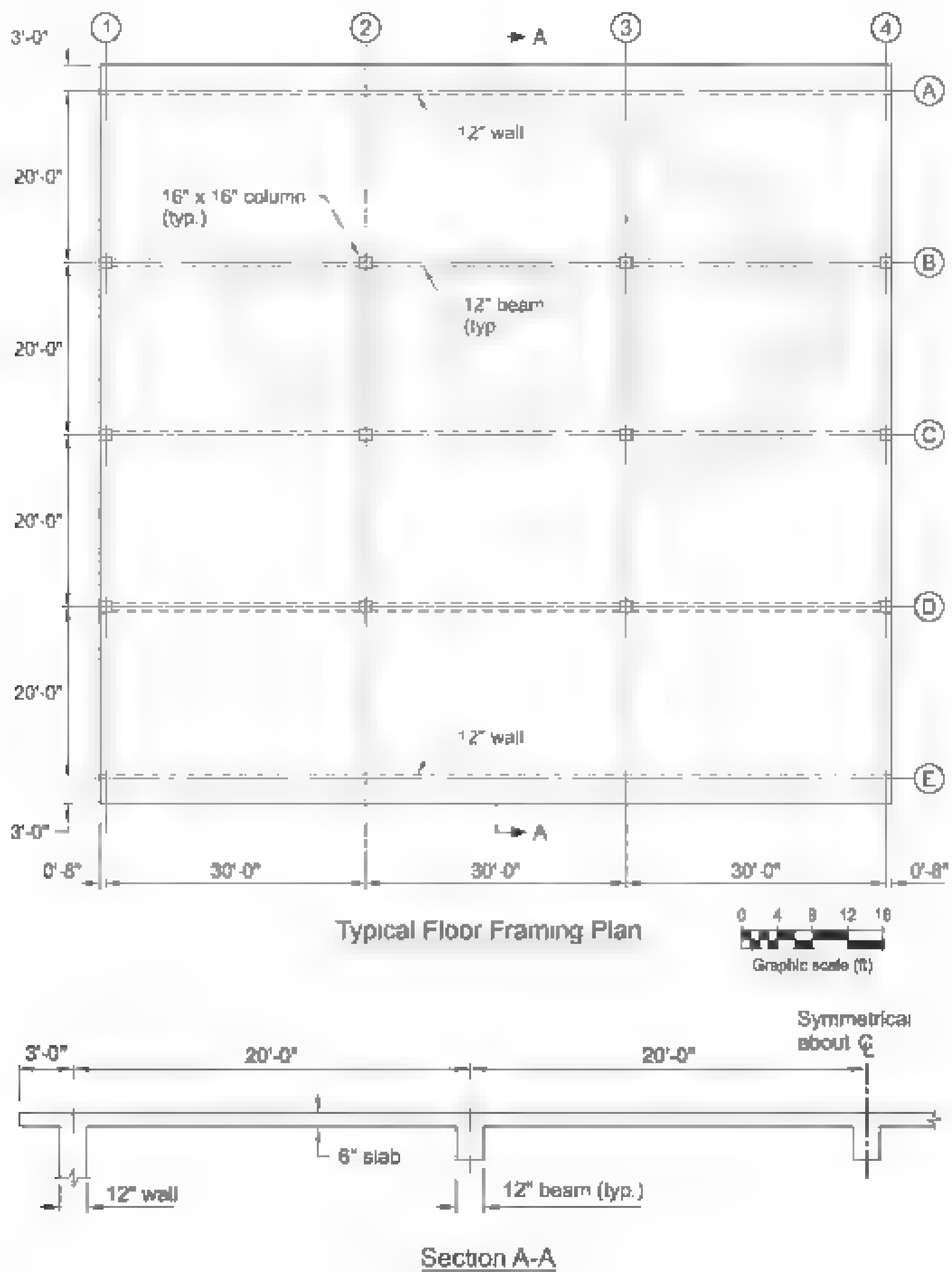

Fig. E3.1—Plan and section of four-span one-way PT slab.

ACI 318	Discussion	Calculation
Step 1: Geometry		
7.3.2	Code limitations on span-to-depth ratios do not apply to PT slabs and deflections must be checked. If service stress conditions are properly addressed in post-tensioned slab systems, then deflections rarely control the design. The *Post-Tensioning Manual*, 2006, sixth edition Chapter 9, Table 9.3, suggests a ratio limit of $\ell/48$. For this example, this ratio gives a slab thickness of	 $(20\text{ ft})(12\text{ in./ft})/48 = 5.0\text{ in.}$ This example uses a 6 in. thick slab
Step 2: Loads and load patterns		
7.4.1.1	For hotel occupancy, the design live load is 40 psf per Table 4-1 in ASCE/SEI 7. Self weight of 6 in. slab produces a 75 psf dead load. To account for load from ceilings, partitions, HVAC systems, etc., add 10 psf as miscellaneous dead load. The slab resists gravity only and is not part of a lateral-force-resisting system, except to act as a diaphragm.	$D = 75\text{ psf} + 10\text{ psf} = 85\text{ psf}$ The required strength equations to be considered are:
5.3.1	$U = 1.4D$ $U = 1.2D + 1.6L$	$U = 1.4(85) = 119\text{ psf}$ $U = 1.2(85) + 1.6(40) = 102 + 64 = 166\text{ psf}$ **Controls**
7.4.1.2 6.4.2	Both ASCE/SEI 7 and the Code provide guidance for addressing live load patterns. Either approach is acceptable. The Code allows the use of the following two patterns (Fig. E3.2). Factored dead load is applied on all spans and factored live load is applied as follows: (a) Maximum positive M_u near midspan occurs with factored L on the span and on alternate spans. (b) Maximum negative M_u at a support occurs with factored L on adjacent spans only.	

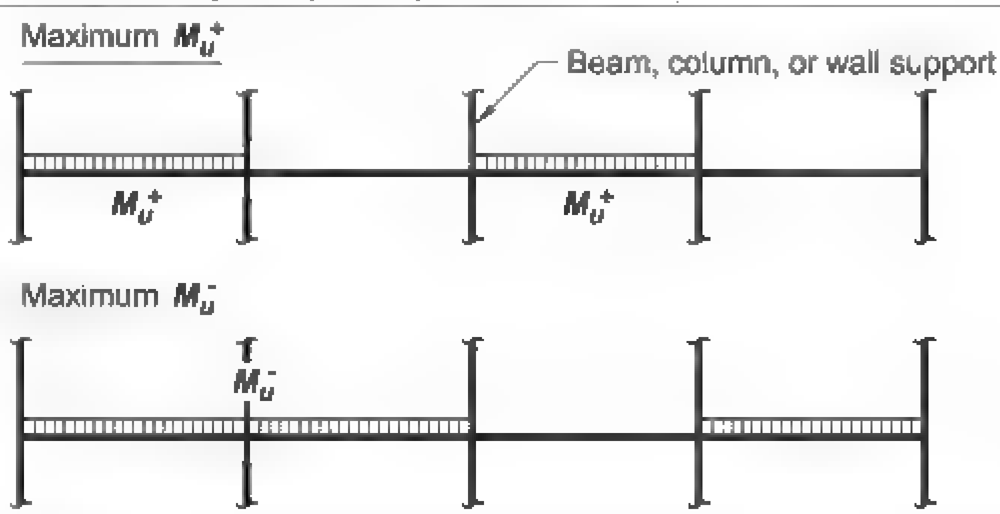

Fig. E3.2—Live load pattern used in elastic analysis of slab for unbalanced loads.

Step 3. Concrete and steel material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements. The designer determines the durability classes. Please refer to Chapter 4 of this design Manual for an in-depth discussion of the Categories and Classes.	
	ACI 301 is a reference specification that is coordinated with the Code. ACI encourages referencing ACI 301 into job specifications	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Code Chapter 19 requirements are satisfied
	There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor	Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi
7.2.2.2	The reinforcement must satisfy Chapter 20	
	In this example, unbonded, 1/2 in. single-strand tendons are assumed.	By specifying the reinforcement shall be in accordance with ACI 301-10, the PT type and strength, and reinforcing bar grade (and any coatings), Chapter 20 requirements are satisfied In this example, assume grade 60 bar and no coatings for flexural strength and shrinkage and temperature crack control
	The designer determines the grade of bar and if the reinforcement should be coated by epoxy or galvanized, or both	
20.3	The Code requires strand material to be 270 ksi, low relaxation (ASTM A416)	
20.3.2.5.1	The U.S. industry usually stresses, or jacks, monostrand to impart a force equal to the least of $0.80f_{pu}$ and $0.94f_{py}$. Immediate and long-term losses will reduce this force. For ASTM A416 strand f_{py} = $0.9f_{pu}$, in which case, $0.80f_{pu}$ will control the limit on jacking stresses.	The jacking force per individual strand is (270 ksi)(0.8)(0.153 in.^{2}) = 33 kip. This is immediately reduced by seating and friction losses, and elastic shortening of the slab. Long term losses will further reduce the force per strand. Refer to Commentary R20.3.2.6 of the Code.
		The design prestress force is usually expressed in terms of kip per foot of slab width. To estimate the tendon spacing, an effective prestress force of 26.5 kip per strand is commonly used for preliminary design purposes and will be used in this example. Refer to ACI 423.10R for a comprehensive treatment on the estimation of prestress losses

Step 4. Slab analysis

6.3 System is braced by shear walls and moment resisting frames. Assume slab is braced and that moment effects in slab caused by lateral loads may be ignored.

Modeling assumptions.
Slab will be designed as Class U. Consequently, use the gross moment of inertia of the slab in the analysis.

Assume the supporting beams have no torsional resistance and act as a knife edge support.

Only the slab at this level is considered.

6.6 The analysis performed should be consistent with the overall assumptions regarding the role of the slab within the building system. Because the lateral force-resisting system only relies on the slab to transmit diaphragm forces, a first order analysis is adequate.

Analysis approach
Tendon profile is arranged in a parabolic drape to equilibrate the distributed loads from self-weight and live load (Fig. E3.3). The prestress force magnitude and eccentricity from the slab centroid are selected to balance a portion of the self-weight. The unbalanced load is then analyzed using structural analysis software to determine the net effect on stresses. The balanced load (w_p) can be calculated using:

$$w_p = 8Fa/\ell^2$$

where F is the effective PT force and a is the tendon drape (average of the two high points minus the low point). In this example the PT force is assumed constant for all spans, but the equivalent load varies due to different tendon drapes.

20.5.1.3.2 The strand profile is arranged to maximize the eccentricity, which reduces the demand for PT to balance the gravity load. At the exterior support an eccentricity of 0.25 in. is chosen to balance the cantilever load. At the interior supports and midspans, the maximum possible eccentricity is chosen (1 in. cover).

required clear cover = 3/4 in.
cover to center of tendon
= 3/4 in. + 0.5 in./2 = 1 in.

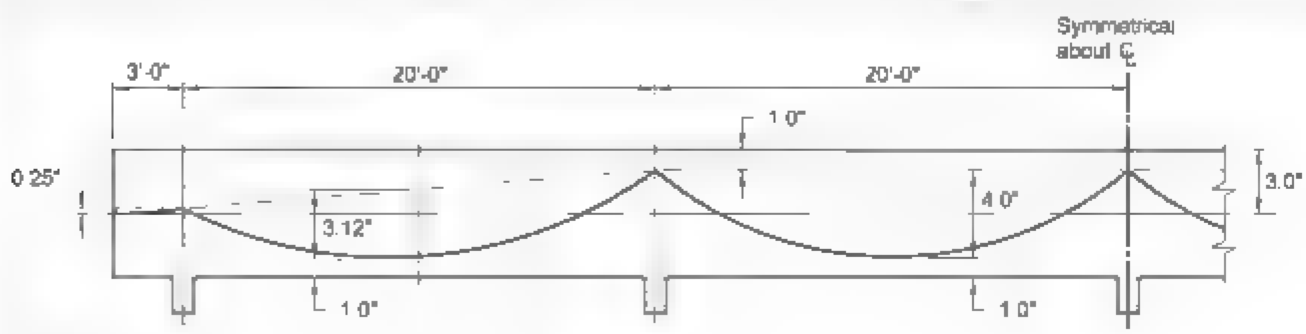

Fig. E3.3—Post-tensioning strand profile.

Step 5. Slab stress limits		
7.3.4.1 24.5.2.1	This example demonstrates the design of a Class U slab, that is, when full service load results in concrete tension stress not exceeding $7.5\sqrt{f_c'}$. The same slab analysis model is used for service and strength conditions. To verify that the concrete tensile stresses are less than $7.5\sqrt{5000 \text{ psi}}$, the net service moments are used to calculate the tensile stresses at the face of supports. The following design parameters were used in this example: (a) The PT force (F) provides an average effective slab compressive stress of at least 125 psi (9 kip/ft), and (b) The combination of PT force and strand profile provides an equivalent upward distributed load w_p to balance 75 percent of the slab weight, or 56 psf (Fig. E3.4). The basic equation for concrete tensile stress is: $f_t = M/S - F/A$, where M is the net service moment. At midspan of the exterior span, the drape is 3.12 in. (Fig. E3.3). The required effective prestress force is calculated from $w_p = 8Fa/\ell^2$ where $w_p = 0.056$ kip/ft²	$7.5\sqrt{5000 \text{ psi}} = 530$ psi Service load $w_s = 85$ psf + 40 psf = 125 psf $S = (12 \text{ in.})(6 \text{ in.})^2/6 = 72 \text{ in.}^3$ (section modulus), $A = (12 \text{ in.})(6 \text{ in.}) = 72 \text{ in.}^2$ (gross slab area per foot) $F = \dfrac{(0.056 \text{ kip/ft})(20 \text{ ft})^2}{8(3.12 \text{ in.})/(12 \text{ in./ft})} = 10.8$ kip/ft

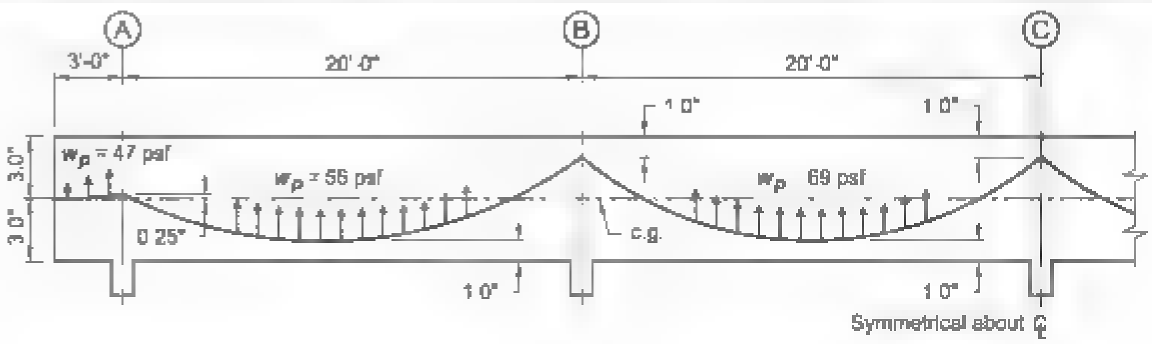

Fig. E3.4—Unbalanced load calculation (psf).

7.3.4 Subtract balanced load from total service load to determine unbalanced load (psf).

	Location from left to right along the slab		
Service loads	Cantilever	Span AB	Span BC
Total load	125	125	125
Balanced load	47	56	69
Unbalanced load	78	69	56

7.3.4	The slab stresses are determined from an elastic analysis of moments caused by unbalanced load minus the effect of the axial compression. Slab axial compressive stress due to the effective prestress force F	 F/A = (10.8 kip/ft)(1000 lb/kip)/(6 in.)(12 in./ft) = 150 psi

7.3.4 Net tensile stress is computed as follows from the results of the analysis using unbalanced loads. Positive stress (+) tension and negative stress ( ) compression

	Location from left to right along the slab			
	Exterior support	Midspan AB	Support B	Midspan BC
Unbalanced moments, ft-kip/ft	0.2	1.5	2.2	[illegible]1
Net tensile stress, psi	33 15 = 7	250 50 = [illegible]00	367 150 = **217**	183 [illegible]5 = 33

7.4.2.1 The cantilever moment at the support centerline is:
$w_{net}\ell^2/2$ (0.078 psf)(3 ft)^2/2 0.351 kip-ft/ft
The Code permits the design slab moment to be calculated at the face of the support: 0.2 kip-ft/ft
The moments for the interior supports are calculated at the faces of interior supports.

7.3.4.1 The maximum slab tensile stress (217 psi) calculat
24.5.2.1 ed for an average PT force of 10.8 kip/ft is less than

$$7.5\sqrt{f_c'} = 530 \text{ psi} > 217 \text{ psi}$$

19.2.3.1 Slab is Class U according to the Code Table 24.5.2.1

Step 6: Deflections		
7.3.2.1	Code Section 7.3.2.1 refers to Code Section 24.2, "Deflections due to service-level gravity loads," for allowable stiffness approximations to calculate immediate and time-dependent (long-term) deflections. Section 24.2.2 provides maximum permissible calculated deflections.	
24.2.3.8	Because slab is Class U, use I_g in analysis to determine deflections.	$I_g = \dfrac{(12\text{ in.})(6\text{ in.})^3}{12} = 216\text{ in.}^4$
	The balanced portion of the total load is offset by the camber from the prestressing, which results in a zero net deflection. Unbalanced load, however, will result in short- and long-term deflections that must be checked against Table 24.2.2. The following equation, which can be downloaded from the Reinforced Concrete Design Handbook Design Aid – Analysis Tables, https://www.concrete.org/MNL1721Download1, provides an approximate maximum deflection of the slab span with the largest unbalanced load (69 psf). Deflections could also be obtained from the software used for structural analysis of the unbalanced load. Deflections rarely control the design of PT slabs.	
	$\Delta_{max} = 0.0065w\ell^4/EI$	$\Delta_{max} = \dfrac{(0.0065)(69\text{ psf})(240\text{ in.})^4}{(4{,}030{,}000\text{ psi})(216\text{ in.}^4)/(12\text{ in./ft})} = 0.14\text{ in.}$ Expressed as a ratio, $\ell/\Delta = 240\text{ in.}/0.14\text{ in.} = 1700$
	The additional time-dependent deflection can be approximated by the immediate deflection due to sustained load multiplied by two (refer to Scanlon and Suprenant, 2011, "Estimating Two-Way Slab Deflections," *Concrete International*, V. 33, No. 7, July, pp. 24-29).	
24.2.2	Assume that no portion of the live load is sustained and that the floor is supporting nonstructural elements likely to be damaged by large deflections. Calculate the immediate deflection based on a total sustained load of 85 psf reduced by the balanced load of 56 psf.	$(0.14\text{ in.})(85\text{ psf} - 56\text{ psf})/(69\text{ psf}) = 0.06\text{ in.}$
	Sum the immediate and time-dependent components of deflection that will affect nonstructural elements and check against the limit imposed by Table 24.2.2.	$0.14\text{ in.} + 2(0.06) = 0.26\text{ in.}$ $\ell/\Delta = 240\text{ in.}/0.26\text{ in.} \approx 920$ The deflection ratios are much less than the limit of $\ell/480$, so deflections are satisfied without more detailed calculations.

One-Way Slabs

Step 7, Required moment strength	
7.4.2	The gravity design moments, including pattern loading, are shown in Fig. E3.5

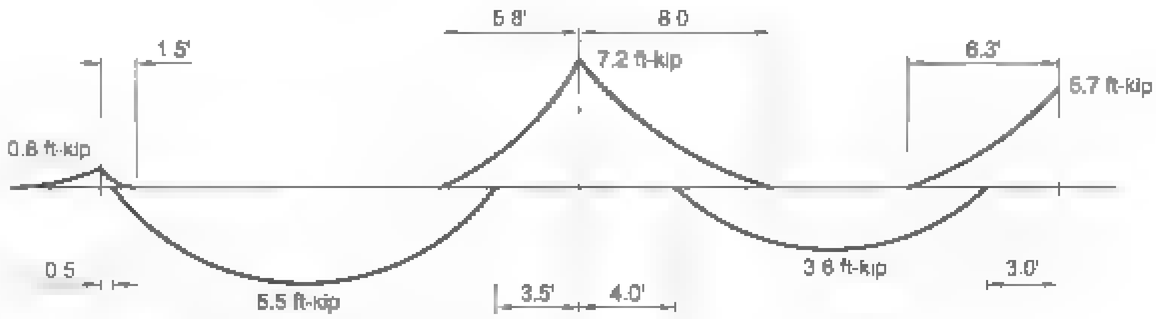

Fig. E3.5—Moment envelope due to factored loads

7.4.1.3 5.3.11	The Code requires that moments due to reactions induced by prestressing (secondary moments) be included with a load factor of 1.0. Secondary moments are calculated at each support as: $M_2 = M_{pt} - Fe$ The secondary moment diagram is linear between supports (Fig. E3.6)
7.4.1.3	The PT secondary moments are

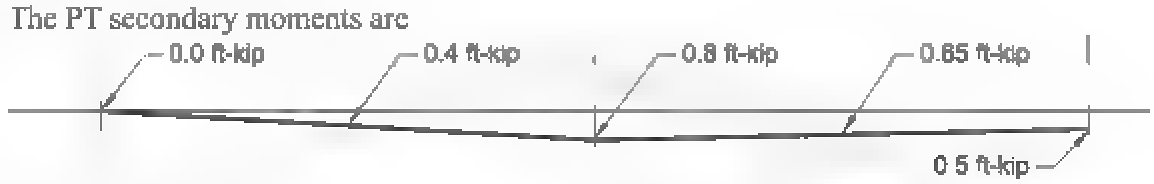

Fig. E3.6—Secondary moments due to post-tensioning

7.4.2.1 5.3.1b 5.3.11	The required moment strength envelope is the factored combination of gravity and secondary moments as shown in Fig. E3.7

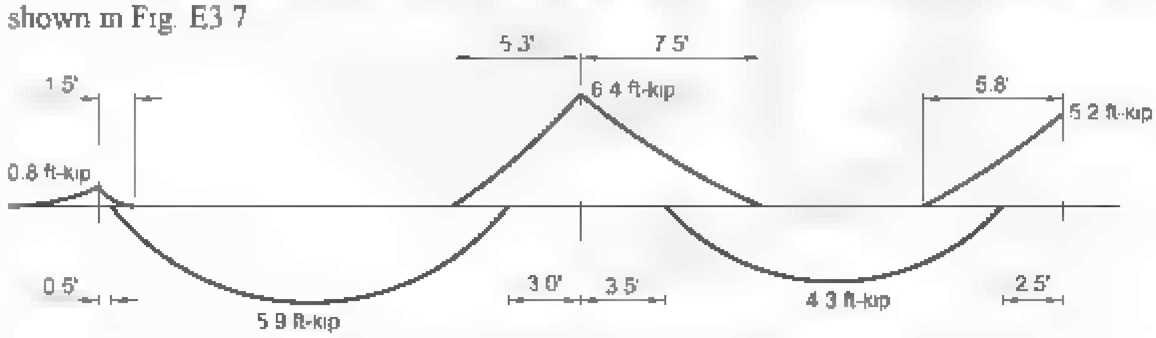

Fig. E3.7—Moment envelope including factored effect from gravity and secondary moments

Required strength (Gravity only)	Face of exterior support	Midspan AB	Face of support B	Midspan BC	Face of support C
	Location from left to right along the slab				
Factored moment at face, ft-kip	-0.8	5.5	7.2	3.6	5.7
M_2, ft-kip	0.0	0.4	0.8	0.65	0.5
M_u, ft-kip	-0.8	5.9	-6.4	4.3	5.2

Step 8, Nonprestressed bonded reinforcement for flexural strength		
7.7.4.2	If the PT tendons do not provide sufficient design strength, then it is more efficient to supplement this strength with bonded (nonprestressed) mild steel reinforcement rather than to add PT tendons. If required for strength, then bonded reinforcement must satisfy the detailing requirements of 7.7.3. If not required for strength, then minimum bonded reinforcement (7.6.2.3) must be detailed in accordance with 7.7.4.4. Compute the design moment strength considering tendons alone and compare to the required moment strength	
7.5.2.1 20.3.2.4	7.5.2.1 refers to 22.3 for the calculation of ϕM_n. Section 22.3 refers to 22.2 for calculation of M_n. Section 22.2.4 refers to 20.3.2.4 to calculate f_{ps}. The span-to-depth ratio is 240/6 = 40, so the following equation applies The reinforcing bar and tendons are usually at the same height at the support and at midspan	$f_{ps} = f_{se} + 10{,}000 + \dfrac{f_c'}{300\rho_p}$
20.3.2.4	Each single unbonded tendon is stressed to the value prescribed by the supplier. Friction losses cause a variation of f_{se} along the tendon length, but for design purposes, f_{se} is usually taken as the average value	The tendon supplier usually calculates f_{se}, and 175,000 psi is a common value. The force per strand is therefore 175,000 psi × 0.153 in.^{2} = 26,800 lb
	The required effective force per foot of slab is 10.8 kip/ft, so the spacing of tendons is	26.8 kip/10.8 kip × 12 in./ft = 29.7 in., or 2 ft-5 in.
	The value of A_{ps} is therefore, The value of ρ_p in $A_{ps}/(b \times d_p)$	A_{ps} = (0.153 in.^{2})(12 in./ft)/(29 in.) = 0.063 in.^{2}/ft ρ_p = 0.063 in.^{2}/ft / 60 in^2 = 0.0011 $f_{ps} = 175{,}000\text{ psi} + 10{,}000\text{ psi} + \dfrac{5000\text{ psi}}{0.294} = 202{,}000\text{ psi}$
20.3.2.4	f_{ps} limit as follows (a) f_{se} + 30,000 and (b) $f_{py} = 0.9f_{pu}$	(a) 175,000 psi + 30,000 psi = 205,000 psi and (b) (0.9)(270,000 psi) = 243,000 psi f_{ps} = 202,000 psi controls
	The compression block depth is therefore. $a = \dfrac{A_{ps}f_{ps}}{0.85f_c'(12\text{ in./ft})}$	$a = \dfrac{(0.063\text{ in.}^2)(202{,}000\text{ psi})}{(0.85)(5000\text{ psi})(12\text{ in./ft})} = 0.25\text{ in.}$
	Note that the effective depth is 5 in. at critical locations, except at the exterior joint where the tendon is positioned at slab middepth. At this location, d = 3 in. $\phi M_n = \phi A_{ps}f_{ps}(d - a/2)$	ϕM_n = (0.9)(0.063 in.^{2}/ft)(202,000 psi)(3 in. − 0.13 in.) = 2.74 ft-kip/ft

	Location from left to right along the slab				
	Face of exterior support	Midspan AB	Face of support B	Midspan BC	Face of support C
ϕM_n, only tendons, ft-kip	2.74	4.63	4.63	4.63	4.63
M_u, ft-kip	0.8	5.9	5.5	4.3	4.6

Because almost all the design moments are greater than ϕM_n when considering the tendons alone, reinforcement cutoff locations must be calculated.

Step 9: Minimum flexural reinforcement

7.6.2.3 The minimum area of flexural reinforcing bar per foot is a function of A_{ct}, which is the portion of the cross section that is in tension. The area is then the product of the unit width and distance between the tension face and centroid.

$A_{s,min} = 0.004A_{ct} = 0.004 \times 12 \text{ in.} \times 3 \text{ in.} = 0.15 \text{ in.}^2/\text{ft}$

Step 10: Design moment strength

7.5.1 The two common strength inequalities for one-way slabs, moment and shear, are noted in Code Section 7.5.1.1.

Because section must be tension-controlled, the strength reduction factor is 0.9.

7.5.2 Determine if supplying the minimum area of reinforcing bar is sufficient to achieve a design strength that exceeds the required strength.

Comparing this value with the required moment strength M_u indicates that the minimum reinforcement plus the tendons supply enough tensile reinforcement for slab to resist the factored loads at all locations.

22.3.2.1 In prestressed concrete members, mild steel reinforcement meeting 20.2.1 can be assumed to have reached yield stress.

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f_c'(12 \text{ in./ft})}$$

Set the section's concrete compressive strength equal to steel tensile strength, and rearrange for compression block depth a.

$$a = \frac{(0.063 \text{ in.}^2)(202{,}000 \text{ psi}) + (0.15 \text{ in.}^2)(60{,}000 \text{ psi})}{(0.85)(5{,}000 \text{ psi})(12)}$$

$a = 0.43$ in.

$$M_n = \phi\left[A_{ps}f_{ps} + A_s f_y\right]\left(d - \frac{a}{2}\right)$$

$$= 0.9[0.063 \times 202{,}000 + 0.15 \times 60{,}000](5 - 0.22)$$

$$= 7.7 \text{ ft-kip/ft}$$

Therefore, minimum reinforcement provides adequate strength. **OK**

Step 11: Shrinkage and temperature reinforcement		
7.6.4.2	To resist shrinkage and temperature stresses perpendicular to the slab span, it is typical to use tendons rather than mild reinforcement in one-way post-tensioned slabs. To calculate the number and spacing of temperature tendons, the Code allows the designer to consider the effect of beam tendons on the slab.	
24.4.4.1	Assuming that the beam is 12 in. x 30 in. and that it has an effective post-tensioning force of 189 kip, determine if the minimum prestress requirement is satisfied.	$A_{slab\ area}$ = (6 in.)(20 ft)(12 in./ft) + (12 in.)(24 in.) = 1726 in.^{2}
7.7.6.3	This amount is, therefore, sufficient to meet the Code minimum of 100 psi. The Code also has three spacing requirements which apply: Provide at least one tendon on each side of the beam. If temperature tendon spacing does not exceed 4.5 ft, additional reinforcing bar is not needed, but if temperature tendon spacing exceeds 4.5 ft, supplemental reinforcement is required along the edge of the slab adjacent to tendon anchors. Spacing above 6 ft is prohibited. In this example, temperature tendons, starting at 4 ft from the beam, are specified at 4 ft on center. No supplemental edge reinforcement is needed.	σ = (189,000 lb)/(1726 in.^{2}) = 109 psi
Step 12: Maximum spacing of deformed reinforcement		
7.7.2.3	The maximum spacing of deformed reinforcement in a PT slab is the lesser of 3h and 18 in.	The area of deformed reinforcement must be at least 0.15 in.^{2}/ft; use No. 4 bar at 16 in. on center, which also satisfies the maximum spacing requirement.

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Step 13: Design shear strength

Shear reinforcement is not typically used in one-way slabs so all of the shear strength is provided by the concrete contribution ($\phi V_n = \phi V_c$).

7.6.3.1 Minimum shear reinforcement is required when $V_u > \phi V_c$

22.5.5.1c Use appropriate equation from Table 22.5.5.1c

$$V_c = \left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d$$

21.2.1 Strength reduction factor for shear from Table 21.2.1b

$\phi = 0.75$

Effective depth to centroid of reinforcement

$d = 6\text{ in.} - 0.75\text{ in.} - 0.5 \times 0.5\text{ in.} = 5\text{ in.}$

Consider only bonded nonprestressed flexural reinforcement and use No. 4 at 16 in. spacing. Use largest anticipated spacing to ensure shear strength check will cover all reinforcement conditions.

$s = 16\text{ in.} = 1.33\text{ ft}$

$$A_s = \frac{0.31\text{ in.}^2}{s} = \frac{0.31\text{ in.}^2}{1.33\text{ ft}} = 0.233\text{ in.}^2 \text{ per unit width of slab}$$

Unit slab width of 12 in.

$b_w = 12\text{ in.}$

2.2 Reinforcement ratio of flexural reinforcement

$$\rho_w = \frac{A_s}{b_w d}$$

$$\rho_w = \frac{0.233\text{ in.}^2}{12\text{ in.}(5\text{ in.})} = 0.00388$$

Axial load is zero

$N_u = 0$

22.5.5.1.3 Size effect factor

$$\lambda_s = \sqrt{\frac{2}{1+0.1d}}$$

$$\lambda_s = \sqrt{\frac{2}{1+0.1(5)}} = 1.15 \quad \text{use } \lambda_s = 1.0$$

$\lambda = 1.0$

$$V_c = \left[8(1.0)(1.0)(0.00388)^{1/3}\sqrt{5000}\right](12)(5)\frac{1}{1000}$$

$V_c = 4.24$ kip/ft

$\phi V_c = 0.75(4.24) = 3.18 > V_u = 2.0$ kip/ft **OK**

Design shear strength from concrete contribution is more than the factored shear. Slab thickness is adequate.

Step 14. Top bar cutoff at exterior support		
7.7.4.4	At this location, mild steel reinforcement is not necessary to satisfy flexural strength requirements (7.7.4.2). Consequently, these bars can be detailed in accordance with 7.7.4.4.1a. Bar length of $\ell_n/6$ on each side of face of support. Extend bar to end of cantilever to avoid cutting off in potential tension zone and provide hook for development and good detailing practice. At other locations, mild steel reinforcement is required to satisfy flexural strength requirements. Therefore, cutoff requirements of 7.7.3 must be satisfied in these locations. <u>Balcony considerations</u> The architect usually specifies a slab recess of about 0.75 in. at the exterior wall of residential units to guard against water intrusion. In addition, the architect usually specifies a balcony slope of about 1/4 in./ft. These two details result in a slab thickness at the edge of 4.5 in. Balcony considerations are discussed in detail by Suprenant in "Understanding Balcony Drainage," *Concrete International*, Jan. 2004.	bar length left = (3 ft − 0.5 ft)/6 = 5 in. bar length right = (20 ft − 1 ft)/6 = 38 in. <u>Solution</u> The top bar length is 36 in. − 2 in. (balcony minus clear cover) plus 6 in. (half of the beam width) plus 38 in. = 78 in. Use a length of 6 ft 8 in. <u>Trim bar at the outside edge</u> The PT suppliers usually require two No. 4 continuous "back-up" bars behind the anchorages, about 2 to 3 in. from the edge. These bars can also limit widths of possible cracks due to unexpected restraint, drying shrinkage, or other local issues. At the edge of the balcony, it is recommended to hook the top flexure bars around the continuous edge bars.

Step 15: Bottom bar cutoff in exterior span		
	The inflection points for positive moments are 0.5 ft from the exterior support centerline and 3.0 ft from the first interior support centerline.	
7.7.3.3	Bar cutoffs. Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	
7.7.3.4	Continuing flexural tensile reinforcement must have an embedment length not less than ℓ_d beyond the point where bent or terminated tensile reinforcement is no longer required to resist flexure.	Note: the development length of a No. 4 black bar in an 6 inch slab with 0.75 in. cover is:
7.7.1.2 25.4.2.4	$\ell_d = \left(\dfrac{3}{40}\dfrac{f_y}{\lambda\sqrt{f'_c}}\dfrac{\psi_t\psi_e\psi_s}{\left(\dfrac{c_b+K_{tr}}{d_b}\right)}\right)d_b$	$\ell_d = \left(\dfrac{3}{40}\dfrac{60{,}000\text{ psi}}{1.0\sqrt{5000\text{ psi}}}\dfrac{1.0\times1.0\times0.8}{\left(\dfrac{1.0+0}{0.5}\right)}\right)0.5 = 13\text{ in.}$
7.7.3.5	Flexural tensile reinforcement must not be terminated in a tensile zone unless (a), (b), or (c) is satisfied.	$V_u < (2/3)\phi V_n$ at the cutoff point. $V_u = 2000$ lb and $\phi V_n = 3180$ lb (refer to Step 11) $(2/3)\phi V_n = 2120$ lb $> V_u = 2000$ lb therefore, **OK** Note that (b) and (c) do not apply
7.7.3.8.2	At least one-fourth the maximum positive moment reinforcement must extend along the slab bottom into the continuous support a minimum of 6 in.	Extend bottom reinforcement minimum 6 in. into the supports
7.7.3.8.3	At points of inflection, d_b for positive moment, tensile reinforcement must be limited such that ℓ_d for that reinforcement satisfies Code Eq. (7.7.3.8.3b). $\ell_d < M_n/V_u + \ell_a$	

7.7.3.3	Bars close to their maximum spacing limit may not be cut off within the tensile zones because the maximum reinforcing bar spacing would then be exceeded. All bottom bars must extend at least 6 in. beyond the positive moment inflection points	greater of d = 5 in. and $12d_b$ = 12(0.5 in.) = 6 in.
7.7.3.5	Because all bottom bars will be extended past the inflection points, 7.7.3.5 is not applicable	
7.7.3.8.2	The Code requires that at least 25 percent of bottom bars be full length, extending 6 in. into the support	
	Check bar size M_n is: $M_n = [A_s f_y]\left(d - \frac{a}{2}\right) + [A_{ps} f_{ps}]\left(d_p - \frac{a}{2}\right)$	M_n = (0.15 in.^{2}/ft)(60,000 psi)(5 in. – 0.22 in.) + [(0.063 in.^{2}/ft)(202,000 psi)(4.8 in. – 0.22 in.)] = 101 in.-kip/ft
	The elastic analysis indicates that V_u at inflection point is 2.0 kip/ft. The term ℓ_a is 6 in.	
	$\ell_d \leq M_n/V_u + \ell_a$	$\ell_d < \frac{M_n}{V_u} + 5\text{ in.} = \frac{101\text{ in.-kip}}{2.0\text{ kip}} + 5 = 54\text{ in.}$ > 13 in., therefore, No. 4 bar is **OK**. Because the cut off location is within a foot of the left support, extend all bottom bars through the left support and then to the end of the cantilever to support shrinkage and temperature bars in the balcony. For field placing simplicity, specify all bottom bars in this span also extend 6 in. into the right support
Step 16. Top bar cutoff at the first interior support		
	The inflection points for negative moment at the first interior support are 5.3 ft to the left and 7.5 ft to the right of support centerline	
7.7.3.3	Bar cutoffs All bars must extend beyond the inflection point at least d (5 in.) or 12 × 0.5 in.	Therefore, 6 in.
7.7.3.8.4	At least 1/3 of the bars must extend beyond the inflection point at least: However, the top bars at the exterior joint will extend from the end of the cantilever, past the support and into the span. Because the reinforcing bar is at wide spacing, no percentage of bars (as permitted by Section 7.7.3.8) can be cut off in the tension zone	d = 5 in. $12d_b$ = 6 in. $\ell_n/16$ = (19 ft × 12 in./ft)/16 = 14.25 use 15 in. Therefore, the top bar length is 5.3 ft (inflection) plus an extension of 15 in. plus 7.5 ft plus 15 in. A practical length for top bars is 16 ft

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Step 17: Bottom bar cutoff in interior span		
	The inflection points for positive moments are 3.5 ft from the left support centerline and 2.5 ft from the right interior support centerline.	
7.7.3.3	<u>Bar cutoffs</u> Bars close to their maximum spacing limit may not be cut off within the tensile zones because the maximum reinforcing bar spacing would then be exceeded. All bottom bars must extend at least 6 in. beyond the positive moment inflection points.	
7.7.3.5	Because all bottom bars will be extended past the inflection points, 7.7.3.5 is not applicable.	
7.7.3.8.2	The Code requires that at least 25 percent of bottom bars be full length, extending 6 in. into the support.	Minimum bottom bar length: 20 ft (span) − 3.5 ft (left infl.) − 2.5 ft (right infl.) + 2(6 in.)(extension beyond infl.) = 15 ft Extend every fourth bar into the support
Step 18: Top bar cutoff at middle support, C		
	The inflection points for negative moments are 5.8 ft from the support centerline on both sides.	
7.7.3.3	<u>Bar cutoffs</u> Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply supported spans and at free ends of cantilevers.	$d = 5$ in. $12d_b = 6$ in. use 6 in.
7.7.3.8.4	At least one-third the negative moment reinforcement at a support must have an embedment length beyond the point of inflection at least the greatest of d, $12d_b$, and $\ell_n/16$.	$d = 5$ in. $12d_b = 6$ in. $\ell_n/16 = (19\text{ ft} \times 12\text{ in./ft})/16 = 14.25$ use 15 in.
	Because the reinforcing bar is already at maximum spacing, no percentage of bars (as permitted by Code Section 7.7.3.5 or 7.7.3.8) can be cut off in the tension zone.	
		<u>Solution</u> The top bar length is two times (5.8 ft (inflection) plus an extension of 15 in. at each end). A practical length for top bars is 15 ft.

Step 19: Tendon anchorage		
	There are requirements both for anchorage zones (the reinforced concrete around the anchorage) and for the anchorages themselves.	
7.7.4.3.1	Post-tensioned anchorage zones must be designed and detailed in accordance with Code Section 25.9	The concrete around the anchorage is divided into a local zone and a general zone. For monostrand anchorages, the local zone reinforcement, according to Code, "shall meet the bearing resistance requirements of ACI 423.7." ACI 423.7 limits the bearing stresses an anchorage can impose on the concrete, unless the monostrand anchorage is tested to perform, as well as those meeting those stresses. All U.S. manufacturers test their anchorages to satisfy this requirement. For the general zone, Section 25.9.4.4.6a requires two "back up" bars for monostrand anchorages at the edge of the slab, and Section 25.9.3.2(b) is not applicable to this example.
7.7.4.3.2	Post-tensioning anchorages and couplers must be designed and detailed in accordance with 25.8	The information in Code Section 25.8 provides performance requirements for the design of PT anchorages. These only apply to the anchorage design, so the engineer rarely (if ever) is concerned about Section 25.7.
Step 20: Shrinkage and temperature tendons		
7.7.6.3	There are 4 temperature tendons evenly spaced per span (at 4 ft 0 in. O/C). Because the spacing is less than 4 ft 6 in., no additional edge reinforcement is required by the Code. The commentary recommends placing temperature tendons so that the resultant is within the kern of the slab (middle 2 inches). Anchors are usually attached to the outside forms at mid-height of the slab and longitudinally supported directly by the flexural tendons in such a manner as to meet this recommendation.	
Step 21: Slab integrity steel		
7.7.7	Provide structural integrity reinforcement in the slab to protect overall structural stability.	
4.10.2.1	Table 4.10.2.1 indicates that the use of integrity reinforcement is one-way prestressed slabs is not required.	Structural integrity is already provided in slab with the use of the continuous PT tendons.

Step 22. Detailing

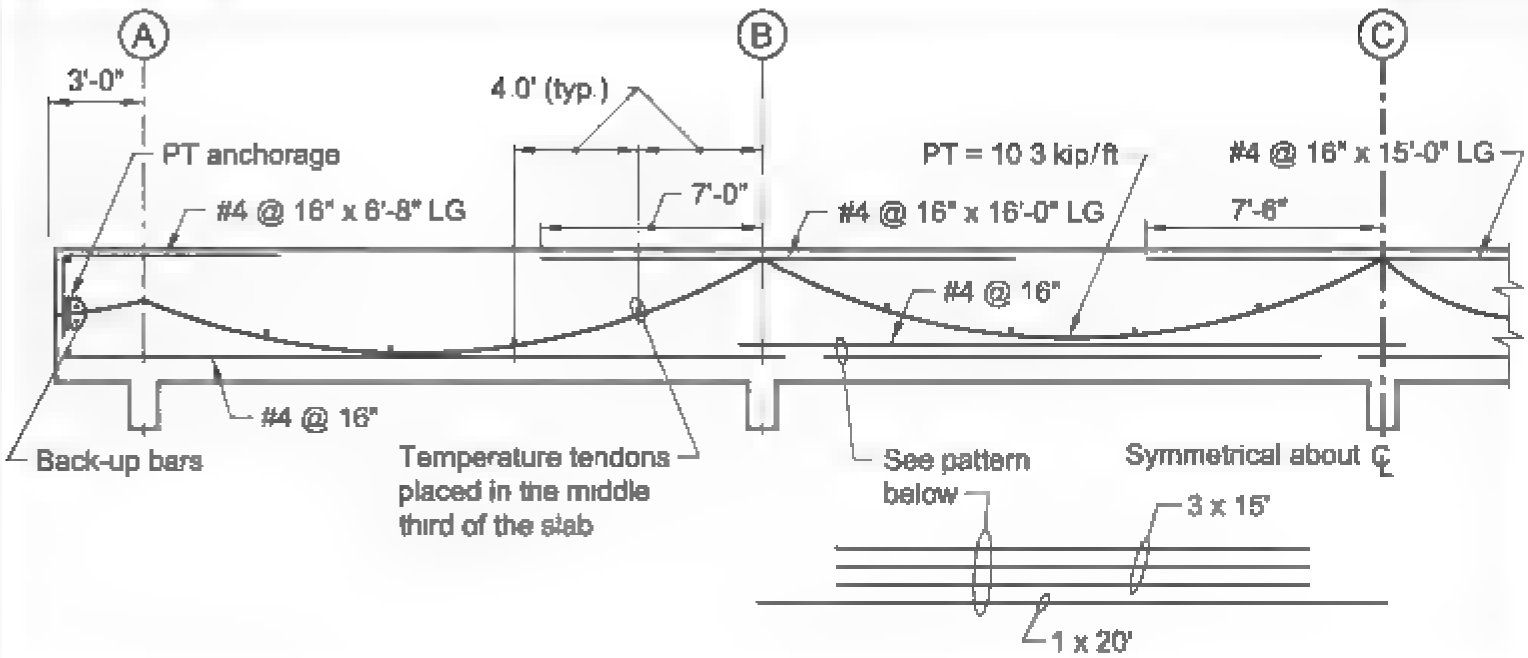

Fig. E3.8—Slab reinforcement.

CHAPTER 6—TWO-WAY SLABS

6.1—Introduction

Two-way slab systems are usually used in buildings with columns that are approximately evenly spaced, creating a span length in one direction that is within a factor of 2 in the perpendicular direction. Structural concrete two-way slabs, which have been constructed for over 100 years, have taken many forms. The basic premise for these forms is that the slab system transmits the applied loads directly to the supporting columns through internal flexural and shear resistance.

Two-way slabs are designed in accordance with Code Chapter 8 for strength and serviceability. This chapter covers cast-in-place concrete two-way slab systems with nonprestressed or prestressed reinforcement or both. While the Code allows for use of either bonded or unbonded tendons, the typical practice in the U.S. is to use unbonded prestressed reinforcement.

At the preliminary design stage, with spans given by the architect, the designer determines the loads, reinforcement type (prestressed or nonprestressed), and slab thickness. The preliminary concrete strength is based on experience and the Code's exposure and durability provisions.

6.2—Analysis

Code Section 8.2.1 allows the designer to use any analysis procedure that satisfies equilibrium and geometric compatibility, as long as design strength and serviceability requirements are met. This same section along with Code Section 6.2.4.1 allow the use of the Direct Design Method (DDM) or the Equivalent Frame Method (EFM) for gravity load analysis. These are well-established methods of analysis that have been in use for many years. Code editions from 1971 to 2014 contained detailed provisions for the use of these methods. The provisions were recently removed from the Code because they are only two of several methods that are available for the analysis of two-way slabs and their application is covered in available textbooks and other ACI guides such as ACI 421.3R. DDM and EFM are discussed briefly in this chapter of the Manual and are also used as part of the two-way slab examples to assist in demonstrating the design procedures.

The commentary notes that while the analysis of a slab system is important, the design results should not deviate far from common practice, unless it is justified based on the reliability of the calculations used in the analysis.

6.2.1 *Direct Design Method*—DDM is a simplified method of analysis that has several geometric and loading limitations. Nonprestressed reinforced flat plates, flat slabs, and waffle slabs can all be designed by this method. In previous versions of the Code, prestressed slabs were not permitted to be designed by DDM. The results of the DDM are the approximate magnitude and distribution of slab moments, both along the span and transverse to it. The coefficients that distribute the total static moment in the design panel to the column and middle strips are based on papers by Corley, Jirsa, Sozen, and Siess (Corley et al. 1961; Jirsa et al. 1963, 1969; Corley and Jirsa 1970). The total static moment is determined assuming that the reactions are along the faces of the support perpendicular to the span considered. Once the total static moment is determined, it is then distributed to negative and positive moment areas of the slab. From there, it is further distributed to the column strip and middle strips. The designer uses these moments to calculate the flexural reinforcement area in the direction being designed. The designer needs to perform calculations in both directions to determine two-way slab reinforcement. The DDM also provides the design shear at each column.

6.2.2 *Equivalent Frame Method*—EFM is more broadly applicable to a range of slab geometries than are appropriate for DDM. Flat plates, flat slabs, and waffle slabs as well as prestressed slabs can all be designed by this method. The EFM assumptions used to calculate the effective stiffness of the slab, torsional beams, and columns at each joint are based on papers by Corley, Jirsa, Sozen, and Siess (Corley et al. 1961; Jirsa et al. 1963, 1969; Corley and Jirsa 1970). EFM models a three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in the plane of the frames. The original analysis method used with EFM was the moment distribution method; however, any linear elastic analysis method is valid. The analysis calculates design moments and shears along the length of the model. For nonprestressed slabs, EFM uses DDM coefficients to distribute the total moments into column strips and middle strips. For prestressed slabs, the slab strip is from the middle of one bay to the middle of the next bay, and is designed in flexure as a wide, shallow beam.

6.2.3 *Finite Element Method*—A great variety of FEA computer software programs are available, including those that perform static, dynamic, elastic, and inelastic analysis. In general, nonuniform column placement or unusual two-way slab geometry can be accommodated. Finite element models could have beam-column elements that model structural framing members along with plane stress elements, plate elements, and shell elements, brick elements, or both, that are used to model the floor slabs, mat foundations, diaphragms, walls, and connections. The model mesh size selected should be capable of determining the structural response in sufficient detail. Any set of reasonable assumptions for member stiffness is allowed.

6.3—Service limits

6.3.1 *Minimum thickness*—For nonprestressed flat plates and flat slabs, the Code allows the designer to either calculate slab deflections or simply satisfy a minimum slab thickness (Code Section 8.3.1). Most flat slab and flat plate designs simply conform to the minimum thickness criteria and, therefore, designers do not usually calculate deflections for nonprestressed reinforced two-way slabs. For prestressed slabs, the Code does not provide a minimum span-to-depth

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ratio, but rather requires that both immediate and time-dependent deflections be calculated in accordance with Code Section 24.2 and checked against the limits in Code Section 24.2.2. For typical post-tensioned (PT) construction, however, span-to-depth ratios in the range of 37 to 45 will provide satisfactory structural performance.

6.3.2 *Deflections*— Deflections must be calculated for nonprestressed slabs less than minimum thickness or with long-to-short span ratios exceeding 2.0 and for all prestressed slabs. For slabs that resist a heavy live load or for waffle slabs, deflections should also be calculated. Deflections can be calculated by EFM, FEA, or classical methods. For EFM, the slab system is modeled in both directions, and the calculated deflection at midspan of a panel is the sum of the column strip deflection and the perpendicular middle strip deflection (refer to the crossing beam method [ACI 435R]).

The calculated deflections must not exceed the limits in Section 24.2 of the Code. For most buildings, the limit of $\ell/480$ for long-term deflections usually controls. This limit applies when nonstructural elements are likely to be damaged by large deflections and applies to the part of the deflection that occurs after attachment of the nonstructural elements.

Note that the bar spacing necessary to limit crack width, timing of form removal, concrete quality, timing of construction loads, and other construction variables all can affect the actual measured deflection. These variables should be considered when assessing the accuracy of deflection calculations. In addition, creep over time will increase the immediate deflections.

If PT slab span-to-thickness ratios are kept between 37 and 45, then calculated slab deflections are usually within the Code allowable limits. The Code limits the maximum service concrete tensile stress to below cracking stress, so deflection calculations use the gross slab properties.

6.3.3 *Concrete service stress*—Nonprestressed slabs are designed for strength but do not have limitations placed on concrete service flexural stress.

For prestressed slabs, the analysis of concrete flexural tension stresses is a critical part of the design. Code Section 8.3.4.1 requires that prestressed slabs be designed as Class U, which limits the net concrete flexural tensile stress to $6\sqrt{f_c'}$ and allows the use of gross section properties for deflection calculations. This generally results in small deflections that rarely control the design. At positive moment sections, Code Section 8.6.2.3 requires minimum bonded reinforcement if the concrete tensile stress exceeds $2\sqrt{f_c'}$. Bottom bonded reinforcement is often not required due to the low net tensile stresses typically found in prestressed slabs. In addition, Section 8.6.2.1 requires the average axial compressive stress in both directions due to post-tensioning to be at least 125 psi.

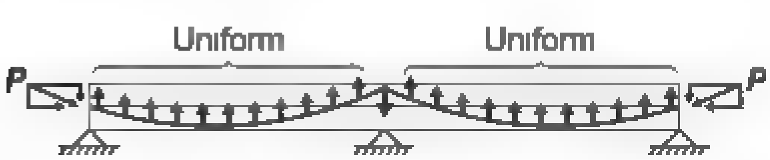

Fig. 6.3.3—Load balancing concept.

Before the slab flexural stresses in a design strip can be calculated, the tendon profile needs to be defined. The profile and the tendon force are directly related to the slab forces and moments created by the effective prestress force. A common approach to calculate slab moments is to use the "load balancing" concept, where the profile is usually the maximum practical considering cover requirements, the tendon profile is parabolic, the parabola has an angular "break" at the column centerlines, and that the tendon terminates at middepth of the slab edge (refer to Fig. 6.3.3).

The load balancing concept assumes the tendon exerts a uniform upward "load" along the parabolic length, and a point load down at the support. These loads are then combined with the gravity loads, and the analysis is performed with a net load. Figure 6.3.3 shows the commonly used simplification of the tendon profile. The real tendon profile is smooth with reverse parabolas over the interior supports rather than sharp bends. To conform to the Code stress limits, the designer can use an iterative approach or a direct approach. In the iterative approach, the tendon profile is defined and the tendon force is assumed. The analysis is executed, flexural stresses are calculated, and the designer then adjusts the profile or force or both, depending on results and design constraints.

6.4—Shear strength

Two-way slabs must have adequate one-way shear strength in each design strip (assuming the slab is a wide, shallow beam) and adequate two-way shear strength at each column. The discussion for the nominal one-way shear strength are the same as provided in Chapter 7 (Beams) of this Manual and is not reproduced herein.

6.4.1 *Punching shear strength*—Two-way shear strength, also called punching shear strength, is considered a critical strength for two-way slabs. Nominal punching shear strength is based on the slab's concrete strength, geometry, and shear reinforcement when provided. The effect of the slab's flexural reinforcement on punching shear strength is ignored.

6.4.2 *Critical section*—The punching shear failure shape (Fig. 6.4.1) is usually a truncated cone or pyramid-shape surface around the column. To determine the punching shear strength, the Code defines simplified critical section as a vertical section extending through the slab at a distance $d/2$ from the face of the column, where d is the slab's effective depth. In Fig. 6.4.2, the perimeter of the critical section is $b_o = 2[(c_1 + d) + (c_2 + d)]$. The critical section to calculate concrete shear stress is then $b_o d$.

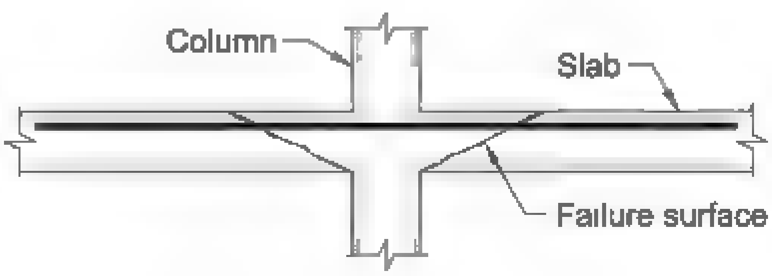

Fig. 6.4.1—Punching shear failure.

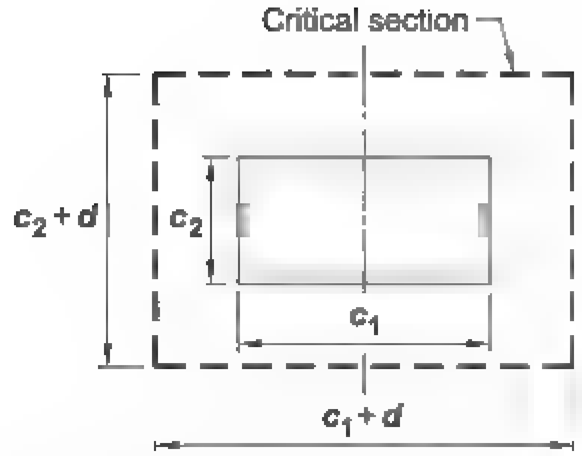

Fig. 6.4.2—Critical section geometry.

Table 6.4.3—Calculation of v_c for two-way shear (Code Table 22.6.5.2)

	v_c
Least of:	$4\lambda_s\lambda\sqrt{f_c'}$
	$\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'}$
	$\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'}$

Note: β is the ratio of long side to short side of the column and α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns. λ_s is the size effect factor given in Code Section 22.5.5.1.3.

6.4.3 *Calculation of nominal shear strength*—Punching shear strength limits are defined in terms of stress. As shown in Table 6.4.3, the shear stress limit for a nonprestressed reinforced slab is the least of the three expressions. The size effect factor is calculated using the following equation

$$\lambda_s = \sqrt{\frac{2}{1+\frac{d}{10}}} \leq 1$$

where d is the effective depth of the reinforcement (in.). Shear strength of slabs less than approximately 12 in. are not affected by this factor. For deeper slabs, however, the shear strength is reduced to account for the size effect in shear strength for deep members that do not have shear reinforcement.

The Code punching shear strength limit for prestressed slabs is usually slightly higher than that of nonprestressed reinforced slabs. For prestressed, two-way members, v_c is permitted to be the lesser of (a) and (b) (Code Eq. (22.6.5.5a) and (22.6.5.5b))

$$v_c = \left(3.5\lambda\sqrt{f_c'} + 0.3f_{pc}\right) + \frac{V_p}{b_o d} \quad (22.6.5.5a)$$

$$v_c = \left(1.5 + \frac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'} + 0.3f_{pc} + \frac{V_p}{b_o d} \quad (22.6.5.5b)$$

where α_s is the same as in Table 6.4.3; the value of f_{pc} is the average of f_{pc} in the two directions, limited to 500 psi, V_p is the vertical component of the effective prestress force crossing the critical section, and the value of $\sqrt{f_c'}$ is limited to 70 psi. For prestressed two-way slabs, the designer can use Eq. (22.6.5.5a) and (22.6.5.5b) unless the column is closer to a discontinuous edge than four times the slab thickness h. For many edge columns, this requires shear strength to be calculated using the equations in Table 6.4.3. Because of the shallow depth of most prestressed slabs, many engineers conservatively ignore the $V_p/b_o d$ component when calculating v_c.

The Code also requires the engineer to consider the effect of slab openings close to columns. Such openings, which are commonly used for heating, ventilating, and air conditioning (HVAC), and plumbing chases, will reduce the shear strength. The Code requires a portion of b_o enclosed by straight lines projecting from the centroid of the column and tangent to the boundaries of the opening to be ignored when calculating the area of the critical section that contributes to shear strength (Code Section 22.6.4.3).

6.5—Calculation of required shear strength

The factored punching shear stress at a column, v_u, is the total of two components: 1) direct shear stress, v_{uv}, and 2) shear stresses due to moments transferred from the slab to the column. The two stress diagrams are added and the total is the required shear stress diagram at the critical section.

Direct shear stress v_{uv} is calculated by $v_{uv} = V_u/b_o d$. To calculate the shear stresses due to slab bending, the Code first stipulates that a percentage of the unbalanced slab moment at the column, M_{sc}, should be resisted by slab flexure within a limited width over the column. The remaining percentage of M_{sc} is assumed to be transferred to the slab by eccentricity of shear. The following two sections of Code Chapter 8 state that:

8.4.2.2.2 The fraction of factored slab moment resisted by the column, $\gamma_f M_{sc}$, shall be assumed to be transferred by flexure, where γ_f shall be calculated by

$$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

8.4.4.2.2 The fraction of M_{sc} transferred by eccentricity of shear, $\gamma_v M_{sc}$, shall be applied at the centroid of the critical section in accordance with Section 8.4.4.1, where

$$\gamma_v = 1 - \gamma_f$$

Under the conditions given in Table 8.4.2.2.4 of the Code, the value of γ_f can be increased, which then decreases the fraction of M_{sc} required to be transferred by eccentricity of shear. These modified values do not apply to prestressed slabs.

The slab shear stresses due to the unbalanced moment transferred to the column by eccentricity of shear is calculated by $\gamma_v M_{sc} c/J_c$, where c is the distance from b_o to the crit-

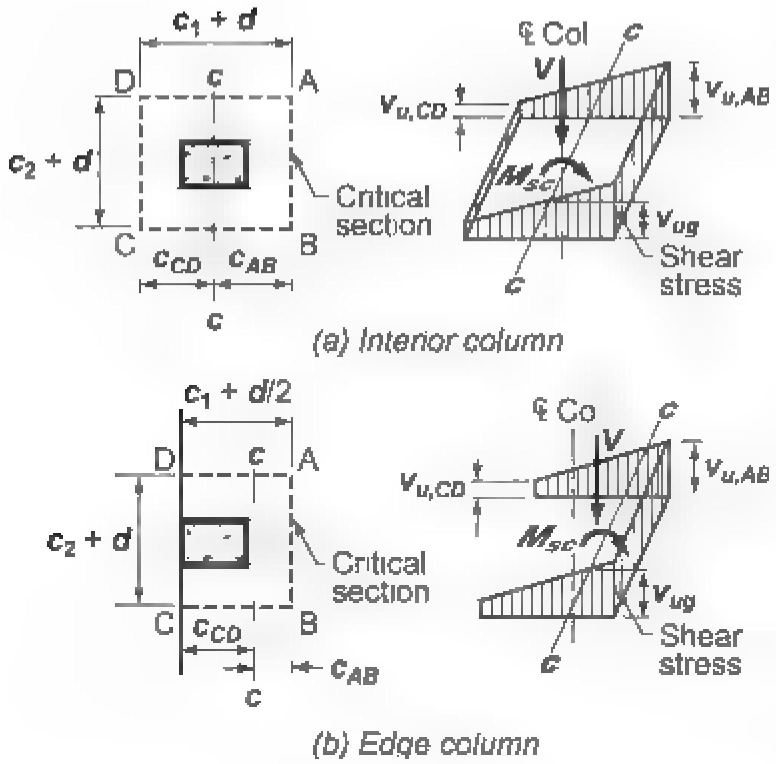

Fig. 6.5—Assumed distribution of shear stress (Code Section R8.4.4.2.3).

ical section centroid, and J_c is the polar moment of inertia of the critical section about its centroidal axis. The combination of this flexural term with the shear term v_{uv} produces the factored shear stress diagram shown in Fig. 6.5. If the maximum factored shear stress from this combination, $v_{u,AB}$, does not exceed the design shear strength in terms of stress, ϕv_n, then the slab's concrete shear strength is adequate. If the maximum total factored shear stress exceeds the design shear stress, the slab thickness near the column can be increased by using, for example, shear capitals (Code Section 8.2.5) or shear reinforcement can be added.

At times, the design of a two-way slab requires point loads to be considered, such as wheel loads in parking garages.

These result in local shear slab stresses, and the slab's punching shear strength in that area needs to be verified.

6.6—Design of shear reinforcement

Shear reinforcement can be provided to increase the slab's nominal shear strength close to a column. Assuming the shear reinforcement is uniformly spaced, shear strength is first checked at the first critical section at $d/2$ beyond the column face including the contribution of shear reinforcement. The shear strength is then checked at $d/2$ beyond the outermost peripheral line of shear reinforcement, without the contribution of shear reinforcement. In slabs without shear reinforcement, v_c is usually $4\sqrt{f'_c}$; however, for slab sections with shear reinforcement, the concrete contribution to shear strength is limited to the values in Table 6.6a.

There is an upper limit to a slab's nominal shear strength even with shear reinforcement, as shown in Table 6.6b. The Code states this limit in terms of the maximum factored two-way shear stress, v_u, calculated at a critical section.

Note that the use of stirrups as slab shear reinforcement is limited to slabs with an effective depth d that satisfy (a) and (b):

Table 6.6a—Maximum v_c for two-way members with shear reinforcement (Code Table 22.6.6.1)

Type of shear reinforcement	Critical sections	v_c		
Stirrups	All		$2\lambda_s\lambda\sqrt{f'_c}$	(a)
Headed shear stud reinforcement	According to 22.6.4.1	Least of (b), (c), and (d):	$3\lambda_s\lambda\sqrt{f'_c}$	(b)
			$\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c}$	(c)
			$\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c}$	(d)
	According to 22.6.4.2		$2\lambda_s\lambda\sqrt{f'_c}$	(e)

Notes: λ_s is the size effect factor given in Code Section 22.5.5.1.3. β is the ratio of long to short sides of the column, concentrated load, or reaction. α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns.

Table 6.6b—Maximum v_u for two-way members with shear reinforcement (Code Table 22.6.6.3)

Type of shear reinforcement	Maximum v_u at critical sections defined in 22.6.4.1
Stirrups	$\phi 6\sqrt{f'_c}$
Headed shear stud reinforcement	$\phi 8\sqrt{f'_c}$

(a) d is at least 6 in.

(b) d is at least $16d_b$, where d_b is the diameter of the stirrups

The use of shear studs is not limited by the slab thickness, but the studs must fit within the geometric envelope. The overall height of the shear stud assembly needs to be at least the thickness of the slab minus the sum of (a) through (c):

(a) Concrete cover on the top flexural reinforcement

(b) Concrete cover on the base rail

(c) One-half the bar diameter of the flexural tension reinforcement

6.7—Flexural strength

After the designer calculates the factored slab moments, the required area of flexural reinforcement over the slab width is calculated with the same behavior assumptions as a beam.

6.7.1 *Calculation of required moment strength*—There are two calculations for required moment strength for two-way slabs. The first calculation is to determine factored moments over the entire panel in the positive and negative moment areas. For nonprestressed reinforced slabs, the slab analysis should provide the distribution of panel factored moments to the column strip and middle strip.

For prestressed slabs, effects of reactions induced by prestressing (secondary moments) should be included. The slab's secondary moments are a result of the column's vertical restraint of the slab against the effective prestress force at each support. Because the prestress force and drape are determined during the service stress checks, secondary moments can be quickly calculated by the load-balancing analysis.

A simple way to calculate the secondary moment is to subtract the tendon force times the tendon eccentricity (distance from the NA) from the total balance moment, expressed mathematically as $M_2 = M_{bal} - P \times e$

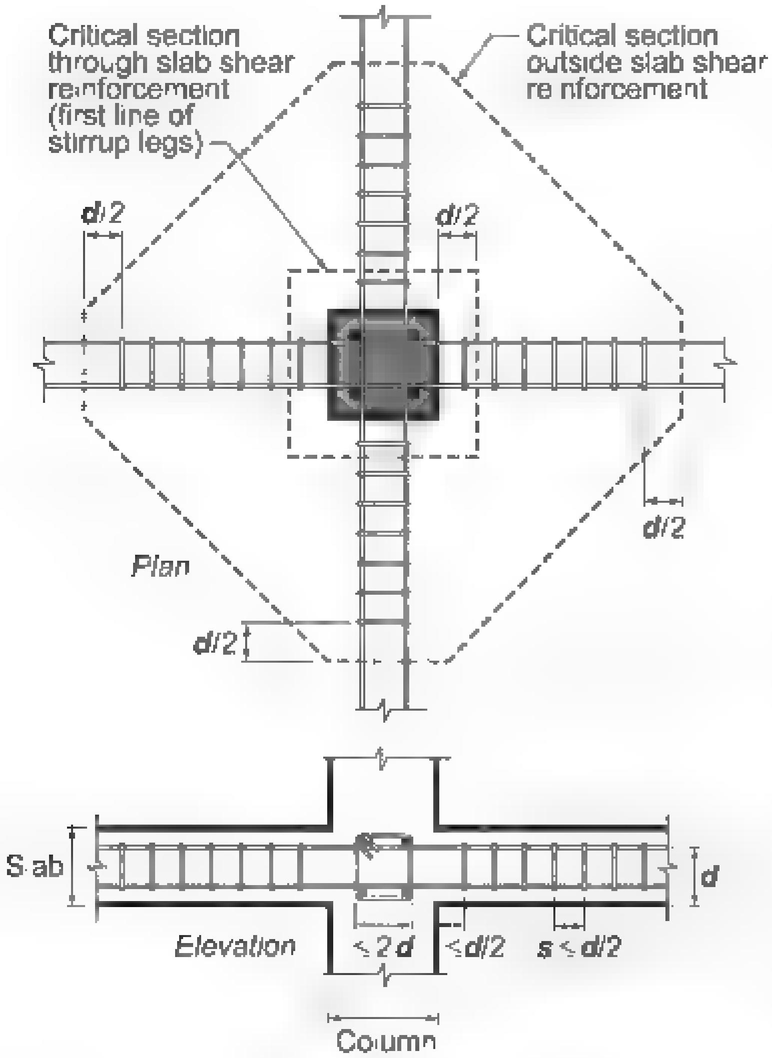

Fig. 6.8.1—Arrangement of stirrup shear reinforcement, interior column (Code Fig. R8.7.6d).

Table 6.8.1—First stirrup location and spacing limits (Code Table 8.7.6.3)

Direction of measurement	Description of measurement	Maximum distance or spacing, in.
Perpendicular to column face	Distance from column face to first stirrup	$d/2$
	Spacing between stirrups	$d/2$
Parallel to column face	Spacing between vertical legs of stirrups	$2d$

The second calculation is to determine $\gamma_f M_{sc}$ at each slab-column joint. The value of M_{sc} is the difference between the design moments on either side of the column

6.7.2 *Calculation of design moment strength*—In a nonprestressed slab, the required reinforcement area A_s resisting the column and middle strip's negative and positive M_u is usually placed uniformly across each strip. The required reinforcement area A_s resisting $\gamma_f M_{sc}$ must be placed within a width b_{slab}

For PT slabs, the tendons are usually placed in a banded pattern, in which all tendons in one direction are gathered together in a band that follows the column lines. In the orthogonal direction, the tendons are uniformly distributed over the width of the slab. The slab flexural strength calculations for tendons (with f_{ps} determined from Section 20.3.2.4 of the Code and substituted for f_y in the M_n equation) in the banded direction and in the uniform direction are the same, regardless of the tendon's horizontal location within the slab

For prestressed slabs, the reinforcement area A_s resisting the panel's negative M_u is usually placed only in the area surrounding the column. The sum of A_s and A_{ps} resisting $\gamma_f M_{sc}$ must be placed within the width b_{slab} per Code Section 8.4.2.2.3. If the panel reinforcement already within b_{slab} is not sufficient, designers usually add A_s to increase the flexural strength

For PT slabs, the A_{ps} provided to limit concrete service tensile stresses will usually be sufficient to also resist the panel's positive M_u

6.8—Shear reinforcement detailing

6.8.1 *Stirrups*—If stirrups are provided to increase shear strength, the Code provides limits on their location and spacing in Table 6.8.1

The related Code Fig. R8.7.6d as shown in Fig. 6.8.1 of this Manual shows the two critical section locations.

6.8.2 *Shear studs*—If shear studs are provided to increase shear strength, the Code provides limits on shear stud locations and spacing in Table 6.8.2

The related Code Fig. R8.7.7 as shown in Fig. 6.8.2 shows the two critical section locations.

6.9—Flexure reinforcement detailing

6.9.1 *Nonprestressed reinforced slab reinforcement area and placing*—Code Section 8.6.1.1 requires a minimum area of flexural reinforcement $A_{s,min}$ of $0.0018A_g$. If more than the

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Table 6.8.2—Shear stud location and spacing limits (Code Table 8.7.7.1.2)

Direction of measurement	Description of measurement	Condition		Maximum distance or spacing, in.
Perpendicular to column face	Distance from column face to first peripheral line of shear studs	All		$d/2$
	Constant spacing between peripheral lines of shear studs	Nonprestressed slab with	$v_u \le \phi 6\sqrt{f_c'}$	$3d/4$
		Nonprestressed slab with	$v_u > \phi 6\sqrt{f_c'}$	$d/2$
		Prestressed slabs conforming to Code Section 22.6.5.4		$3d/4$
Parallel to column face	Spacing between adjacent shear studs on peripheral line nearest to column face	All		$2d$

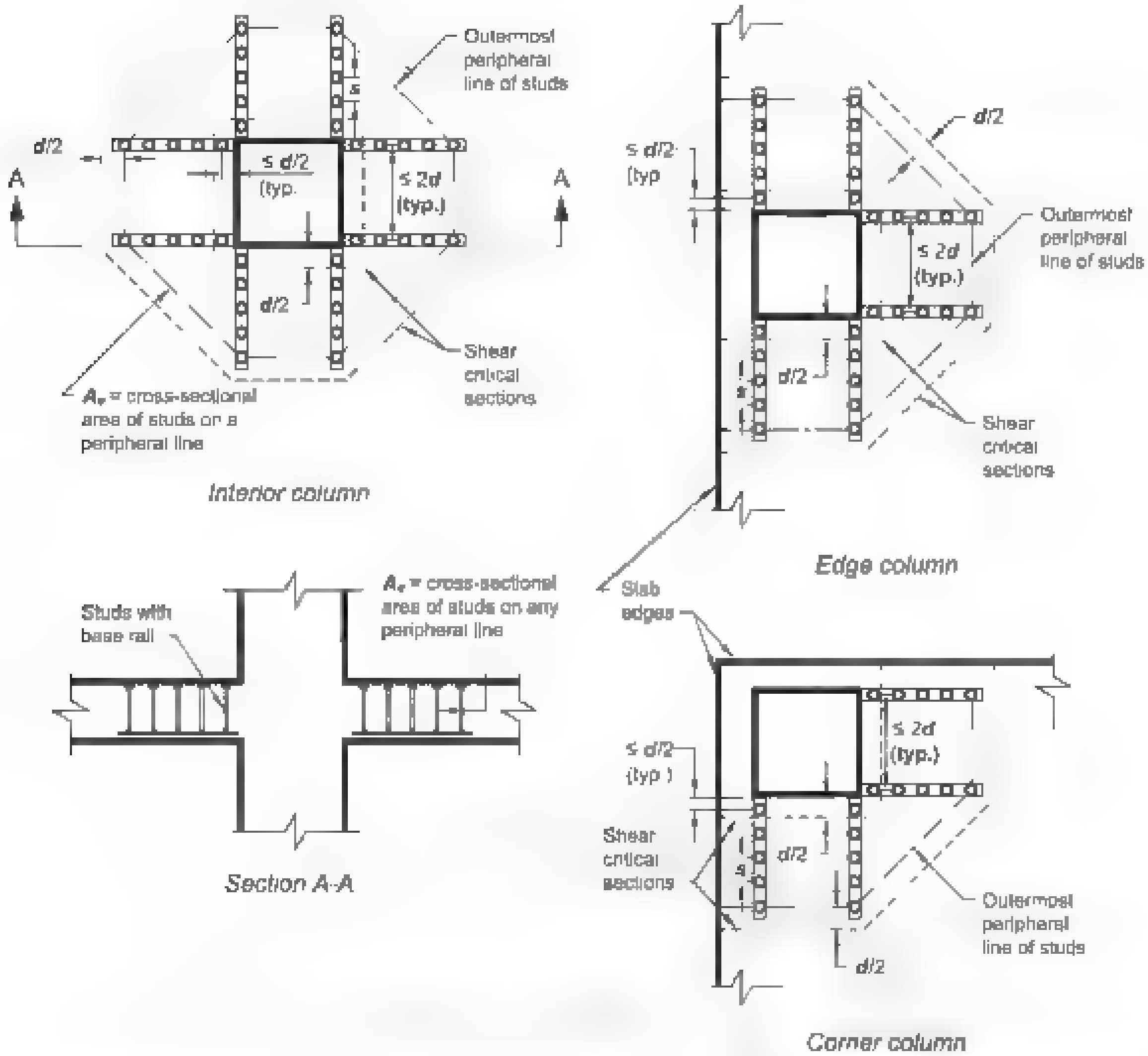

Fig. 6.8.2—Typical arrangements of headed shear stud reinforcement and critical sections (Code Fig. R8.7.7).

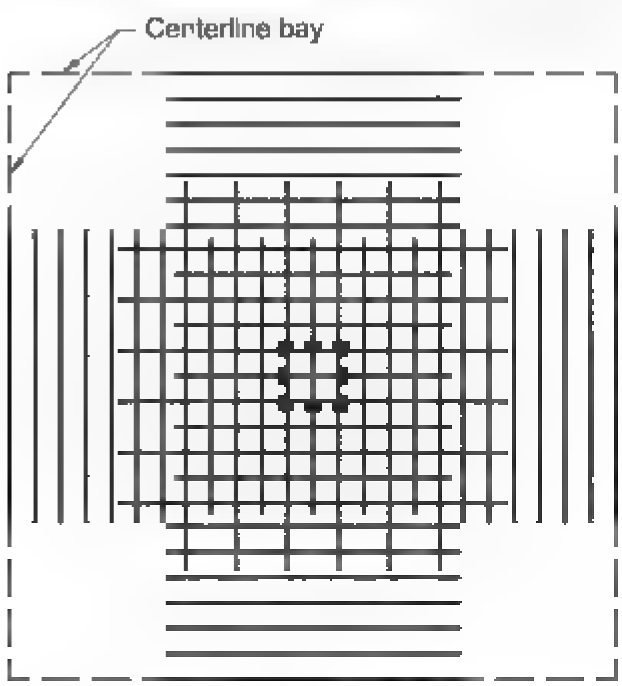

Fig. 6.9.1—Arrangement of minimum reinforcement near the top of a two-way slab (Code Fig. R8.6.1.1).

minimum area is required by analysis, that reinforcement area must be provided. If $v_{uv} > \phi 2\lambda_s\lambda\sqrt{f'_c}$, then the possibility of a flexure-driven punching failure increases if the following $A_{s,min}$ is not satisfied

$$A_{s,min} = \frac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y}$$

This limit was developed for an interior column, such that the factored shear force on the critical section for shear equals the shear force associated with local yielding at the column faces.

Two-way slab flexural reinforcement is placed in top and bottom layers. For nonprestressed reinforced two-way slabs without beams, Fig. 6.9.1 (Code Fig. R8.6.1.1) provides a typical layout of column strip and middle strip top bars. Code Fig. 8.7.4.1.3(a) provides the minimum reinforcing bar extensions, lap locations, and the minimum A_s at various sections. If the panel geometry is rectangular rather than square, the outer layer is usually placed parallel to the longer span.

6.9.2 *Corners*—Corner restraint, created by walls or stiff beams, induces slab moments in the diagonal direction and perpendicular to the diagonal. These moments are in addition to the calculated flexural moments. Additional reinforcement per Code Section 8.7.3 is required for this condition.

6.9.3 *Post-tensioned slab – Bonded reinforcement area and placing*—Over each column region, the Code requires an area of flexural reinforcing bar of at least $0.0075A_{cf}$ in each direction, placed within 1.5h of the outside of the column. The Code also requires bonded reinforcement in positive moment areas if the calculated service tensile flex

ural stress in other areas (usually midspan at the bottom) exceeds $2\sqrt{f_c'}$ or if required for strength. Bottom bar placement is at the discretion of the designer.

The Code allows for reduced top and bottom minimum bonded reinforcement lengths if the PT tendons provide all of the required design strength. The top bars must extend at least $\ell/6$ on each side of the column. The bottom bars (if needed) must be at least $\ell/3$ and be centered at the maximum moment. The shorter lengths often control under typical spans and loadings. If the sectional strength using only the area of PT tendons is insufficient to satisfy design strength, then the minimum top and bottom bar lengths are the same as that of a nonprestressed reinforced slab.

6.9.4 *Post-tensioned slab: Tendon area and placing*—A minimum of 125 psi axial compression in each direction is required in a PT two-way slab. PT tendons are usually placed in two orthogonal directions. In this configuration, the Code allows banding of tendons in one direction and in the other direction the tendon spacing is uniform across the design panel, within the spacing limits of 8*h* and 5 ft. This layout is predominant in the United States. The Code also requires at least two tendons to be placed within the column reinforcement cage in either direction for overall building integrity.

6.9.5 *Slab openings*—For relatively small slab openings, trim reinforcing bar usually limits crack widths that can be caused by geometric stress concentrations and provides adequate strength. For larger openings, a local increase in slab thickness as well as additional reinforcement may be necessary to provide adequate serviceability and strength.

REFERENCES

American Concrete Institute (ACI)

ACI 435R-95(00)—Control of Deflection in Concrete Structures

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Jirsa, J. O., Sozen, M. A., and Siess, C. P., 1969, "Pattern Loadings on Reinforced Concrete Floor Slabs," *Proceedings*, ASCE, V. 95, No. ST6, June, pp. 1117-1137.

Two-Way Slabs

6.10—Examples

Two-way Slab Example 1: *Two-way slab design using direct design method (DDM)—Interior Frame*

This two-way slab is nonprestressed without interior beams between supports. This example designs the interior strip along grid line B. Material properties are selected based on the code requirements of Chapters 5 and 6, engineering judgment, and locally available materials. Lateral loads are resisted by shear walls, therefore, the design is for gravity loads only. Diaphragm design is not considered in this example.

Given:

Uniform loads—

Self-weight dead load is based on concrete density including reinforcement at 150 lb/ft^3

Superimposed dead load D = 0.015 kip/ft^2

Live load L = 0.100 kip/ft^2

Material properties—

f_c' = 5000 psi

f_y = 60,000 psi

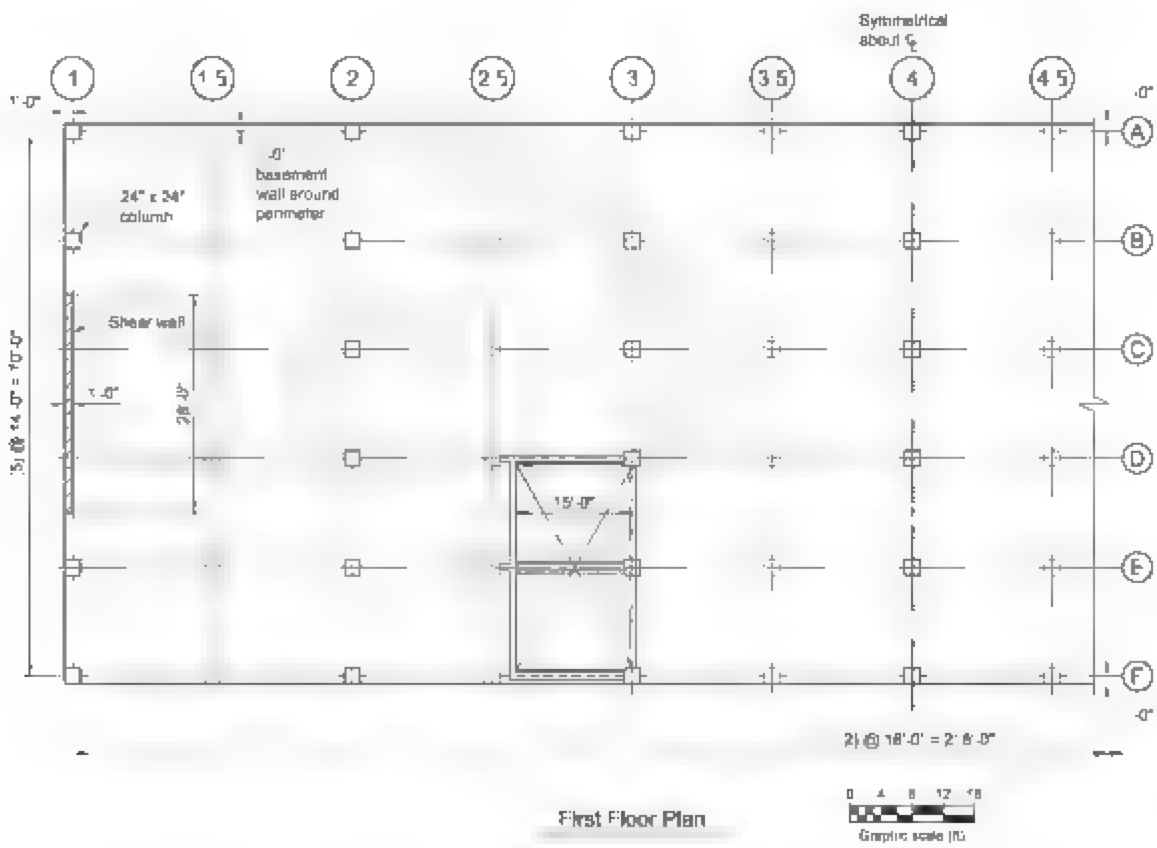

Fig. E1.1—First floor plan

ACI 318	Discussion	Calculation
Step 1 Geometry		
8.2.1	This slab is designed using the Direct Design Method (DDM), which is detailed in Section 8.10 of ACI 318-14. DDM details have been removed from the Code, but is still permitted to be used according to Section 8.2.1. Check the limitations of DDM from ACI 318-14.	There are at least three continuous spans in each direction so Section 8.10.2.1 is satisfied.
	The slab geometry satisfies the limits of Sections 8.10.2.1 through 8.10.2.4, which allows the use of DDM.	The successive spans are the same lengths so Section 8.10.2.2 is satisfied. The ratio of the longer to the shorter panel dimension is 1.29 so Section 8.10.2.3 is satisfied. Columns are not offset through the slab so Section 8.10.2.4 is satisfied.
	The uniform design loads satisfy the limits of Sections 8.10.2.5 and 8.10.2.6 to allow use of DDM.	All design loads are distributed uniformly and due to gravity only so Section 8.10.2.5 is satisfied. Ratio of unfactored live load to unfactored dead load is approximately 100/102.5 = 0.98. This ratio is less than 2, so Section 8.10.2.6 is satisfied. There are no supporting beams so Section 8.10.2.7 is not applicable. This example does not include drop panels or shear caps so Sections 8.2.4 and 8.2.5 are not applicable.
8.3.1.1	Check the slab thickness for deflection control.	Using Table 8.3.1.1 with f_y = 60,000 psi, without drop panels, and assuming the wall performs as a stiff edge beam, the minimum thickness for the exterior panel is $\frac{\ell_n}{33} = \frac{192 \text{ in.}}{33} = 5.8 \text{ in.}$ The minimum thickness for the interior panels is calculated using the same table and the result is the same as that for the exterior panel. The remainder of the building is using a slab thickness of 7 in., therefore, use 7 in. The slightly thicker than necessary slab aids with both deflections and shear strength.
8.3.1.3	No concrete floor finish is placed monolithically with the slab or composite with the floor slab.	
8.3.2	Calculated deflections are not required because the slab thickness-to-span ratio satisfies Section 8.3.2.1.	

Step 2. Load and load patterns		
8.4.1.1	The load factors are provided in Table 5.3.1.	The load combination that controls is $1.2D + 1.6L$. Because Section 8.10.2.6 (ACI 318-14) is satisfied in Step 1, pattern loading is already considered in the DDM moment coefficients
Step 3. Initial two-way shear check		
22.6.5.1	Before performing detailed calculations, it is often beneficial to perform an approximate punching shear check. This check should reduce the probability of having to repeat the calculations shown in this example. This check uses the following limits on the ratio of the design shear strength to the effects of shear stress based on direct shear stress alone ($\phi v_n/v_{uv}$) For interior columns $\phi v_n/v_{uv} > 1.2$ For edge columns $\phi v_n/v_{uv} > 1.6$ For corner columns $\phi v_n/v_{uv} > 2.0$ If these ratios are not exceeded, it is possible that the slab will not satisfy two-way shear strength requirements. The design slab could be thickened, drop panels added, or other options for adding two-way shear strength may be considered	For ϕv_n, the calculations here are discussed in Step 10 more fully $v_n = 4\sqrt{f_c'} = \frac{4\sqrt{5000}}{1000}\text{ ksi} = 0.283\text{ ksi}$ $v_n = \left(2 + \frac{4}{\beta}\right)\sqrt{f_c'} = \frac{6\sqrt{5000}}{1000}\text{ ksi} = 0.424\text{ ksi}$ $v_n = \left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = \frac{3.89\sqrt{5000}}{1000}\text{ ksi} = 0.275\text{ ksi*}$ *controls $\phi v_n = 0.75 \times 0.275\text{ ksi} = 0.206\text{ ksi}$ For v_{uv}, the calculations here are discussed in Step 7 more fully: $v_{uv} = \frac{V_u}{b_o d}$ $V_u = \left(14\text{ ft} \times 18\text{ ft} - \frac{29.6\text{ in.} \times 29.6\text{ in.}}{144}\right) \times \frac{283}{1000}\frac{\text{kip}}{\text{ft}^2}$ $V_u = 70\text{ kip}$ $v_{uv} = \frac{70\text{ kip}}{118.4\text{ in.} \times 5.6\text{ in.}} = 0.106\text{ ksi}$ $\phi v_n/v_{uv} = 0.206/0.106 = 1.94 > 1.2$ ∴ proceed Note that due to the basement wall supporting the exterior perimeter of the slab, the punching shear for the edge and corner columns will not need to be checked in this example.
Step 4: Analysis—Direct design method moment		
ACI 318.14 8.4.1.3 8.4.1.4 8.4.1.5 8.4.1.6 8.10.3.1	The geometry of the design is shown in Fig. E1.2	The design strip is bounded by the panel center line on each side of the column line and consists of a column strip and two half middle strips.

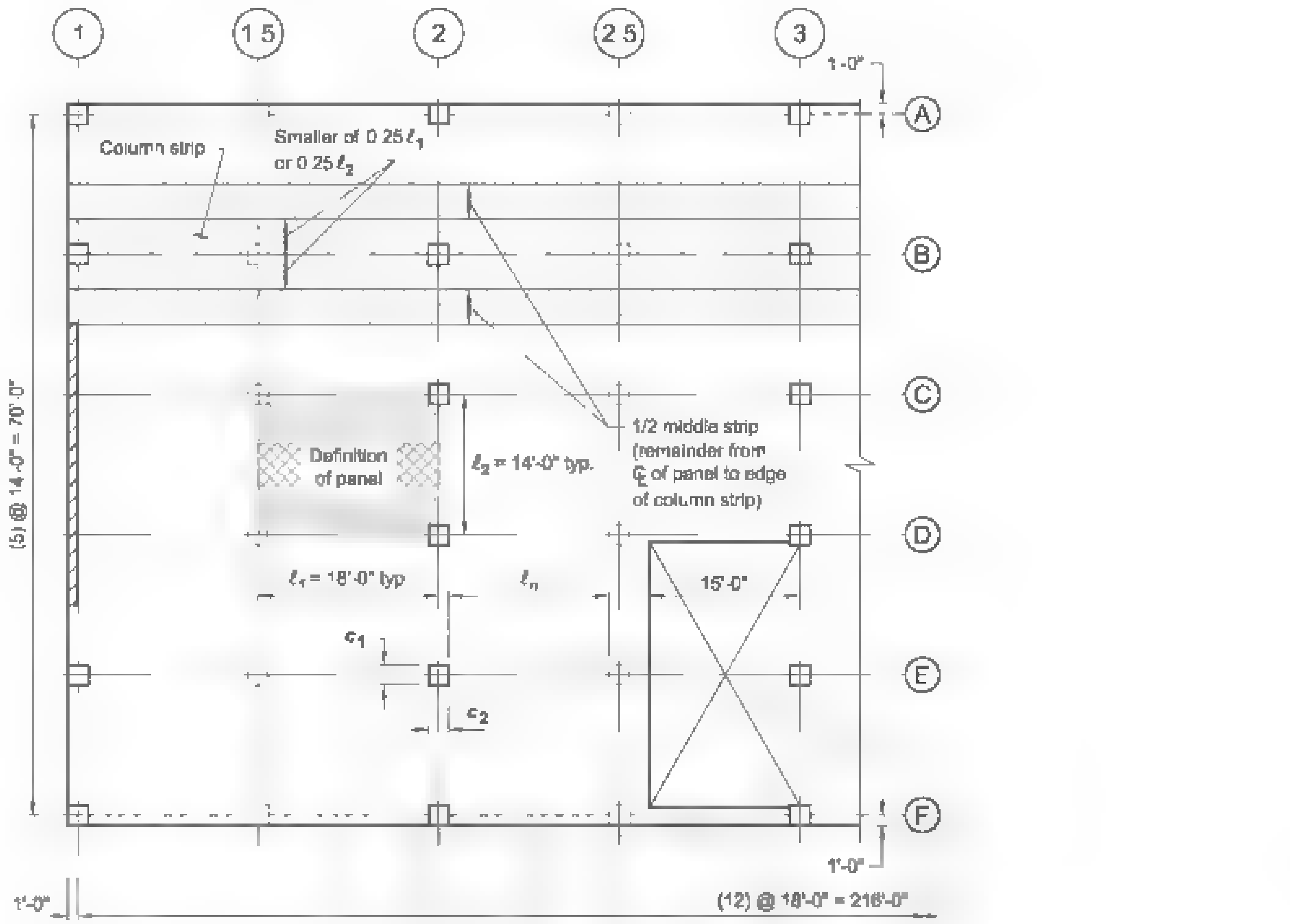

Fig. E1.2—Geometry definitions of panels and strips.

8.4.1.9	The lateral loads in this building are assumed to be resisted by shear walls with the slabs only acting as diaphragms between the shear walls. The slab is assumed not to contribute flexural resistance to lateral loads.	Because all lateral loads are assumed to be resisted by shear walls, the slab's flexural analysis is not combined with the lateral load analysis.
ACI 318-14 8.10.1, 8.10.2	The slab is eligible for design by DDM as shown in Step 1.	
ACI 318-14 8.10.3.1, 8.10.3.2.1, 8.10.3.2.2, 8.10.3.2.3	Refer to Fig. E1.2 for slab span lengths in both directions and column dimensions in plan.	
ACI 318-14 8.10.3.2	The DDM calculates a total panel M_o and then uses coefficients to determine maximum positive and negative design moments. In this example, all spans have ℓ_n = 16 ft. If spans vary, M_o must be calculated for each span length.	$M_o = \dfrac{q_u \ell_2 \ell_n^2}{8}$ Long span ℓ_n = 16 ft ℓ_2 = 14 ft $q_u = 1.2 \times (DL_{super} + DL_{slab,sw}) + 1.6 \times LL$ = 283 psf $M_o = \dfrac{q_u \ell_2 \ell_n^2}{8}$ = 127 ft-kip
ACI 318-14 8.10.4	Distribute M_o in the end span, from A to B.	

ACI 318-14 8.10.4.1 8.10.4.2 8.10.4.3 8.10.4.4 8.10.4.5	Table 8.10.4.2 gives the M_u distribution coefficients for the slab panel. In Table 8.10.4.2, this example uses the fully restrained column of the table. The reason for this is that the combined member of the wall and column is much stiffer than the slab and little rotation is expected at the slab-to-wall connection. Section 8.10.4.3 gives the option of modifying the factored moments by up to 10 percent, but that allowance is not used in this example. Section 8.10.4.4 indicates the negative moments are at the face of the supporting columns. Section 8.10.4.5 requires that the greater value of the two interior negative moments at the first interior column controls the design of the slab.	In Table 8.10.4.2, for the exterior edge being fully restrained Negative M_u at face of exterior column = $0.65M_o$ = 83 ft-kip Maximum positive $M_u = 0.35M_o$ = 45 ft-kip Negative M_u at face of first interior column = $0.65M_o$ = 83 ft-kip
ACI 318-14 8.10.5	Proportion the total panel factored moments from 8.10.4 to the column and middle strips for the end span, from A to B.	
ACI 318-14 8.10.5.1	After distributing the total panel negative and positive M_u as described earlier in Section 8.10.4, Table 8.10.5.1 then proportions the interior negative M_u assumed to be resisted by the column strip.	In Table 8.10.5.1, ℓ_2/ℓ_1 = 14/18 = 0.778 and α_f = 0 Therefore, the top line of the table controls $M_{u,int.neg,cs}$ = 0.75 × 83 ft-kip = 63 ft-kip
ACI 318-14 8.10.5.2	After distributing the total panel M_u as described earlier in Section 8.10.4, Table 8.10.5.2 then proportions the exterior negative M_u assumed to be resisted by the column strip.	In Table 8.10.5.2, ℓ_2/ℓ_1 = 14/18 = 0.778 and α_f = 0 Assuming the wall behaves as a beam, C is calculated to determine β_t using Eq. (8.10.5.2(a) and (b)) $\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$ $C = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3}$ x = 10 in. y = 120 in. C = 37,900 in.^{4} $E_{cb} = E_{cs}$ $I_s = \frac{bh^3}{12} = \frac{168\text{ in.} \times (7\text{ in.})^3}{12}$ = 4802 in.^{4} $\beta_t = \frac{37,900}{2 \times 4802} = 3.9$
ACI 318-14 8.10.5.5	After distributing the total panel M_u as described earlier in Section 8.10.4, Table 8.10.5.5 then proportions the positive M_u assumed to be resisted by the column strip.	In Table 8.10.5.5, ℓ_2/ℓ_1 = 14/18 = 0.778 and α_f = 0 Therefore, the top line of the table controls $M_{u,pos,cs}$ = 0.60 × 45 ft-kip = 27 ft-kip
ACI 318-14 8.10.6	The total panel M_u from 8.10.4 is distributed into column strip moments and middle strip moments. The middle strip M_u is the portion of the total panel M_u not resisted by the column strip.	Determine the amounts distributed to the middle strips. Subtract the amounts distributed to the column strips in Section 8.10.5 from the panel M_u calculated in Section 8.10.4. $M_{u,int.neg,ms}$ = 83 ft-kip − 63 ft-kip = 20 ft-kip $M_{u,ext.neg,ms}$ = 83 ft-kip − 63 ft-kip = 20 ft-kip $M_{u,pos,ms}$ = 45 ft-kip − 27 ft-kip = 18 ft-kip

ACI 318-14 8.10.7.3	The gravity load moment transferred between slab and edge column by eccentricity of shear is $0.3M_o$	If there was no wall supporting the exterior edge of the slab, this moment would be used to calculate the two-way shear in the slab at the exterior column in 8.5. However, because of the wall, two-way shear does not apply to the design at the exterior column.
ACI 318-14 8.10	Repeat the M_u calculations for the interior span	The results are shown for interior panels, with the same negative M_u at either end of the panel $M_{u,neg,cs}$ = 63 ft-kip $M_{u,neg,ms}$ = 20 ft-kip $M_{u,pos,cs}$ = 27 ft-kip $M_{u,pos,ms}$ = 18 ft-kip Refer to Fig. E1.3 for final distribution along this column line. The middle strip moments are split into two half-middle strips, one on either side of the column strip.

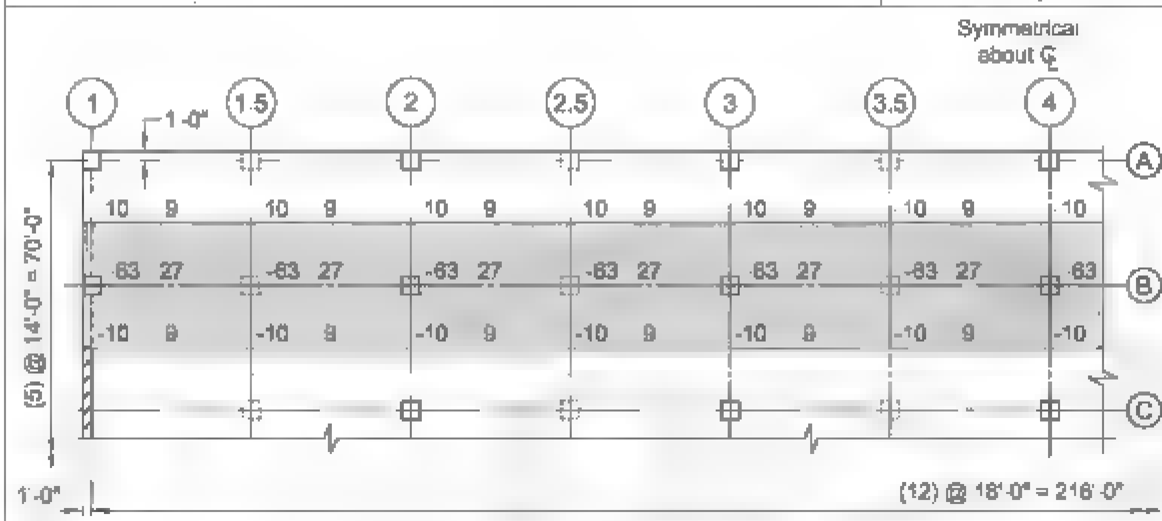

Fig. E1.3—Final moment distribution.

Step 5: Required strength—Factored slab moment resisted by the column		
8.4.2.2	Slab negative moments at a column can be unbalanced, that is, different on either side of the column. This difference in slab moments, M_{sc}, must be transferred into the column, usually by a combination of flexure or shear. Equation (8.4.2.2.2) calculates a factor that determines the fraction of M_{sc} transferred by flexure. In this example, the permitted modifications to this factor are not used.	The columns are square, so $b_1/b_2 = 1$ $\gamma_f = \dfrac{1}{1+\dfrac{2}{3}\sqrt{\dfrac{b_1}{b_2}}} = 0.6$
8.4.2.2.3 8.4.2.2.5	The effective slab width to resist $\gamma_f M_{sc}$ is the width of the column plus 1.5h of the slab on either side of the column. Section 8.4.2.2.5 requires sufficient reinforcement within the effective slab width to resist $\gamma_f M_{sc}$.	This concentration of reinforcement within the effective slab width is considered during the detailing of the column-slab joint in Section 8.5. Figure E1.4 shows undistributed total panel moments. The moment diagram is symmetric about the axis of the column in the center of the building (108 ft). Note that using this moment diagram will result in a net zero M_{sc}. The DDM uses an artificial unbalanced load condition in ACI 318-14 Section 8.10.7 to avoid an unconservative design for two-way shear.

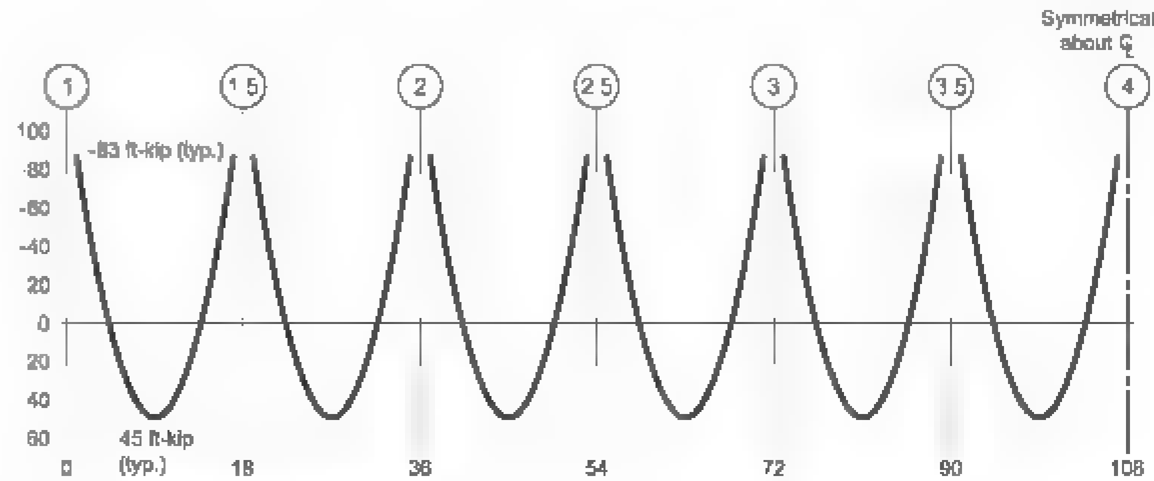

Fig. E1.4—Total panel moments.

ACI 318-14 8.10.7 8.10.7.1 8.10.7.2	M_{sc} to satisfy the DDM provisions at an interior column is calculated by Eq. (8.10.7.2) $M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q_{Du}'\ell_2'(\ell_n')^2]$ where the ' indicates the shorter span. When the spans are the same as in our example, the ' simply indicates the next span.	At an interior column $M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q_{Du}'\ell_2'(\ell_n')^2]$ $M_{sc} = 0.07[(0.123\text{ kip/ft}^2 + 0.5(0.160\text{ kip/ft}^2))(14\text{ ft})(16\text{ ft})^2 - 0.123\text{ kip/ft}^2(14\text{ ft})(16\text{ ft})^2] = 20.1\text{ ft-kip}$
ACI 318-14 8.10.7.3	M_{sc} to satisfy the DDM provisions at an exterior column is calculated by Section 8.10.7.3: $M_{sc} = 0.3M_o$	At an exterior column $M_{sc} = 0.3M_o$ $M_{sc} = 0.3(127\text{ ft-kip}) = 38.1\text{ ft-kip}$
ACI 318-14 8.4.2.2.2 8.4.4.2.2	M_{sc} is required to be transferred through both flexure and two-way shear into the column. The two-way shear calculations are discussed in Step 7. The amount of steel required to transfer $\gamma_f M_{sc}$ into the column via flexure is determined. The flexural reinforcement determined in later steps is allowed to be used to meet the required A_s in this step. Therefore, at Step 13, the required reinforcement from this step will be checked.	At interior columns $\gamma_f = 0.6$ $M_{sc} = 20.1$ ft-kip $\gamma_f M_{sc} = (0.6)(20.1\text{ ft-kip})$ $\gamma_f M_{sc} = 12.1$ ft-kip Using the method described in Step 8, the amount of flexural steel required within $1.5h$ of the column is $A_s = 0.49\text{ in.}^2/45\text{ in.}$ or $0.13\text{ in.}^2/\text{ft}$ At exterior columns $\gamma_f = 0.6$ $M_{sc} = 38.1$ ft-kip $\gamma_f M_{sc} = (0.6)(38.1\text{ ft-kip})$ $\gamma_f M_{sc} = 22.9$ ft-kip Using the method described in Step 8, the amount of flexural steel required within $1.5h$ of the column is $A_s = 0.94\text{ in.}^2/45\text{ in.}$ or $0.25\text{ in.}^2/\text{ft}$
Step 6: Required strength—Factored one-way shear		
8.4.3 8.4.3.1 8.4.3.2	One-way shear rarely controls over two-way shear in the design of a two-way slab, but it must be checked. In this section, V_u is determined. In Step 9, it is verified that the slab shear strength, ϕV_n, is sufficient to resist V_u.	Figure E1.5 shows one-way shears. The shear diagram is symmetric about the axis of the column at the center of the building (108 ft). $V_u = 32$ kip

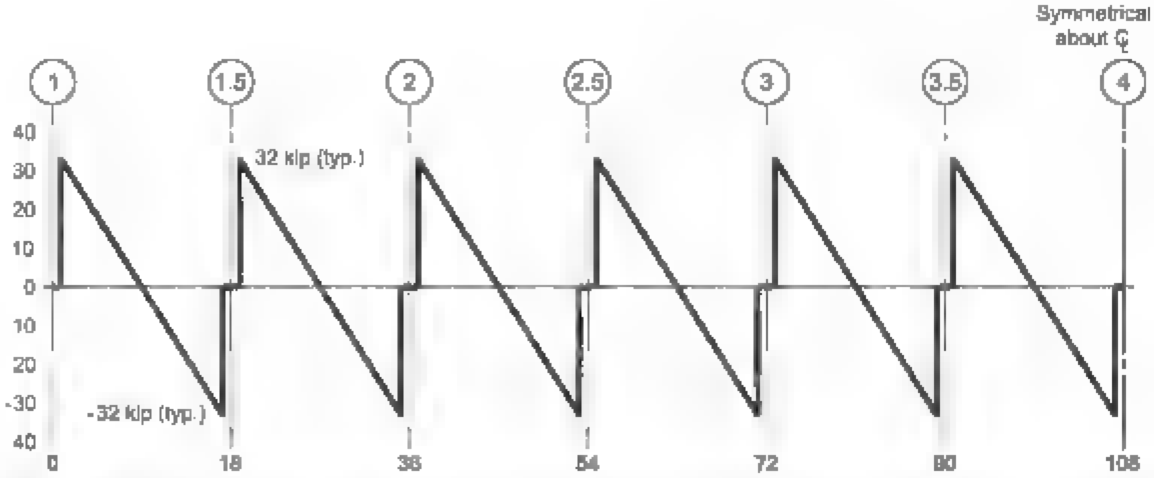

Fig. E1.5—Shear diagram

Step 7: Required strength — Factored two-way shear

8.3.1.4	Stirrups are not used as shear reinforcement in this example	
8.4.4.1 22.6.4	Determine the critical section for two-way shear without shear reinforcement. Calculate b_o at an interior column	
	$b_o = 2 \times (c_1 + d) + 2 \times (c_2 + d)$	$b_o = 2 \times (24\text{ in.} + 5.6\text{ in.}) + 2 \times (24\text{ in.} + 5.6\text{ in.})$ $b_o = 118.4\text{ in.}$
	where d is the average effective depth (Fig. E1.6) and this example assumes No. 5 bars when determining d. Cover is assumed to be 0.75 in. per Table 20.6.1.3.1	 Fig. E1.6—Average slab effective depth Figure E1.7 shows two-way critical sections, b_o, at an interior column

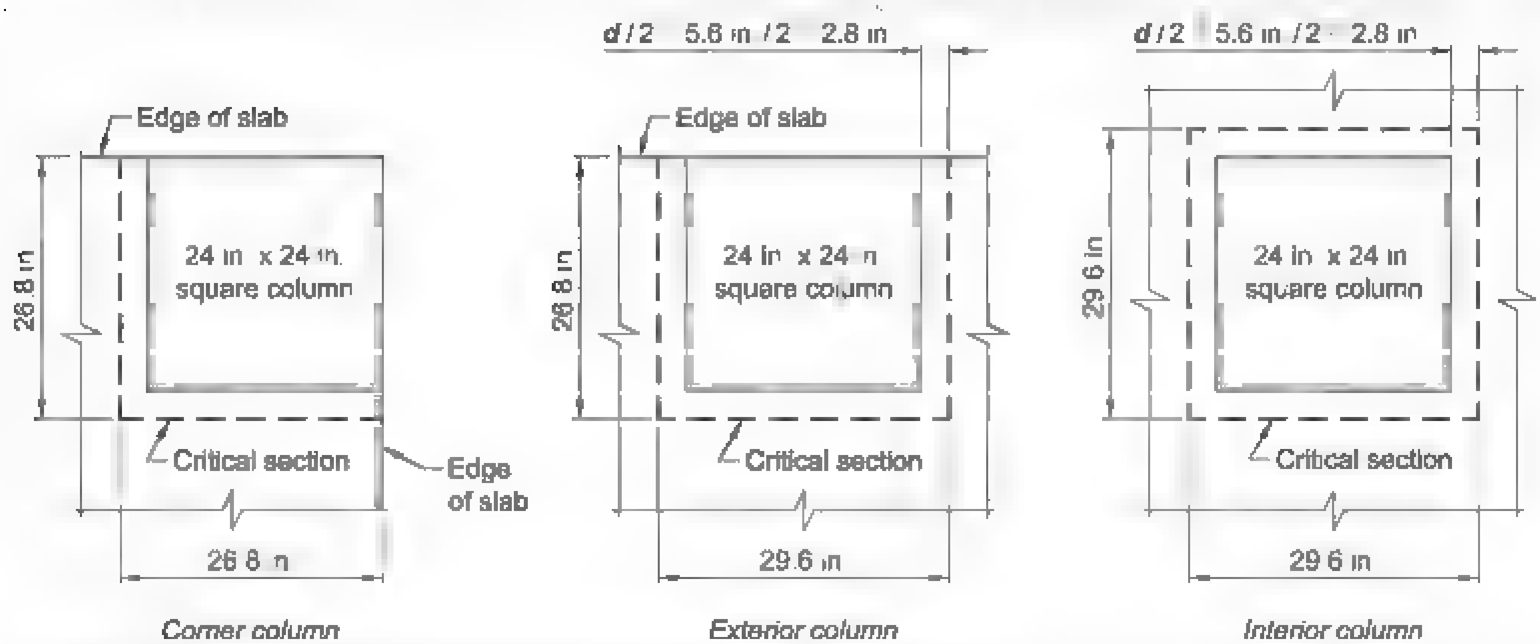

Fig. E1.7—Two-way shear-critical section locations

Two-Way Slabs

8.4.4.2 8.4.4.2.1	Determine v_{uv} due to direct slab shear stress.	Calculate the direct shear stress at the interior column with full factored load on all spans $v_{uv} = \dfrac{V_u}{b_o d}$ $V_u = \left(14\text{ ft}\times 18\text{ ft} - \dfrac{29.6\text{ in.}\times 29.6\text{ in.}}{144}\right)\times\dfrac{283\text{ kip}}{1000\text{ ft}^2}$ $V_u = 70\text{ kip}$ $v_{uv} = \dfrac{70\text{ kip}}{118.4\text{ in.}\times 5.6\text{ in.}} = 0.106\text{ ksi}$
8.4.4.2.1 8.4.4.2.2	Determine the slab shear stress due to moment.	Calculate the shear stress due to moments at an interior column $\gamma_v = 0.4$ $M_{sc} = 20.1\text{ ft-kip}$ $c_{AB} = 14.8\text{ in.}$ $J_c = 97688\text{ in.}^4$ $\dfrac{\gamma_v M_{sc} c_{AB}}{J} = 0.015\text{ ksi}$
8.4.4.2.3	Calculate v_u by combining the two-way direct shear stress and the stress due to moment transferred to the column via eccentricity of shear	$v_u = v_{uv} + \dfrac{\gamma_v M_{sc} c_{AB}}{J_c}$ Calculate the design shear stress at an interior column $v_u = 0.106\text{ ksi} + 0.015\text{ ksi} = 0.121\text{ ksi}$ Note that these calculations are conservative. M_{sc} assumes that some live load is not present to produce unbalanced moments, but v_{uv} assumes that full live load and dead load are present.

Step 8. Design strength: Reinforcement required to resist factored moments		
8.5.1 8.5.2 8.4.2.2.5	There are many methods available to determine the flexural reinforcement required at all sections within the span in each direction. To determine the amount of flexural steel required, this example solves the following quadratic equation: $\phi M_n = \phi\left(A_s f_y\left(d - \frac{a}{2}\right)\right)$ $\phi M_n = \phi\left(A_s f_y\left(d - \frac{A_s f_y}{2\times 0.85 b f_c'}\right)\right)$ $\omega = \frac{A_s f_y}{b d f_c'}$ $\phi M_n = \phi(b d f_c' \omega(d - 0.59\omega d))$ $\phi M_n = \phi(b d^2 f_c' \omega(1 - 0.59\omega))$ Set $\phi M_n = M_u$ and solve for ω.	ϕ is assumed to be 0.9 for flexure as the slab is lightly reinforced. Using the moments shown in Fig. E1.8 and E1.9 for the column strip and middle strip, respectively, to determine the reinforcement required at each location. Reinforcement in an exterior panel: Column strip at the columns: $M_u = 63$ ft-kip Solving the quadratic equation gives: $\omega = 0.0664$ $A_s = \frac{\omega b d f_c'}{f_y}$ $A_s = \frac{(0.0664)(84\text{ in.})(5.6\text{ in.})(5000\text{ psi})}{60{,}000\text{ psi}}$ $A_s = 2.61\text{ in.}^2$ Column strip at midspan: $M_u = 27$ ft-kip Solving the quadratic equation gives: $\omega = 0.0278$ $A_s = \frac{\omega b d f_c'}{f_y}$ $A_s = \frac{(0.0278)(84\text{ in.})(5.6\text{ in.})(5000\text{ psi})}{60{,}000\text{ psi}}$ $A_s = 1.09\text{ in.}^2$ Using the same method, the following can be found: <u>Exterior Panels:</u> Column strip at column line: $A_s = 2.61\text{ in.}^2$ Middle strip at column line: $A_s = 0.81\text{ in.}^2$ Column strip at midspan: $A_s = 1.09\text{ in.}^2$ Middle strip at midspan: $A_s = 0.73\text{ in.}^2$

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		Interior Panels. Column strip at column lines $A_s = 2.61$ in.^{2} Middle strip at column lines. $A_s = 0.81$ in.^{2} Column strip at midspan $A_s = 1.09$ in.^{2} Middle strip at midspan $A_s = 0.73$ in.^{2}

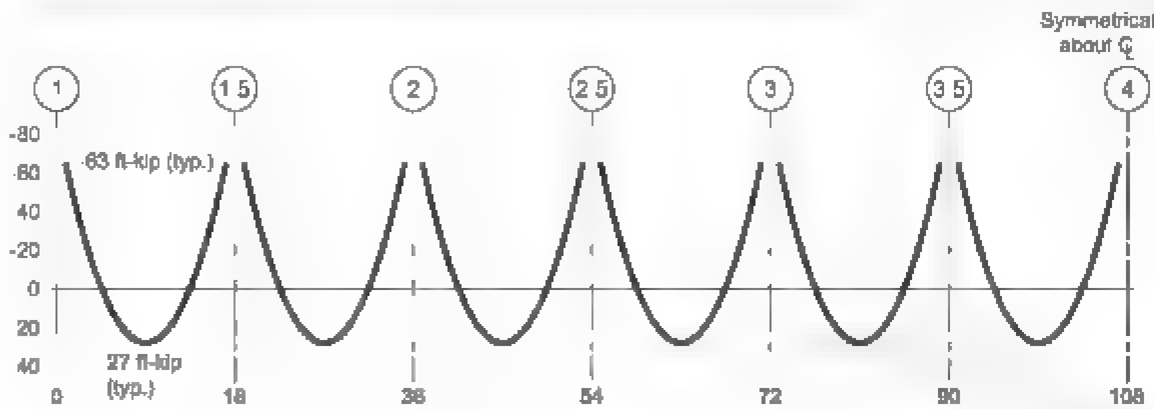

Fig. E1.8—Column strip moment diagram

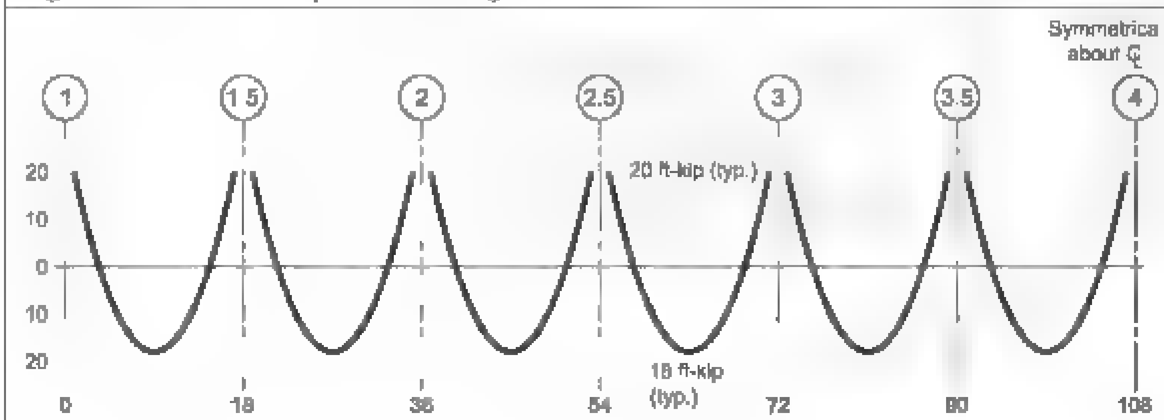

Fig. E1.9—Middle strip moment diagram

Step 9: Design strength—One-way shear		
8.5.3.1.1 22.5 22.5.5	Check one-way shear strength where the critical section extends in a plane across the entire slab width. One-way shear does not usually control in two-way slab systems nor is shear reinforcement typically provided for such. Check one-way design shear strength considering only the concrete contribution ($\phi V_n = \phi V_c$)	
22.5.5.1c	When no shear reinforcement is provided, use equation from Table 22.5.5.1c $V_c = \left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right]b_w d$	
21.2.1	Strength reduction factor for shear from Table 21.2.1b	$\phi = 0.75$
	Effective depth to centroid of reinforcement	$d = 7\text{ in.} - 0.75\text{ in.} - 0.625\text{ in.} = 5.63\text{ in.}$
	Use the least amount of flexural reinforcement provided to ensure shear strength check will cover all reinforcement conditions. Reinforcement is on a unit width basis	$A_s = 0.73\text{ in.}^2$
2.2	Reinforcement ratio of flexural reinforcement relative to the unit slab width of 12 in.	
	$\rho_w = A_s/b_w d$	$\rho_w = \dfrac{0.73\text{ in.}^2}{12\text{ in.}(5.6\text{ in.})} = 0.01086$
	Axial load is zero	$N_u = 0$
22.5.5.1.3	Size effect factor $\lambda_s = \sqrt{\dfrac{2}{1+0.1d}}$ $\lambda_s \leq 1.0$	$\lambda_s = \sqrt{\dfrac{2}{1+0.1(5.6)}} = 1.132$; use $\lambda_s = 1.0$ $\lambda = 1.0$ $V_c = \left[8(1.0)(1.0)(0.01086)^{1/3}\sqrt{5000}\right](168)(5.6)\dfrac{1}{1000}$ $V_c = 118$ kip $\phi V_c = 0.75(118) = 88.5 > V_u = 32$ kip **OK** Design shear strength from concrete contribution is more than twice the factored shear. Slab thickness is adequate.

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Step 10: Design strength—Two-way shear		
22.6.5.1	Determine two-way shear strength contributed by concrete to find if shear reinforcement is required	
22.6.5.2	Determine the nominal two-way shear strength. Strength is represented in terms of shear stress (v_c) and is the least of the following: $4\lambda_s\lambda\sqrt{f'_c}$ $\left(2+\dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c}$ $\left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c}$	
22.5.5.1.3	Size effect factor $\lambda_s = \sqrt{\dfrac{2}{1+0.1d}}$ $\lambda_s \leq 1.0$	$\lambda_s = \sqrt{\dfrac{2}{1+0.1(5.63)}} = 1.13$ $\lambda_s \leq 1.0$ Upper limit on size effect controls $\lambda_s = 1.0$ $4(1.0)(1.0)\sqrt{5000\text{ psi}} = 283\text{ psi}$ $\left(2+\dfrac{4}{1.0}\right)(1.0)(1.0)\sqrt{5000\text{ psi}} = 424\text{ psi}$ $(2+1.89)(1.0)(1.0)\sqrt{5000\text{ psi}} = 275\text{ psi}$ **Controls** $\phi v_n = 0.75(275\text{ psi}) = 206\text{ psi}$ This is greater than the required strength for interior columns of 0.121 ksi from Step 7, therefore, two-way shear at interior columns is okay. Two-way shear reinforcement is not required at these locations.

Step 11: Reinforcement limits – Minimum flexural reinforcement in nonprestressed slabs		
8.6.1.1	Minimum area of flexural reinforcement is required to ensure ductile failure mode. For two-way slabs, the quantity is the same as that required for shrinkage and temperature in 24.4.3.2. All of the reinforcement, however, must be placed in the tension face	
8.6.1.2	For nonprestressed slabs, determine if there is a possibility of a flexure-driven punching failure. If so, then minimum reinforcement may be greater than 0.0018. Determine if:	
	$v_{uv} > \phi 2\lambda_s\lambda\sqrt{f'_c}$	$\phi = 0.75$
22.5.5.1.3		$\lambda_s = \sqrt{\frac{2}{1+0.1(5.63)}} = 1.13$
		$\lambda_s = 1.0$ Upper limit on size effect controls
		$\lambda = 1.0$
		$0.75(2)(1.0)(1.0)\sqrt{5000\text{ psi}} = 106$ psi
	From calculation for interior column punching shear from Step 7	$v_{uv} = 106$ psi
		OK. No need to increase minimum flexural reinforcement
		$A_{s,min} = 0.0018 \times A_g$ $A_{s,min} = 0.0018 \times 7\text{ in.} \times 14\text{ ft} \times (12\text{ in./ft.})$ $A_{s,min} = 2.12\text{ in.}^2$
		This minimum area of reinforcement is split evenly between the column and middle strips, therefore 1.06 in.^2 per strip
		Minimum flexural reinforcement controls at the middle strip of all panels
Step 12: Reinforcement detailing – General requirements		
8.7.1 8.7.1.1 20.6.1	Concrete cover, development lengths, and splice lengths are determined in these sections	Concrete cover requirements are provided in Table 20.5.1.3.1. The slab is not exposed to weather or in contact with the ground. Assuming No. 5 bars for reinforcement, the specified cover is 0.75 in.

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8.7.1.2 25.4	Development length is needed to determine splice length	
25.4.3	Determine required development length using simplified formulas from Table 25.4.2.3 for No. 6 bars and smaller, and for clear spacing of bars at least $2d_b$ and clear cover at least d_b	$d_b = 0.625$ in. < 0.75 in. clear cover $2.2d_b = 1.25$ in. < bar spacing
25.4.3.1	$\ell_d \geq \left(\dfrac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f_c'}}\right) d_b$ $\ell_d > 12$ in.	$\lambda = 1.0$
25.4.3.2	ψ_t = casting position ψ_e = epoxy ψ_g = reinforcement grade	Bars are cast with less than 12 in. of fresh concrete below the bars. $\psi_t = 1.0$ Bars are uncoated $\psi_e = 1.0$ Bars are Grade 60 $\psi_g = 1.0$ Required development length $\dfrac{60{,}000\text{ psi}(1.0)(1.0)(1.0)}{25(1.0)\sqrt{5000\text{ psi}}} 0.625\text{ in.} = 21.2\text{ in.}$ use 22 in.
8.7.1.3 25.5	It is likely that splices will be required during construction. Allowable locations for splices are shown in ACI 318 Fig. 8.7.4.1.3	Lap splice lengths are determined in accordance with Table 25.5.2.1. The provided A_s does not exceed the required A_s by a substantive amount. Therefore, class B splices are required. $\ell_{st} = 1.3 \times 21.2$ in. $= 27.5$ in. use $\ell_{st} = 28$ in.

Step 13: Reinforcement detailing Spacing requirements		
8 7 2 8 7 2 1 25 2 1 8 7 2 2	Minimum and maximum spacing limits are determined. The bar spacing for design strength is also reviewed	Minimum spacing is determined in accordance with Section 25 2 1 Minimum spacing is 1 in., d_b, and (4/3) d_{agg}. Assuming that the maximum nominal aggregate size is 1 in., then the minimum clear spacing is 1 33 in With a No. 5 bar, this equates to a minimum spacing of approximately 2 in Maximum spacing is limited by Section 8.7 2 2 At critical sections, the maximum spacing is the lesser of $2h$ (2 × 7 in.) and 18 in., so 14 in controls. All other sections, the critical spacing is the lesser of $3h$ (3 × 7 in.) and 18 in., so 18 in. However, because all of the bars cross a critical section, use a maximum spacing of 14 in. for all sections Assuming No. 5 bars are used, the spacing for the different areas of the slab are as follows. <u>All spans.</u> Column strip at column line 2.61 in.^{2}/0 31 in.^{2} = nine No 5 bars over 7 ft spacing is 9 in Middle strip at column line 0 81 in.^{2}/0.31 in.^{2} = six No. 5 bars over 7 ft spacing is 14 in. (maximum spacing controls over minimum area in the middle strip) Column strip at midspan 1 09 in.^{2}/0.31 in.^{2} = six No. 5 bars over 7 ft - spacing is 14 in. (maximum spacing controls over strength requirements at this location) Middle strip at midspan 0 73 in.^{2}/0.31 in.^{2} = six No. 5 bars over 7 ft – spacing is 14 in. (maximum spacing controls over minimum area in the middle strip)
8 4 2 2 2	This is a check to verify that the reinforcement amounts required to transfer the fraction of factored slab moment via flexure are satisfied using the design slab reinforcement.	The minimum requirements for all column strips is a 14 in. spacing of No. 5 bars. This equates to 0.26 in.^{2}/ft. This meets or exceeds the 0 13 in.^{2} ft and 0 26 in.^{2}/ft required from Step 5, therefore, Section 8.4.2.2.2 is satisfied. Note that if this had not been met, additional steel would have been required to be placed within the effective slab width as defined in Section 8 4.2 2 3
Step 14: Reinforcement detailing Reinforcement termination		
8 7 4.1	Reinforcement lengths and extensions are at least that required by Fig. 8 7.4 1.3 of ACI 318	Use ACI 318, Fig. 8 7 4.1 3 to determine reinforcement lengths. The figure for final layout of reinforcement in these panels shows the design lengths

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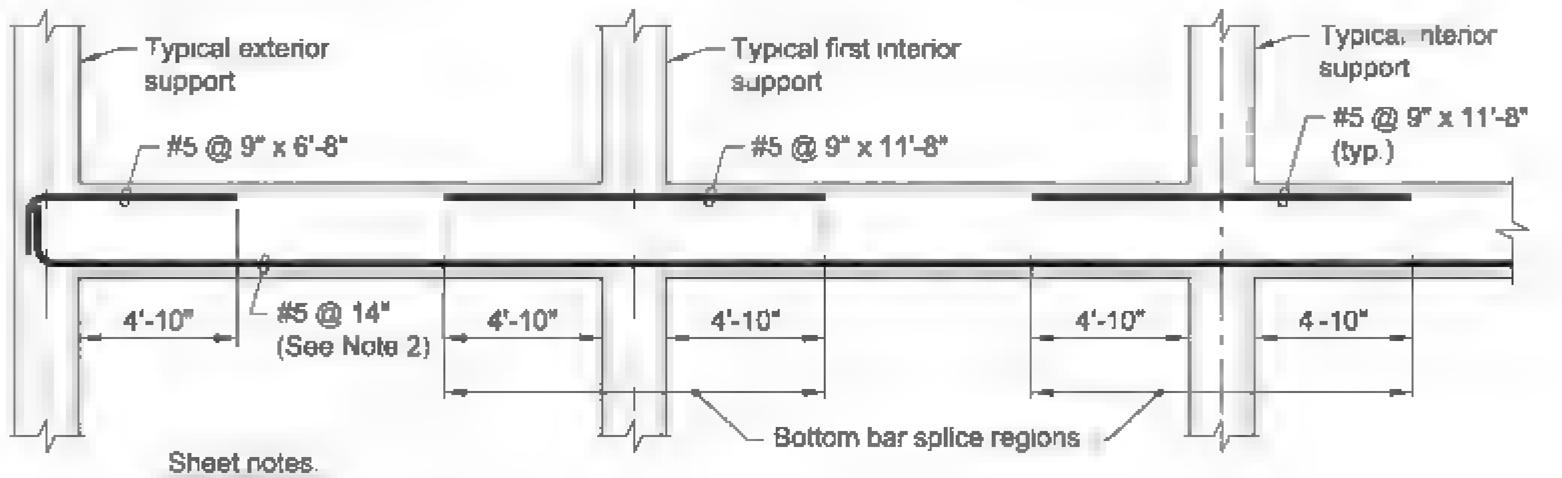
Step 15: Reinforcement detailing Structural integrity		
8.7.4.2	Structural integrity for a two-way slab is met by satisfying ACI 318-14 detailing provisions.	Section 8.7.4.2.1 is met when reinforcement is detailed in accordance with Fig. 8.7.4.1.3 (ACI 318) Section 8.7.4.2.2 requires that at least two of the column strip bottom bars pass through the column inside the column reinforcement cage
Step 16 Slab-column joints		
8.2.7 15.2.9 15.3.2 15.5	Joints are designed to satisfy Chapter 15 of ACI 318. Slab-column connections transferring moment must satisfy strength and detailing requirements of Chapter 8, 15.3.2, and 22.6.	The specified concrete strength of the slab and columns are identical and therefore, 15.2.2 and 15.5 are met Chapter 8 and 22.6 requirements are addressed in Steps 7 and 10 of this example. Section 15.3.2.1 applies to columns along the exterior of the building. The tie spacing determined from 25.7.2 for the column design will likely be larger than the joint depth of 7 in. If that is the case, then only one tie is required within the slab-column joint in exterior columns
Step 17: Summary tables of required A_s		

A_s required, column strip, in.^{2}				
	Exterior bays		Interior bays	
	Column lines	Midspan	Column lines	Midspan
Strength	2.6	09	2.6	09
Minimum	1.06	1.06	1.06	1.06
Maximum spacing, assuming No. 5 bars	1.86	86	86	86
A_s required, middle strip, in.^{2}				
	Exterior bays		Interior bays	
	Column lines	Midspan	Column lines	Midspan
Strength	0.81	0.73	0.81	0.73
Minimum	1.06	1.06	1.06	1.06
Maximum spacing, assuming No. 5 bars	1.86	86	86	86

Note: The highlighted cells indicate the required reinforcement that controls design.

Step 18: Summary sketches of required bars
Figures E1.10 and E1.11 are summary sketches of required bars for this example

Column strip
Interior 12-span design panel



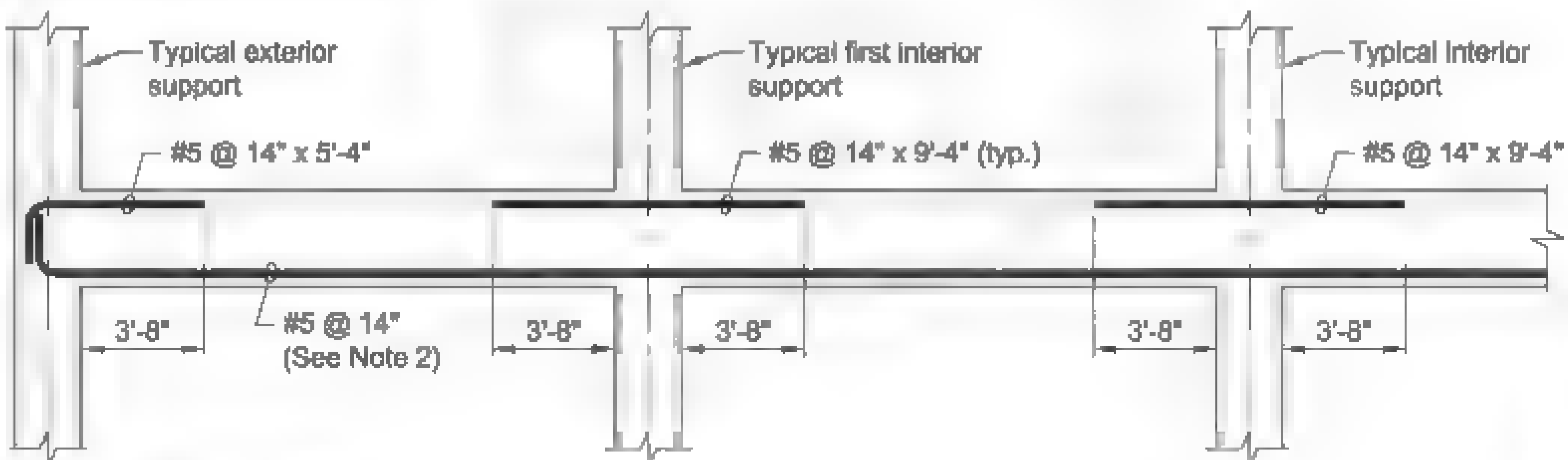
Sheet notes:

1. – Indicates standard hook. Standard hooks may be 90 or 180 degree hooks. Standard hook bars may be inclined up to 45 degrees to allow hook to fit in slab.

2. Minimum of two bottom bars must pass within longitudinal column bars.

Fig. E1.10—Summary sketch of required bars, column strip.

Middle strip
Interior 12-span design panel



Sheet notes:

1. – Indicates standard hook. Standard hooks may be 90 or 180 degree hooks. Standard hook bars may be inclined up to 45 degrees to allow hook to fit in slab.

2. Bottom bar splices permitted in same region as column strip.

Fig. E1.11—Summary sketch of required bars, middle strip.

Two-way Slab Example 2: *Equivalent Frame Method (EFM)—Interior frame*

This example is an interior column strip along grid line B in a nonprestressed two-way slab without beams between supports. This example uses the moment distribution method to determine design moments, but any method for analyzing a statically indeterminate structure can be used. This example uses the Hardy column analogy to determine the structural stiffness for the members analyzed.

Given:

Uniform loads—

Self-weight dead load is based on concrete density including reinforcement at 150 lb/ft^3

Superimposed dead load D = 0.015 kip/ft^2

Live load L = 0.100 kip/ft^2

Material properties—

f_c' = 5000 psi

f_y = 60,000 psi

Thickness of slab t = 7 in.

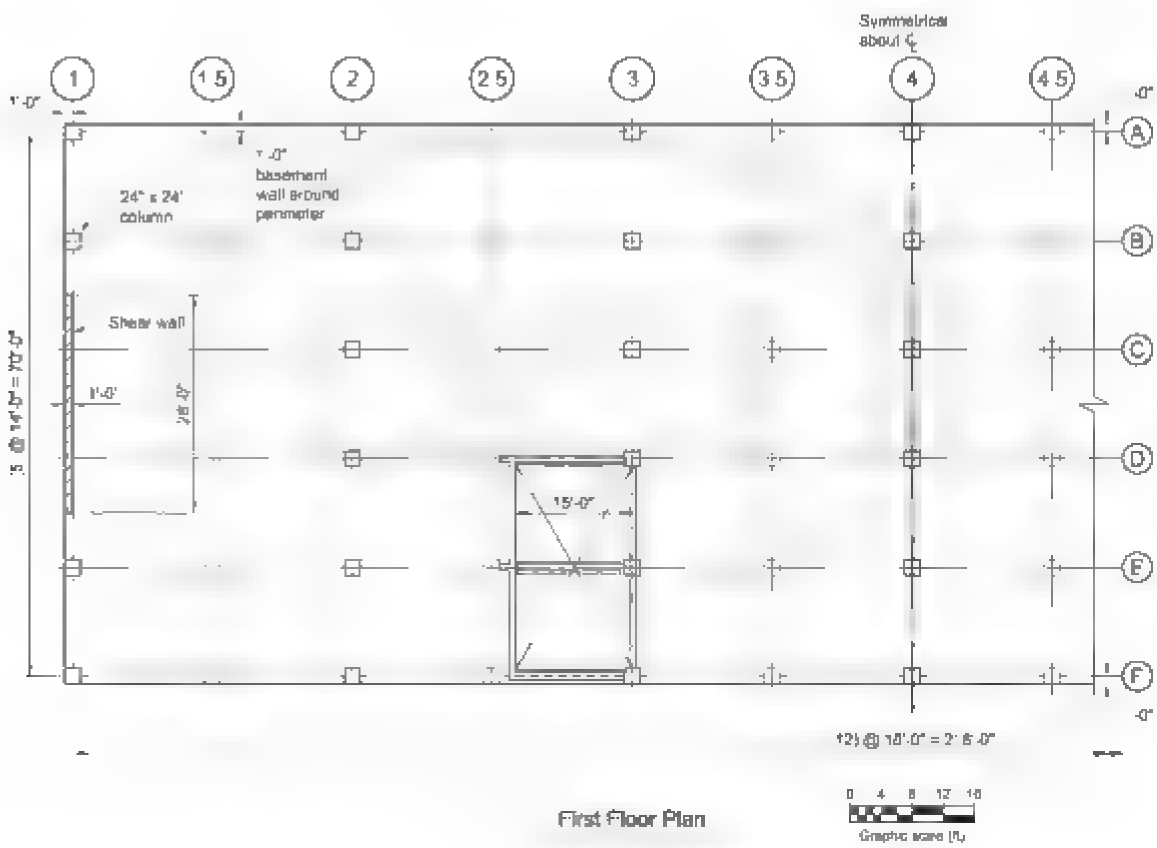
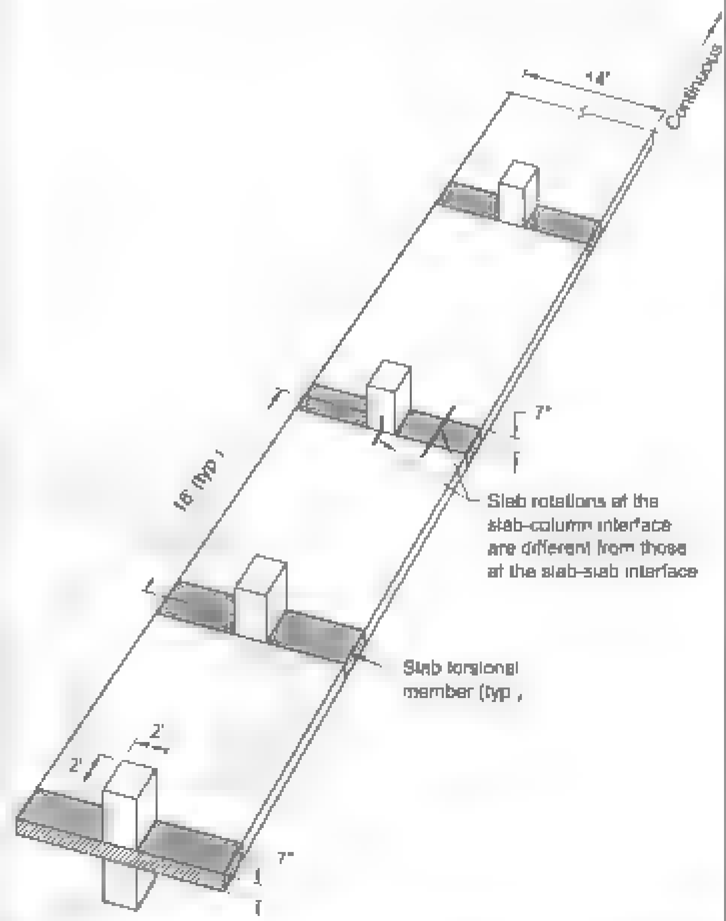


Fig. E2.1—First-floor plan.

ACI 318	Discussion	Calculation
Step 1. Geometry		
8.2.1	The slab is designed using the Equivalent Frame Method (EFM), which is detailed in Section 8.11 of ACI 318-14. EFM details have been removed from the Code, but is still permitted to be used according to Section 8.2.1. The Code also indicates that the equivalent frame method is limited in application to orthogonal frames subject to gravity loads only.	
ACI 318-14 8.11.1 8.11.2	Figure E2.2 shows the slab-beam strip and the attached torsional members of the equivalent frame model. A key element of the EFM is that, unlike a beam and column frame, in a slab and column frame some of the unbalanced moments can redistribute around the column into the next span regardless of the stiffness of the columns. The EFM softens the columns to simulate this effect on slab moments by incorporating the flexibility of the slab torsional member in the equivalent column stiffness.	 *Fig. E2.2—Equivalent frame strip.*
Step 2. Analysis: Equivalent column stiffness		
ACI 318-14 8.11.3 8.11.4 8.11.5	Use the equivalent frame geometry and design aids to determine the equivalent column stiffness, moment coefficient, and carry-over factor for use in the moment distribution method. (Corley and Jirsa, 1970, "Equivalent Frame Analysis for Slab Design," *ACI Journal Proceedings*, V. 67, No. 11, Nov., pp. 875-884).	

ACI 318-14 8.11.3 8.11.5

To determine the equivalent column stiffness, K_{ec}, the stiffness of the torsional member intersecting with the column, K_t, is needed at each intersection. K_t is determined using an equation given in Commentary Section R8.11.5 of ACI 318-14. The effects of cracking on K_t are neglected.

$$K_t = \sum \frac{9E_{cs}C}{\ell_2\left(1-\frac{c_2}{\ell_2}\right)^3}$$

$\ell_2 = 14$ ft $= 168$ in.

$E_{cs} = E_{cc}$

$c_2 = 24$ in.

This example uses the same concrete strength throughout the structure, so the modulus of elasticity is also considered equal. This simplifies the calculations.

Interior column torsional members

For the torsional member at the interior columns and the slab portion of the torsional member at the exterior columns,

$x = 7$ in.

$y = 24$ in.

and

$$C = \sum\left(1-0.63\times\frac{x}{y}\right)\frac{x^3y}{3}$$

$$C = \left(1-0.63\times\frac{7\text{ in.}}{24\text{ in.}}\right)\frac{(7\text{ in.})^3\times 24\text{ in.}}{3}$$

$C = 2240$ in.^{4}

$K_t = 191$

for each side of the column. Therefore, the total torsional member stiffness at an interior column is $K_t = 2 \times 191 = 382$

The basement wall is monolithic with the column and provides substantial stiffness to the exterior equivalent column, but the wall rotation will be greater than the column rotation so the torsional stiffness of the wall will be considered along with its flexural stiffness. The basement wall dimension, $y = 113$ in., in the calculations here is the distance from the bottom of the slab being designed to the top of the mat foundation.

Exterior column torsional members

For the wall portion of the torsional member at the exterior columns,

$x = 12$ in.

$y = 113$ in.

And the total C for the exterior column torsional members is

$$C = \sum\left(1-0.63\times\frac{x}{y}\right)\frac{x^3y}{3}$$

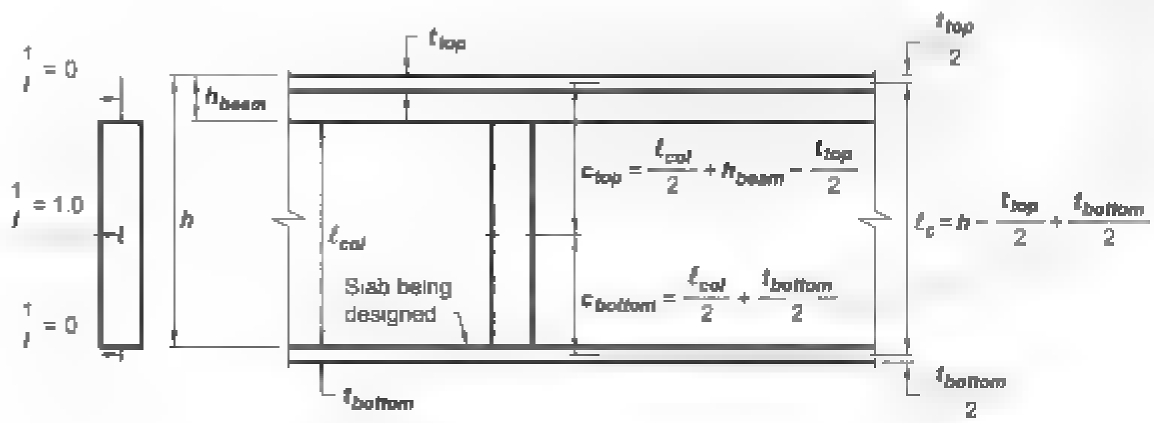
$$C = \left(1-0.63\times\frac{12\text{ in.}}{113\text{ in.}}\right)\frac{(12\text{ in.})^3\times 113\text{ in.}}{3} + 2240\text{ in.}^4$$

$C = 60{,}733$ in.^{4}

$K_t = 5357$

for each side of the column. Therefore, the total torsional member stiffness at an exterior column is $K_t = 2 \times 5357 = 10{,}714$

ACI 318-14 8.11.4	To determine the equivalent column stiffness, K_{ec}, the stiffness coefficients for the columns above and below the slab are needed at each intersection. Because the slab thickness, column heights, and foundation thickness geometry is uniform, K_{ctop} and K_{cbot} are consistent at each interior joint in this design strip. K_{ctop} and K_{cbot} are determined using the Hardy column analogy (K. Wang, *Intermediate Structural Analysis*, McGraw-Hill, New York, 1983). Note that if Fig. E2.3 and E2.4 were combined, it provides a section cut through the basement slab being designed in this example. The bottom-most slab in Fig. E2.4 is the mat foundation while the upper-most beam and slab in Fig. E2.3 is the first floor above the entrance/lobby level of the structure. Please refer to the short discussion at the end of this example regarding an alternate method for determining the stiffness for the columns, beams, and slabs.	$K = \frac{k_c \times E_{cc} \times I_c}{\ell_c}$ The following values are used in the calculations for K_{ctop} and correspond to Fig. E2.3: t_{top} = 7 in. h_{beam} = 2.5 ft ℓ_{col} = 15.5 ft h = 18 ft t_{bottom} = 7 in. K_{ctop} is determined using the geometry from Fig. E2.3 $k_{ctop} = \ell\left(\frac{1}{A_a} + \frac{Mc}{I_a}\right)$ $A_a = \ell_{col} = 15.5$ $I_a = \frac{A_a}{12} = \frac{15.5^3}{12} = 310.3$ $M_{bot} = 1.0c_{bottom} = 8.04$ $c = c_{bottom} = 8.04$ $k_{ctop} = 18 \times \left(\frac{1}{15.5} + \frac{8.04^2}{310.3}\right) = 4.91$ $K_{ctop} = \frac{k_{ctop} \times I_{col}}{\ell_c} = \frac{4.91 \times 24^4}{12 \times 18(12)} = 629$

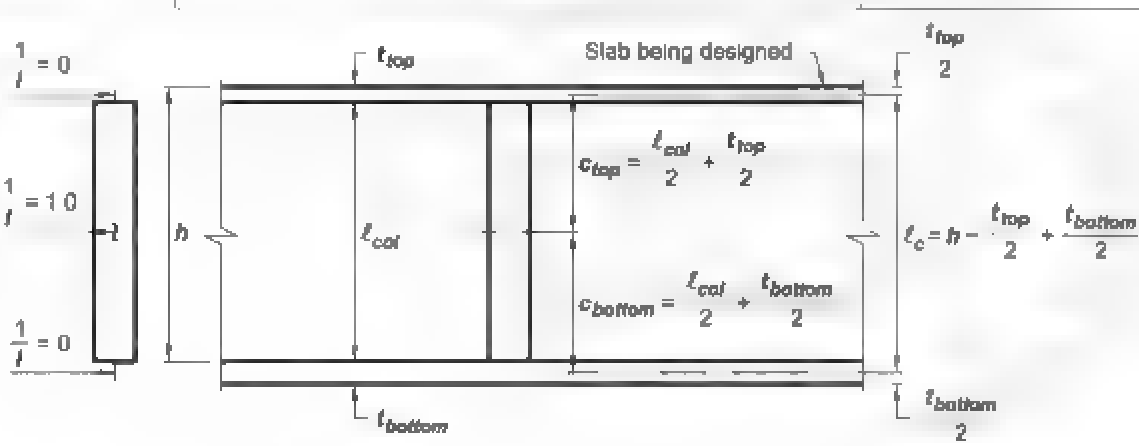


c_{bottom} in this figure is used to determine K_{ctop} for the slab being designed

Figure for Hardy column analogy to determine K_{ctop}

Fig. E2.3—Hardy column analogy for the columns above the slab being designed.

ACI 318-14 8.11.4		The following values are used in the calculations for K_{cbot} and correspond to Fig. E2.4. $t_{top} = 7$ in. $\ell_{col} = 9.42$ ft $h = 10$ ft $t_{bottom} = 3.5$ ft (assumed mat foundation thickness) K_{cbot} is determined using the geometry from Fig. E2.4 $k_{bot} = \ell\left(\frac{1}{A_a} + \frac{Mc}{I_a}\right)$ $A_a = \ell_{col} = 9.42$ $I_a = \frac{A_a^3}{12} = \frac{9.42^3}{12} = 69.6$ $M_{bot} = 1.0c_{top} = 5$ $c = c_{top} = 5$ $k_{bot} = 11.46 \times \left(\frac{1}{9.42} + \frac{5^2}{69.6}\right) = 5.33$ $K_{cbot} = \frac{k_{bot} \times I_{col}}{\ell_c} = \frac{5.33 \times 24^4}{12 \times 11.46(12)} = 1072$



c_{top} in this figure is used to determine K_{cbot} for the slab being designed

Figure for Hardy column analogy to determine K_{cbot}

Fig. E2.4—Hardy column analogy for the columns below the slab being designed.

ACI 318-14 8.11.4	To determine the equivalent column stiffness, K_{ec}, combine the torsional beam stiffness with the column stiffness's determined above	<u>Exterior column</u> $K_{ec} = \dfrac{1}{\dfrac{1}{\sum K_c} + \dfrac{1}{K_t}}$ $K_{ec} = \dfrac{1}{\dfrac{1}{629+1072} + \dfrac{1}{10714}}$ $K_{ec} = 1468$ <u>Interior column</u> $K_{ec} = \dfrac{1}{\dfrac{1}{\sum K_c} + \dfrac{1}{K_t}}$ $K_{ec} = \dfrac{1}{\dfrac{1}{629+1072} + \dfrac{1}{382}}$ $K_{ec} = 312$
Step 3: Analysis—Slab stiffness		
ACI 318-14 8.11.2	The slab stiffness is determined using the Hardy column analogy	<u>Slab panel (refer to Fig. E2.5)</u> $c_1 = c_2 = 2$ ft = 24 in. $\ell_s = 18$ ft = 216 in. $\ell_2 = 14$ ft = 168 in. $\ell_n = 16$ ft = 192 in. $M = 18$ ft/2 = 9 ft $c = 18$ ft/2 = 9 ft $I_{col} = \left(1 - \dfrac{c_2}{\ell_2}\right)^2 = \left(1 - \dfrac{2}{14}\right)^2 = 0.7347$ $I_s = \dfrac{bh^3}{12} = \dfrac{168 \times 7^3}{12} = 4802$ $I_a = \dfrac{\ell_n^3}{12} + \dfrac{\dfrac{c_1+c_2}{2}}{1/I_{col}}\left(\dfrac{\ell_n}{2} + \dfrac{c_1/2}{2}\right)^2$ $I_a = \dfrac{16^3}{12} + \dfrac{\dfrac{2+2}{2}}{1.361}\left(\dfrac{16}{2} + \dfrac{2/2}{2}\right)^2$ $I_a = 448$ $A_a = \ell_n + \dfrac{c_1}{2}(I_{col}) + \dfrac{c_2}{2}(I_{col})$ $A_a = 16 + \dfrac{2}{2}(0.7347) + \dfrac{2}{2}(0.7347)$ $A_a = 17.53$ $k_s = \ell_s\left(\dfrac{1}{A_a} + \dfrac{Mc}{I_a}\right)$ $k_s = 18\left(\dfrac{1}{17.53} + \dfrac{9^2}{448}\right)$ $k_s = 4.281$ $K_s = \dfrac{k_s \times I_s \times E_{cs}}{\ell_s} = \dfrac{4.281 \times 4802}{18 \times 12} = 95$

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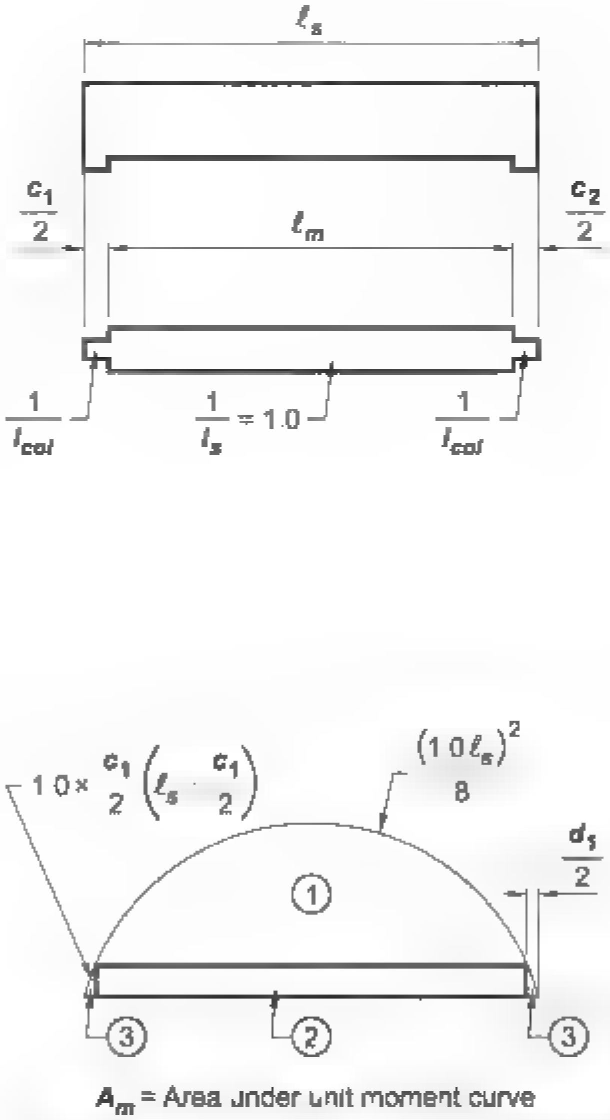
		$C.O.F=\dfrac{\ell_s\left(\dfrac{1}{A_u}-\dfrac{Mc}{I_u}\right)}{k_s}$ $C.O.F=\dfrac{-18\left(\dfrac{1}{17.53}-\dfrac{9^2}{448}\right)}{k_s}$ $C.O.F=\dfrac{2.228}{4.255}=0.524$ $A_m=A_1+A_2+2\times A_3$ $A_1=\dfrac{2}{3}\ell_u\times\left(\dfrac{\ell_s^2}{8}-\dfrac{c_1}{2}\times\left(\ell_s-\dfrac{c_1}{2}\right)\right)$ $A_1=\dfrac{2}{3}\times16\times\left(\dfrac{18^2}{8}-\dfrac{1}{2}\times(18-1)\right)$ $A_1=341.3$ $A_2=\dfrac{c_1}{2}\times\left(\ell_s-\dfrac{c_1}{2}\right)\times\ell_u$ $A_2=\dfrac{1}{2}\times(18-1)\times16=136$ $A_3=\dfrac{c_1}{2}\times\left(\ell_s-\dfrac{c_1}{2}\right)\times\dfrac{c_1}{2}\times\dfrac{1}{2}$ $A_3=\dfrac{1}{2}\times(18-1)\times1\times\dfrac{1}{2}=4.25$ $A_m=341.33+136+2\times4.25$ $A_m=481.6$ $FEM=\dfrac{A_m}{A_u\ell_s^2}=\dfrac{481.6}{17.53\times18^2}=0.085$



c_1 = width of near column in direction being checked

c_2 = width of far column in direction being checked

ℓ_2 = span width in other direction (into the page)

$$I_{col} = \left(1 - \frac{c_2}{\ell_c}\right)^2$$

$$M = \frac{\ell_s}{2}$$

$$C = \frac{\ell_s}{2}$$

$$I_s = \frac{\ell_n^3}{12} + \frac{2 \times \frac{c_2}{2}}{I_{col}}\left(\frac{\ell_n}{2} + \frac{\frac{c_2}{2}}{2}\right)^2$$

$$A_s = \ell_n + \frac{c_1}{2}\left(\frac{1}{I_{col}}\right) + \frac{c_2}{2}\left(\frac{1}{I_{col}}\right)$$

$$k_s = \left(\frac{1}{A_s} + \frac{MC}{I_s}\right)\ell_s$$

$$C.O.F. = \frac{\left(\frac{1}{A_s} - \frac{MC}{I_s}\right)\ell_s}{k_s}$$

$$FEM = \frac{A_m}{A_s \ell_s^2}$$

Fig. E2.5—Section properties.

Step 4: Analysis—Moment distribution

ACI 318-14 8.11.1	This example uses moment distribution with pattern live load in accordance with Section 6.4.3 of the Code. Loading all spans simultaneously does not necessarily produce the maximum flexural stresses in the slab. Therefore, in Section 6.4.3, live load patterns are defined for use with two-way slab systems. Figure E2.6 shows examples of the different live load patterns considered in the code. When reduced to face of support, these results are comparable to the DDM analysis in Example 1.	The moment distribution in Fig. E2.7 shows the first four column lines when full live load is applied to all spans. The structure is symmetrical and repeats from column line 2.5 through column line 5.5. The moment distributions for the different live load patterns are not included here but have been incorporated into the example. The moment diagram (Fig. E2.8) and shear diagram (Fig. 2.9) show the final results considering the live load patterns per Section 6.4.3.3 of the Code. The shear and moment diagrams in Fig. E2.8 and E2.9 are determined using known moments at the end of the slab along with known loads on the slab. The numerical values shown on these diagrams are the maximums determined at each location from the live load patterns discussed in Section 6.4.3.3. Note that the numerical values shown on these diagrams are the moments and shears reduced to the moments at the face of the columns, not at the midline of the columns as shown in the moment distribution (Fig. E2.7).

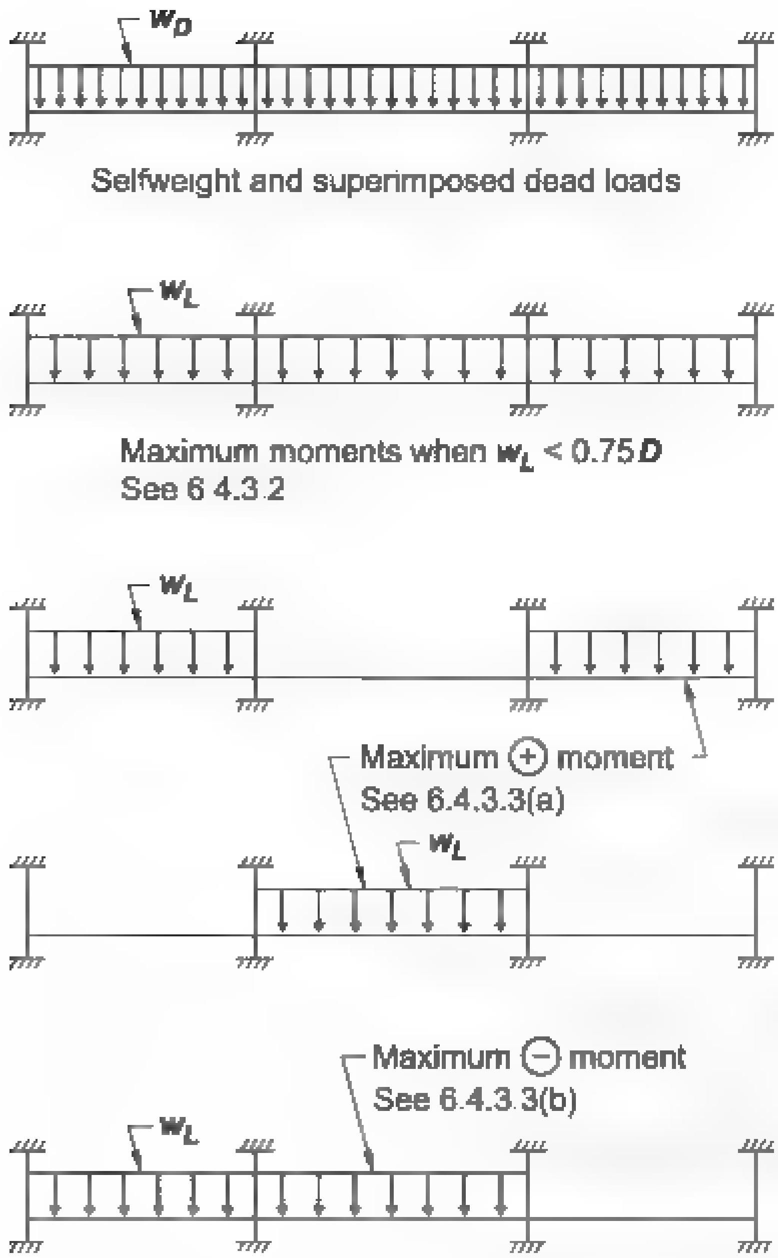


Fig E2 6—Code live load patterns, example uses 6 4.3 3(a) and (b)

Moment distribution example

Given:

w_u	0.283 k/ft
$\ell_{external}$	216 in
$\ell_{internal}$	216 in
ℓ_2	168 in

Column line	1			1.5			2			2.5
K_{ec}	1468			312			312			312
$K_{s,left}$	0			95			95			95
$K_{s,right}$	95			95			95			95
ΣK	1563			502			502			502
Moment coefficient, M	0.085			0.085			0.085			0.085
C.O.F., C	0.52	>>	<<	0.52	>>	<<	0.52	>>	<<	0.52
Slab Distribution Factor		0.06	0.19		0.19	0.19		0.19	0.19	
Column Distribution Factor	0.94			0.62			0.62			0.62
FEM		109	109		109	109		109	109	
bal	102	7	0	0	0	0	0	0	0	0
carryover		0	4		0	0		0	0	
bal	0	0	1	2	1	0	0	0	0	0
carryover		1	0		0	1		0	0	
bal	1	0	0	0	0	0	1	0	0	0
carryover		0	0		0	0		0	0	
bal	0	0	0	0	0	0	0	0	0	0
Balanced moment at Column Centerline	103	−103	112	−2	−110	108	[illegible]	109	109	0

Fig. E2.7—Moment distribution: example partial distribution with all spans with full live load.

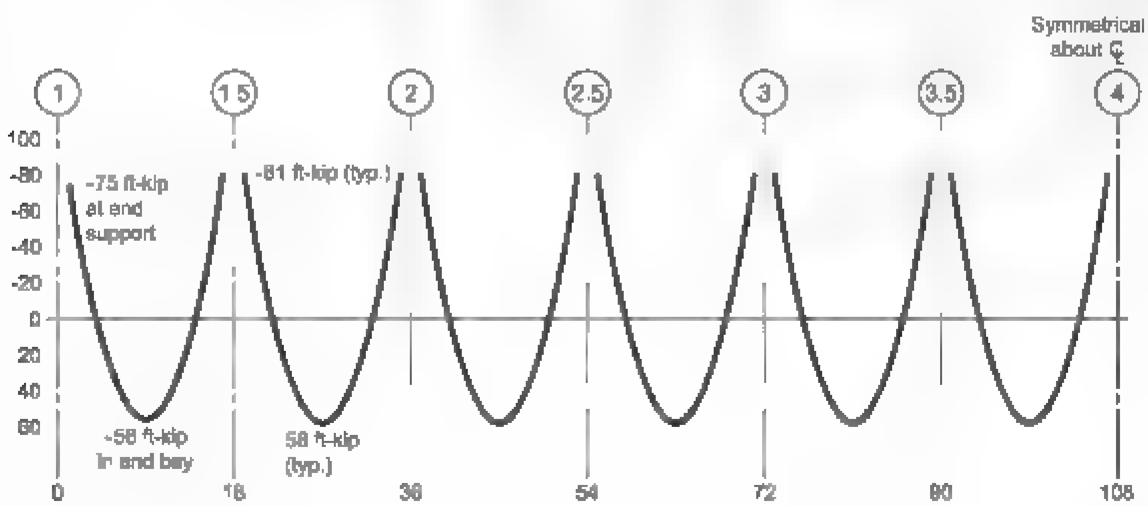


Fig. E2.8—Moment diagram maximum values at face of support and midspan.

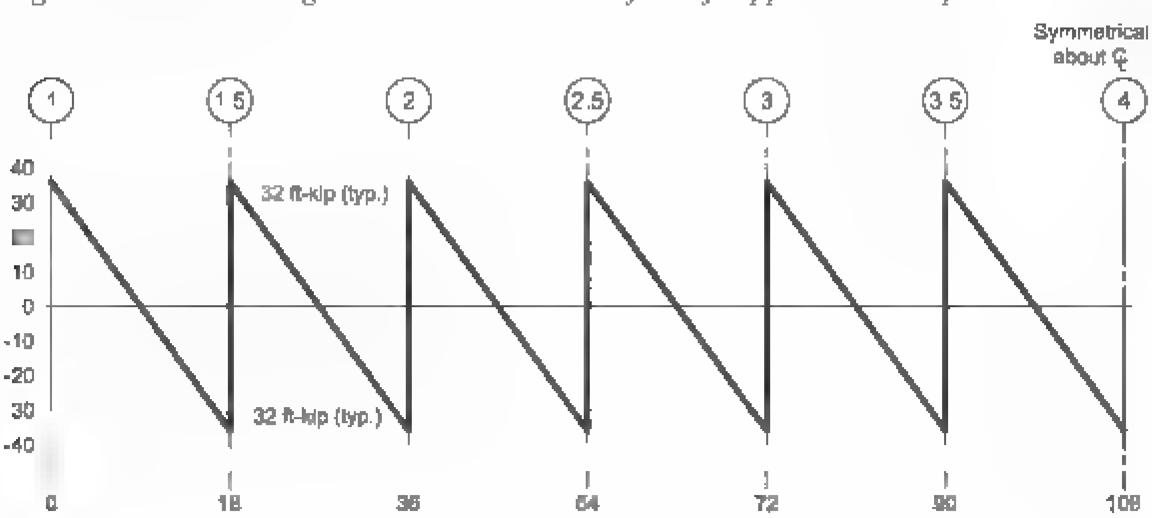


Fig. E2.9—Shear diagram maximum values at face of support.

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Step 5. Design

ACI 318-14 8.11.6.6	Moments determined from the EFM may be distributed to the column and middle strips in accordance with the Direct Design Method (DDM) in Section 8.10 of ACI 318-14. Continuing on, the design solution follows a similar method as the direct design method.	Refer to the DDM in the Two-way Slab Example of this Manual for this procedure.

Alternative method for determining stiffness coefficients for use in the moment distribution calculations
ACI 318-14, Section 6.3.1.1 states that:
"Relative stiffnesses of members within structural systems shall be based on reasonable and consistent assumptions."
This provision allows the designer to use any set of reasonable assumptions for determining the stiffnesses of the members in a two-way slab system in the EFM. In this example, the Hardy column analogy was used. An alternative method is suggested in the following discussion.

Given that Table 6.6.3.1.1(a) of ACI 318-14 will be used to account for the effects of cracking and the approximations in Table 6.6.3.1.1, detailed calculations for k_c to include the effects of rigid ends on the column stiffness are not warranted. (The effects of rigid ends are small compared to the effects of cracking.) Therefore, take $k_c = 4.0$ and:

Columns: $I_c = 0.7I_g = 0.7\frac{24(24)^3}{12} = 19{,}353\text{ in.}^4$

Walls: $I_w = 0.35I_g = 0.35\frac{168(12)^3}{12} = 8467\text{ in.}^4$

Slabs: $I_s = 0.25I_g = 0.25\frac{168(7)^3}{12} = 1200\text{ in.}^4$

Using these stiffness values to determine K_c to use for moment distribution calculations. Note that because all of the concrete strengths are the same, the modulus of elasticity is assumed equal to 1 ksi in this example.

Upper column:
$$K_c = \frac{k_c E_{cc} I_c}{\ell_c}$$
$$K_c = \frac{(4)(1)(19{,}353)}{216} = 358$$

Lower column:
$$K_c = \frac{k_c E_{cc} I_c}{\ell_c}$$
$$K_c = \frac{(4)(1)(19{,}353)}{137.5} = 563$$

Walls (neglecting torsion action with the column):
$$K_c = \frac{k_c E_{cc} I_w}{\ell_c}$$
$$K_c = \frac{(4)(1)(8467)}{137.5} = 246$$

Slabs:
$$K_s = \frac{k_s E_{cs} I_s}{\ell_1}$$
$$K_s = \frac{(4)(1)(1200)}{216} = 22$$

Combining these values with K_t (Step 2 torsional members from this Example), and using the resulting stiffness values in the moment distribution along with the fixed end moments (without modification of the fixed end moment factor—that is, FEM = $(1/12)(w\ell^2)$), gives results that are approximately 5 percent different from the values shown in the example above.

Two-way Slab Example 3: *Post-tensioned two-way slab design*

This two-way slab is a prestressed solid slab roof without beams between supports. The strength of the slab is checked and two-way shear reinforcement at the external columns is designed. Material properties were selected based on the code requirements of Chapters 5 and 6, engineering judgment, and known available materials.

Given:

Uniform loads—

Superimposed dead load $D = 0.015$ kip/ft^2

Roof live load $L = 0.040$ kip/ft^2

Material properties—

$f_c' = 5000$ psi

$f_y = 60{,}000$ psi

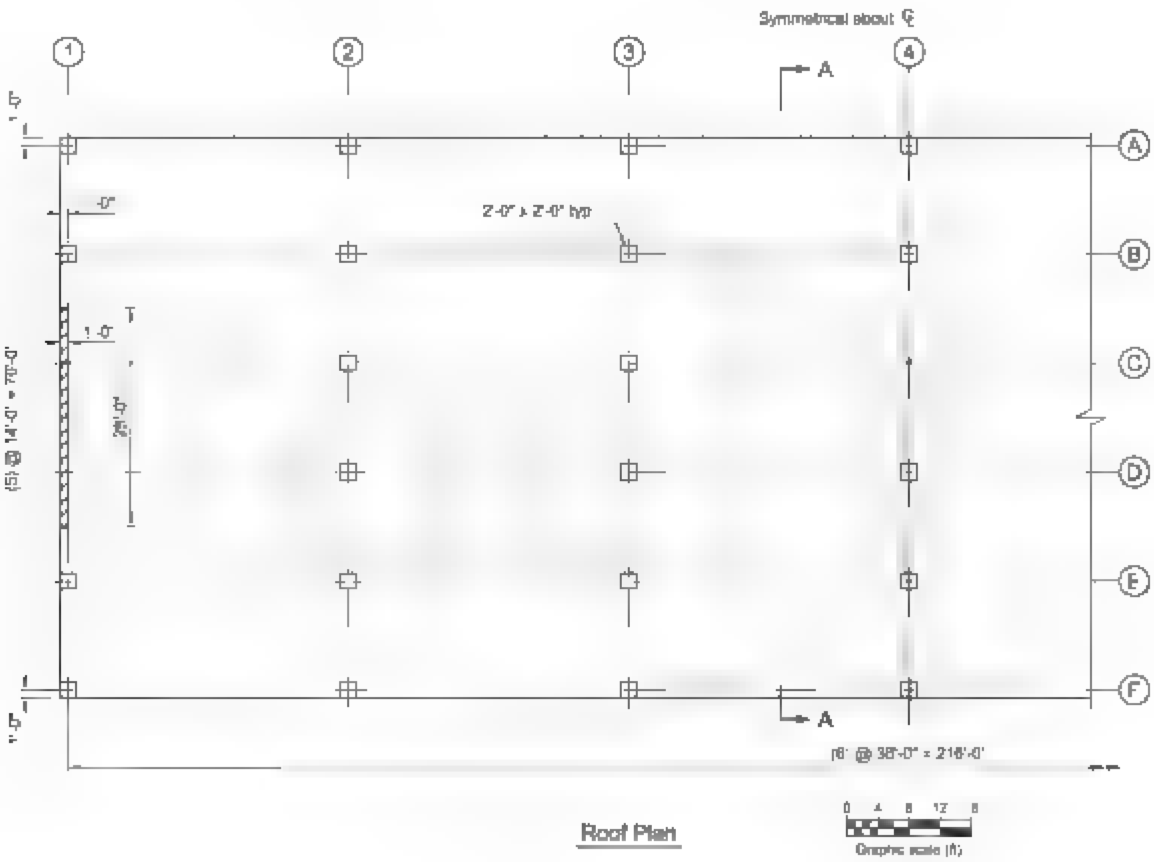

Fig. E3.1—Roof plan.

Two-Way Slabs

ACI 318	Discussion	Calculation
Step 1: Geometry		
8.3.2	In the direction taken, there are six spans of 36 ft. The slab is supported by 24 in. square columns. The ACI 318 span-to-depth ratios do not apply to post-tensioned (PT) slabs. Span depth ratios between 40 and 50 are typically reasonable for two-way slab designs (Nawy G., 2006, *Prestressed Concrete: A Fundamental Approach, Fifth edition*, Pearson Prentice Hall, New Jersey, 945 pp.) Use a ratio of 45 to set the initial thickness of the slab.	$45 \leq \frac{\ell}{t}$ $\ell = 432$ in. $t = 9.6$ in. Use a thickness of 10 in.
Step 2: Load and load patterns		
8.4.1.2	Loading all spans simultaneously does not necessarily produce the maximum flexural stresses in the slab. Therefore, in Section 6.4.3 of the Code, live load patterns are defined for use with two-way slab systems. Figure E3.2 shows examples of the different live load patterns considered in the Code. Section 6.4.3.2 is applicable for this example because the roof live load is less than 75 percent of the combined dead loads.	

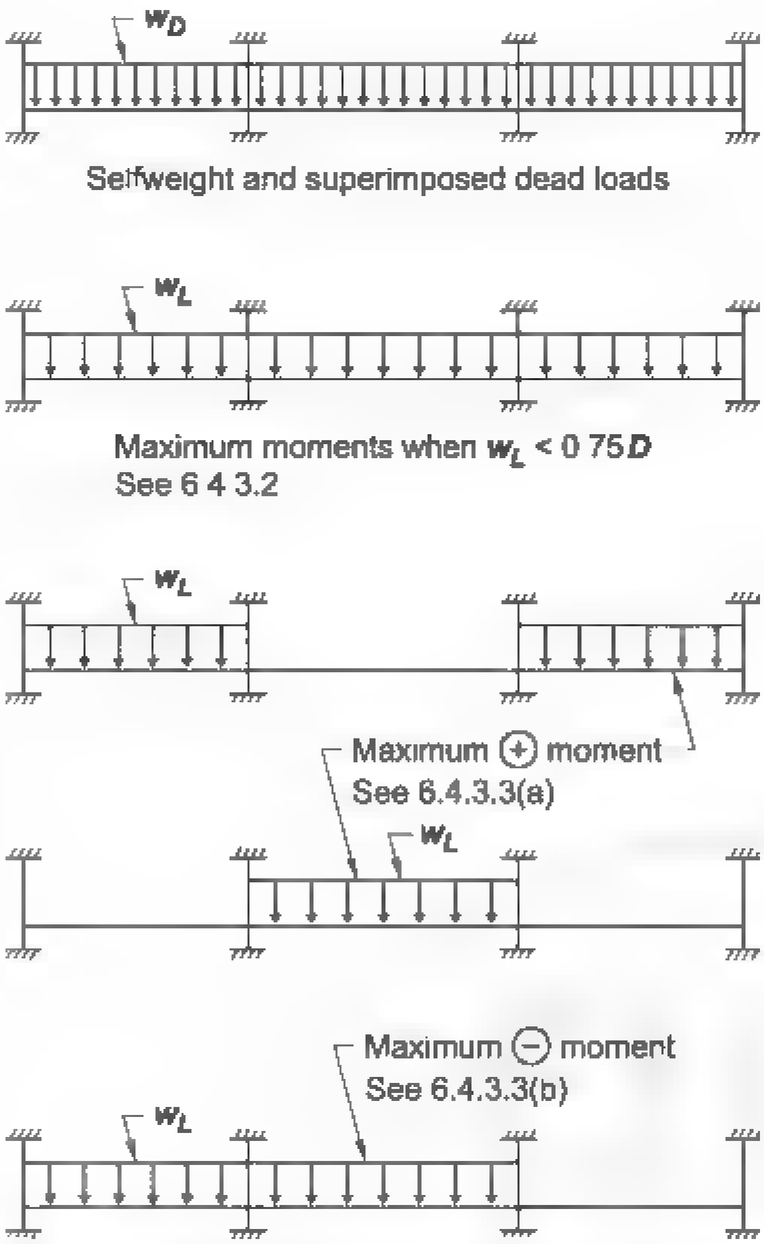

Fig. E3.2—Code live load patterns, example uses 6.4.3.2.

Step 3. Concrete and steel material requirements		
8.2.6.1	The mixture proportion must satisfy the durability requirements of Chapter 19 (ACI 318) and structural strength requirements. The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that correlates with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 (ACI 318) requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi.

8.2.6.2	The reinforcement must satisfy Chapter 20 of ACI 318. In this example, unbonded, 1/2 in. single strand tendons are assumed. The designer specifies the grade of bar and whether the reinforcing bar should be coated by epoxy or galvanized, or both. In this case, assume grade 60 bar and no coatings.	Chapter 20 (ACI 318) requirements are satisfied by specifying that the reinforcement shall be in accordance with ACI 301. This includes the PT type and strength, and reinforcing bar grade and any coatings for the reinforcing bar.
20.3	The Code requires strand material to be 270 ksi, low relaxation (ASTM A416). The U.S. industry usually stresses, or jacks, monostrand to impart a force equal to $0.80f_{pu}$, which is the maximum allowed by the Code.	The jacking force per individual strand is: $270\text{ ksi} \times 0.8 \times 0.153\text{ in.}^2 = 33\text{ kip}$ This is immediately reduced by seating and friction losses, and elastic shortening of the slab. Long-term losses will further reduce the force per strand. Refer to R20.3.2.6 of the Code.
	The final stress after all losses is usually between 60 to 64 percent of the specified tensile strength of low relaxation strands.	The design prestress force is usually expressed in terms of kip per foot of slab width. To estimate the tendon spacing, an effective prestress force of 26.5 kip per strand is commonly used for preliminary design purposes and will be used in this example. Refer to ACI 423.10R for a comprehensive treatment of the estimation of prestress losses.

Step 4: Analysis		
6.6	The analysis performed should be consistent with the overall assumptions about the role of the slab within the building system. Because the lateral force resisting system relies on the slab to transmit axial forces, a first order analysis is adequate.	<u>Modeling assumptions</u> According to 8.3.4.1, two-way slabs must be designed as Class U, which allows the use of gross section properties in the analysis for both service stresses and deflections. Service tension stresses in concrete must satisfy this equation
8.3.4.1		$f_t \le 6\sqrt{f'_c}$
	Although gravity moments are calculated independent of PT moments, the same model is used for both.	In practice, software programs utilizing the equivalent frame method or finite element analysis techniques are typically used for analysis. <u>Analysis approach</u> To analyze the flexural effects of post-tensioning on the concrete slab under service loads, the tendon drape is arranged to be parabolic with a discontinuity at the support centerline as shown below. This imparts a uniform uplift over each span when tensioned. The magnitude of the uplift, w_p, or "balanced load," in each span of a prismatic member is calculated as $w_p = \dfrac{8Fa}{\ell^2}$ where F is effective PT force and a is tendon drape (average of the two high points minus the low point). In this example the PT force is assumed constant for all spans, but the uplift force varies due to different tendon drapes. Figure E3.3 shows the tendon profile assumed in this example.

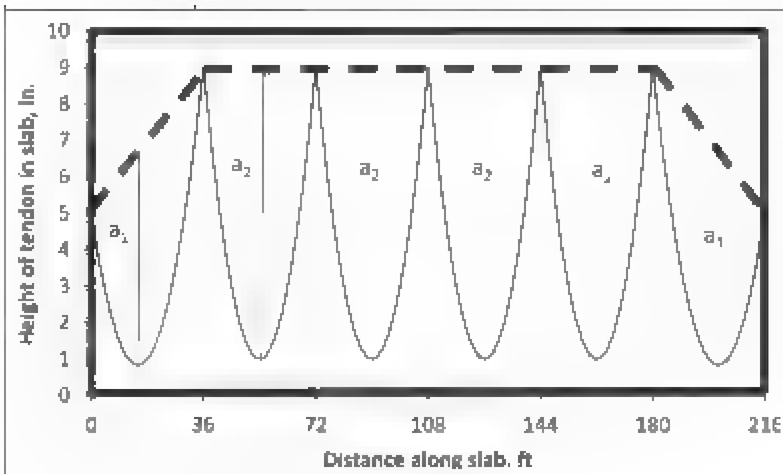

Fig. E3.3—Tendon profile

$$a_1 = \frac{(5\text{ in.} - 1\text{ in.}) + (9\text{ in.} - 1\text{ in.})}{2} = 6\text{ in.}$$

$$a_2 = \frac{(9\text{ in.} - 1\text{ in.}) + (9\text{ in.} - 1\text{ in.})}{2} = 8\text{ in.}$$

Two-Way Slabs

Step 5. Analysis—Slab stress limits		
8.3.4.1	The code requires a Class U slab assumption; that is, a slab under full service load with a concrete tension stress not exceeding $6\sqrt{f_c'}$. The slab analysis model for the service condition is the same as for the nominal condition	For 5000 psi concrete, this limit is $6\sqrt{5000\text{ psi}} = 424\text{ psi}$
	To verify that the concrete tensile stresses are less than $6\sqrt{5000\text{ psi}}$, the net service moments and tensile stresses at the face of supports are needed. This example assumes two parameters:	The basic equation for concrete tensile stress is $f_t = M/S - F/A$, where M is the net service moment, $S = \dfrac{bt^2}{6} = \dfrac{(12\text{ in.})(10\text{ in.})^2}{6} = 200\text{ in.}^3$ (section modulus),
8.6.2.1	(a) the PT force provides a F/A slab compressive stress of at least 125 psi (15 kip/ft) (b) the combination of PT force and profile provides a uplift force w_p of at least 75 percent of the slab weight, or 94 psf	and $A = bt = 12\text{ in.} \times 10\text{ in.} = 120\text{ in.}^2$ (gross slab area per foot)
	Solve for F:	At the exterior support, the drape is $a = 6$ in. The equation for $0.094\text{ kip/ft} = \dfrac{8Fa}{l^2} = \dfrac{8F(6\text{ in.})}{(36\text{ ft})^2(12\text{ in./ft})^2}$ $F = 30.5$ kip/ft

Service loads	Location from left to right along the span					
	First midspan	Second midspan	Third midspan	Fourth midspan	Fifth midspan	Sixth midspan
Gravity uniform load, psf	180	180	180	180	180	180
PT Uniform uplift, psf	94	126	26	126	126	94
Net load, psf	86	54	54	54	54	86

	Using the above information and performing an equivalent frame analysis (refer to Two-way Slab Example 2), the following maximum service moments in the slab are determined. Negative moment is maximum at the face of the first interior support and is 8.1 ft-kip/ft. Positive moment is maximum at midspan of the first and sixth spans and is 5.3 ft-kip/ft.	Use these moments to determine the stresses at service load. At the face of the first interior support, $f_t = -\dfrac{P}{A} + \dfrac{M}{S}$ $f_t = \left(-\dfrac{30.5\text{ kip}}{120\text{ in.}^2} + \dfrac{8.1\text{ ft-kip}}{200\text{ in.}^3} \times \dfrac{12\text{ in.}}{\text{ft}}\right)\left(\dfrac{1000\text{ lb}}{1\text{ kip}}\right)$ $f_t = 232\text{ psi} \leq 424\text{ psi}$ ∴ **OK** For the positive moment at midspan, it is usually desirable to avoid additional reinforcement required by Section 8.6.2.3. To avoid this, the tensile stresses in the slab should not exceed $2\sqrt{5000\text{ psi}} = 141\text{ psi}$ $f_t = -\dfrac{P}{A} + \dfrac{M}{S}$ $f_t = \left(-\dfrac{30.5\text{ kip}}{120\text{ in.}^2} + \dfrac{5.3\text{ ft-kip}}{200\text{ in.}^3} \times \dfrac{12\text{ in.}}{\text{ft}}\right) \times \left(\dfrac{1000\text{ lb}}{1\text{ kip}}\right)$ $f_t = 64\text{ psi} \leq 141\text{ psi}$ **OK**

Step 6: Analysis Deflections		
8.3.2	The two-way slab chapter refers the user to Section 24.2.2 (ACI 318) that states, "Deflections due to service-level gravity loads…" for allowable stiffness approximations to calculate immediate and time-dependent (long term) deflections. Section 24.2.2 provides maximum allowed span-to-deflection ratios. Section 24.2.3.8 permits using I_g to calculate deflections for Class U slabs. Commentary Section R24.2.3.3 of the Commentary alerts the designer that calculations for deflections of two-way slabs is challenging. This example determines the deflections in one direction and doubles it for the effect from the other direction. This is not an accurate assumption, but it should give conservative and reasonable results. Note that excessive deflections are generally not experienced in PT slabs and do not typically control the design.	The example assumes the deflections in each direction are identical and combines them to give the maximum deflection at the midpoint of the slab. Deflections are checked in the long direction of the slab. Deflections due to the uniform live load are checked in Section 24.2.2, therefore, the uniform live load only is applied in this deflection calculation. The analysis is approximate due to several simplifying assumptions, but it provides a reasonable result. $\Delta_{max} = \dfrac{0.0065w\ell^4}{EI} = \dfrac{0.0065(0.040/12)(432)^4}{4030(1000)} = 0.19$ in. Assuming twice this to account for the two-way action of the slab, $\Delta_{max} = 2 \times 0.19$ in. $= 0.38$ in. Expressed as a ratio, $\ell/\Delta = 432/0.38 = \ell/2400$. This is much less than the limit of $\ell/180$, so deflection limits are satisfied.
Step 7: Analysis Balanced, secondary, factored, and design moments		
	Balanced moments are determined using the PT Uniform uplift load from the table of service loads in Step 5 and performing an equivalent frame analysis (refer to Two-way Slab Example 2). Secondary moments are determined using balanced moments and primary moments. Factored moments are determined using the factored load combinations required by code and performing an equivalent frame analysis (refer to Two-way Slab Example 2). Design moments are determined by subtracting the secondary moments from the factored moments. The following table gives the balanced, secondary, factored, and design moments at the face of supports across the slab section. Figure E3.4 shows the design moments as determined by the equivalent frame analysis. The slab is symmetrical about the third column.	The moment curve below the table is the full design moment curve with critical section moments shown on the curve. The design moments shown are at the face of supports and at the point of maximum positive moment in the span.

	Location from left to right along the span (sym about Col 4)					
	Col 1 (ext)	Col 2 (ext)	Col 2 (int)	Col 3 (int)	Col 3 (int)	Col 4 (int)
Balanced moment, M_{bal}, ft-kip/ft	6.3	10	11.1	11.6	11.6	11.4
Eccentricity, e, in.	0	4	4	4	4	4
Primary moment, M_1, ft-kip/ft	0	10.2	10.2	10.2	10.2	10.2
Secondary moment, $M_2 = M_{bal} - M_1$, ft-kip/ft	6.3	-0.2	0.9	1.4	1.4	1.2
Factored load moment, M_u', ft-kip/ft	16.1	23.4	21.6	21	21.2	21.3
Design moment, $M_u = M_u' - M_2$, ft-kip/ft	9.8	23.6	20.7	19.6	19.8	20.1

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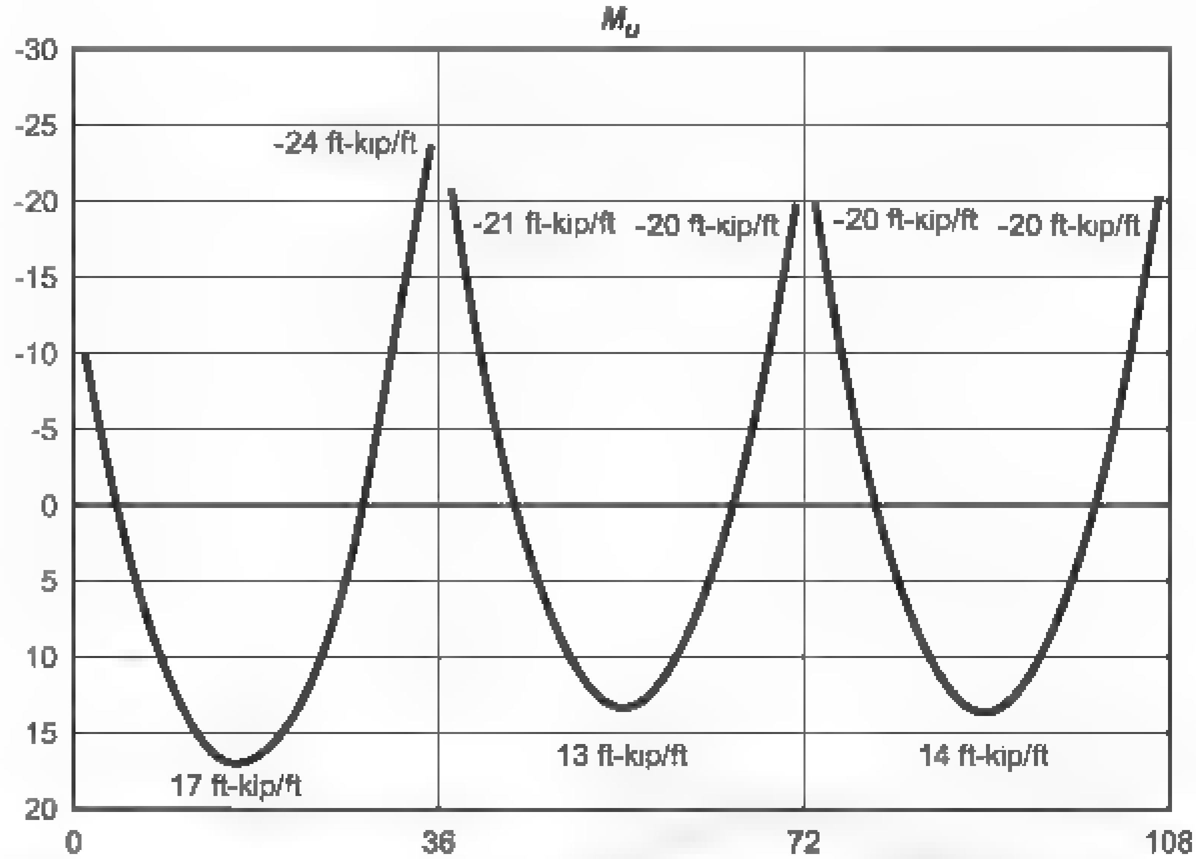

Fig. E3.4—Moment diagram (negative moment at face of column).

Step 8: Required strength – Calculate required A_s

8.7.5.2	<u>Check flexure strength considering PT tendons</u> If the PT tendons alone provide the necessary design strength, $\geq \phi M_n$, then the code permits reinforcement to be detailed with shorter cut-off lengths. If the PT tendons alone do not provide the design strength, then the reinforcement is required to conform to standard lengths. The reinforcing bar and tendons are usually at the same position near the support and midspan.	The depth of the equivalent stress block, a, is calculated by $a = \frac{A_{ps} f_{ps}}{0.85 f'_c (12\ \text{in./ft})}$ where A_{ps} is the tendon area per foot of slab. Section 8.5.2.1 refers to Section 22.3 of the Code for the calculation of ϕM_n. Section 22.3 refers to Section 22.2 for calculation of M_n. Section 22.2.4 refers to Section 20.3.2.4 to calculate f_{ps}. The span-to-depth ratio is 432/10 = 43, so the below equation applies $f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p}$
20.3.2.4	Each single unbonded tendon is stressed to the value prescribed by the supplier. The value of f_{se} (effective stress in the strand) varies along the tendon length due to friction losses (ACI 423.10R), but for design purposes, f_{se} is usually taken as the average value.	The tendon supplier usually calculates f_{se}, and 175,000 psi is a common value. The force per strand is therefore 175,000 psi x 0.153 in.^{2} = 26,800 lb. The required effective force per foot of slab is 30.5 kip/ft, so the spacing of tendons is 26.8 kip/30.5 kip/ft)(12 in./ft) = 10.5 in. The value of A_{ps} is therefore 0.153 in.^{2} × 12/10.5 = 0.175 in.^{2}/ft. The value of ρ_p is $A_{ps}/(b \times d_p)$ = 0.175/108 in.^{2} = 0.00162 $f_{ps} = 175{,}000 + 10{,}000 + \frac{5000}{0.486} = 195{,}000\ \text{psi}$ This value has upper limits of f_{se} + 30,000 (= 205,000 psi) and f_{py} (= 0.9f_{pu}, or 242,900 psi from commentary), so the design value of f_{ps} is 195,000 psi.

22.2.2.4.1 — Note that the effective depth is 9 in. at critical locations, except at the exterior joint.

The compression block depth is therefore

$$a = \frac{A_{ps}f_{ps}}{0.85f_c'(12\text{ in./ft})} = \frac{0.175 \times 195{,}000}{0.85 \times 5000 \times 12} = 0.67\text{ in.}$$

For 3/4 in. cover: $M_n = \phi A_{ps}f_{ps}(d - a/2) = 0.9 \times 0.175 \times 195{,}000 \times (9 - 0.34) = 266{,}000$ in.-lb/ft
$= 22$ ft-kip/ft

	Location from left to right along the span						
	Face of exterior support	First midspan	Face of second support	Second midspan	Face of third support	Third midspan	Face of fourth support
M_n, only tendons, ft-kip/ft	20	22	22	22	22	22	22
M_u, ft-kip/ft	10	17	24	13	21	14	20

M_n considering the tendons alone are greater than the design moments except at the face of the second support. The reinforcement required to resist the moments at the face of the second support are required to satisfy the detailing requirements of Section 7.7.3 of the Code while minimum reinforcing bar lengths can be used at all other locations.

Step 9: Required strength—Minimum area of bonded reinforcement

8.6.2.3 — The minimum area of flexural reinforcing bar per foot is a function of the slab's cross sectional area. A_{cf} is based on the greater cross sectional area of the slab-beam strips of the two orthogonal equivalent frames intersecting at a column of a two-way slab.

$A_{s,min} = 0.00075 \times A_{cf} = 0.00075 \times 10\text{ in.} \times 12\text{ in.} = 0.09\text{ in.}^2/\text{ft}$

Step 10: Required strength—Design moment strength of combined prestressing steel and bonded reinforcement

8.5.2 — Determine if supplying the minimum area of reinforcing bar is sufficient to achieve a design strength that exceeds the required strength.

Set the section's concrete compressive strength equal to steel tensile strength, and rearrange for compression block depth a:

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85f_c'(12)}$$

$$a = \frac{0.175 \times 195{,}000 + 0.09 \times 60{,}000}{0.85 \times 5000 \times 12}$$

$a = 0.78$ in.

For 3/4 in. cover

$$M_n = \phi\left[A_{ps}f_{ps} + A_s f_y\right]\left(d - \frac{a}{2}\right)$$

$$= 0.9\left[0.175 \times 195{,}000 + 0.09 \times 60{,}000\right](9 - 0.78)$$

$= 292{,}000$ in.-lb

$= 24.3$ ft-kip

Comparing this value with the required moment strength M_u indicates that the minimum reinforcement plus the tendons supply enough tensile reinforcement for the slab to resist the factored loads at all locations.

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Step 11. Analysis: Distribute moments to column and middle strips		
8.7.2.3	Tests and research have shown that for uniformly loaded structures variations in tendon distribution does not alter the deflection behavior or the capacity for the same total prestressing steel percentage. Section 8.7.2.3 provides specific guidance regarding tendon distribution that allows the use of banded tendon distribution in one direction.	The deflection behavior and capacity differences are not dependent upon the distribution of tendons. It can be extrapolated that distribution of moments to the column and middle strips is unnecessary.

Step 12. Required strength: Factored one-way shear		
8.4.3 8.4.3.1 8.4.3.2	One-way shear rarely controls thickness design of a two-way slab, but it must be checked. In this section, one-way shear load on the structure is determined.	Figure E3.5 shows one-way shear diagram with the one-way shear reduced to the face of support. Check maximum factored shear. V_u = 58 kip/14 ft = 4.1 kip/ft

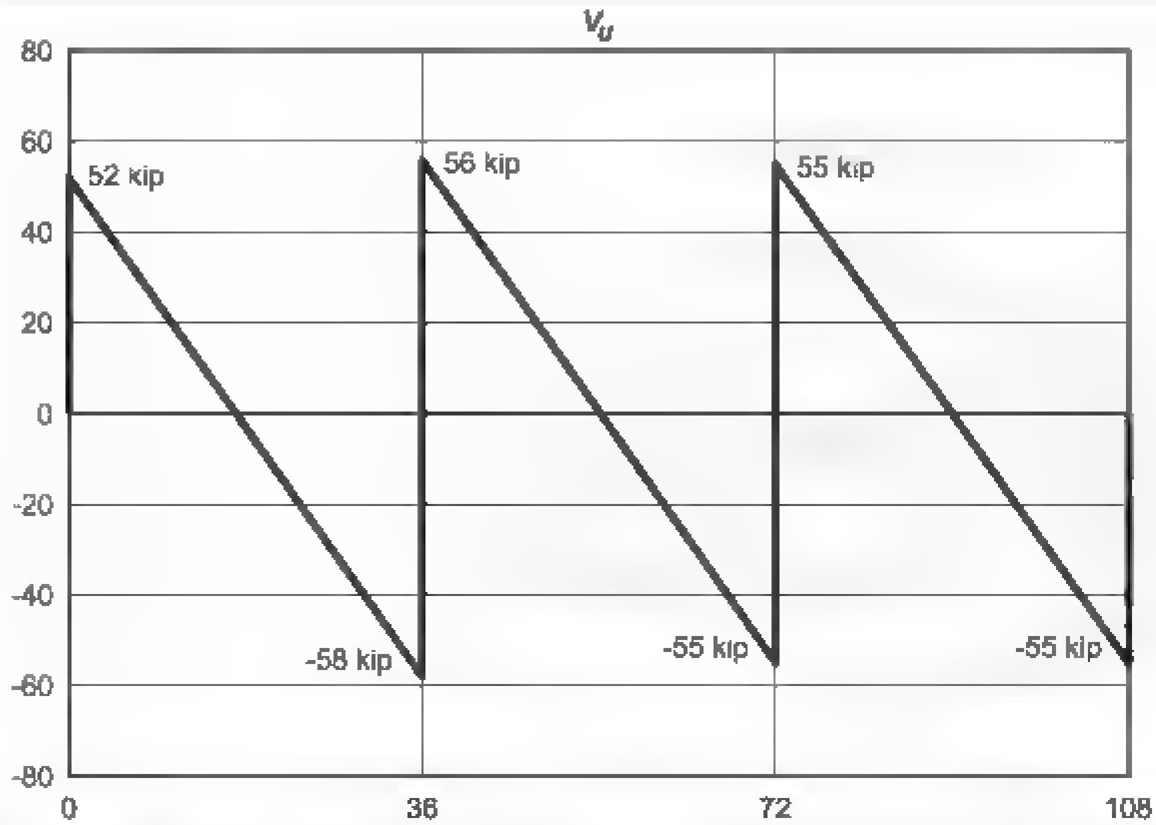

Fig. E3.5—Shear diagram.

Step 13. Required strength: Factored two-way shear		
8.3.1.4	No stirrups are to be used as shear reinforcement.	
8.4.4.1 22.6.4	Determine the location and length of the critical section for two-way shear assuming that shear reinforcement is not required. Figure E3.6 shows this examples critical sections. Note that only the exterior and interior columns are calculated in this example.	Exterior columns: d = 10 in. – 1 in. = 9 in. $b_o = 2\times\left(c_1+\frac{d}{2}\right)+(c_2+d)$ $b_o = 2\times\left(24\text{ in.}+\frac{9\text{ in.}}{2}\right)+(24\text{ in.}+9\text{ in.})$ $b_o = 90$ in. Interior columns: $b_o = \left(c_1+\frac{d}{2}\right)+\left(c_2+\frac{d}{2}\right)$ $b_o = 2\times(24\text{ in.}+9\text{ in.})+2\times(24\text{ in.}+9\text{ in.})$ $b_o = 132$ in.

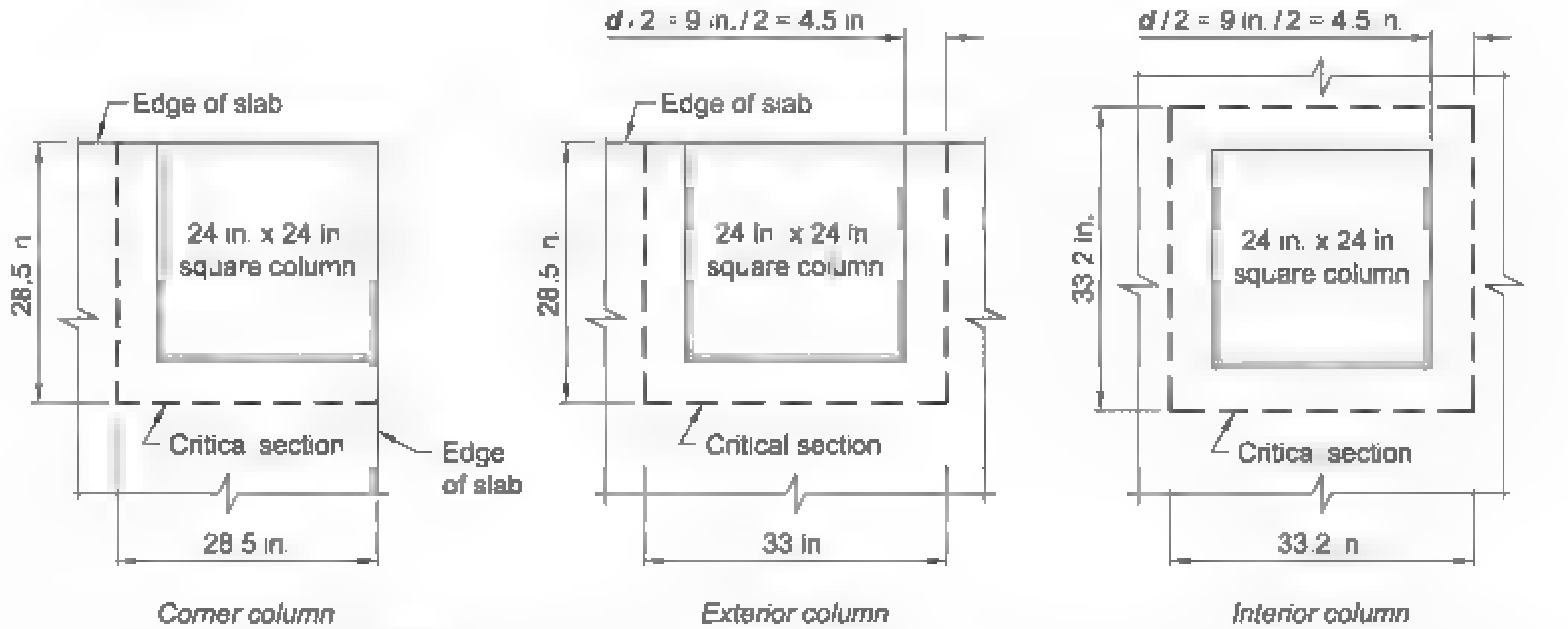

Fig. E3.6—Two-way shear critical section locations.

8.4.4.2, 8.4.4.2.1	Determine factored slab shear stress due to gravity loads v_u	Direct slab shear stress on slab critical section at the exterior columns $v_u = \dfrac{V_u}{b_o d}$ $V_u = \left(14\text{ ft}\times 18\text{ ft} - \dfrac{33\text{ in.}\times 28.5\text{ in.}}{144}\right)\times\dfrac{232}{1000}\dfrac{\text{kip}}{\text{ft}^2}$ $V_u = 57\text{ ksi}$ $v_u = \dfrac{57\text{ ksi}}{90\text{ in.}\times 9\text{ in.}} = 0.070\text{ ksi}$ Direct slab shear stress on slab critical section at the interior columns $v_u = \dfrac{V_u}{b_o d}$ $V_u = \left(36\text{ ft}\times 14\text{ ft} - \dfrac{(33\text{ in.})^2}{144}\right)\times\dfrac{232}{1000}\dfrac{\text{kip}}{\text{ft}^2}$ $V_u = 115\text{ kip}$ $v_u = \dfrac{115\text{ kip}}{132\text{ in.}\times 9\text{ in.}} = 0.097\text{ ksi}$

8.4.4.2.1 8.4.4.2.2	Determine the slab shear stress due to factored slab moment resisted by column	Shear stress on slab due to moments at exterior columns. $\gamma_v = 0.4$ $M_{sc} = 10\text{ ft-kip/ft} \times 14\text{ ft}$ $c_{AB} = 9.02\text{ in.}$ $J_c = 76383\text{ in.}^4$ $\dfrac{\gamma_v M_{sc} c_{AB}}{J} = 0.079\text{ ksi}$ Shear stress on slab due to moments at interior columns. $\gamma_v = 0.4$ $M_{sc} = 24 - 21\text{ ft-kip/ft} \times 14\text{ ft}$ $c_{AB} = 16.5\text{ in.}$ $J_c = 219633\text{ in.}^4$ $\dfrac{\gamma_v M_{sc} c_{AB}}{J} = 0.015\text{ ksi}$
8.4.4.2.3	Determine v_u by combining results from the two-way direct shear and the moment transferred to the column via eccentricity of shear	$v_u = v_{uv} + \dfrac{\gamma_v M_{sc} c_{AB}}{J}$ Exterior columns $v_u = 0.070\text{ ksi} + 0.079\text{ ksi} = 0.149\text{ ksi}$ Corner columns. $v_u = 0.097\text{ ksi} + 0.015\text{ ksi} = 0.112\text{ ksi}$
Step 14. Design strength – One-way shear		
8.5.3.1.1 22.5	Shear reinforcement is not typically used in one-way slabs so all of the shear strength is provided by the concrete contribution ($\phi V_n = \phi V_c$)	
22.5.6.2	For prestressed concrete, if the prestress level satisfies the code minimum, then the concrete contribution to shear is calculated from this equation $V_c = 2\sqrt{f'_c}bd$ Check effective prestress level using the following equation $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_sf_y)$	Use an effective prestress for strands of 26.5 kip/strand and a spacing of 10 in. $A_{ps}f_{se} = 175\text{ ksi}(0.153\text{ in.}^2)\left(\dfrac{12\text{ in.}}{10\text{ in.}}\right) = 32.1\text{ kip}$ $0.4(A_{ps}f_{pu} + A_sf_y)$ $0.4\left[270\text{ ksi}(0.153\text{ in.}^2)\left(\dfrac{12\text{ in.}}{10\text{ in.}}\right) + 0.09\text{ in.}^2(60\text{ ksi})\right] = 22.0\text{ kip}$ $A_{ps}f_{se} > 0.4(A_{ps}f_{pu} + A_sf_y)$ **OK** $V_c = (2\sqrt{5000}\text{ psi})(12\text{ in.})(9\text{ in.}) = 15.3\text{ kip}$ $\phi V_c = 0.75(15.3) = 11.48\text{ kip} > V_u = 4.1\text{ kip}$ **OK** Design shear strength from concrete contribution is adequate.

Step 15: Design strength Two-way shear		
22.6.5.1	Determine two-way shear strength contributed by concrete to find if shear reinforcement is required	
22.6.5.2	Determine the nominal two-way shear strength. Strength is represented in terms of shear stress (v_c) and is the least of the following: $4\lambda_s\lambda\sqrt{f_c'}$ $\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'}$ $\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'}$	
22.5.5.1.3	Size effect factor: $\lambda_s=\sqrt{\frac{2}{1+0.1d}}$ $\lambda_s\leq 1.0$	$\lambda_s=\sqrt{\frac{2}{1+0.1(6)}}=1.2$ $\lambda_s = 1.0$ Upper limit on size effect controls $\lambda = 1.0$ Exterior column $4(1.0)(1.0)\sqrt{5000}$ psi = 283 psi **Controls** $\left(2+\frac{4}{1.0}\right)(1.0)(1.0)\sqrt{5000}$ psi = 424 psi $(2+2.73)(1.0)(1.0)\sqrt{5000}$ psi = 334 psi $\phi v_n = 0.75(283\text{ psi}) = 212$ psi This is greater than the required strength for interior coluns of 0.149 ksi from Step 6, therefore, two-way shear at interior columns is okay. Two-way shear reinforcement is not required at this location. Interior column $4(1.0)(1.0)\sqrt{5000}$ psi = 283 psi **Controls** $\left(2+\frac{4}{1.0}\right)(1.0)(1.0)\sqrt{5000}$ psi = 424 psi $(2+2.5)(1.0)(1.0)\sqrt{5000}$ psi = 318 psi $\phi v_n = 0.75(283\text{ psi}) = 212$ psi This is greater than the required strength for interior columns of 0.112 ksi from Step 6, therefore, two-way shear at interior columns is okay. Two-way shear reinforcement is not required at this location.

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Step 16: Reinforcement detailing—General requirements		
8.7.1 8.7.1.1 20.5.1	Concrete cover, development lengths, and splice lengths are determined in these sections	Concrete cover is determined using Table 20.5.1.3.2 (ACI 318). The bottom of this slab is not exposed to weather or in contact with the ground. The specified cover is 0.75 in.
8.7.1.2 25.4	Development length is used for splice length determination assuming No. 5 bars.	
25.4.3	Determine required development length using simplified formulas from Table 25.4.2.3 for No. 6 bars and smaller, and for clear spacing of bars at least $2d_b$ and clear cover at least d_b.	d_b = 0.625 in. < 0.75 in. clear cover $2d_b$ = 1.25 in. < bar spacing
25.4.3.1	$\ell_d \geq \left(\frac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f'_c}}\right)d_b$ ℓ_d > 12 in.	$\lambda = 1.0$
25.4.3.2	ψ_t = casting position ψ_e = epoxy ψ_g = reinforcement grade	Bars are cast with less than 12 in. of fresh concrete below the bars. $\psi_t = 1.0$ Bars are uncoated $\psi_e = 1.0$ Bars are Grade 60 $\psi_g = 1.0$ Required hook development length $\frac{6000\text{ psi}(1.0)(1.0)(1.0)}{25(1.0)\sqrt{5000\text{ psi}}}(0.625\text{ in.}) = 21.2\text{ in.}$ use 22 in.
8.7.1.3 25.5	It is likely that splices are required during construction. Allowable locations for splices are shown in ACI 318, Fig. 8.7.4.1.3	Lap splice lengths are determined in accordance with Table 25.5.2.1 (ACI 318). The provided A_s is not more than two times larger than the required A_s. Therefore, class B splices are required. ℓ_{st} = 1.3 × 21.2 in. = 27.5 in. use ℓ_{st} = 28 in.
Step 17: Reinforcement detailing—Spacing requirements		
8.7.2 8.7.2.1 25.2.1 8.7.2.3	Minimum and maximum spacing requirements are determined. The bar spacing for required strength reinforcement are also reviewed	Minimum spacing is determined in accordance with Section 25.2.1. Minimum spacing is 1 in., d_b, and $(4/3)d_{agg}$. Assuming that the maximum nominal aggregate size is 1 in., then the minimum clear spacing is 1.33 in. With a No. 5 bar, this equates to a minimum center-to-center spacing of approximately 2 in. Maximum spacing is controlled by Section 8.7.2.3. Assuming that this direction is banded, the maximum spacing requirements of Section 8.7.2.3 are not applicable to this direction. The tendons in the orthogonal direction are limited to a maximum spacing of 5 ft.

Step 18: Reinforcement detailing – Reinforcement termination		
8.7.5.2 8.7.5.5	Reinforcement termination is controlled by Section 8.7.5.2	Bonded nonprestressed reinforcement is required for flexure in one location and Section 8.7.5.2 controls termination of the minimum bonded reinforcement in that location. When the termination location is determined per Section 8.7.5.2, it is approximately 6 in. beyond the face of support. This termination location is within the minimum lengths of Section 8.7.5.5. Therefore, the termination locations indicated in Section 8.7.5.5 satisfy the termination location required by Section 8.7.5.2 for the locations requiring bonded nonprestressed reinforcement for flexural strength.
Step 19: Reinforcement detailing – Structural integrity		
8.7.5.6	Structural integrity is met using detailing	Requirement Section 8.7.4.2.2 is met when at least two of the PT tendons pass through the column inside the column reinforcement cage. In this direction, banding of the post-tensioning tendons makes this a simple requirement to satisfy.
Step 20: Slab-column joints		
8.2.7 15.2.9 15.3.2 15.5	Joints are designed to satisfy Chapter 15 of ACI 318. Slab-column connections transferring moment must satisfy strength and detailing requirements of Chapter 8, 15.3.2, and 22.6	The concrete strength of the slab and columns are identical and therefore, Sections 15.2.2 and 15.5 are met. Chapter 8 and 22.6 requirements are addressed in Steps 7 and 10 of this example. Section 15.3.2.1 applies to columns along the exterior of the building. The tie spacing determined from 25.7.2 for the column design will likely be larger than the joint depth of 10 in. If that is the case, then only one tie is required within the slab-column joint in exterior columns.
	Note: Design post-tensioning is 30.5 kip/ft One strand every 10 in. with this profile. Minimum bonded reinforcement required is 0.09 in.^{2}/ft. Figure E3.7 shows the final configuration of the slab.	Assuming (26.5 kip/strand)(1 strand/x in.) = (30.5 kip/ft)/(12 in.) No. 4 bars: 0.2 in.^{2}/(x in.) = 0.09 in.^{2}/0.09 = 26.7 in. → No. 4 bar every 2 ft

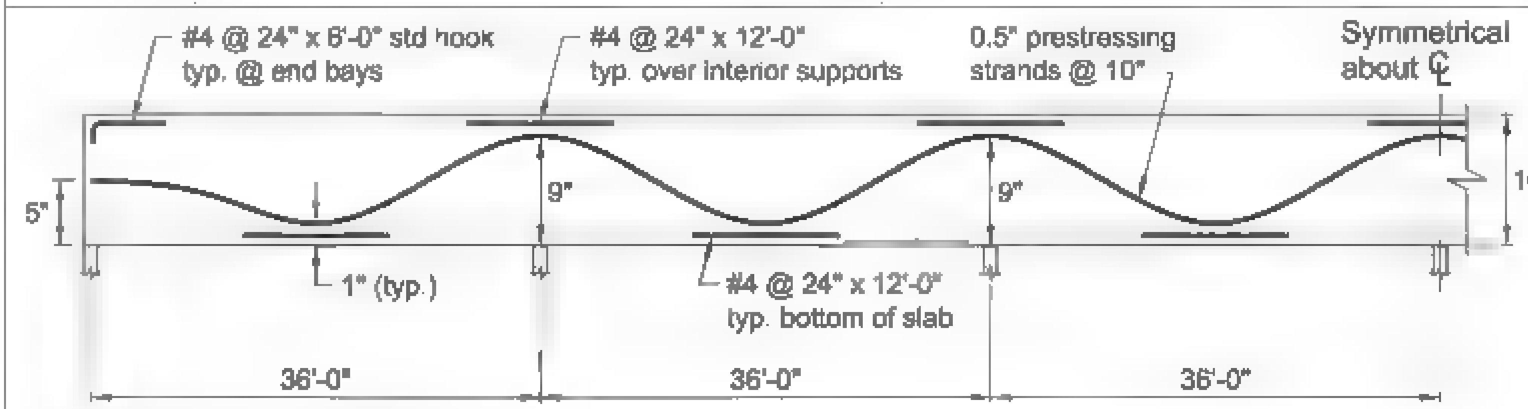


Fig. E3.7—Reinforcement detailing.

Note: A minimum of two unbonded PT strands must be placed in both directions through the column cage at each supporting column.

CHAPTER 7—BEAMS

7.1—Introduction

Structural beams resist gravity and lateral loads, and any combination thereof, and transfer these loads to girders, columns, or walls. Code Chapter 9 applies to both nonprestressed and prestressed beams as well as composite beams. Composite beams are composed of elements constructed in separate placements that are connected such that they act as a single unit. Special provisions are included in the chapter covering one-way joists (Section 9.8) and deep beams (Section 9.9). Deep beams are also addressed in Code Chapter 23, Strut-and-Tie Method.

Beams are designed in accordance with Code Chapter 9 for strength and serviceability. Beams are assumed to be approximately horizontal, with rectangular or T-shaped (a stem and a flange) cross sections. The flange width of T-shaped beams is geometrically limited by Code Sections 6.3.2 for flexure and 9.2.4.4 for torsion, respectively. The flange is assumed to contribute to the beam's flexural and torsional strength.

Beams, either nonprestressed or prestressed, that are monolithic with the floor framing, can be considered laterally braced. For beams that are not monolithic with the floor, Code Section 9.2.3.1 provides guidance on the spacing of lateral bracing.

For cast-in-place construction, connections to other members is covered in Code Chapter 15 and for precast members connections are covered in Code Section 16.2.

7.2—Service limits

7.2.1 *Beam depth*—The engineer determines the beam's concrete strength, steel strength, and other material characteristics to achieve the design performance criteria for strength and service life.

After defining the material properties and the beam's design loads, the engineer chooses the beam's dimensions. These are either provided by architectural constraints, attained from experience, or reached by assuming a depth and width and then adjusting iteratively until the beam design meets the designated constraints. Beam depth is addressed in Code Table 9.3.1.1, which applies if a beam is nonprestressed, not supporting concentrated loads along its span, and not supporting or attached to partitions that may be damaged by deflections.

For prestressed beams, the Code does not provide a minimum span-to-depth ratio, but rather requires that both immediate and time-dependent deflections be calculated in accordance with Code Section 24.2 and checked against the limits in Code Section 24.2.2. For a superimposed live load in the range of 60 to 80 lb/ft^2, a usual span-to-depth ratio is in the range of 20 to 30. Table 9.3 of *The Post-Tensioning Manual* (Post Tensioning Institute [PTI] 2006) lists span-to-depth ratios for different members that have been found from experience to provide satisfactory structural performance.

The slab thickness is considered as part of the overall beam depth if the beam and slab are monolithic or if the slab is composite with the beam in accordance with Code Chapter 16.

7.2.2 *Deflections*—For all prestressed beams and nonprestressed beams that have depths less than those in Code Table 9.3.1.1, deflections must be calculated. For unusually heavy loads—usually one- or two-way slabs subjected to above 100 lb/ft^2—or for unusual configurations such as heavy concentrated loads, it is prudent to calculate deflections. Equations for calculating deflections can be found in Volume 3 of this Manual, *ACI Reinforced Concrete Design Handbook Design Aid – Analysis Tables*. The calculated deflections should not exceed the limits in Code Table 24.2.2, after consideration of time-dependent deflections. Chapter 14 of this Manual includes several examples of deflection calculations using design aids for T- and L-shaped cross section beams.

7.2.3 *Reinforcement strain limits and concrete service stress*

7.2.3.1 *Strain limits*—Nonprestressed beams with a design axial force less than 10 percent of gross sectional strength ($< 0.1f_c'A_g$) must be designed so that they are tension controlled (Code Section 9.3.3), which requires the net tensile strain in the extreme tension reinforcement to be at least 0.005 for Grade 60 reinforcement. For higher grade reinforcement the limit is

$$\varepsilon_t > \varepsilon_{ty} + 0.003 \qquad (7.2.3.1a)$$

where ε_{ty} is the yield strain of the reinforcement. This limitation does not apply to prestressed beams and effectively provides an upper limit to the quantity of reinforcement to ensure yielding behavior in case of overload. This provision is included in the 2019 Code to accommodate use of higher grades of reinforcement. Codes prior to 2019 specified a minimum strain limit of 0.004 for nonprestressed flexural members.

The Code contains no explicit concrete or steel service stress limits for nonprestressed beams. For prestressed beams, however, permissible concrete service stresses are addressed in Code Section 24.5.3.

For prestressed beams, the analysis of concrete flexural tension stresses is a critical part of the design. Code Section 9.3.4.1 provides provisions to classify beams presented previously, as U (uncracked), C (cracked), or T (transition).

7.2.3.2 *Concrete stresses in prestressed beams*—Before the beam flexural stresses can be calculated, the tendon profile needs to be defined. The position of the tendon in the section and the force imparted by the tendon creates axial and flexural stress in the section that must be limited to avoid overstressing the concrete. These stresses can be calculated on a section basis using first principles. Alternatively, "load balancing"—a method commonly used to design post-tensioned construction—can be used to determine the stresses. In this method, the transverse force applied by the tendon to the concrete section is used to balance a portion of the load due to self-weight. For instance, a parabolically

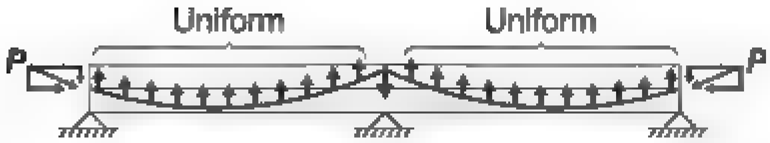


Fig. 7.2.3.2—Load balancing concept.

draped tendon such as that shown in Fig. 7.2.3.2 will balance a uniformly distributed load caused by the self-weight of the slab or beam. The abrupt change in tendon profile required at supports is typically idealized as an angular "break" at the column centerlines. Tendon anchors are usually positioned at section centroid when terminating at the end of a member. Load other than the "balanced load" will cause flexural stresses that must be controlled to avoid overstressing the concrete. To ensure the most effective use of the prestressing reinforcement, the maximum possible tendon eccentricity is typically used with due consideration to the shape of the profile and concrete cover requirements.

7.3—Analysis

Beams can be analyzed by any method satisfying equilibrium and geometric compatibility, provided design strength and serviceability requirements are satisfied. Code Chapter 6 allows for nonprestressed beams satisfying the conditions of Section 6.5.1 to use a simplified approximate method to calculate the design moment and shear forces in beams at the face of support and at midspan. Redistribution of design moments calculated by this method is not permitted.

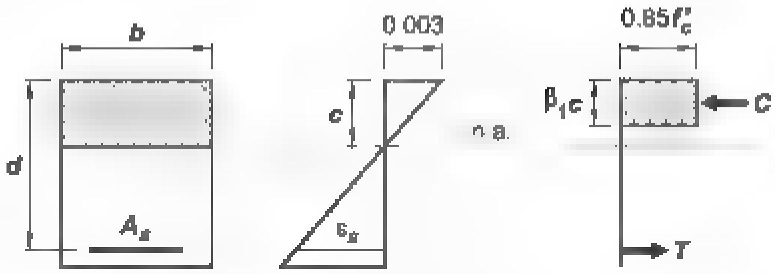
Beam moments, shear, and deflections along the beams' length are commonly calculated from classical elastic structural analysis methods. The supplement to this Manual, *ACI Reinforced Concrete Design Handbook Design Aid – Analysis Tables*, provides equations to calculate moment and shear forces at beam supports and midspan for various boundary and loading conditions. The moment of inertia and modulus of elasticity values used in these equations are addressed in the Code. Redistribution of elastic moments calculated by a classical method is permissible for members with sufficient ductility.

The engineer can also use finite element software to calculate moments, shear, and deflections along the beams' length. The moment of inertia and modulus of elasticity values used in the finite element model should be carefully considered to obtain realistic deflections and design forces. Redistribution of elastic moments calculated by an elastic finite element method is permissible for members with sufficient ductility.

7.4—Design strength

Beams resist self-weight and applied loads, which can result in beam flexure, shear, torsion, and axial force. At each section along a beam's length, the design strength is at least equal to the factored load effects, mathematically expressed as $\phi S_n \geq S_u$.

7.4.1 *Flexure*—Reinforced concrete beam design for flexure typically involves a sectional design that satisfies the conditions of static equilibrium and strain compatibility across the depth of the section.



$\beta_1 = 0.85$ for $f'_c \leq 4000$ psi

$\beta_1 = 0.85 - 0.05(f'_c - 4000)/1000 \geq 0.65$ for $f'_c > 4000$ psi

Fig. 7.4.1—Assumed strain and stress at nominal flexural strength.

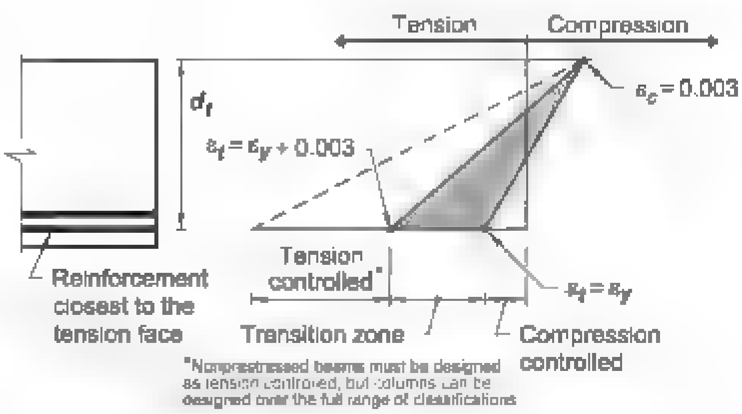


Fig. 7.4.1.1—Strain distribution.

Following are the assumptions for strength design method listed in Code Section 22.2; five of these are highlighted as follows:

(a) Strains in reinforcement and concrete are directly proportional to the distance from neutral axis (plane sections remain plane after loading).

(b) Maximum concrete compressive strain in the extreme compression fibers is 0.003 in./in.

(c) Stress in reinforcement varies linearly with strain up to the specified yield strength f_y. The stress remains constant beyond this point as strains continue increasing. The strain hardening of steel is ignored.

(d) Tensile strength of concrete is neglected.

(e) Concrete compressive stress distribution is assumed to be rectangular (Fig. 7.4.1).

7.4.1.1 *Nominal* (M_n) *and design flexural strength* (ϕM_n)—M_n is calculated from internal forces using an assumed ultimate usable concrete compressive strain capacity of 0.003 in./in. Beam ductility depends on the strain level in the extreme tension reinforcement at nominal flexural strength (Code Section 21.2.2), which is used to classify the beam as tension-controlled, $\varepsilon_t \geq \varepsilon_{ty} + 0.003$, compression-controlled, $\varepsilon_t \leq \varepsilon_{ty}$, or transition, $\varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003$. ε_t is the strain in the layer of steel closest to the tension face as illustrated in Fig. 7.4.1.1. ε_{ty} is the yield strength of the reinforcement and, for Grade 60 reinforcement, can be assumed to be 0.002. Before ACI 318-14, the compression-controlled strain limit was defined as 0.002 for Grade 60 reinforcement and all prestressed reinforcement, but it was not explicitly defined for other types of reinforcement. In the 2019 Code, to accommodate nonprestressed reinforcement of higher

grades, the tension-controlled limit was changed from 0.002 to ε_{ty} + 0.003. Test data cited by Mast (1992) show that, when using reinforcement with strengths higher than Grade 60, this expression will ensure that tension-controlled beam designs will have adequate ductility.

Reinforced concrete beams behave in a ductile manner by limiting the area of reinforcement such that the tension reinforcement yields before concrete crushes. Tension-controlled beam sections have a restricted amount of reinforcement, which improves the likelihood of ductile behavior at nominal strength. This allows redistribution of stresses and sufficient steel yielding to warn against an imminent failure. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members (if factored axial compression is less than $0.1f_c'A_g$). Beginning with the 2019 Code, however, this limit was revised to require that the section be tension-controlled (Code Section 9.3.3.1.)

The basic design inequality is that the factored moment must not exceed the design flexure strength, mathematically expressed as $M_u \le \phi M_n$.

7.4.1.2 *Rectangular sections with only tension reinforcement*—Nominal moment strength of a rectangular section with nonprestressed and prestressed tension reinforcement is calculated from the internal force couple shown in Fig. 7.4.1. The area of reinforcement is calculated from the

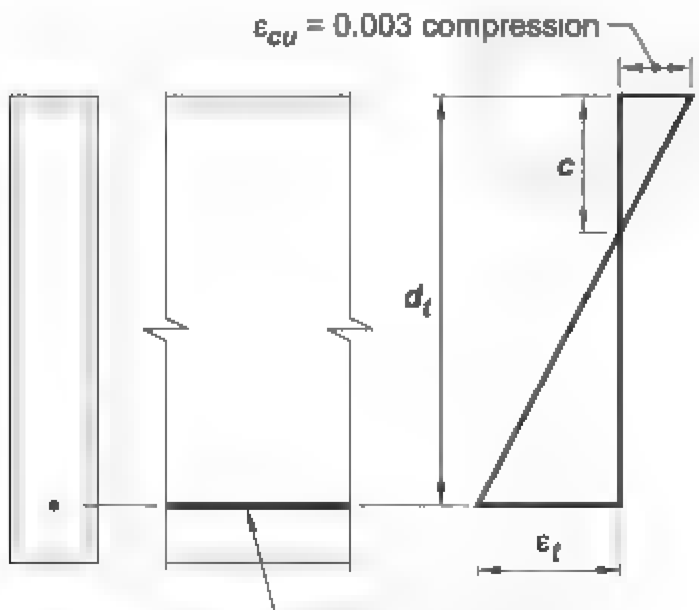


Fig. 7.4.1.2—Strain distribution and net tensile strain in a nonprestressed beam.

equilibrium of forces. It is assumed that tension steel yields before concrete reaches the assumed compression strain limit of 0.003 in./in. Accordingly, from equilibrium, set steel strength equal to concrete strength

$$T = C \qquad (7.4.1.2a)$$

Substituting the corresponding components for T and C

$$A_s f_y + A_{ps} f_{ps} = 0.85 f_c' \beta_1 c b \qquad (7.4.1.2b)$$

where f_{ps} is calculated in Code Section 20.3. Assume that $a = \beta_1 c$ and rearrange expressions

$$a = \beta_1 c = \frac{A_s f_y + A_{ps} f_{ps}}{0.85 f_c' b} \qquad (7.4.1.2c)$$

Take moments about the concrete resultant, and M_n is calculated as

$$M_n = T\left(d - \frac{\beta_1 c}{2}\right) = (A_s f_y)\left(d - \frac{a}{2}\right) + (A_{ps} f_{ps})\left(d_p - \frac{a}{2}\right) \qquad (7.4.1.2d)$$

For reinforced concrete sections with a single layer of tension reinforcement, $d = d_t$ and $\varepsilon_s = \varepsilon_t$ (Fig. 7.4.1.2). The stress block geometric parameter β_1 is between 0.85 and 0.65. For concrete strengths higher than 8000 psi, the value of β_1 should be reviewed (Ozbakkaloglu and Saatcioglu 2004; Ibrahim and MacGregor 1997). For nonprestressed beams, the $A_{ps}f_{ps}$ term in Eq. (7.4.1.2c) and (7.4.1.2d) of this Manual is deleted.

7.4.1.3 *Rectangular sections with tension and compression reinforcement*—Generally, beams are designed with tension reinforcement only. To add moment strength, designers can increase the tension reinforcement area or the beam depth. The cross-sectional dimensions of some applications, however, are limited by architectural or functional considerations, and additional moment strength can be provided by adding an equal area of tension and compression reinforcement. The internal force couple adds to the sectional moment strength without changing the section's ductility. In such cases, the total moment strength consists of adding two fictitious moments: M_1 = moment strength from the tension reinforcement-concrete compression couple; and M_2 = moment strength

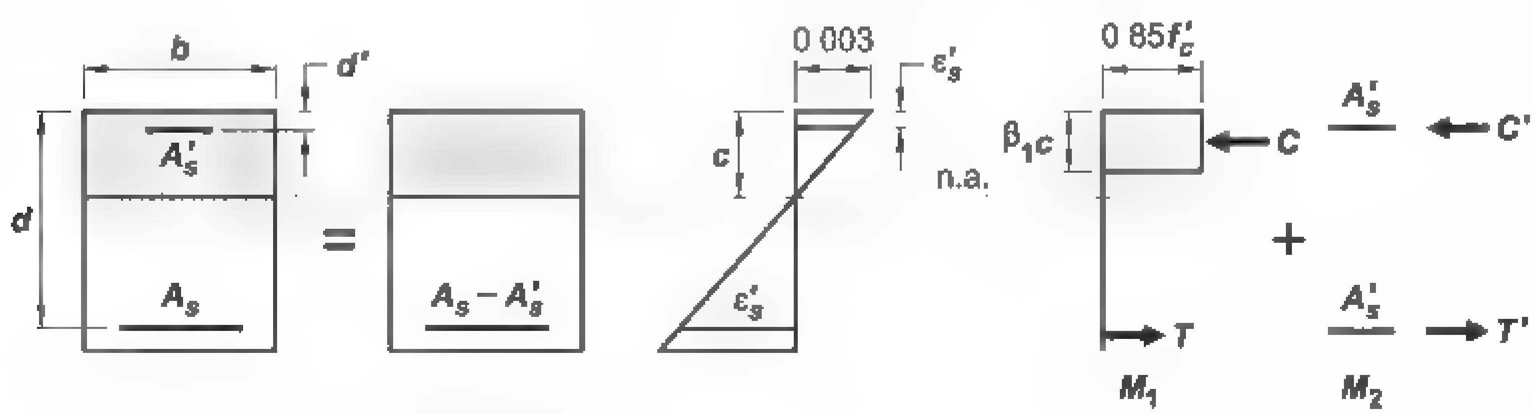


Fig. 7.4.1.3—Forces in a doubly reinforced concrete beam.

from the additional tension reinforcement-compression reinforcement couple, assuming both sets of reinforcement yield, as illustrated in Fig. 7.4.1.3

$$M_n = M_1 + M_2 \tag{7.4.1.3a}$$

where M_1 is determined by summing moments about the resultant of the compressive force, C

$$M_1 = f_y(A_s - A_s')\left(d - \frac{\beta_1 c}{2}\right) \tag{7.4.1.3b}$$

M_2 is determined by summing moments about the centroid of the tension reinforcement and assuming that the compression reinforcement has yielded

$$M_2 = A_s'(f_y - f_c')(d - d') \tag{7.4.1.3c}$$

The term $f_y - f_c'$ accounts for the slight reduction in the area of the compressive stress block due to the area occupied by the compression steel.

Because the steel couple does not require an additional concrete force, adding more tension steel does not create an over-reinforced section as long as an equivalent area is added in the compression zone. The underlining assumption in calculating the steel force couple is that the steel in compression yields at nominal strength, developing a force equal to the tensile yield strength. This assumption is true in most heavily reinforced sections because the compression Grade 60 steel (0.002 in./in. yield strain) is near the extreme compression fiber, which will strain to 0.003 in./in. at nominal strength.

Depending on the location of the compression reinforcement within the overall strain diagram, it is possible that the compression reinforcement has less strain than 0.002 at nominal strength and, therefore, does not yield. The designer, in this case, increases the compression reinforcement area proportional to the ratio of yield strain to compression steel strain. The strain in the compression steel, ε_s', can be computed from Fig. 7.4.1.3 as $\varepsilon_s' = \varepsilon_s(c - d')/(d - c)$, once ε_s is determined for sections with tension reinforcement to assess if the compression steel yields at nominal strength.

7.4.1.4 *T-beams*—Cast-in-place and many precast concrete slabs and beams are monolithic, so the slab contributes to the beam's flexural stiffness, resulting in a T-beam. The flange width of a T-beam is the effective width of the slab, as defined in Code Section 6.3.2.1, and the rectangular beam forms the web. Precast double T-beams also benefit from an increase in beam stability during construction.

The flange width in most T-beams is significantly wider than the web width (Fig. 7.4.1.4a). For a lightly reinforced section, this often places the neutral axis of the nominal strain diagram within the flange depth. T-beams are analyzed the same as rectangular sections, with section width equal to the effective flange width.

In heavily reinforced T-beams, the area of tension reinforcement in the web (required by the applied moment) brings the neutral axis below the flange, which places part of the compression zone in the web. In such a case, the total moment strength consists of: 1) tension steel force equal to the flange concrete compression force; and 2) the remaining tension steel force equal to the web concrete compression force. The flexural strength of a T-beam can then be expressed as

$$M_u < \phi M_n = \phi(M_{nf} + M_{nw}) \tag{7.4.1.4a}$$

where

$$\phi M_{nf} = \phi\left[0.85 f_c' b h_f\left(d - \frac{h_f}{2}\right)\right] \tag{7.4.1.4b}$$

$$\phi M_{nw} = \phi A_s f_y\left(d - \frac{a}{2}\right) + \phi A_p f_{ps}\left(d_p - \frac{a}{2}\right) \tag{7.4.1.4c}$$

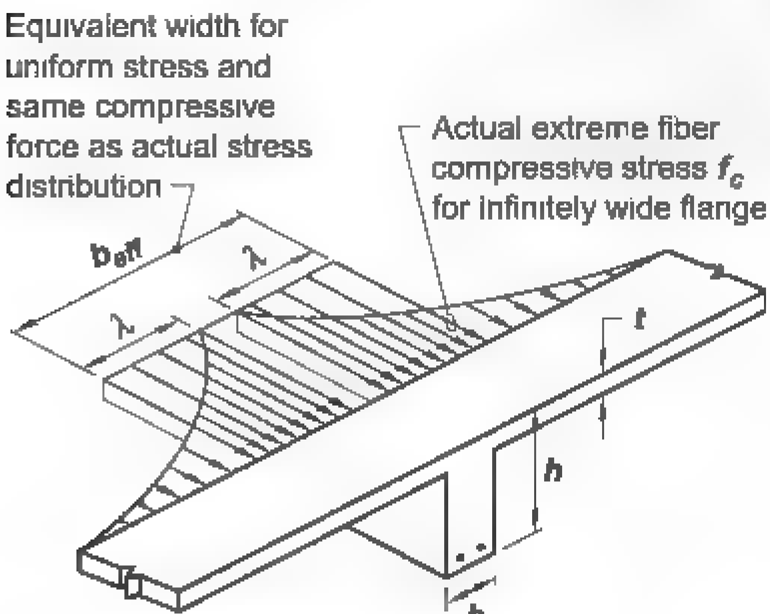


Fig. 7.4.1.4a—Equivalent stress distribution over flange width.

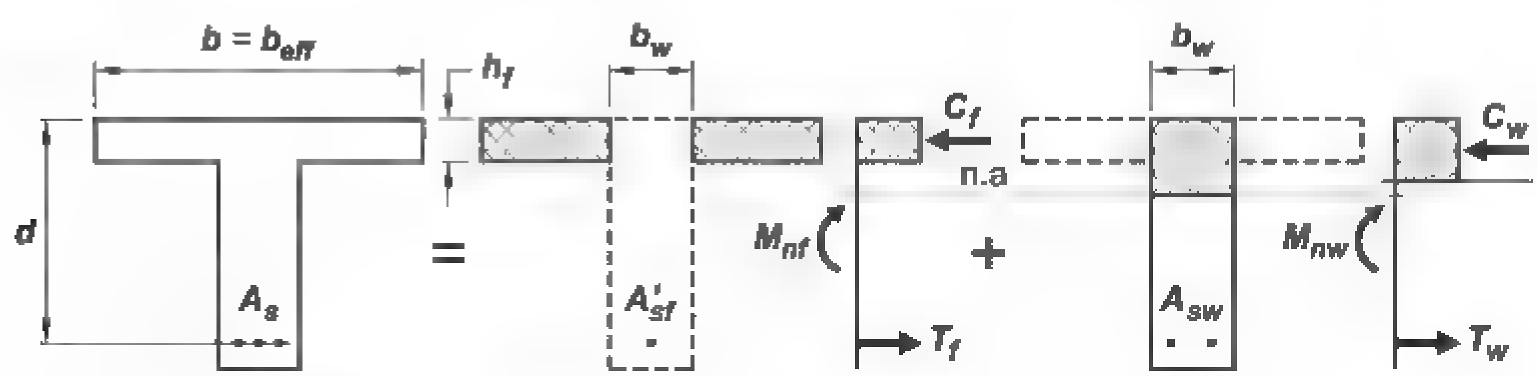


Fig. 7.4.1.4b—T-beam behavior.

Many engineers calculate M_{nf} first from equilibrium to find the area of total tension steel needed to balance the flange concrete. The M_{nw} is then calculated assuming a rectangular cross section as shown in Fig. 7.4.1.4b.

For continuous, statically indeterminate, post-tensioned (PT) beams, effects of reactions induced by prestressing (secondary moments) need to be included per Code Section 5.3.11. The beam's secondary moments are a result of the column's vertical restraint of the beam against the PT "load" at each support. Because the post-tensioning force and drape are determined during the service stress checks, secondary moments can be quickly calculated by the "load-balancing" analysis concept.

A simple way to calculate the secondary moment is to subtract the tendon force times the tendon eccentricity (distance from the neutral axis) from the total balance moment, expressed mathematically as $M_2 = M_{bal} - P \times e$.

7.4.1.5 *Minimum flexural reinforcement*—Nonprestressed reinforcement in a section is effective only after concrete has cracked. If the beam's reinforcement area is insufficient to provide a nominal strength larger than the cracking moment, the section cannot sustain its loads upon cracking. This level of reinforcement can be calculated under light loads or beams that are, for architectural and other functional reasons, much larger than required for strength. To protect against potentially brittle behavior immediately after cracking, the Code requires a minimum area of tension reinforcement (refer to Code Section 9.6.1.1).

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \qquad (7.4.1.5a)$$

but $A_{s,min}$ needs to be at least $200 b_w d / f_y$.

For statically determinate beams, where the T-beam flange is in tension, the reinforcement required to provide a nominal strength above the cracking moment is approximately twice that required for rectangular sections. Therefore, b_w in Eq. (7.4.1.5a) is replaced by the smaller of $2b_w$ or the flange width (Code Section 9.6.1.2). However, when the steel area provided in every section of a member is sufficient to provide flexural strength at least one-third greater than required by analysis, the minimum steel area need not apply (Code Section 9.6.1.3). This exception prevents requiring excessive reinforcement in overlarge beams.

For prestressed beams with bonded prestressed reinforcement, the minimum reinforcement area is that required to develop a design moment at least equal to 1.2 times the cracking moment (Code Section 9.6.2.1).

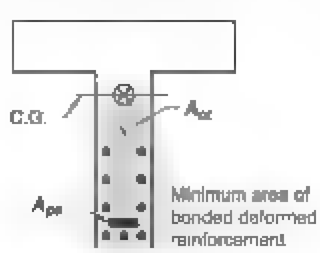


Fig. 7.4.1.5—Area of minimum bonded deformed longitudinal reinforcement distribution.

$$\phi M_n \geq 1.2 M_{cr} \qquad (7.4.1.5b)$$

For prestressed beams with unbonded tendons, abrupt flexural failure immediately after cracking does not occur because there is no strain compatibility between the unbonded strands and the surrounding concrete. Therefore, for unbonded tendons, the code only requires a minimum steel area of $0.004A_{ct}$. These bars should be uniformly distributed over the precompressed tensile zone, close to the extreme tension fibers (Fig. 7.4.1.5).

7.4.2 *Shear*—Unreinforced concrete shear failure is brittle. This behavior is prevented by providing adequate shear reinforcement that intercepts the assumed inclined cracks. For beams with uniform load, shear is maximum at a support, and decreases linearly to zero near at the midspan. In regions of high moment, flexural cracks form perpendicular to the longitudinal tension reinforcement. In regions with moderate moment and shear, the flexural cracks tend to grow into the web of the beam, where the principal tension stresses are at an angle of approximately 45 degrees to the beam axis; these are flexure-shear cracks (Fig. 7.4.2). In regions with large shear, diagonal cracks will initiate in the web; these are web-shear cracks.

7.4.2.1 *Shear strength*—Concrete beams are designed to resist shear and torsion and to ensure ductile behavior at the nominal condition. Shear strength at any location along a beam is calculated as the combination of concrete shear strength, V_c, and the steel shear reinforcement, V_s (Code Section 22.5.1.1). The nominal concrete shear strength V_c is based on the shear required to cause inclined cracking. Prior to the 2019 Code, the concrete contribution to shear strength could be calculated using $V_c = 2\lambda\sqrt{f'_c}b_w d$. In the 2019 Code, these provisions were changed to include the size effect factor and to incorporate the effect of flexural reinforcement. V_c must be calculated using the equations shown in Table 7.4.2.1, where ρ_w is the flexural reinforcement ratio and λ_s is the size effect factor calculated using the effective depth d as follows

$$\lambda_s = \sqrt{\frac{2}{1+\frac{d}{10}}} \leq 1 \qquad (7.4.2.1)$$

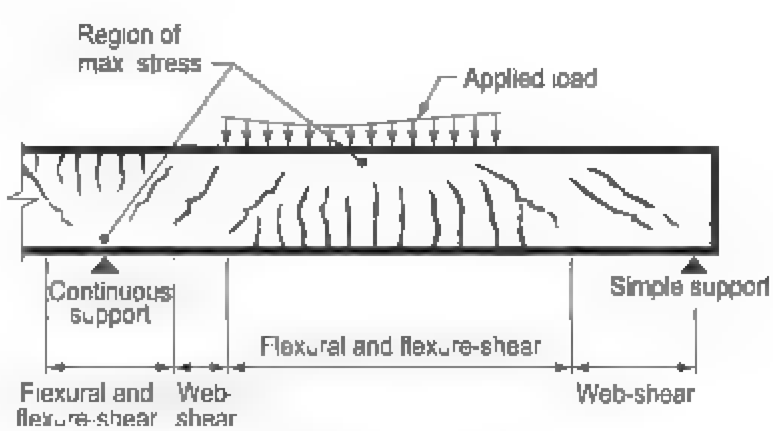


Fig. 7.4.2—Types of cracking in concrete beams (Code Section R22.5.6.3).

Table 7.4.2.1—V_c for nonprestressed members (Code Table 22.5.5.1)

Criteria		V_c	
$A_v \geq A_{v,min}$	Either of	$\left[2\lambda\sqrt{f'_c} + \dfrac{N_u}{6A_g}\right]b_w d$	(a)
		$\left[8\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \dfrac{N_u}{6A_g}\right]b_w d$	(b)
$A_v < A_{v,min}$		$\left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \dfrac{N_u}{6A_g}\right]b_w d$	(c)

Notes: Axial load, N_u, is positive for compression and negative for tension; V_c shall not be taken less than zero.

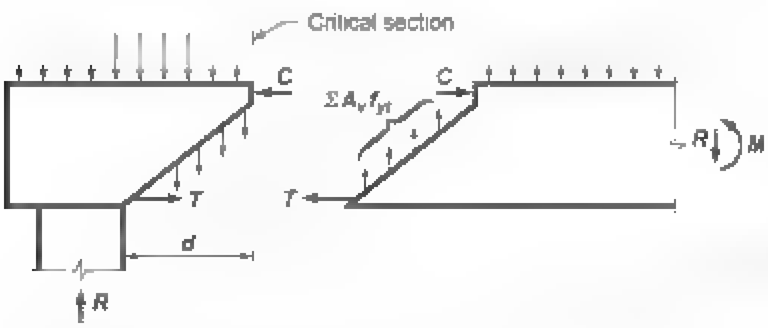


Fig. 7.4.2.1a—Free body diagrams of the end of a beam (Code Fig. R9.4.3.2a).

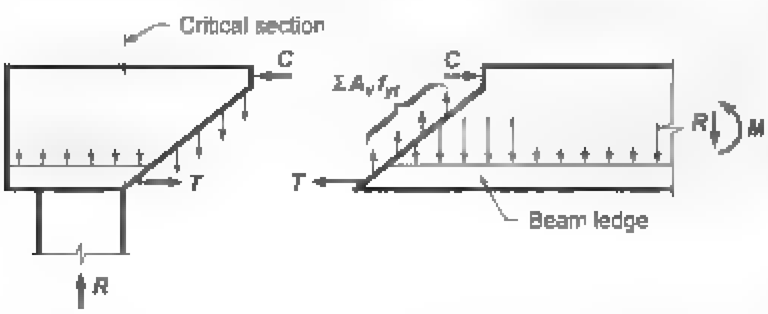


Fig. 7.4.2.1b—Location of critical section for shear in a beam loaded near bottom (Code Fig. R9.4.3.2b).

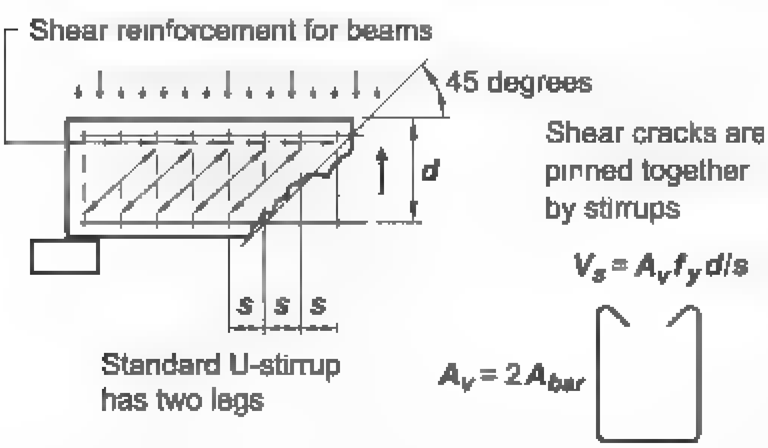


Fig. 7.4.2.1c—Shear reinforcement.

Without shear reinforcement, testing (Kuchma et al. 2019) has indicated that the measured concrete contribution to shear strength does not increase in direct proportion with member depth and could be less than predicted by the traditional equation for concrete contribution. To account for this effect, the size effect factor must be applied to any member that does not contain minimum shear reinforcement. This will result in a reduction of V_c for effective depths greater than 10 in. Increasing flexural reinforcement, however, tends to improve the concrete contribution and is reflected in the term ρ_w. The value of A_s used in this term may be taken as the sum of the areas of longitudinal bars located more than two-thirds of the overall member depth away from the extreme compression fiber. Note that for a beam resisting tension, V_c cannot be smaller than zero.

The nominal concrete shear strength, V_c, for prestressed beams (defined as $A_{ps}f_{ps} \geq 0.4(A_{ps}f_{pu} + A_s f_y)$ can be calculated using simplified equations in Code Table 22.5.6.2, but need not be less than $2\lambda\sqrt{f'_c}b_w d$. The more detailed approach to calculate V_c for prestressed beams is to use the lesser of flexure-shear cracking term, V_{ci} (Code Section 22.5.6.3.1) and web-shear cracking term, V_{cw} (Code Section 22.5.6.3.2).

The critical section for factored shear (or required shear strength), V_u, can be calculated at a distance d from a support face for the usual support condition (Fig. 7.4.2.1a). For other support conditions, or if a concentrated load is applied within the distance d from the support, the required shear strength is taken at the support face Fig. 7.4.2.1b).

Beam shear reinforcement usually takes the form of U-shaped stirrups (Fig. 7.4.2.1c) or closed stirrups. The Code uses the plastic truss analogy in which diagonal compression struts are assumed to occur at 45 degrees from horizontal and the stirrups are vertical tension ties. The longitudinal reinforcement is the tension chord, and concrete is the compression chord.

For design, the tension force in each stirrup leg is assumed to be its yield strength times the leg area. Beam stirrups usually have two vertical legs. Consequently, the area of each stirrup is $A_v = 2 \times$ (leg area). The nominal shear strength provided by shear reinforcement is then calculated by $V_s = A_v f_{yt} d/s$. Designers usually calculate the required V_s and then determine the stirrup size and spacing, so the equation is oftentimes rearranged as $A_v/s = V_c/(f_y d)$.

7.4.2.2 *Designing shear reinforcement*—When designing a beam for shear, the need for minimum reinforcement is typically checked to determine the lower-bound requirement. For practical purposes, stirrups are generally required to provide support to longitudinal bars to maintain their position during bar installation and concrete placement. Consequently, it is impractical to eliminate beam stirrups completely, so it is recommended to provide stirrups in all cast-in-place beams. For nonprestressed beams, minimum shear reinforcement, A_v, must be provided where $V_u > \phi\lambda\sqrt{f'_c}b_w d$, with the exception of some beam types noted in the following. Stirrup bar size, configuration, and spacing that satisfies the minimum requirement can be selected at this point. As noted in the previous section, V_c depends on whether minimum shear reinforcement is present.

Once minimum shear reinforcement has been established, the spacing of the stirrups is adjusted to satisfy the strength requirements. Using a strength reduction factor ϕ for shear is 0.75, the required stirrup spacing to satisfy strength requirement is

$$s \leq A_v f_{yt} d/(V_u/\phi - V_c) \qquad (7.4.2.2)$$

where the term $V_u/\phi - V_c$ represents the required nominal shear strength provided by shear reinforcement. Stirrup size and spacing should be selected such that V_s is greater than this value. Stirrup spacing along the beam is limited based on the required V_s, as shown in Table 7.4.2.2.

There are limited exceptions to the aforementioned general rules given in Code Section 9.6.3.1. For example, a beam shallower than 24 in. that is cast integral with a slab and has a width b_w more than twice the thickness h does not require minimum shear reinforcement as long as the design concrete shear strength exceeds the required shear strength.

A type of ribbed floor slab, known as a joist system, is often constructed without shear reinforcement in the joist ribs. A joist system's relative dimensional limits, such as slab thickness, rib width, and rib spacing, are provided in Code Section 9.8.1. If the ribbed floor system does not conform to all the code limits (such as a skip joist system), the system needs to be designed as a beam and slab system.

Table 7.4.2.2—Shear reinforcement requirements

Condition	Spacing		Code Section
$V_u \le \phi\lambda\sqrt{f'_c}b_w d$	No shear reinforcement required		9.6.3.1
$V_s \le 4\sqrt{f'_c}b_w d$	Nonprestressed	$s \le \frac{d}{2} \le 24$ in.	9.7.6.2.2
	Prestressed	$s \le \frac{3h}{4} \le 24$ in.	
$V_s > 4\sqrt{f'_c}b_w d$	Nonprestressed	$s \le \frac{d}{4} \le 12$ in.	9.7.6.2.2
	Prestressed	$s \le \frac{3h}{8} \le 12$ in.	
$V_u > \phi V_c + \phi 8\sqrt{f'_c}b_w d$	Increase cross section		22.5.1.2

Code Section 9.6.3.4 sets lower limits on the A_v to ensure that stirrups do not yield upon shear crack formation. The value of A_v must exceed the larger of $0.75\sqrt{f'_c}b_w s/f_{yt}$ and $50b_w s/f_{yt}$. The first quantity governs if $f'_c > 4440$ psi. When calculating the concrete contribution to shear strength, the Code limits the term $\sqrt{f'_c}$ to 100 psi, which corresponds to a maximum concrete compressive strength of 10,000 psi. Code Section 22.5.3.2 allows the value of $\sqrt{f'_c}$ to be greater than 100 psi if the reinforced and prestressed beam has shear reinforcement per Code Sections 9.6.3.4 and 9.6.4.2. Refer to the Code commentary on this section for more information.

7.4.3 *Torsion*—Beam torsion (or twisting) creates sectional shear stresses that increase from zero stress at the beam's sectional center to the maximum at the section perimeter. The Code design provisions are based on the use of a thin-walled tube space truss analogy in which it is assumed that the shear stress is concentrated around the section perimeter. Once torsional cracking has occurred, torsional strength is provided primarily by the closed stirrups placed at the section perimeter. Empirical expressions developed from this analogy for torsional strength are provided in Code Section 9.5.4.1. The torsion shear stress adds to the gravity shear stress on one vertical face but subtracts from it on the opposite vertical face (Fig. 7.4.3a). Refer also to Fig. 7.4.3a for the definitions of section properties.

When designing for torsion, the engineer needs to distinguish between statically determinate (an uncommon condition) and statically indeterminate torsion (most common condition).

Statically determinate (or equilibrium) torsion is the condition where the equilibrium of the structure requires the beam's torsional resistance—that is, the torsional moment cannot be reduced by internal force redistribution to other members. If inadequate torsional reinforcement is provided to resist this type of torsion, the beam cannot resist the applied factored torsion.

Definitions

A_{cp} = area enclosed by outside perimeter of section

A_o = gross area enclosed by shear flow path

A_{oh} = area enclosed by centerline of closed tie

P_{cp} = outside perimeter of concrete section

P_h = perimeter of centerline of closed tie

h_b = projection of beam above or below the slab whichever is greater

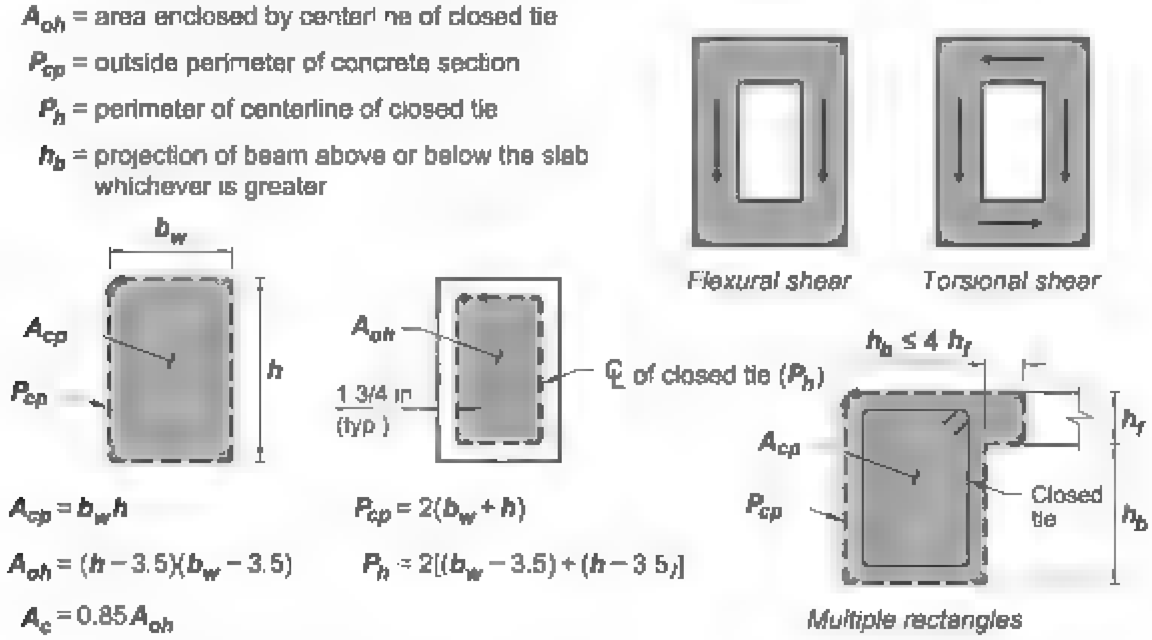


Fig. 7.4.3a—Torsion strength definitions of section properties.

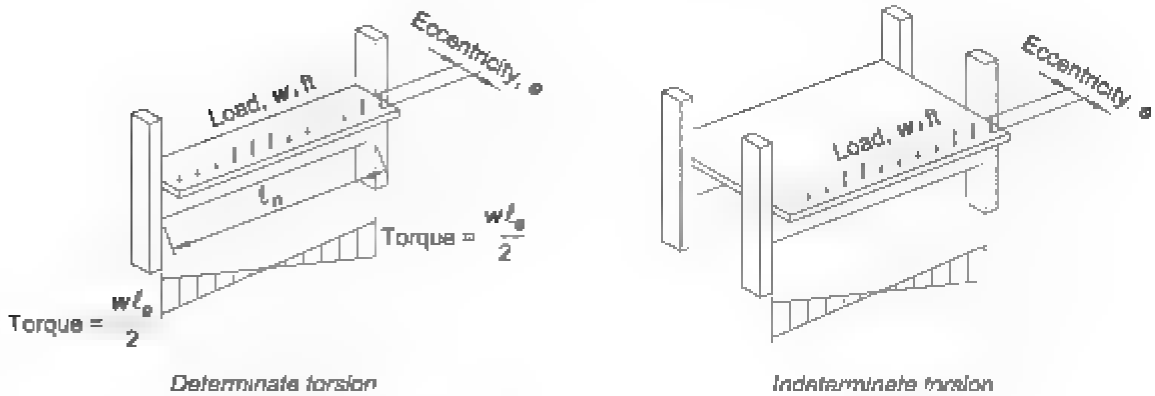


Fig. 7.4.3b—Determinate and indeterminate torsion.

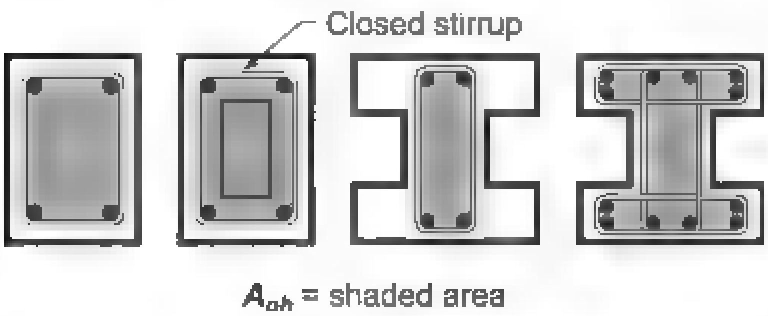


Fig. 7.4.3c—Determining A_{oh}.

Statically indeterminate (or compatibility) torsion is the condition where, if the beam loses its ability to resist torsion, the moment is able to be redistributed, equilibrium is maintained, and the torsion load is safely resisted by the rest of the structural system. Torsional moments can be redistributed after beam cracking if the member twisting is resisted by compatibility of deformations with the connected members.

In Fig. 7.4.3b(a), the determinate beam must resist the eccentric load ($w_u e$) on the ledge to columns through beam torsion.

In Fig. 7.4.3b(b), the eccentric load can be resisted by torsion of the beam or by slab flexure. In other words, if the beam loses torsional stiffness, the slab can resist the eccentric loading effects through flexure.

A beam's cracking torque, T_{cr}, is calculated without consideration of torsion reinforcement.

$$T_{cr} = 4\lambda\sqrt{f'_c}\left(A_{cp}\right)^2 / p_{cp} \tag{7.4.3a}$$

The Code assumes that torques less than 1/4 of T_{cr} will not cause a structurally significant reduction in the shear strength and thus is ignored. The Code limits $\sqrt{f'_c}$ to a maximum of 100 psi, which corresponds to 10,000 psi concrete strength. This limit is based on available research. Code Eq. (22.7.7.1a), provides an upper limit to the torque resistance of a concrete beam.

$$T_{max} = 17\left(A_{oh}\right)^2 \lambda\sqrt{f'_c} / p_h \tag{7.4.3b}$$

where A_{oh} is concrete area enclosed by centerline of the outermost closed transverse torsional reinforcement (Fig. 7.4.3c).

7.4.3.1 *Torsion reinforcement*—Concrete beams reinforced for torsion per Code are ductile and thus will continue to twist after reinforcement yields. The Code specifies beam reinforcement that resists torsion be closed stirrups and longitudinal bars located around the section periphery. Torsion cracks are assumed at angle θ from the member axis, so the torsion strength from closed stirrups is calculated as

$$T_n = \frac{2A_o A_t f_{yt}}{s}\cot\theta \tag{7.4.3.1a}$$

where A_o is the gross area enclosed by torsional shear flow path, in.^{2}; A_t is the area of one leg of a closed stirrup, in.^{2}; and f_{yt} is the yield strength of transverse reinforcement, psi. The Code specifies that angle θ must be greater than 30 degrees and less than 60 degrees; for simplicity in design, use θ = 45 degrees. Solid concrete sections should be large enough to resist the shear stresses due to factored shear V_u and torsion T_u within the upper limits given by Code Eq. (22.7.7.1a).

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \tag{7.4.3.1b}$$

Where stirrups are required for torsion in addition to shear, Code Section 9.6.4.2 requires that the area of two legs of a closed stirrup ($A_v + 2A_t$) must exceed $0.75(b_w s / f_{yt})$ and $50b_w s / f_{yt}$.

Longitudinal spacing of the closed stirrups must not exceed $p_h/8$, or 12 in. The spacing between the longitudinal bars around the section periphery must not exceed 12 in.

Code Section 9.7.5.1 requires that the longitudinal bar area, A_ℓ, be distributed around the section perimeter. Code Section 9.6.4.3 requires a minimum area of longitudinal reinforcement $A_{\ell,min}$ be the lesser of (a) and (b).

(a) $\frac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}$

(b) $\frac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{25b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}$

The torsion strength from longitudinal bars is calculated as

$$T_n = \frac{2A_o A_\ell f_y}{p_h}\tan\theta \tag{7.4.1.3c}$$

7.5—Temperature and shrinkage reinforcement

Refer to Chapter 5, One-way slabs, for information.

7.6—Detailing

Detailing of longitudinal reinforcement includes determining the bar size(s), bar spacing around the perimeter, bar lengths, and bar cutoff locations. Stirrup details includes determining bar size, spacing, and bend configuration.

7.6.1 *Reinforcement placement*—To limit crack widths, it is preferable to use a larger number of small bars, as opposed to fewer large bars.

7.6.1.1 *Minimum spacing of longitudinal reinforcement*. Longitudinal reinforcement should be placed at spacing that allows for proper placement of concrete. Table A-3 of the *ACI Reinforced Concrete Design Handbook Design Aid Analysis Tables* shows the ACI 318-14 minimum spacing requirements for beam reinforcement.

7.6.1.2 *Concrete protection for reinforcement*—The reinforcement should be protected against corrosion and aggressive environments by a sufficiently thick concrete cover (Code Section 20.5.1.3.1), as indicated in *ACI Reinforced Concrete Design Handbook Design Aid Analysis Tables*. The engineer should also consider the beam's required fire rating when determining concrete cover (Code Section 4.11.2 and ACI 216.1-14(19)). Considering cover, reinforcement should be placed as close to the concrete surface as practicable to maximize the lever arm for internal moment strength and to restrain crack widths.

7.6.1.3 *Reinforcement in a T-beam flange*—Where a T-beam flange is in tension due to flexure, all tension reinforcement required for negative moment strength should be located within the lesser of the effective flange width and $\ell_n/10$ (Code Section 24.3.4). Common practice is to place more than half of the reinforcement over the beam web. This requirement is intended to limit slab crack widths that can result from widely spaced reinforcement. When 1/10 of the span is smaller than the effective width, additional reinforcement satisfying Code Section 24.4.3.1 should be provided in the outer portions of the flange to minimize wide cracks in these slab regions.

7.6.1.4 *Maximum spacing of flexural reinforcement*—Beams reinforced with few large bars could experience cracking between the bars, even when the required tension reinforcement area is provided and the sectional strength is adequate. To limit crack widths to acceptable limits for various exposure conditions, Code Section 24.3.2 specifies a maximum spacing, s, for reinforcement closest to the tension face. The spacing limit is the lesser of the two equations that follow.

$$s \leq 15\left(\frac{40{,}000}{f_s}\right) - 2.5c \qquad (7.6.1.4)$$

In the aforementioned equation, c_c is the least distance from the reinforcement surface to the tension face of concrete, and f_s is the service stress in reinforcement. The service stress, f_s, can be calculated from strain compatibility analysis under unfactored service loads or may be taken as $2/3f_y$. Note that Eq. (7.6.1.4) does not provide sufficient crack control for beams subject to very aggressive exposure conditions or designed to be watertight. For such conditions, further investigation is warranted (Code Section 24.3.5).

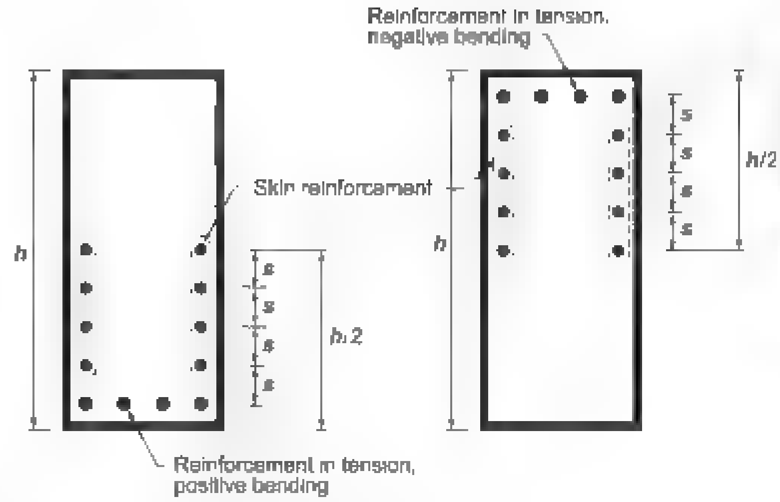


Fig. 7.6.1.5—Skin reinforcement for beams and joists with h *> 36 in. (Code Section R9.7.2.3)*

7.6.1.5 *Skin reinforcement*—In deep beams, cracks may develop near the beam's middepth, between the neutral axis and the tension face. Therefore, the Code requires beams with a depth h > 36 in. to have "skin reinforcement" with a maximum spacing of s, as defined in Eq. (7.6.1.4) and illustrated in the *ACI Reinforced Concrete Design Handbook Design Aid Analysis Tables* (refer to Fig. 7.6.1.5 and Code Section 9.7.2.3). For this case, c_c is the least distance from the skin reinforcement surface to the side face. ACI 318 does not specify a required steel area as skin reinforcement. Research indicates that No. 3 to No. 5 bar sizes or welded wire reinforcement with a minimum area of 0.1 in.^{2}/ft provide sufficient crack control (Frosch 2002).

7.6.2 *Shear reinforcement*—Stirrup bar size is usually a No. 3, No. 4, or No. 5, because larger bar sizes can be difficult to bend. Note that stirrup spacing less than 3 in. can create difficulties in placing concrete. Therefore, some engineers increase the stirrup spacing by doubling the stirrups (refer to Fig. 7.6.2(d)). For wider beams, stirrup spacing across the width is limited to ensure uniform transfer of diagonal compression across the beam web. Where required $V_s \leq 4\sqrt{f_c'}b_wd$, s_w is limited to d for nonprestressed beams and $3h/2$ for prestressed beams. Where required $V_s > 4\sqrt{f_c'}b_wd$, s_w is decreased to $d/2$ for nonprestressed beams and $3h/4$ for prestressed beams (Fig. 7.6.2(d)).

7.6.3 *Torsion reinforcement*—The detailing requirements for beams resisting torsion are listed in Code Sections 9.7.5 and 9.7.6, for longitudinal and transverse reinforcement, respectively. The longitudinal bars are distributed around the stirrup perimeter, with at least one longitudinal bar placed in each corner (Code Section 9.7.5.1). To resist torsion, the stirrup ends are closed with 135-degree hooks (Fig. 7.6.2(c) and (e) and 7.6.3). A 135-degree hook may be replaced by a 90-degree hook where the stirrup end is confined and restrained against spalling by a slab or flange of a T-beam (refer to Fig. 7.6.2(a), (b), and (d)). Splicing stirrups is not acceptable for torsion reinforcement (Fig. 7.6.3).

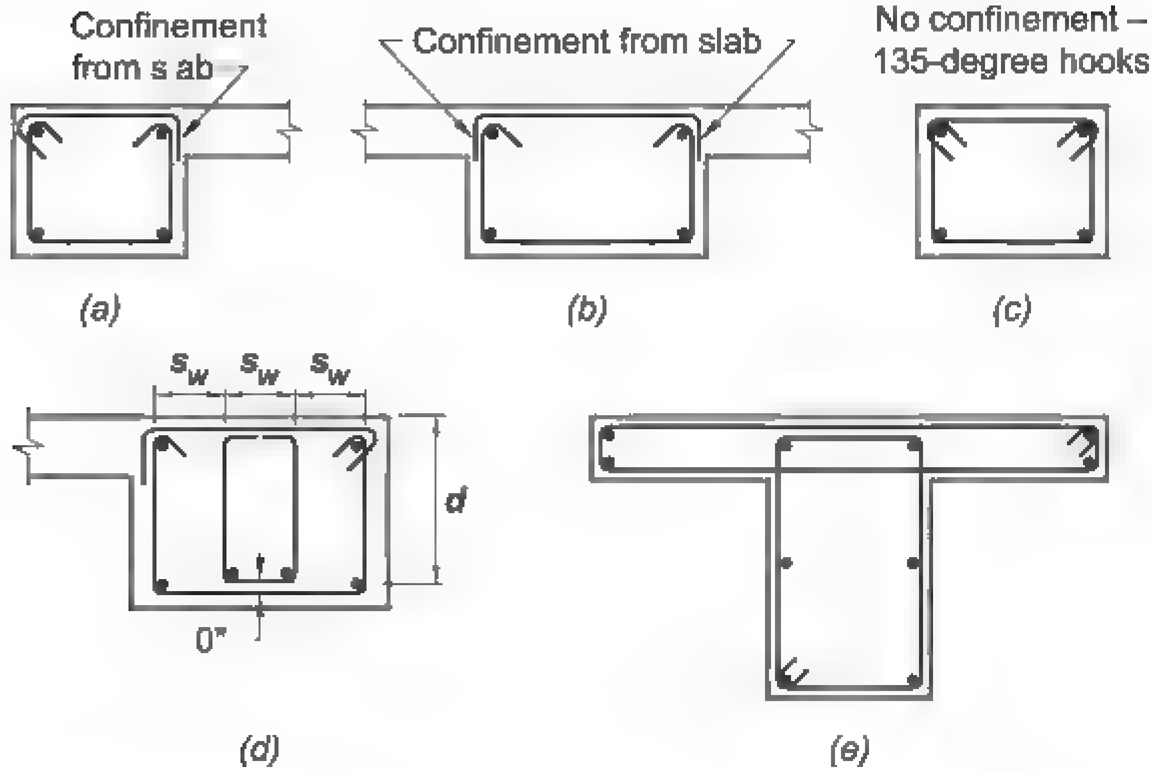


Fig. 7.6.2—Longitudinal reinforcement distributed around beam perimeter with closed stirrups.

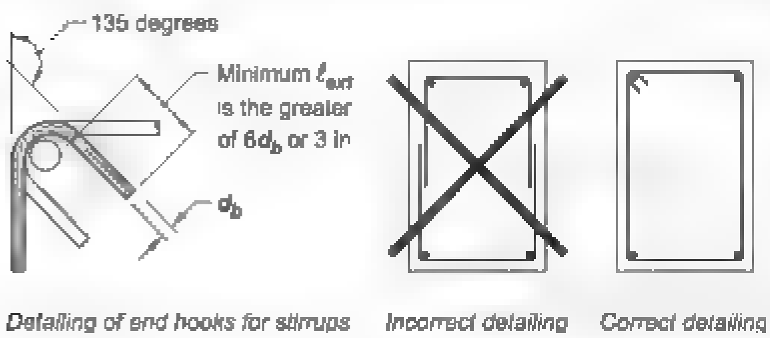


Fig. 7.6.3—Detailing of closed stirrups for torsion.

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American Concrete Institute (ACI)

ACI 216.1-14(19)—Code Requirements for Determining Fire Resistance of Concrete and Masonry Construction Assemblies

ACI MNL-17DA-21—Reinforced Concrete Design Handbook Design Aid – Analysis Tables, https://www.concrete.org/MNL1721Download

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7.7—Examples

Beam Example 1. *Continuous interior beam*

Design and detail an interior, continuous, six-bay beam, built integrally with a 7 in. slab.

__Given:__

Load—

Service additional dead load D = 15 psf
Service live load L = 65 psf
Beam and slab self-weights are given below

Material properties—

f_c' = 5000 psi (normalweight concrete)
f_y = 60,000 psi
λ = 1.0 (normalweight concrete)

Span length: 36 ft
Beam width: 18 in.
Column dimensions: 24 in. x 24 in.
Tributary width: 14 ft

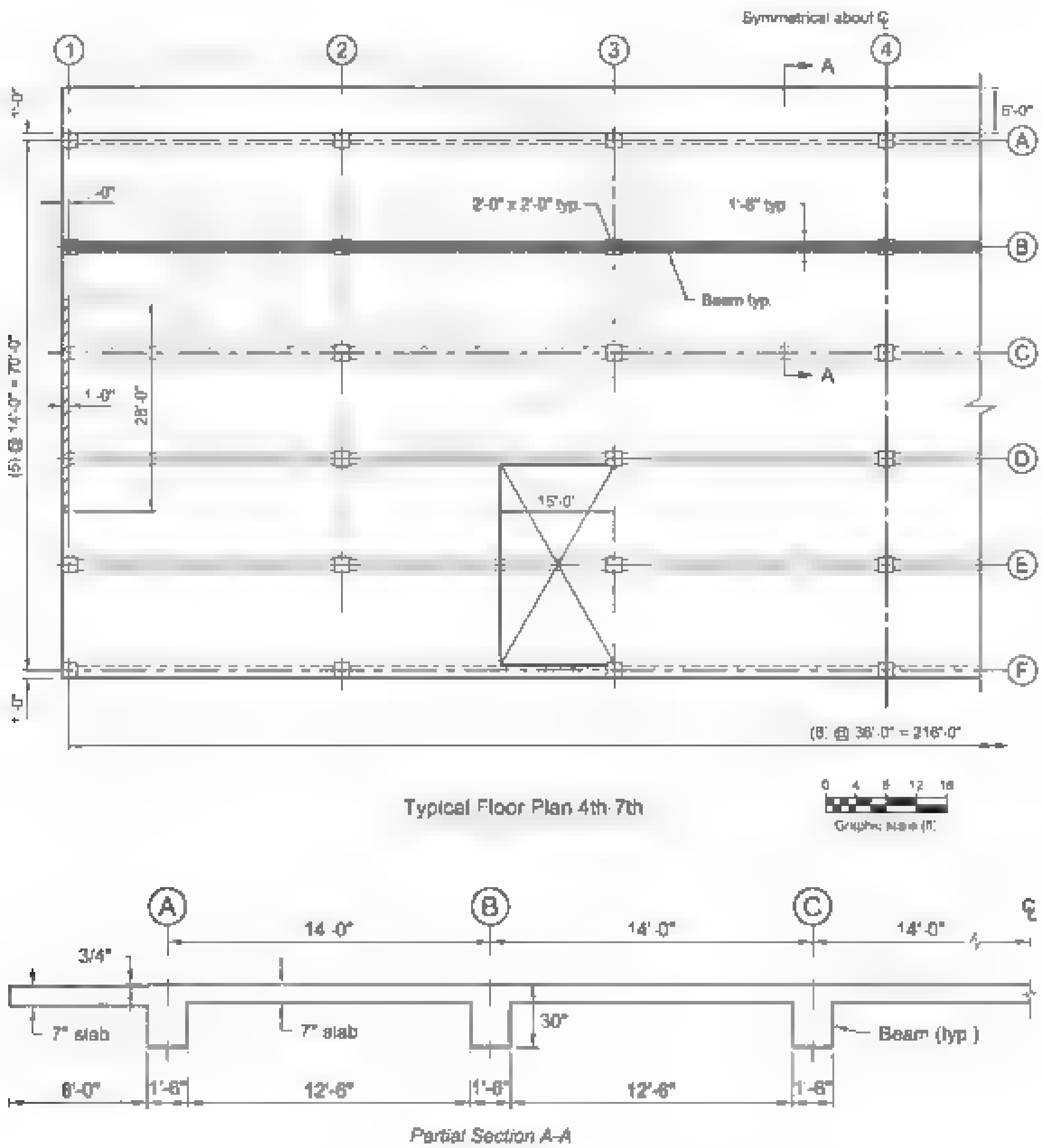


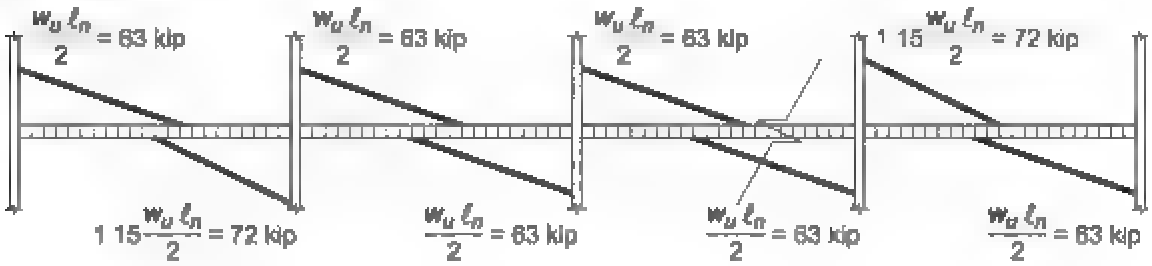
Fig. E1.1—Framing plan and partial section showing six-span interior beam.

ACI 318	Discussion	Calculation
Step 1. Material requirements		
9.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 (ACI 318) and structural strength requirements. The designer determines the durability classes. Chapter 2 of this Manual addresses an in-depth discussion of the Categories and Classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture shall be in accordance with ACI 301-20 and providing the exposure classes, Chapter 19 (ACI 318) requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi. Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26.
Step 2. Beam geometry		
9.3.1.1	Beam depth If the depth of a beam satisfies Table 9.3.1.1, ACI 318 permits a beam to be designed without having to check deflections, as long as the beam is not supporting or attached to partitions or other construction likely to be damaged by large deflections. Otherwise, beam deflections must be calculated and satisfy the deflection limits in Section 9.3.2 of ACI 318.	The beam has four continuous spans, so the controlling condition for beam depth is one end continuous. $h = \frac{\ell}{18.5} = \frac{(36\text{ ft})(12\text{ in./ft})}{18.5} = 23.35\text{ in.}$ Use 30 in. A deeper section is selected so all beams will have the same depth.
	Self weight Beam: Slab:	 $w_b = [(18\text{ in.})(30\text{ in.})/(144)](0.150\text{ kip/ft}^3) = 0.56\text{ kip/ft}$ $w_s = (14\text{ ft} - 18\text{ in./12})(7\text{ in./12})(0.150\text{ kip/ft}^3) = 1.1\text{ kip/ft}$
 9.2.4.2 6.3.2.1	Flange width The beam is placed monolithically with the slab and will behave as a T-beam. The flange width on each side of the beam is obtained from Table 6.3.2.1. Each side of web is the least of $\begin{cases} 8h_{slab} \\ s_w/2 \\ \ell_n/8 \end{cases}$ Flange width $b_f = \ell_n/8 + b_w + \ell_n/8$	 8(7 in.) = 56 in. (14 ft)(12)/2 = 84 in. ((36 ft)(12 in./ft) – 24 in.)/8 = 51 in. **Controls** b_f = 51 in. + 18 in. + 51 in. = 120 in.

Step 3: Loads and load patterns		
	The service live load is 50 psf in offices and 80 psf in corridors per Table 4-1 in ASCE/SEI 7. This example will use 65 psf as an average as the actual layout is not provided. A 7 in. slab is a 87.5 psf service dead load. To account for the weight of ceilings, partitions, HVAC systems, etc., add 15 psf as miscellaneous dead load.	
5.3.1	The beam resists gravity only and lateral forces are not considered in this problem.	
	$U = 1.4D$	$U = 1.4(0.56\ \text{kip/ft} + 1.1\ \text{kip/ft} + (15\ \text{psf})(14\text{ft})/1000)$ $= 2.6\ \text{kip/ft}$
	$U = 1.2D + 1.6L$	$U = 1.2(2.6\ \text{kip/ft})/1.4 + 1.6((65\ \text{psf})(14\ \text{ft})/1000)$ $= 3.7\ \text{kip/ft}$ **Controls**
	Note: Live load is not reduced as permitted by ASCE/SEI 7 in this example.	
Step 4: Analysis		
9.4.3.1	The beams are built integrally with supports, therefore, the factored moments and shear forces (required strengths) are calculated at the face of the supports.	
9.4.1.2	Chapter 6 of ACI 318 permits several analysis procedures to calculate the required strengths.	
6.5.1	The beam's required strengths can be calculated using approximations per Table 6.5.2 of ACI 318, if the conditions in Section 6.5.1 are satisfied.	
	(a) Members are prismatic (b) Loads uniformly distributed (c) $L < 3D$ (d) There are at least two spans	Beams are prismatic Satisfied (no concentrated loads) 65 psf < 3(87.5 psf + 15 psf + Beam SW) satisfied Actual 6 spans > 2 spans
	Difference between two spans does not exceed 20 percent.	Beams have equal lengths
		All five conditions are satisfied, therefore, the approximate procedure is used.

Using ℓ_n = 34 ft for all bays results in the following and moment and shear forces at face of columns.

6.5.4 *Shear diagram*



6.5.2 *Moment diagram*

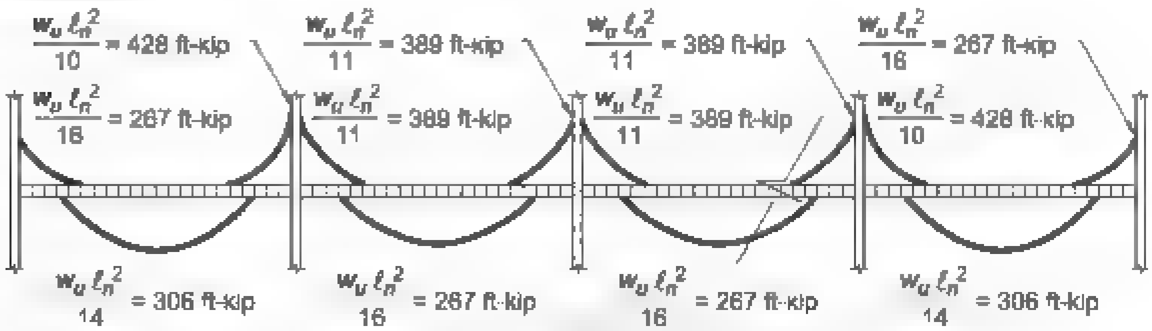


Fig. E1.2—Shear and moment diagrams.

6.5.3 Note:

The moments calculated using the approximate method cannot be redistributed in accordance with Section 6.6.5.1.

Moment diagram drawn on the tension side of the beam.

Step 5. Moment design		
9.3.3.1	Limiting steel strain restricts the amount of reinforcement to ensure warning of failure by excessive deflection and cracking. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members. Beginning with the 2019 Code, this limit is revised to require that the section be tension-controlled.	$\varepsilon_{ty} = \dfrac{f_y}{E_s} = \dfrac{60{,}000\text{ psi}}{29{,}000{,}000\text{ psi}} \cong 0.002$ $\varepsilon_t \geq \varepsilon_{ty} + 0.003 = 0.002 + 0.003 = 0.005$
21.2.1(a)	Because section must be tension-controlled, the strength reduction factor is 0.9	Beam must be tension-controlled in accordance with Table 21.2.2 $\phi = 0.9$
	Determine the effective depth assuming No. 3 stirrups, No. 7 longitudinal bars, and 1.5 in. cover:	$d = 30\text{ in.} - 1.5\text{ in.} - 0.375\text{ in.} - 0.875\text{ in.}/2 = 27.6\text{ in.}$
20.5.1.3.1	One row of reinforcement $d = h - \text{cover} - d_{tie} - d_b/2$	use $d = 27.5$ in.
22.2.2.1	The concrete compressive strain at nominal moment strength is calculated at $\varepsilon_{cu} = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and is approximately 10 to 15 percent of the concrete compressive strength. ACI 318 neglects the concrete tensile strength to calculate nominal strength. Determine the equivalent concrete compressive stress at nominal strength:	
22.2.2.3 22.2.2.4.1	The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distribution of $0.85f_c'$ with a depth of:	
22.2.2.4.3	$a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3. For $f_c' = 5000$ psi	$\beta_1 = 0.85 - \dfrac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$
22.2.1.1	Find the equivalent concrete compressive depth a by equating the compression force to the tension force within the beam cross section: $C = T$ $0.85f_c'ba = A_sf_y$ For positive moment: $b = b_f = 120$ in. For negative moment: $b = b_w = 18$ in.	$0.85(5000\text{ psi})(b)(a) = A_s(60{,}000\text{ psi})$ $a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(120\text{ in.})} = 0.118A_s$ $a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(18\text{ in.})} = 0.784A_s$

Beams

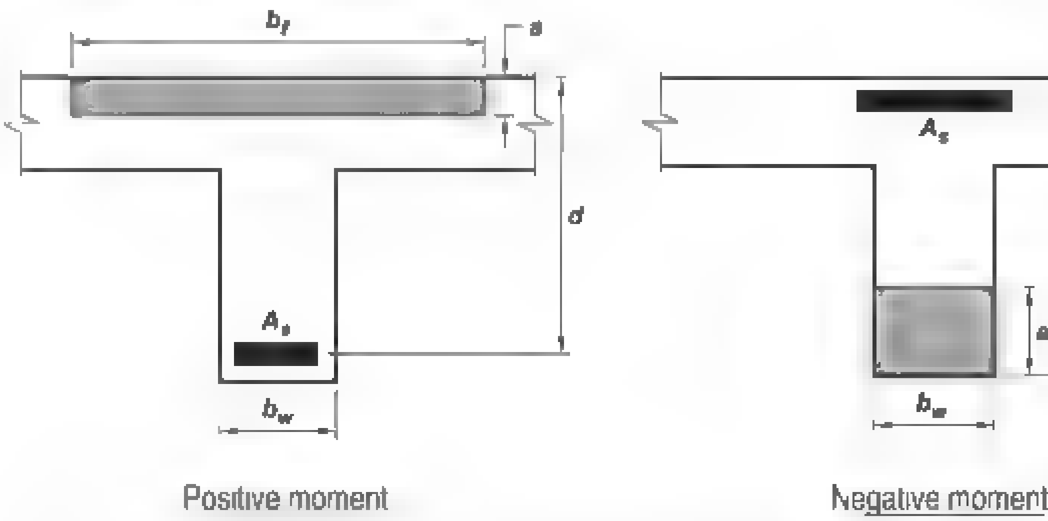


Fig. E1.3—Section compression block and reinforcement locations.

	The beam is designed for the maximum flexural moments obtained from the approximate method above.
	The first interior support will be designed for the larger of the two moments.
9.5.1.1	The beam's design strength must be at least equal to the required strength at each section along its length $\phi M_n \geq M_u$ $\phi V_n \geq V_u$
9.5.2.1	Beam is not subjected to axial force, therefore, assume $P_u < 0.1 f_c' A_g$
22.3	Calculate the required reinforcement area (refer to Fig. E1.2 for design moment values and Fig. E1.4 for moment location) $M_u \leq \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$
21.2.1a	$\phi = 0.9$
	A No. 7 bar has a $d_b = 0.875$ in. and an $A_s = 0.6$ in.^{2} a has been calculated above as a function of A_s
21.2.2 9.3.3.1	Check if the calculated strain exceeds 0.005 in./in. to ensure section is tension-controlled (Fig. E1.5) $a = \frac{A_s f_y}{0.85 f_c' b}$ and $c = \frac{a}{\beta_1}$ where $\beta_1 = 0.8$ (calculated above) Note: $b = 18$ in. for negative moments and 120 in. for positive moments $\varepsilon_t = \frac{\varepsilon_{cu}}{c}(d - c)$

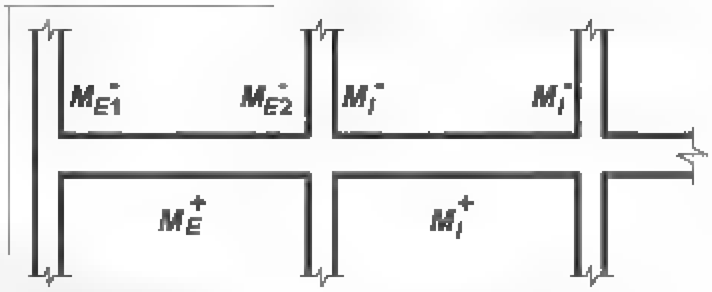


Fig. E1.4—Key to moment, use with table below.

Table 1.1—Required reinforcement

Location	M_u, ft-kip	$A_{s,req'd}$, in.^{2}	Number of No. 7 bars: Req'd	Number of No. 7 bars: Prov.
M_{E1}^-	267	2.23	3.7	4
M_{E2}^-	428	3.65	6.1	7
M_I^-	389	3.30	5.5	6
M_E^+	306	2.49	4.1	5
M_I^+	267	2.17	3.6	4

Note: The beam at the first interior support is designed for the larger of M_{E2} and M_I, refer to Fig. E1.4

Table 1.2—Tension strain in reinforcement

Location	M_u, ft-kip	$A_{s,prov}$, in.^{2}	a, in.	ε_t, in./in.	$\varepsilon_t >$ 0.005?
M_{E1}^-	267	2.40	1.88	0.0313	Y
M_{E2}^-	428	4.20	3.29	0.0[illegible]79	Y
M_I^-	389	3.60	2.82	0.0208	Y
M_E^+	306	3.00	0.35	0.[illegible]66	Y
M_I^+	267	2.40	0.28	0.208	Y

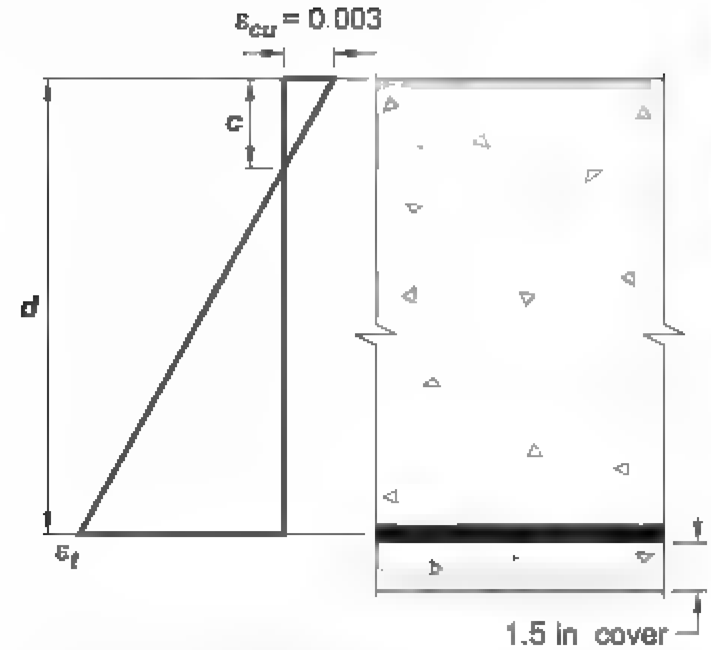


Fig. E1.5—Strain distribution across beam section.

Minimum reinforcement

9.6.1.1 The provided reinforcement must be at least the
9.6.1.2 minimum required reinforcement at every section along the length of the beam.

$$A_s = \frac{3\sqrt{f_c'}}{f_y} b_w d \qquad (9.6.1.2a)$$

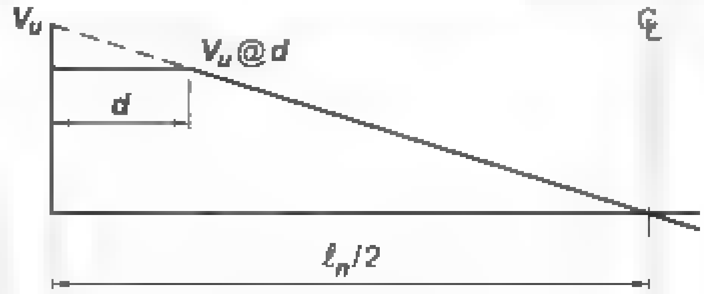
$$A_s = \frac{3\sqrt{5000\text{ psi}}}{60{,}000\text{ psi}}(18\text{ in.})(27.5\text{ in.}) = 1.75\text{ in.}^2$$ **Controls**

$$A_s = \frac{200}{f_y} b_w d \qquad (9.6.1.2b)$$

$$A_s = \frac{200}{60{,}000\text{ psi}}(18\text{ in.})(27.5\text{ in.}) = 1.65\text{ in.}^2$$

Because f_c' > 4444 psi, Eq. (9.6.1.2a) controls.

Required reinforcement areas exceed the minimum required reinforcement area at all locations.

	Exterior spans	
Step 6: Shear design		
9.5.1.1 9.5.3.1 22.5.1.1	<u>Shear strength</u> $\phi V_n \geq V_u$ $V_n = V_c + V_s$	 *Fig. E1.6—Shear at the critical section*
9.4.3.2	Because conditions (a), (b), and (c) of Section 9.4.3.2 are satisfied, the design shear force is taken at critical section at distance d from the face of the support (Fig. E1.6)	
22.5.5.1	2019 Code introduced size effect for shear design in which the shear strength of an element that does not contain shear reinforcement is not directly proportional to its depth. This effect is addressed by incorporating a size effect factor λ_s into the concrete contribution equation. If shear reinforcement is not present, then the concrete contribution to shear strength must be reduced by the size effect factor. If minimum shear reinforcement is provided, then Eq. (22.5.5.1a) can be used to calculate V_c.	
	Minimum shear reinforcement is required where $V_u > \phi\lambda\sqrt{f_c'}b_w d$	$V_{u@d} = (72\text{ kip}) - (3.7\text{ kip/ft})(27.5\text{ in.}/12) = 63.5\text{ kip}$
	For this example, use minimum shear reinforcement over entire length of beam. The concrete contribution to shear strength is then $V_c = 2\sqrt{f_c'}b_w d$ (22.5.5.1a)	$V_c = 2\sqrt{5000\text{ psi}}(18\text{ in.})(27.5\text{ in.})/1000 = 70\text{ kip}$
21.2.1b	Shear strength reduction factor:	$\phi_{shear} = 0.75$ $\phi V_c = (0.75)(70\text{ kip}) = 52.5\text{ kip}$
9.5.1.1	Check if $\phi V_c > V_u$	$\phi V_c = 52.5\text{ kip} < V_{u@d} = 63.5\text{ kip}$ **NG** Therefore, shear reinforcement is required.
	Prior to calculating shear reinforcement, check if the cross-sectional dimensions satisfy Eq. (22.5.1.2):	
22.5.1.2	$V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d)$	$V_u \leq \phi\left(70\text{ kip} + \dfrac{8\sqrt{5000\text{ psi}}(18\text{ in.})(27.5\text{ in.})}{1000\text{ lb/kip}}\right)$
21.2.1b	$\phi = 0.75$	
	$V_u = 63.5\text{ kip}$	63.5 kip < 263 kip **OK**, therefore, section dimensions are satisfactory

 22.5.8.5.1 22.5.8.5.3 22.5.8.5.5	Shear reinforcement Transverse reinforcement satisfying Eq. (22.5.8.5.3) is required at each section where $V_u > \phi V_c$ $V_s \geq \frac{V_u}{\phi} - V_c$	 $\phi V_s > (63.5\text{ kip}) - 52.5\text{ kip} = 11.0\text{ kip}$
	where $V_s = \frac{A_v f_{yt} d}{s}$	Assume a No. 3 bar, two-legged stirrup $\frac{11.0\text{ kip}}{\phi} = \frac{2(0.11\text{ in.}^2)(60{,}000\text{ psi})(27.5\text{ in.})}{s}$
	Calculate maximum allowable stirrup spacing	$s = 24.8$ in. This is a very large spacing and must be checked against the maximum allowed.
	First, does the beam transverse reinforcement value need to exceed the threshold value of $V_s \leq 4\sqrt{f_c'}b_w d$?	$4\sqrt{f_c'}b_w d = 4\left(\sqrt{5000\text{ psi}}\right)(18\text{ in.})(27.5\text{ in.}) = 140.0\text{ kip}$ $V_s = 14.7\text{ kip} < 4\sqrt{f_c'}b_w d = 140.0\text{ kip}$ **OK**
9.7.6.2.2	Because the required shear strength is below the threshold value, the maximum stirrup spacing is the lesser of $d/2$ and 24 in.	$d/2 = 27.5\text{ in.}/2 = 13.8$ in. Use $s = 12\text{ in.} < d/2 = 13.8\text{ in.}$ ∴ **OK**
9.6.3.4	Specified shear reinforcement must be at least: $\frac{A_{v,min}}{s} = 0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$ and $\frac{A_{v,min}}{s} = 50\frac{b_w}{f_{yt}}$	$\frac{A_{v,min}}{s} \geq 0.75\sqrt{5000\text{ psi}}\frac{18\text{ in.}}{60{,}000\text{ psi}} = 0.016\text{ in.}^2/\text{in.}$ **Controls** $\frac{A_{v,min}}{s} \geq 50\frac{18\text{ in.}}{60{,}000\text{ psi}} = 0.015\text{ in.}^2/\text{in.}$ Provided, No. 3 at 12 in. spacing. $\frac{A_v}{s} \geq \frac{2(0.11\text{ in.}^2)}{12\text{ in.}} = 0.018\text{ in.}^2/\text{in.} > \frac{A_{v,min}}{s} = 0.016\text{ in.}^2/\text{in.}$ Spacing satisfies Section 9.6.3.4, therefore, **OK**
Step 7. Reinforcement detailing		
 9.7.2.1 25.2.1	Minimum bar spacing The clear spacing between the horizontal No. 7 bars must be at least the greatest of:	
	Clear spacing greater of $\begin{cases} 1\text{ in.} \\ d_b \\ 4/3(d_{agg}) \end{cases}$	1 in. 0.875 in. 4/3(3/4 in.) = 1 in.
	Assume maximum aggregate size is 0.75 in.	Therefore, clear spacing between horizontal bars must be at least 1.0 in.

9.7.2.2
24.3.4

Tension reinforcement in flanges must be distributed within the effective flange width, b_f = 120 in. (Step 2), but not wider than $\ell_n/10$

$\ell_n/10$ = (34 ft)(12)/10 = 40.8 in. < 120 in., say, 41 in.

Because effective flange width exceeds $\ell_n/10$, additional bonded reinforcement is required in the outer portion of the flange.

Use No. 5 for additional bonded reinforcement.

This requirement is to control cracking in the slab due to wide spacing of bars across the full effective flange width and to protect flange if reinforcement is concentrated within the web width.

For the first interior support, place tension reinforcement per the higher design moment. For moment locations refer to Fig. E1.4.

Table 1.3—Top flange bar distribution

Location	Prov. No. 7	No. 7 in web	No. 7 in $\ell_n/10$*	No. 5 in outer portion*
M_{E1}	4	4		5
M_{E2}	7	5	1	3
M_I	6	4	1	3

*Bars on both sides of the web (Refer to Fig. E1.12—Sections

Exterior span positive moment reinforcement.

Check if five No. 7 bars (resisting positive moment) can be placed in the beam's web per Reinforced Concrete Design Handbook Design Aid – Analysis Tables, which can be downloaded from: https://www.concrete.org/MNL1721Download1. The spacing is calculated below as a demonstration.

$$b_{w,req'd} = 2(\text{cover} + d_{stirrup} + 0.75\text{ in.}) + 4d_b + 4(1\text{ in.})_{min,spacing} \quad (25.2.1)$$

where
$d_{stirrup}$ = 0.375 in.
and d_b = 0.875 in.

$b_{w,req'd}$ = 2(1.5 in. + 0.375 in. + 0.75 in.) + 3.5 in. + 4 in. = 12.75 in. < 18 in. **OK**

Therefore, five No. 7 bars can be placed in one layer in the 18 in. beam web (Fig. E1.7).

Spacing between longitudinal bars.
2.1 in. > 1 in. **OK**

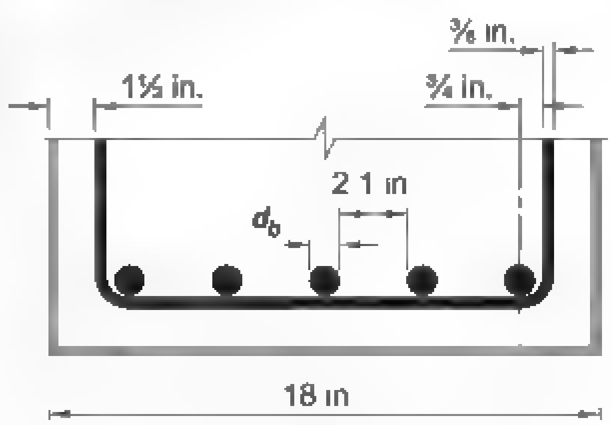


Fig. E1.7—Bottom reinforcement layout.

9.7.2.2
24.3.1

Maximum bar spacing at the tension face must not exceed the lesser of:

24.3.2

$$s \le 15\left(\frac{40{,}000\text{ psi}}{f_s}\right) - 2.5c_c$$

$$s = 15\left(\frac{40{,}000\text{ psi}}{40{,}000\text{ psi}}\right) - 2.5(2\text{ in.}) = 10\text{ in.}$$ **Controls**

and

$$s \le 12\left(\frac{40{,}000\text{ psi}}{f_s}\right)$$

$$s = 12\left(\frac{40{,}000\text{ psi}}{40{,}000\text{ psi}}\right) = 12\text{ in.}$$

where $f_s = 2/3f_y = 40{,}000$ psi

Top reinforcement: 10 in.

24.3.2.1

This limit is intended to control flexural cracking width. Note that c_c is the cover to the No. 7 bar, not to the tie.

Bottom reinforcement:
If bars are not bundled, 2.3 in. spacing is provided (Fig. E1.7), therefore **OK**

Bottom bar length along first span

Calculate the inflection points (Fig. E1.8).

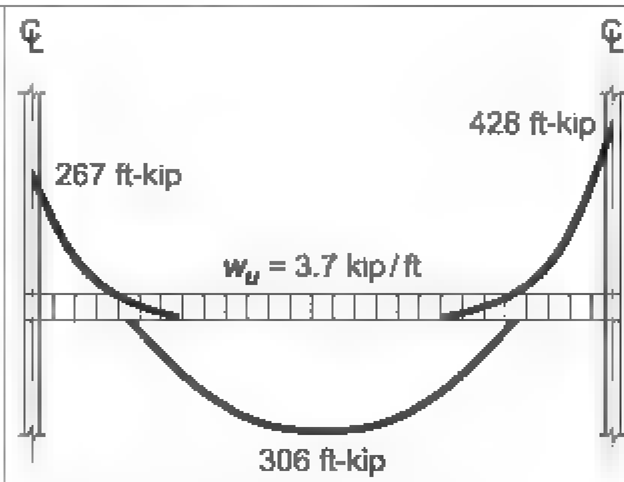
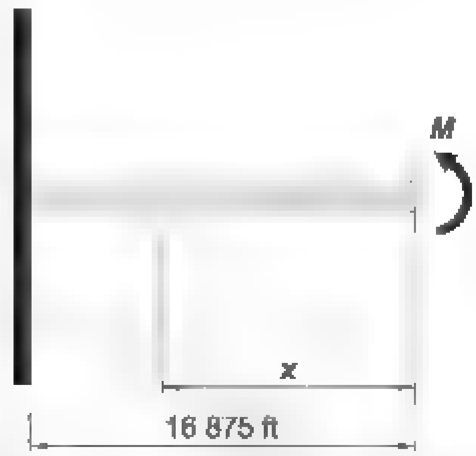


Fig. E1.8—Moment diagram of exterior span.

Inflection point for bottom tension—first span

Assume the maximum positive moment occurs at midspan. From equilibrium, the point of inflection is obtained from the following free body diagram (Fig. E1.9a):

$M_{max} - w_u(x)^2/2 = 0$



(306 ft-kip) − (3.7 kip/ft)$(x)^2/2$ = 0

x = 12.86 ft, say, 13 ft

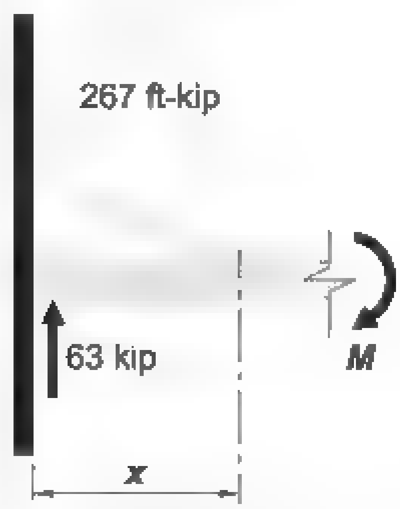
Fig. E1.9a—Inflection point of maximum positive moment.

Inflection point for top tension—first span

Exterior support

Calculate the inflection point for negative moment diagram (Fig. E1.9b):

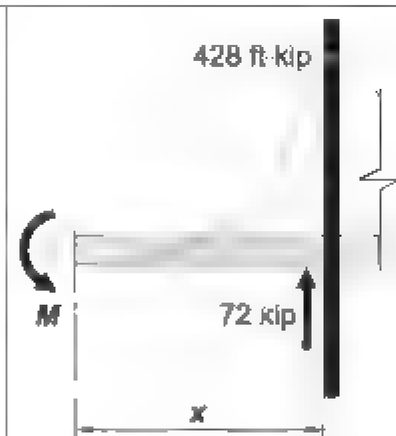
$-M_{max} - w_u(x)^2/2 + V_u x = 0$



(−267 ft-kip) − (3.7 kip/ft)$(x)^2/2$ + (63 kip)x = 0

x = 4.96 ft, say, 5 ft

Fig. E1.9b—Inflection point of exterior negative moment.

	<u>Inflection point for top tension—first interior support</u> Calculate inflection point for the negative moment diagram (Fig. E1.9c) $M_{max} - w_u(x)^2/2 + V_u x = 0$	 $(-428\text{ ft-kip}) - 3.7\text{ kip/ft}(x)^2/2 + (72\text{ kip})x = 0$ $x = 7.32$ ft, say, 7 ft 6 in. *Fig. E1.9c—Inflection point of interior negative moment*
 9.7.1.2 25.4.2.3 25.4.2.5	<u>Development length of No. 7 bar</u> The simplified method is used to calculate the development length of No. 7 bars. $\ell_d = \left(\dfrac{f_y \psi_t \psi_e \psi_g}{20\lambda\sqrt{f_c'}}\right) d_b$ where ψ_t = bar location, $\psi_t = 1.3$ for top bars, because more than 12 in. of fresh concrete is placed below them and $\psi_t = 1.0$ for bottom bars, because not more than 12 in. of fresh concrete is placed below them. ψ_e = coating factor, $\psi_e = 1.0$, because bars are uncoated ψ_g = reinforcement grade factor; $\psi_g = 1.0$ for Grade 60 reinforcement	Top bars $\ell_d = \left(\dfrac{(60{,}000\text{ psi})(1.3)(1.0)(1.0)}{(20)(1.0)\sqrt{5000\text{ psi}}}\right)(0.875\text{ in.}) = 48.3\text{ in.}$ Say, 51 in. = 4 ft 3 in. Bottom bars $\ell_d = \left(\dfrac{(60{,}000\text{ psi})(1.0)(1.0)(1.0)}{(20)(1.0)\sqrt{5000\text{ psi}}}\right)(0.875\text{ in.}) = 37.1\text{ in.}$ Say, 39 in. = 3 ft 3 in.
25.4.10.1	The calculated development lengths could be reduced according to the ratio of $A_{req'd}/A_{prov}$ except as required by Section 25.4.10.2. In this example, development reduction is not applied.	

	<u>First span top bars</u>	
9.7.3.2	<u>Exterior support</u> Bars must be developed at locations of maximum stress and locations along the span where bent or terminated tension bars are no longer required to resist flexure Four No. 7 bars are required to resist the factored negative moment at the exterior column interior face Calculate a distance x from the column face where two No. 7 bars can resist the factored moment.	$-(267\text{ ft-kip})-(3.7\text{ kip/ft})\dfrac{x^2}{2}+(63\text{ kip})(x)$ $= 2(0.5\text{ in.}^2)(0.9)(60\text{ ksi})$ $\times\left(27.6\text{ in.}-\dfrac{2(0.6\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(18\text{ in.})}\right)\left(\dfrac{1}{12}\right)$ $x = 2.05$ ft, say, 24 in. At 2 ft 0 in. from the column face, two No. 7 can be cutoff
9.7.3.3	Bars must extend beyond the location where they are no longer required to resist flexure for a distance equal to the greater of d or $12d_b$.	For No. 7 bars: $d = 27.5$ in. **Controls** $12d_b = 12(0.875\text{ in.}) = 10.5$ in. Therefore, extend the middle two No. 7 bars the greater of the development length (51 in.) and the sum of theoretical cutoff point and d from column face (refer to Fig. E1.10) 24 in. + 27.5 in. = 51.5 in., say, 54 in. **Controls**
9.7.3.8.4	At least one-third of the bars resisting negative moment at a support (two No. 7 > 1/3 of four No. 7) must have an embedment length beyond the inflection point the greatest of d, $12d_b$, and $\ell_n/16$	For No. 7 bars. $d = 27.6$ in. **Controls** $12d_b = 12(0.875\text{ in.}) = 10.5$ in $\ell_n/16 = (36\text{ ft} - 2\text{ ft})/16 = 2.1\text{ ft} = 26$ in. Extend the remainder outside two No. 7 bars, the greater of the development length (51 in.) beyond the theoretical cutoff point (2 ft 0 in.) 51 in. + 24 in. = 75 in. and d = 27.5 in. beyond the inflection point (5 ft 0 in.). 60 in. + 27.5 in. = 87.5 in. > 75 in. **Controls** Therefore, extend bars minimum 87.5 in., increase to, say, 90 in. = 7 ft 6 in. from column face. Refer to Fig. E1.10
	Note: These calculations are performed to present the Code requirements. In practice, the engineer may terminate the two interior No. 7 bars at a distance 4 ft 6 in. from column face and extend the two exterior No. 7 bars the full span length of the beam to support the shear reinforcement stirrups as shown in Fig. E1.11	

	<u>First span top bars</u> <u>Interior support</u>	
9.7.3.2 9.7.3.3	Following the same steps above, seven No. 7 bars are required to resist the factored moment at the first interior column face	
	Calculate a distance x from the column face where four No. 7 bars can be terminated.	$(428\text{ ft-kip}) - (3.7\text{ kip/ft})\dfrac{x^2}{2} + (72\text{ kip})(x) =$ $\dfrac{3(0.6\text{ in.}^2)(0.9)(60\text{ ksi})}{12}\left(27.5\text{ in.} - \dfrac{3(0.6\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(18\text{ in.})}\right)$ $x = 3.19$ ft, say, 39 in. Therefore, extend four No. 7 bars the greater of the development length (51 in.) from column face and d from theoretical cutoff point (39 in.) 39 in. + 27.5 in. = 66.5 in., increase to 69 in. (5 ft 9 in.) 69 in. > 51 in., therefore, extend four No. 7 bars 69 in.
9.7.3.8.4	$d = 27.5\text{ in.} > \ell_n/16 = 25.5\text{ in.} > 12d_b = 10.5\text{ in.}$	Extend the remaining three No. 7 bars the larger of the development length (51 in.) beyond the theoretical cutoff point (38 in.) and $d = 27.5$ in. beyond the inflection point (7 ft 6 in. = 90 in.). The latter controls (Fig. E1.10) 90 in. + 27.5 in. = 117.5 in. > 39 in. + 51 in. = 90 in. **OK**
	Note: These calculations are performed to present the code requirements. In practice, the engineer may terminate the four interior No. 7 bars at a distance d (27.5 in.) beyond the development length from column for a length of (39 in. + 27.5 in. = 66.5 in., say, 5 ft 9 in.). Terminate one No. 7 at 10 ft 0 in. from the support and extend the remaining two exterior No. 7 bars the full span length of the beam to support the shear reinforcement stirrups as shown in Fig. E1.11	
	<u>First span bottom bars</u>	
9.7.3.2 9.7.3.3	Following the same steps above, five No. 7 bars are required to resist the factored moment at the midspan of the exterior span	
	Calculate a distance x from the midspan where two No. 7 bars can resist the factored moment	$(306\text{ ft-kip}) - (3.7\text{ kip/ft})\dfrac{x^2}{2} = 2(0.6\text{ in.}^2)(0.9)(60\text{ ksi})$ $\times\left(27.5\text{ in.} - \dfrac{2(0.6\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(120\text{ in.})}\right)\left(\dfrac{1}{12}\right)$ $x = 10.0$ ft Therefore, extend three No. 7 bars the larger of the development length (39 in.) and a distance d beyond the theoretical cutoff point (10 ft = 120 in.) from maximum moment at midspan 120 in. + 27.5 in. = 147.5 in., say, 12 ft 6 in. from maximum positive moment at midspan (Fig. E1.10). Extend the remaining two No. 7 bars at least the longer of 6 in. into the column or $\ell_d = 39$ in. past the theoretical cutoff point (Fig. E1.10).

9.7.3.8.2	At least one-fourth of the positive tension bars must extend into the column at least 6 in.	Two No. 7 bars out of total five No. 7 will be extended into the column Two No. 7 bars > 1/4 (five No. 7 bars) **OK**
9.7.3.8.3	At the point of inflection, d_b for positive moment tension bars must be limited such that ℓ_d for that bar size satisfies $\ell_d \le \frac{M_n}{V_u} + \ell_a$	Point of inflection occurs at 4 ft from the column face (Fig. E1.9a) $V_u = 63.5\text{ kip} - (3.7\text{ kip/ft})(4\text{ ft}) = 48.2\text{ kip}$ At that location, assume two No. 7 bars are effective:
	where M_n is calculated assuming all bars at the section are stressed to f_y. V_u is calculated at the section. At support, ℓ_a is the embedment length beyond the center of the column. The term ℓ_a is the embedment length beyond the point of inflection, limited to the greater of d and $12d_b$.	$M_n = 2\left(0.6\text{ in.}^2\right)(60\text{ ksi}) \times \left(27.5\text{ in.} - \frac{2(0.6\text{ in.}^2)(60\text{ ksi})}{(2)(0.85)(5\text{ ksi})(120\text{ in.})}\right)$
9.7.3.5	If bars are cutoff in regions of flexural tension, then a bar stress discontinuity occurs. Therefore, the code requires that flexural tensile bars must not be terminated in a tensile zone unless (a), (b), or (c) is satisfied.	$M_n = 1982\text{ in.-kip}$ $\ell_d \le \frac{1982\text{ in.-kip}}{48.2\text{ kip}} + 27.5\text{ in.} = 68.6\text{ in.}$, say, 69 in. This length exceeds $\ell_d = 39$ in., therefore **OK**
	(a) $V_u \le (2/3)\phi V_n$ at the cutoff point (b) Continuing bars provides double the area required for flexure at the cutoff point and the area required for flexure at the cutoff point and $V_u \le (3/4)\phi V_n$. (c) Stirrup or hoop area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance 3/4d from the termination point. Excess stirrup or hoop area shall be at least $60b_ws/f_{yt}$. Spacing s shall not exceed $d/(8\beta_b)$.	(a) At 10 ft and $\ell_n/2 = 17$ ft $V_u = 63\text{ kip} - (3.7\text{ kip/ft})(17\text{ ft} - 10\text{ ft}) = 37.1\text{ kip}$ $\phi V_n = \phi(V_c + V_s)$; ϕV_c is calculated in Step 6 $\phi V_n = 0.75\left(70\text{ kip} + \frac{2(0.11\text{ in.}^2)(60\text{ ksi})(27.5\text{ in.})}{12\text{ in.}}\right)$ $\phi V_n = 75.2$ kip $2/3\phi V_n = 2/3(75.2\text{ kip}) = 50$ kip 50 kip > 37.1 kip, therefore, **OK** Because only one of the three conditions needs to be satisfied, the other two are not checked.
Step 8: Integrity reinforcement		
9.7.7.2	One of the two conditions in 9.7.7.2 must be satisfied.	
	At least one-quarter the maximum positive moment bars, but at least two bars, must be continuous	This condition was satisfied above by extending two No. 7 bars into the support. Also, two bars are more than 1/4 of the provided reinforcement.
	Beam longitudinal bars must be enclosed by closed stirrups along the clear span.	Open stirrups are provided, therefore, the second condition will not be satisfied.
9.7.7.3	Beam structural integrity bars shall pass through the region bounded by the longitudinal column bars.	At least two No. 7 bars are extended through the column longitudinal reinforcement. Therefore, satisfying this condition.

9.7.7.5	Splices are necessary for continuous bars. The bars shall be spliced in accordance with (a) and (b):	splice length = (1.3)(development length)
	(a) Bottom bars (positive moment) shall be spliced at or near the support	ℓ_{st} = 1.3(39 in.) = 50.7 in., say, 4 ft 3 in.
	(b) Top bars (negative moment) shall be spliced at or near midspan	ℓ_{st} = 1.3(51 in.) = 66.3 in., say, 5 ft 9 in.

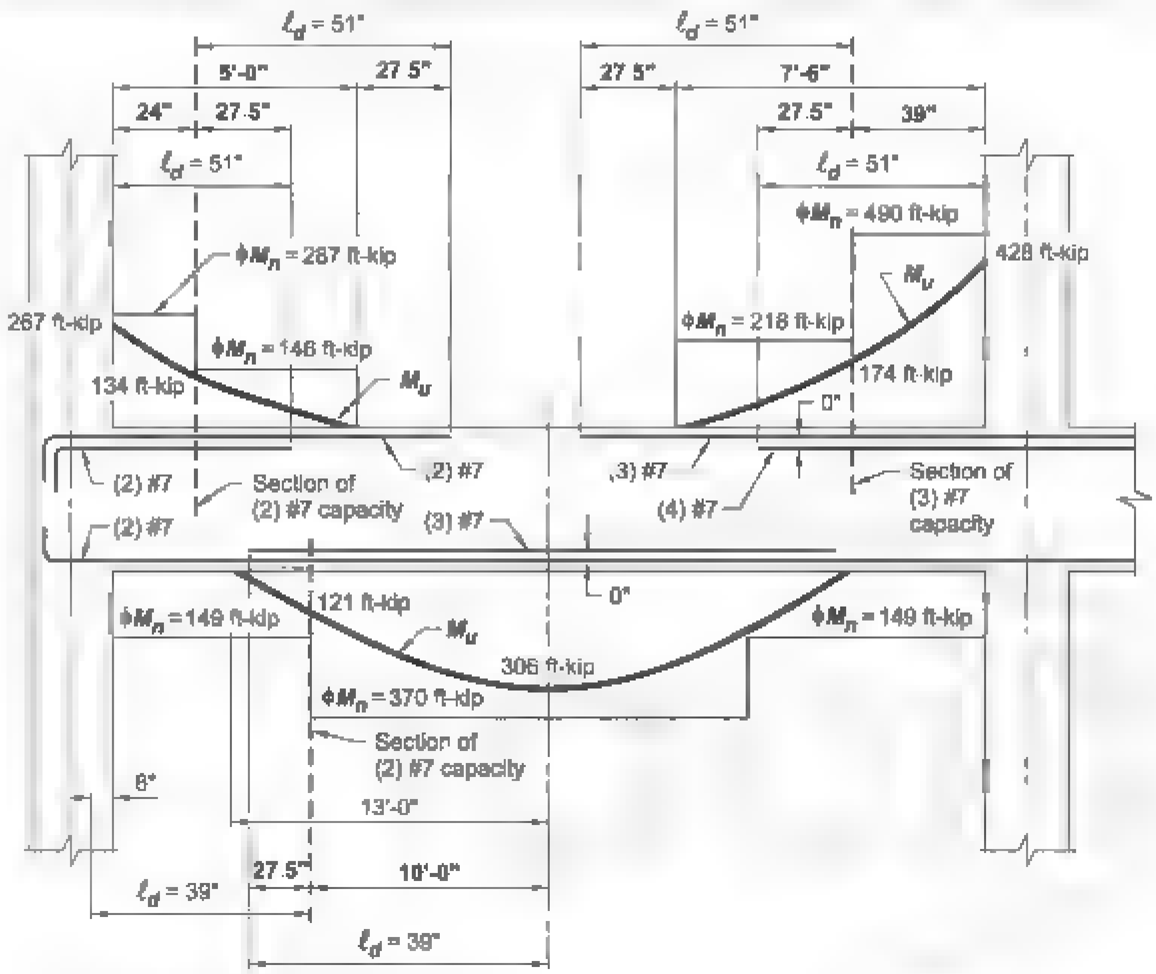


Fig. E1.10—End span bar cutoff locations.

Note: Numbers shown in bold control the bar lengths.

Step 9: Interior spans		
	Flexural bars were calculated above in Step 5.	
	Six No. 7 top bars are required at supports.	
	Four No. 7 bottom bars are required at midspan.	
9.7.6.2.2	Stirrup size and spacing were calculated following Step 6. No. 3 at 12 in. are not required over the full length of the beam, it is, however, good practice to maintain stirrups at 12 in. on center.	

Step 10: Detailing

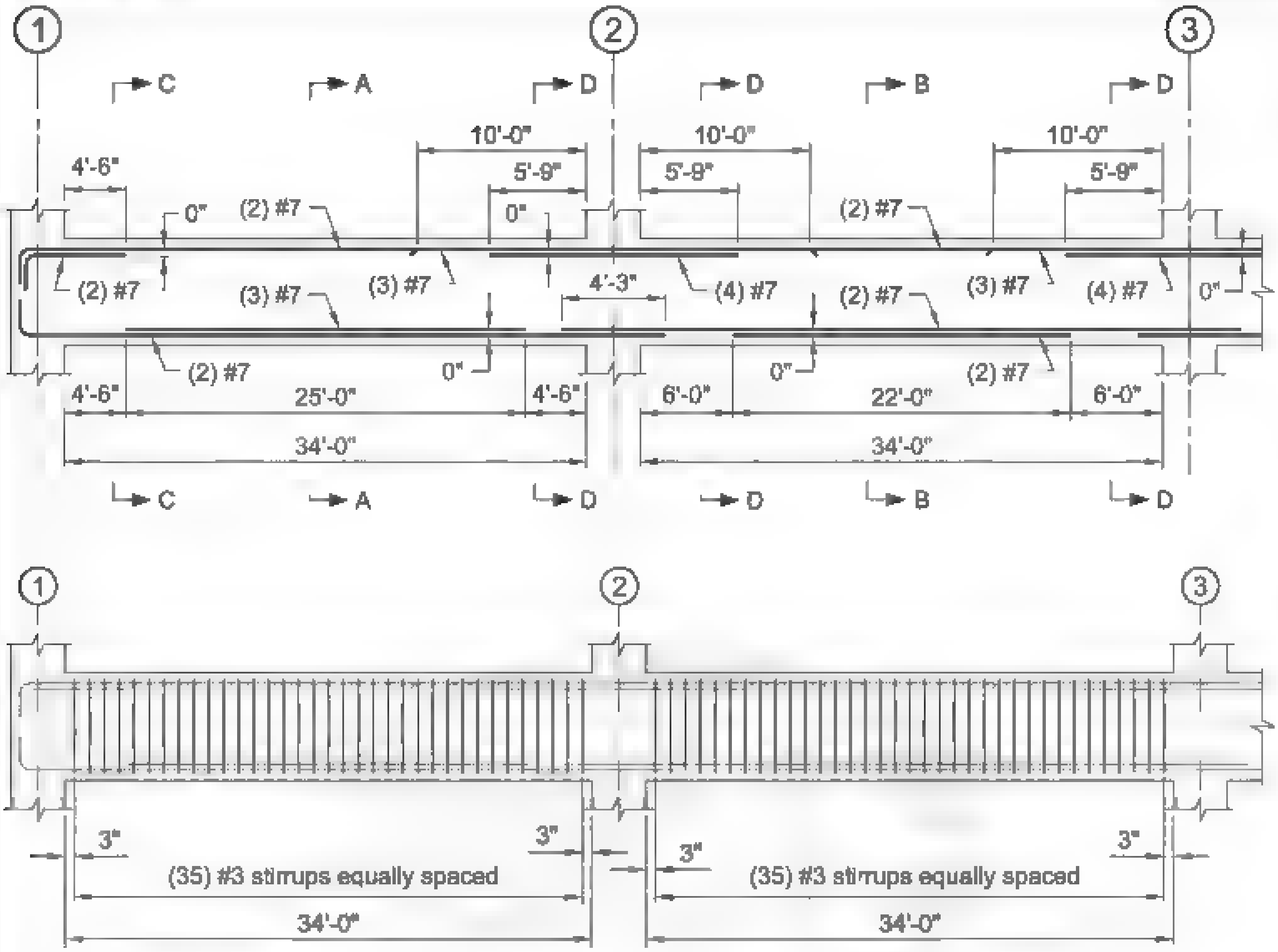


Fig. E1.11—Beam bar details.

Notes:

1. Place first stirrup at 3 in. from the column face. Some designers may choose to place the first stirrup at one-half of the required stirrup spacing from the face of the column, which is acceptable.
2. Total number of stirrups are specified here rather than a stirrup spacing to ensure that the desired number of stirrups are included in the detailed drawings and to ease field inspection.
3. The contractor may prefer to extend two No. 7 top reinforcement over the full beam length rather than adding two No. 5 hanger bars. Bars should be spliced at mid-length.

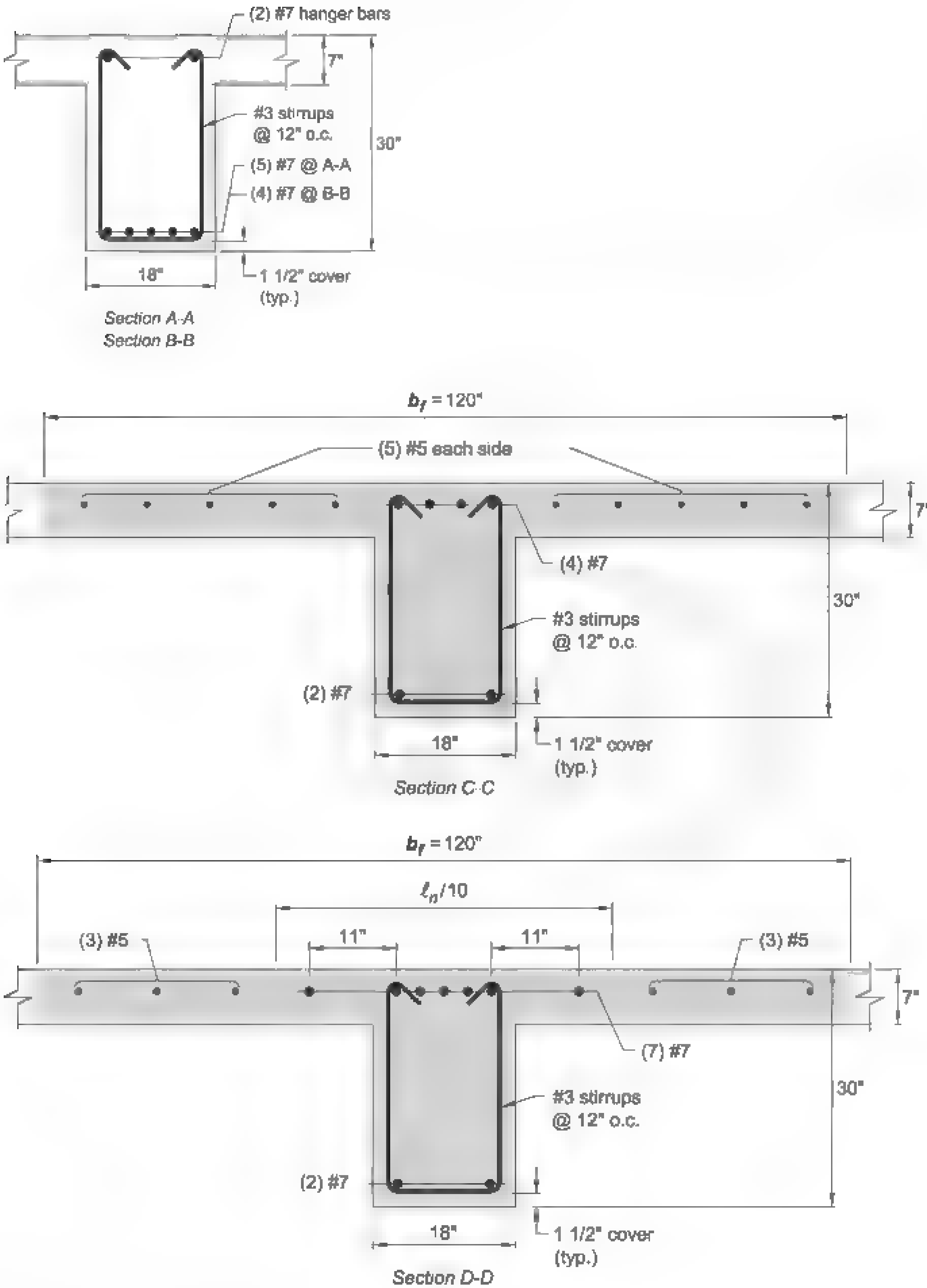


Fig. E1.12—Sections

Note: Refer to Step 7 Table 1.3 for flange negative moment reinforcement placement.

Beam Example 2: *Single interior beam*

Design and detail Beam B1, which frames the slab opening shown in Fig. E2.1. B1 is supported by Beam B2.

Given:

Load—

Service dead load D = 15 psf

Service live load L = 100 psf

Material properties—

f_c' = 5000 psi (normalweight concrete)

f_y = 60,000 psi

Span length = 36 ft

Beam width = 18 in.

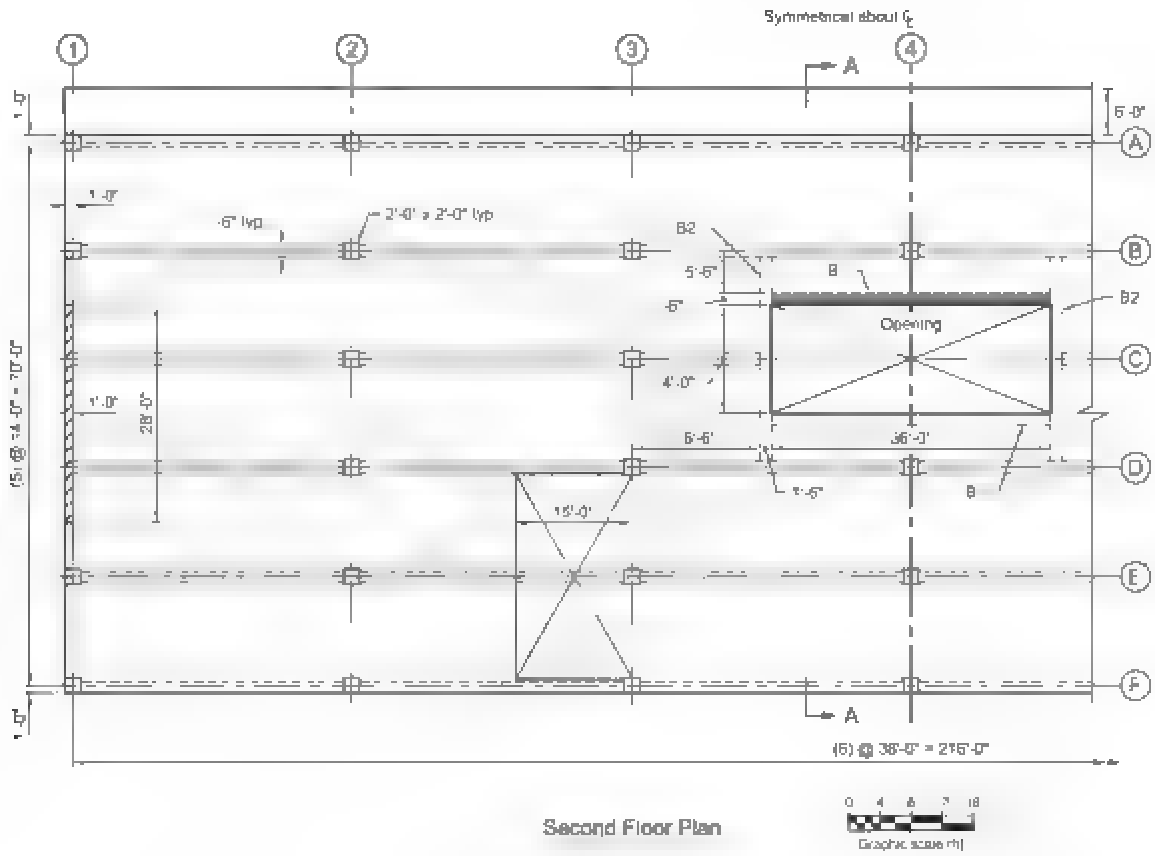


Fig. E2.1—Framing plan showing slab opening and beams framing slab opening.

Beams

ACI 318	Discussion	Calculation
Step 1 Material requirements		
9.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements of ACI 318. The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 (ACI 318) requirements are satisfied Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26
Step 2: Beam geometry		
9.3.1.1	Beam depth ACI 318 permits a beam whose size satisfies Table 9.3.1.1 to be designed without having to check the beam deflection, if the beam is not supporting or attached to partitions or other construction likely to be damaged by large deflections. Otherwise, deflections must be calculated and the deflection limits in Section 9.3.2 must be satisfied The one-span beam is built integrally with the slab and the girders that it frames into.	For a simple supported beam the recommended depth from Table 9.3.1.1: $h = \frac{\ell}{16} = \frac{(36\text{ ft})(12\text{ in./ft})}{16} = 27\text{ in.}$ Use 28 in.
	Self weight Beam: b_w = 18 in. Slab: t = 7 in. thick Tributary width = 4.75 ft/2 = 2.375 ft (Fig. E2.1)	$w_b = (18\text{ in.}/12)(28\text{ in.}/12)(0.150\text{ kip/ft}^3) = 0.53\text{ kip/ft}$ $w_s = (2.375\text{ ft})(7\text{ in.}/12)(0.150\text{ kip/ft}^3) = 0.21\text{ kip/ft}$
9.2.4.2	Flange width The beam is poured monolithically with the slab on one side and will behave as an L-beam. The effective flange width on one side of the beam is obtained from Table 6.3.2.1	
6.3.2.1	One side of web is the least of: $6h_{slab}$; $s_w/2$; $\ell_n/12$ Flange width: $b_f = \ell_n/12 + b_w$	(6)(7 in.) = 42 in. (4.75 ft)(12)/2 = 28.5 in. **Controls** (36 ft(12))/12 = 36 in. b_f = 28.5 in. + 18 in. = 46.5 in.

Step 3. Loads and load patterns		
	The service live load for public assembly is 100 psf per Table 4-1 in ASCE/SEI 7. To account for the weight of ceilings, partitions, HVAC systems, etc., add 15 psf as miscellaneous service dead load. The beam resists gravity load only and lateral forces are not considered in this example.	The superimposed dead load is applied over a tributary width of 4.75 ft/2 + 1.5 ft width of B = 3.875 ft (refer to Fig. E2.1).
5.3.1	$U = 1.4D$	$U = 1.4(0.53\text{ kip/ft} + 0.21\text{ kip/ft} + (15\text{ psf})(3.875\text{ ft})/1000) = 1.12\text{ kip/ft}$
	$U = 1.2D + 1.6L$	$U = 1.2(1.12\text{ kip/ft})/1.4 + 1.6((100\text{ psf})(3.875\text{ ft})/1000) = 1.58\text{ kip/ft}$ **Controls**
	Note: Live load is not reduced per ASCE/SEI 7 in this example.	
Step 4: Analysis		
9.4.3.1	The beam is built integrally with supports, therefore, the factored moments and shear forces (required strengths) are calculated at the face of the supports.	
9.4.1.2	Chapter 6 permits several analysis procedures to calculate the required strengths. For this example, assume an elastic analysis results in the beam moment at the face of each support: $w_u\ell^2/16$	At the face of girders $M_u = w_u\ell^2/16 = (1.58\text{ kip/ft})(36\text{ ft})^2/16 = 128\text{ ft-kip}$
	The total moment is $w_u\ell^2/8$, so the midspan moment is $w_u\ell^2/16$. This distribution assumes the girder remains uncracked. If the girder does crack, its stiffness is greatly reduced, which results in a higher moment at midspan. To be conservative, this example assumes the total beam moment, $w_u\ell^2/8$, is resisted by the positive moment reinforcement and the supports resist $w_u\ell^2/16$.	At the midspan $M_u = w_u\ell^2/8 = (1.58\text{ kip/ft})(36\text{ ft})^2/8 = 256\text{ ft-kip}$
	Beam B1 frames into girders on both ends. Because girders are not as rigid as columns or walls and girders supporting concentrated loads may tend to rotate, the end supports may be considered less than fixed end supports. For a single span beam with fixed end supports, the negative moment at the support would be $(1/12)w_u\ell^2$. For this case, the moment achieved would have to be transferred to the girder efficiently. In real terms, the girder may tend to slightly rotate or may endure cracking which would reduce the rigidity and the fixity of the beam to girder joint. To account for this, assume a moment of $(1/16)w_u\ell^2$ to account for a lower capacity to transfer moment at this connection. Furthermore, considering less than fixed end supports, the positive moment at the mid-span approaches the moment of a simple support beam. So conservatively, use a positive midspan moment of $(1/8)w_u\ell^2$.	

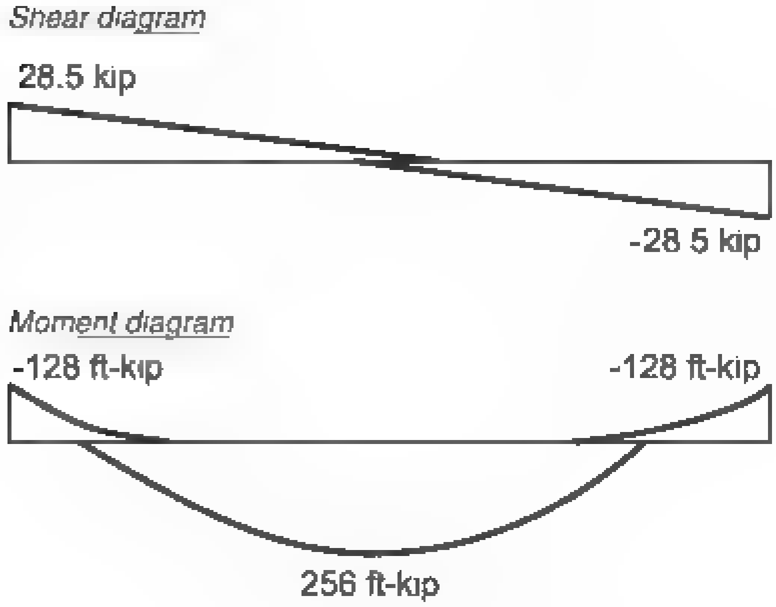


Fig E2 2—Shear and moment envelopes

Step 5. Moment design		
9.3.3.1	Limiting steel strain restricts the amount of reinforcement to ensure warning of failure by excessive deflection and cracking. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members. Beginning with the 2019 Code, this limit is revised to require that the section be tension-controlled.	$\varepsilon_y = \frac{f_y}{E_s} = \frac{60{,}000\text{ psi}}{29{,}000{,}000\text{ psi}} \cong 0.002$ $\varepsilon_t \geq \varepsilon_{ty} + 0.003 = 0.002 + 0.003 = 0.005$
21.2.2	Because the section must be tension-controlled, the strength reduction factor is 0.9.	Beam must be tension-controlled in accordance with Table 21.2.2 $\phi = 0.9$
20.5.1.3.1	Calculate effective depth assuming No. 3 stirrups, No. 6 longitudinal bars, and 1.5 in. cover. The effective depth of one row of longitudinal reinforcement is $d = h - cover - d_{b,stirrup} - d_{b,long}/2$	$d = 28\text{ in.} - 1.5\text{ in.} - 0.375\text{ in.} - 0.75\text{ in.}/2 = 25.75\text{ in.}$, say, $d = 25.7$ in.
22.2.2.1	The concrete compressive strain at which nominal moments are calculated is $\varepsilon_{cu} = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and its value is approximately 10 to 15 percent of the concrete compressive strength. For calculating nominal strength, ACI 318 neglects the concrete tensile strength. Determine the equivalent concrete compressive stress for design.	
22.2.2.3 22.2.2.4.1	The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of nominal strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distribution of $0.85f_c'$ with a depth of: $a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3.	
22.2.2.4.3	For $f_c' = 5000$ psi	$\beta_1 = 0.85 - \frac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$

22.2.1.1	Find the equivalent concrete compressive depth, a, by equating the compression force to the tension force within the beam cross section	
	$C = T$ $0.85f_c'ba = A_sf_y$	At midspan $0.85(5000\text{ psi})(b)(a) = A_s(60,000\text{ psi})$
	For positive moment, $b = b_f = 46.5$ in.	$a = \dfrac{A_s(60,000\text{ psi})}{0.85(5000\text{ psi})(46.5\text{ in.})} = 0.304A_s$
		At support
	For negative moment, $b = b_f = 18$ in.	$a = \dfrac{A_s(60,000\text{ psi})}{0.85(5000\text{ psi})(18\text{ in.})} = 0.784A_s$

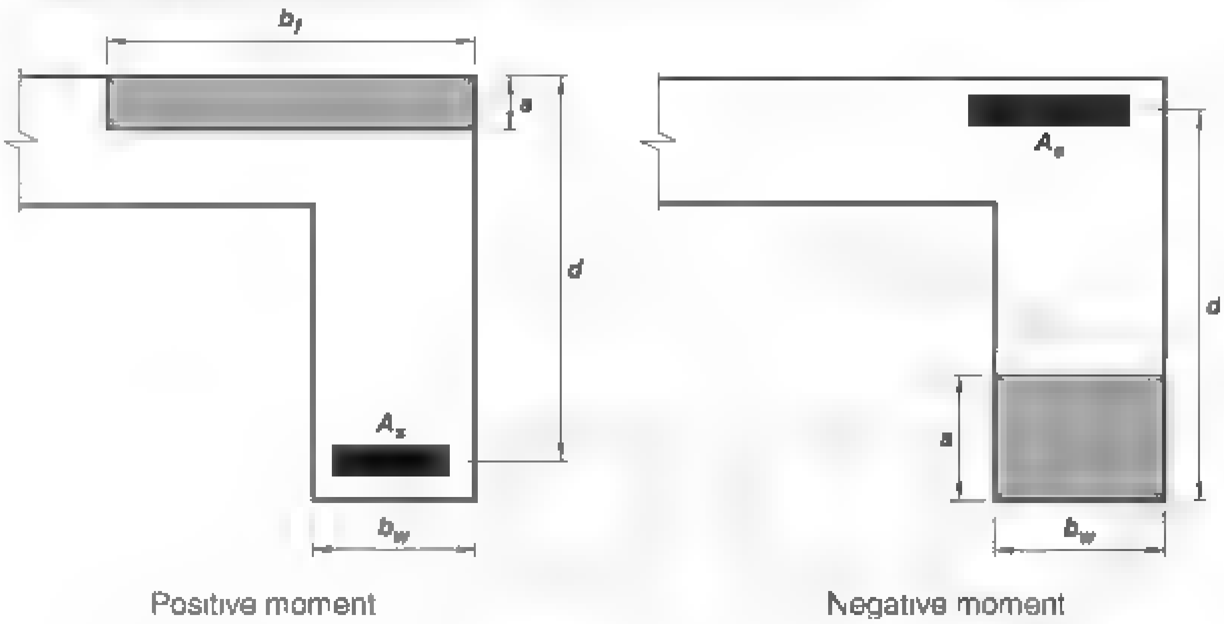
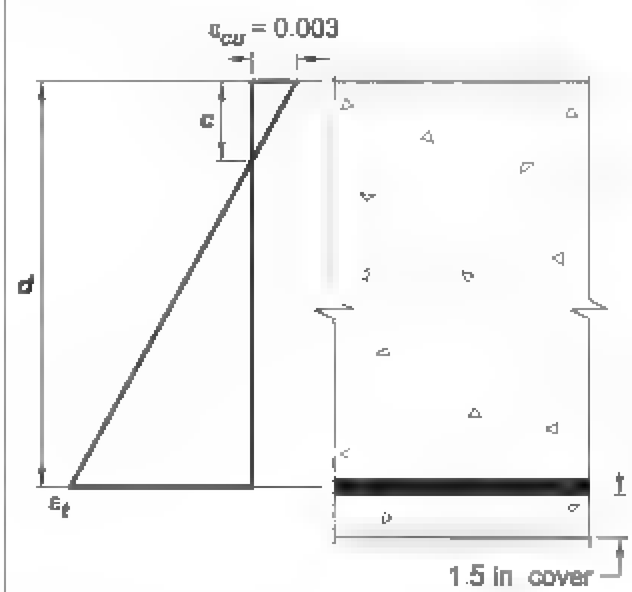
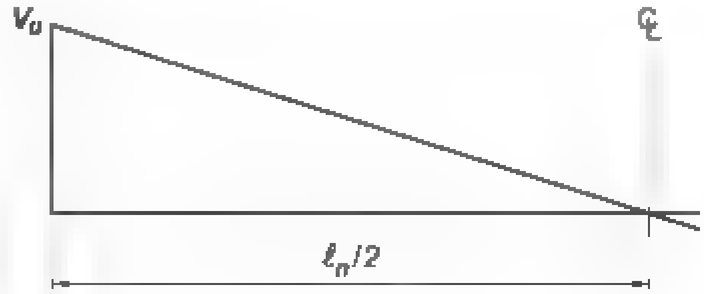
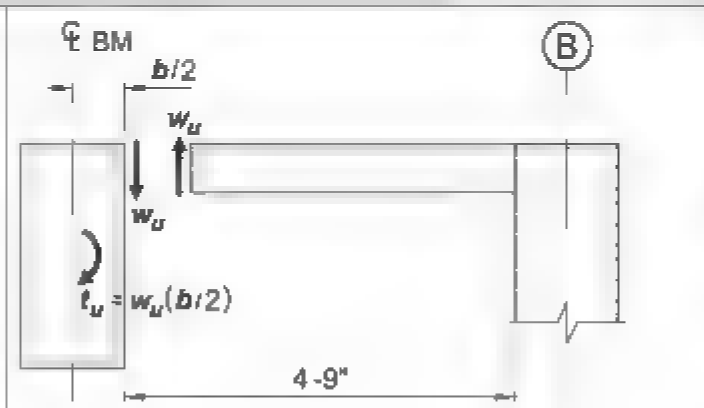
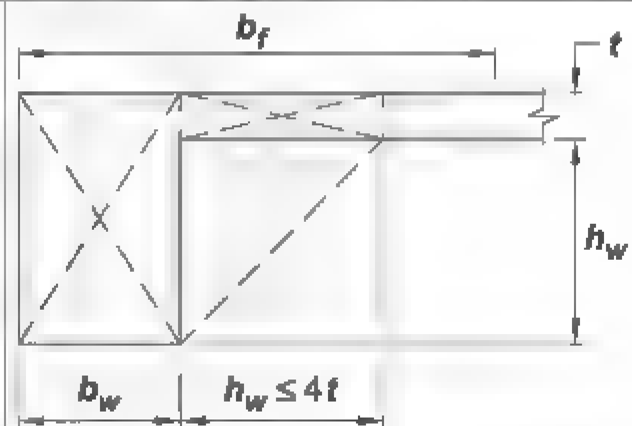


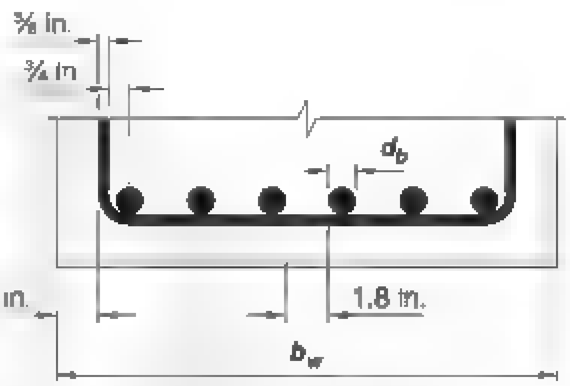
Fig. E2.3—Section reinforcement and compression block at midspan and at support.

	Design the beam for the maximum flexural moment at the midspan and the face of supports.	$M_{u,support} = w_u\ell^2/16 = 128$ ft-kip $M_{u,midspan} = w_u\ell^2/8 = 256$ ft-kip
9.5.1.1	The beam strength must satisfy the following equations at each section along its length $\phi M_n \geq M_u$ $\phi V_n \geq V_u$	
	Calculate required reinforcement area based on the assumptions above	Midspan
	$M_u \leq \phi M_n = \phi A_s f_y\left(d - \dfrac{a}{2}\right)$	$256 \text{ ft-kip} \leq \dfrac{(0.9)(60 \text{ ksi})A_s}{12}\left(25.7 \text{ in.} - \dfrac{0.304A_s}{2}\right)$
	No. 6 bars $d_b = 3/4$ in. and $A_s = 0.44$ in.^{2}	$A_{s,req'd} = 2.24$ in.^{2}, use six No. 6
		Supports $128 \text{ ft-kip} \leq \dfrac{(0.9)(60 \text{ ksi})A_s}{12}\left(25.7 \text{ in.} - \dfrac{0.784A_s}{2}\right)$ $A_s = 1.13$ in.^{2}; use three No. 6
21.2.2 9.3.3.1	Check if calculated strain is greater than 0.005 in./in. (tension-controlled)	
	At midspan $a = \dfrac{A_s f_y}{0.85 f_c' b}$ and $c = \dfrac{a}{\beta_1}$ where $\beta_1 = 0.8$	$a = 0.304A_s = (0.304)(6)(0.44 \text{ in.}^2) = 0.80$ in. $c = a/0.8$ in. $= 1.0$ in. $c < h_f$, therefore, beam section behaves as an L-shape.
	$\varepsilon_t = \dfrac{\varepsilon_c}{c}(d - c)$	$\varepsilon_t = \dfrac{0.003}{1.0 \text{ in.}}(25.7 \text{ in.} - 1.0 \text{ in.}) = 0.074$
	Note that $b = 18$ in. for negative moments and 46.5 in. for positive moments	Section is tension controlled
	At support $a = \dfrac{A_s f_y}{0.85 f_c' b}$ and $c = \dfrac{a}{\beta_1}$ where $\beta_1 = 0.8$	$a = 0.784A_s = (0.784)(3)(0.44 \text{ in.}^2) = 1.03$ in. $c = a/0.8 = 1.29$ in.
	$\varepsilon_t = \dfrac{\varepsilon_{cu}}{c}(d - c)$	$\varepsilon_t = \dfrac{0.003}{1.29 \text{ in.}}(25.7 \text{ in.} - 1.29 \text{ in.}) = 0.057$ Section is tension-controlled

		 *Fig. E2.4—Strain distribution across beam section.*
9.6.1.1 9.6.1.2	Minimum reinforcement ratio The provided reinforcement must be at least the minimum required reinforcement at every section along the length of the beams. (a) $A_s = \dfrac{3\sqrt{f_c'}}{f_y} b_w d$ (b) $A_s = \dfrac{200}{f_y} b_w d$ Because f_c' > 4444 psi, Eq. (9.6.1.2a) controls.	$A_s = \dfrac{3\sqrt{5000\text{ psi}}}{60{,}000\text{ psi}}(18\text{ in.})(25.7\text{ in.}) = 1.63\text{ in.}^2$ **Controls** $A_s = \dfrac{200}{60{,}000\text{ psi}}(18\text{ in.})(25.7\text{ in.}) = 1.54\text{ in.}^2$ At midspan $A_{s,prov} = (6)(0.44\text{ in.}^2) = 2.64\text{ in.}^2 > A_{s,min} = 1.63\text{ in.}^2$ **OK** At support: $A_s = 1.32\text{ in.}^2 < A_{s(min)} = 1.63\text{ in.}^2$ **NG** Therefore, use minimum reinforcement, four No. 6 at support $A_{s,prov\,(supp)} = 1.76\text{ in.}^2 > A_{s(min)} = 1.63\text{ in.}^2$ **OK**

Step 6. Shear design		
	<u>Shear strength</u>	
21.2.1b	Shear strength reduction factor	$\phi_{shear} = 0.75$
9.5.1.1 9.5.3.1 22.5.1.1	$\phi V_n \geq V_u$ $V_n = V_c + V_s$	 *Fig. E2.5—Shear critical section*
9.4.3.2	Design shear force is taken at the face of the support because the vertical reaction causes vertical tension rather than compression (Fig. E2.5). Condition (b), and (c) of Section 9.4.3.2 are satisfied. Condition (c), however, is not satisfied (refer to note at end of this step).	$V_u = 28.5$ kip
22.5.5.1	2019 Code introduced size effect for shear design in which the shear strength of an element that does not contain shear reinforcement is not directly proportional to its depth. This effect is addressed by incorporating a size effect factor for λ_s into the concrete contribution equation. If shear reinforcement is not present, then the concrete contribution to shear strength must be reduced by the size effect factor. If minimum shear reinforcement is provided, then the Eq. 22.5.5.1a can be used to calculate V_c.	
9.6.3.1	Minimum shear reinforcement is required where $V_u > \phi\lambda\sqrt{f_c'}b_w d$ For this example, use minimum shear reinforcement over entire length of beam. The concrete contribution to shear strength is then $V_c = 2\sqrt{f_c'}b_w d$ (22.5.5.1a)	 $V_c = 2\left(\sqrt{5000\text{ psi}}\right)(18\text{ in.})(25.7\text{ in.})/1000 = 65.4\text{ kip}$
	Check if $\phi V_c > V_u$	$\phi V_c = (0.75)(65.4\text{ kip}) = 49\text{ kip}$ $\phi V_c = 49\text{ kip} > V_u = 28.5\text{ kip}$ **OK** Good engineering practice calls for providing minimum shear reinforcement over the full beam span. Provide No. 3 stirrups at 12 in. on center where: 12 in. < $d/2 = 25.7\text{ in.}/2 = 12.8\text{ in.}$ **OK**
	Cross-sectional dimensions are selected to satisfy Eq. (22.5.1.2)	
22.5.1.2	$V_u \leq \phi\left(V_c + 8\sqrt{f_c'}b_w d\right)$	By inspection, this requirement is satisfied.
	Note: Hanger reinforcement may be required in girder B2 as outlined in Commentary R9.7.6.2.1 and discussed in A. H. Mattock and J. F. Shen, 1992, "Joints between Reinforced Concrete Members of Similar Depth," *ACI Structural Journal*, V. 89, No. 3, May-June, pp. 290-295.	

Step 7: Torsion		
		 *Fig. E2.6—Forces transferred from slab to edge beam.*
	Calculate the design load at face of slab to beam connection	$w_u = 1.2(0.21\text{ kip/ft} + (15\text{ psf})(2.375\text{ ft})/1000) + 1.6(0.1\text{ ksf})(4.75\text{ ft})/2$ $w_u = 0.71$ kip/ft
	Calculate the design unit torsion at beam center:	$t_u = (0.76\text{ kip/ft})(9\text{ in.}/12) = 0.53$ ft-kip/ft
	Design torsional force:	$T_u = (0.53\text{ ft-kip/ft})(18\text{ ft}) = 9.6$ ft-kip
22.7.4.1a	Therefore, check threshold torsion T_{th}: $T_{th} = \lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$ $h_w = 28\text{ in.} - 7\text{ in.} = 21\text{ in.}$ where	 *Fig. E2.7—L-beam geometry to resist torsion.*
	$A_{cp} = \sum b_i h_i$ is the area enclosed by outside perimeter of concrete	$A_{cp} = (18\text{ in.})(28\text{ in.}) + (21\text{ in.})(7\text{ in.}) = 651\text{ in.}^2$
	p_{cp} = perimeter of concrete gross area.	$p_{cp} = 2(18\text{ in.} + 21\text{ in.} + 7\text{ in.} + 21\text{ in.}) = 134$ in.
	The overhanging flange dimension is equal to the smaller of the projection of the beam below the slab (21 in.) and four times the slab thickness (28 in.). Therefore, use 21 in. (refer to Fig. E2.7).	$T_{th} = (1.0)\left(\sqrt{5000\text{ psi}}\right)\left(\frac{(651\text{ in.}^2)^2}{134\text{ in.}}\right)$ $T_{th} = 223{,}636$ in.-lb $= 18.6$ ft-kip
21.2.1c	Torsional strength reduction factor: $\phi = 0.75$	$\phi T_{th} = (0.75)(18.6\text{ ft-kip}) = 14.0$ ft-kip $T_u = 9.6\text{ ft-kip} < \phi T_{th} = 14.0$ ft-kip **OK** Torsion reinforcement is not required.

Step 8: Reinforcement detailing		
	<u>Minimum bar spacing</u>	
9.7.2.1 25.2.1	Minimum clear spacing between the horizontal No. 6 bars must be the greatest of:	
	$\text{Greatest of}\begin{cases} 1\text{ in.} \\ d_b \\ 4/3(d_{agg}) \end{cases}$	1 in. 3/4 in. 4/3(3/4 in.) = 1 in.
	Assume maximum aggregate size 3/4 in. Check if six No. 6 bars can be placed in the beam's web:	Therefore, clear spacing between horizontal bars must not be less than 1 in.
	$b_{w,req'd} = 2(\text{cover} + d_{stirrup} + 0.75\text{ in.}) + 5d_b + 5(1\text{ in.})_{min,spacing}$ (25.2.1)	$b_{w,req'd} = 2(1.5\text{ in.} + 0.375\text{ in.} + 0.75\text{ in.}) + 3.75\text{ in.} + 5\text{ in.}$ $= 14\text{ in.} < 18\text{ in.}$ **OK**
	where $d_{stirrup} = 0.375$ in. and $d_b = 0.75$ in.	Therefore, six No. 6 bars can be placed in one layer in the 18 in. beam web with 1.8 in. spacing between bars (Fig. E2.8).  *Fig. E2.8—Bottom reinforcement layout.*
9.7.2.2 24.3.1 24.3.2	Maximum bar spacing at the tension face must not exceed the lesser of:	
	$s = 15\left(\dfrac{40{,}000}{f_s}\right) - 2.5c_c$	$s = 15\left(\dfrac{40{,}000\text{ psi}}{40{,}000\text{ psi}}\right) - 2.5(2\text{ in.}) = 10\text{ in.}$ **Controls**
	$s = 12\left(\dfrac{40{,}000}{f_s}\right)$	$s = 12\left(\dfrac{40{,}000\text{ psi}}{40{,}000\text{ psi}}\right) = 12\text{ in.}$
	The maximum spacing concept is intended to limit flexural cracking widths. Note that c_c is the concrete cover to the flexural bars, not the ties.	12 in. spacing is provided, therefore **OK**

9.7.3 9.7.1.2 25.4.2.3 25.4.2.5	<u>Bar cutoff</u> <u>Development length of No. 6 bar</u> The simplified method is used to calculate the development length of a No. 6 bar: $\ell_d = \left(\frac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f_c'}}\right) d_b$ where ψ_t = bar location, ψ_t = 1.3 for top bars, because more than 12 in. of fresh concrete is placed below them and ψ_t=1.0 for bottom bars, because not more than 12 in. of fresh concrete is placed below them ψ_e = coating factor; ψ_e = 1.0, because bars are uncoated ψ_g = reinforcement grade factor; ψ_g = 1.0 for Grade 60 reinforcement	Top $\ell_d = \left(\frac{(60{,}000\text{ psi})(1.3)(1.0)(1.0)}{(25)(1.0)\sqrt{5000\text{ psi}}}\right)(0.75\text{ in.}) = 33.1\text{ in.}$, say, 36 in. Bottom $\ell_d = \left(\frac{(60{,}000\text{ psi})(1.3)(1.0)(1.0)}{(25)(1.0)\sqrt{5000\text{ psi}}}\right)(0.75\text{ in.}) = 25.5\text{ in.}$, say, 30 in.
9.7.1.3 25.5.2.1	<u>Splice length of No. 6 reinforcing bar</u> Per Table 25.5.2.1 splice length is $(\ell_{st}) = 1.3(\ell_d)$	Top: $1.3\ell_d$ = (1.3)(33.1 in.) = 43.0 in., say, 4 ft 0 in. Bottom: $1.3\ell_d$ = (1.3)(25.4 in.) = 33 in., say, 3 ft 0 in.
	<u>Top tension reinforcement</u> Calculate the inflection point for negative moment diagram $M_{max} - w_u(x)^2/2 + V_u x = 0$	$(-128\text{ ft-kip}) - 1.58\text{ kip/ft}\frac{x^2}{2} + (28.5\text{ kip})x = 0$ x = 5.3 ft, say, 5 ft 6 in. At 5.5 ft from the girder face two No. 6 can be cutoff and the remainder two No. 6 bars will be extended over the full beam span to support stirrups
9.7.3.8.4	At least one-third of the bars resisting negative moment at a support must have an embedment length beyond the inflection point the greatest of d, $12d_b$, and $\ell_n/16$. Two of three No. 6 bars are extended over full beam length and one No. 6 bars is terminated beyond the inflection point a distance equal to the embedment length (27 in.).	For No. 6 bars d = 25.7 in. $12d_b$ = 12(0.75 in.) = 9 in. $\ell_n/16$ = (36 ft)(12)/16 = 27 in. **Controls** 66 in. + 27 in. = 93 in. Therefore, the middle two No. 6 bar will be terminated at 7 ft 9 in. (93 in.) from face of support.

9.7.3.2 9.7.3.3	<u>Bottom tension reinforcement</u> Bars must be developed at points of maximum stress and points along the span where bent or terminated tension bars are no longer required to resist flexure	
	Six No. 6 bars are required to resist the factored moment at the midspan. Two No. 6 bars can resist a factored moment located at a section x from midspan	$(256\text{ ft-kip}) - 1.58\text{ kip/ft}\dfrac{x^2}{2} = 2(0.44\text{ in.}^2)(0.9)(60\text{ ksi})$ $\times\left(25.7\text{ in.} - \dfrac{2(0.44\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(46.5\text{ in.})}\right)$ $x = 14\text{ ft} = 168\text{ in.}$
9.7.3.3	A bar must extend beyond the point where it is no longer required to resist flexure for a distance equal to the greater of d or $12d_b$.	For No. 6 bars 1) $d = 25.7$ in. **Controls** 2) $12d_b = 12(0.75\text{ in.}) = 9$ in. Therefore, extend four No. 6 bars the greater of the development length (30 in.) from the maximum moment at midspan and a distance $d = 25.7$ in. from the theoretical cutoff point. 168 in. + 25.7 in. = 193.7 in., say, 16 ft-6 in. > $d = 25.7$ in. Therefore, extend four No. 6 bars 16 ft-6 in. from midspan.
9.7.3.8.2	A minimum of one-fourth positive tension bars must extend into the support minimum 6 in.	Extend the remaining two No. 6 bars > 1/4 six No. 6 a minimum of 6 in. into the support, but not less than $\ell_d = 30$ in. from the theoretical cutoff point (Fig. E2.7).
	Note: These calculations are performed to present the code requirements. In practice, all longitudinal bottom bars are extended into the support rather than terminating them 1 ft 6 in. from the support as shown by calculations.	
Step 9: Integrity reinforcement		
9.7.7.2	<u>Integrity reinforcement</u> Either one of the two conditions must be satisfied, but not both At least one-fourth the maximum positive moment bars, but not less than two bars must be continuous. Longitudinal bars must be enclosed by closed stirrups along the clear span of the beam	This condition was satisfied above by extending two No. 6 bars into the girders

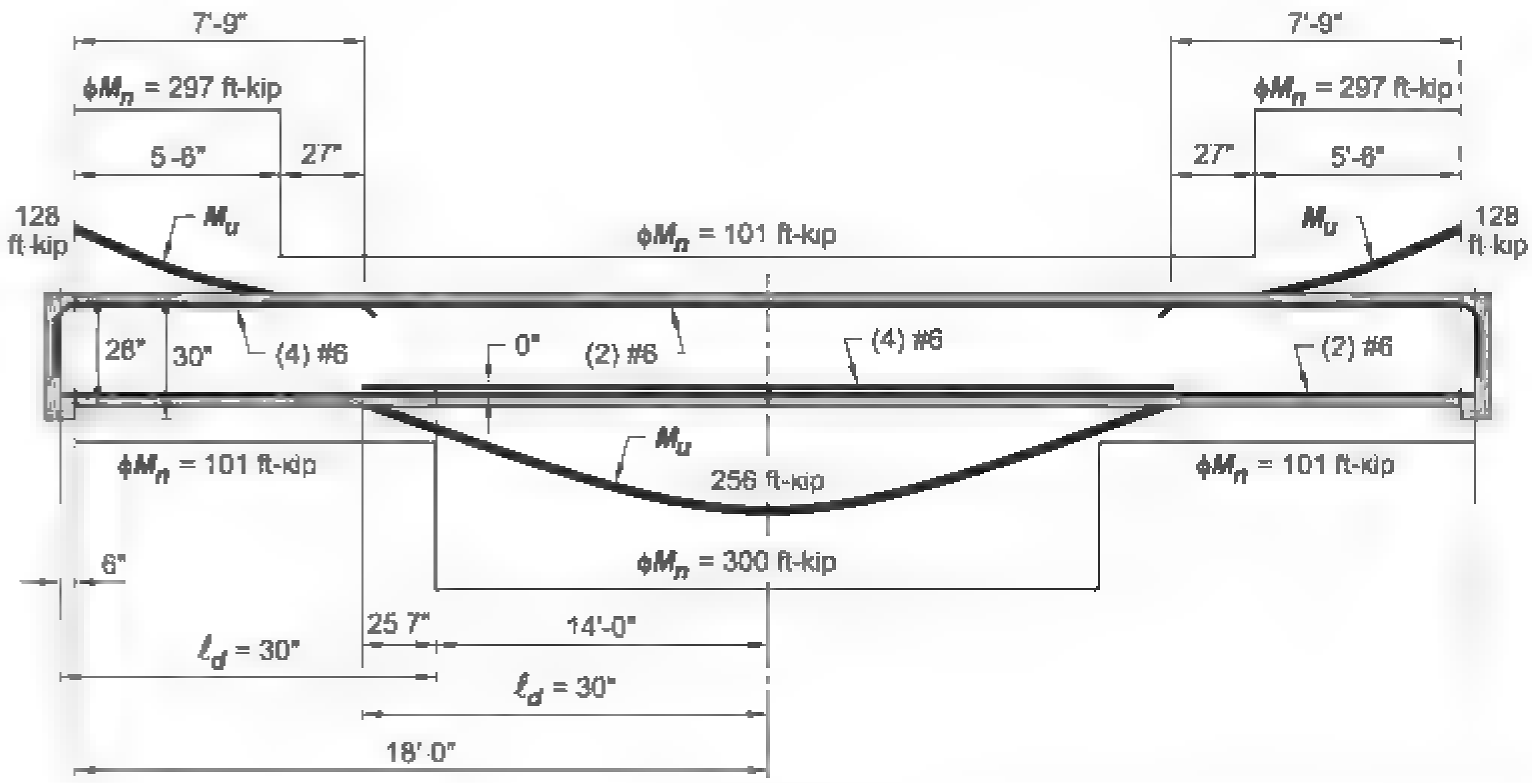
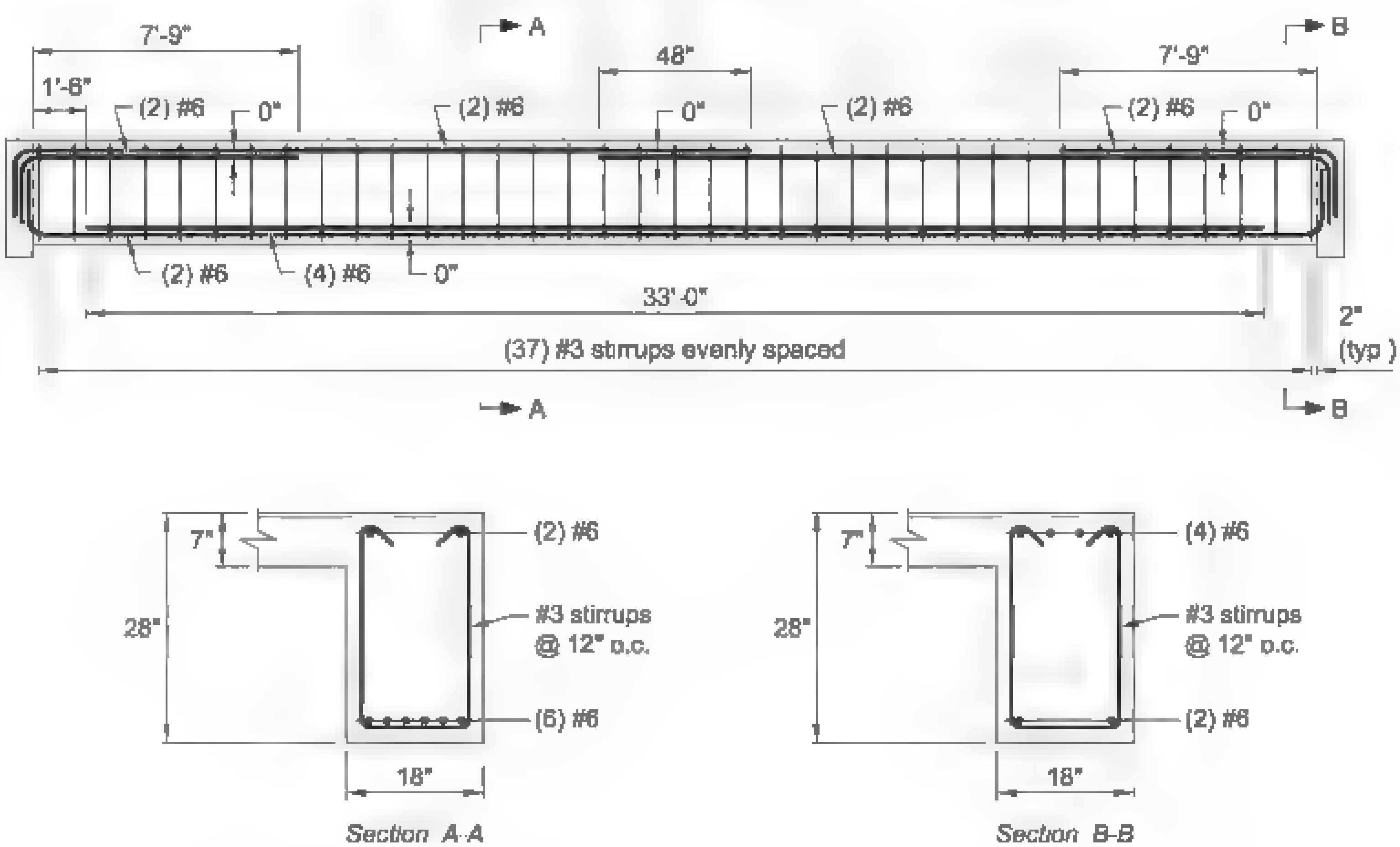


Fig. E2.9—End span reinforcement cutoff locations.

Step 10: Detailing

Final detailing:



Slab reinforcement not shown for clarity

Fig. E2.10—Beam reinforcement details.

Beam Example 3: *Single interior girder beam*

Determine the size of a one-span beam (B2) that frames an opening in the floor. The beam is built integrally with a 7 in. slab of a seven-story building. B2 supports B4 and B1 and is supported by B5. Design and detail the beam.

Given:

Load—

Service dead load $D = 15$ psf

Service live load $L = 100$ psf

Concentrated loads $P_u = 28.5$ kip ($P_D = 14.4$ kip and $P_L = 7$ kip) located 6 ft 3 in. south and north of Column Lines B and D, respectively. (Refer to Example 2.)

Material properties—

$f_c' = 5000$ psi (normalweight concrete)

$f_y = 60{,}000$ psi

Span length, 28 ft

Beam width, 18 in.

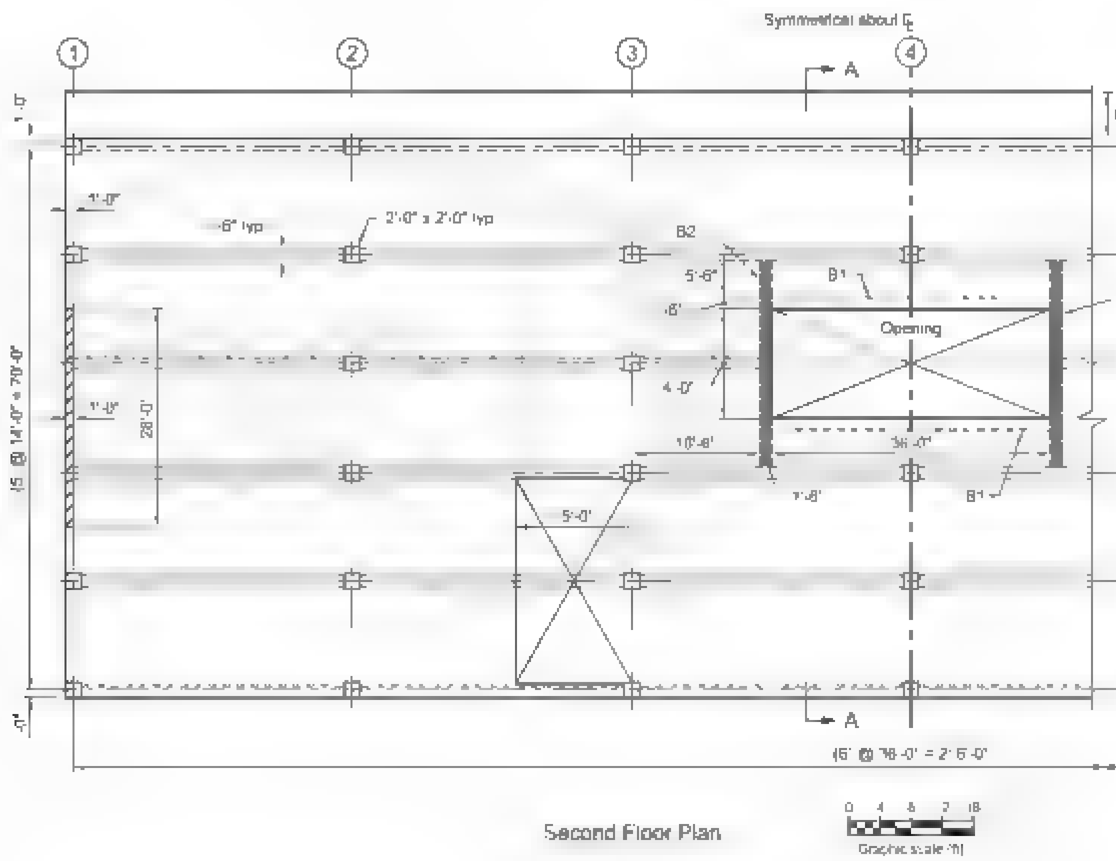
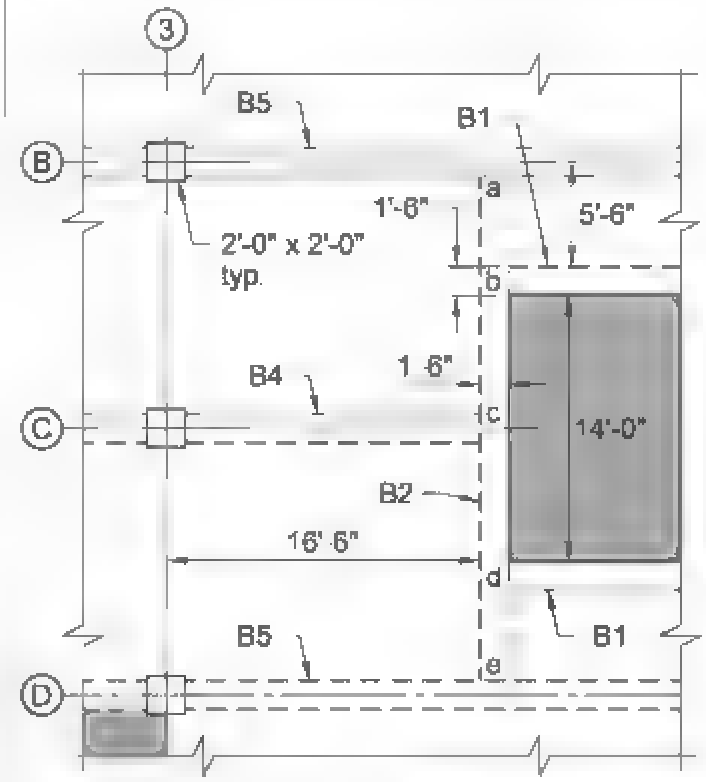


Fig. E3.1—Framing plan showing slab opening and beams framing slab opening.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
9.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements of ACI 318. The designer determines the durability classes. Please refer to Chapter 4 of MNL-17 for an in-depth discussion of the Categories and Classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi. Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26.
Step 2: Beam geometry		
9.3.1.1	Beam depth Beam depth cannot be calculated using Table 9.3.1.1, because two beams frame into it (concentrated loads). For framing simplicity, choose a beam deeper than the beams' depths, framing into it to allow for ease of construction and placement of reinforcement.	Try: $h = 30$ in.
24.2.2	The beam deflection will be checked and compared to Table 24.2.2.	
9.2.4.2	Flange width The beam is monolithic with the slab on one side in the middle over a 14 ft long section and slab on both sides for the remainder of the beam. At the maximum positive moment, the beam will behave as an L-beam. Therefore, the effective flange width on one side of the beam is the least from Table 6.3.2.1.	
6.3.2.1	One side of web is the least of $\left\{\begin{array}{l} 6h_{slab} \\ s_w/2 \\ \ell_n/2 \end{array}\right.$ Therefore, flange width $b_f = \ell_n/12 + b_w$	$(6)(7\text{ in.}) = 42$ in. $(16.5\text{ ft})(12)/2 = 99$ in. $(28\text{ ft}(12) - 18\text{ in.})/12 = 26.5$ in. **Controls** $b_f = 26.5\text{ in.} + 18\text{ in.} = 44.5$ in.
	On both sides of the opening, the beam is placed monolithically with the slab and will behave as a T-beam. The flange width on each side of the beam is obtained from Table 6.3.2.1.	
6.3.1	Each side of web is the least of $\left\{\begin{array}{l} 8h_{slab} \\ s_w/2 \\ \ell_n/8 \end{array}\right.$ Therefore, flange width $b_f = \ell_n/8 + b_w + \ell_n/8$	$8(7\text{ in.}) = 56$ in. $(16.5\text{ ft})(12)/2 = 99$ in. $((28\text{ ft})(12) - 18\text{ in.})/8 = 39.75$ in. **Controls** $b_f = 39.75\text{ in.} + 18\text{ in.} + 39.75\text{ in.} = 97.5$ in.

Step 3. Loads and load patterns		
	Self weights of B2 Beam: beam width b = 18 in.	w_b = [(18 in.)(30 in.)/(144)](0.150 kip/ft³) = 0.56 kip/ft
	Tributary load between Girder B2 and Column Line 3 (refer to Fig. E3.2). The load is transferred to B2 through Beam B4 along Column Line C spanning (15 ft 6 in. clear span) For lobbies and assembly areas, the uniform design live load is 100 psf per Table 4-1 in ASCE/SEI 7. To account for weights from ceilings, partitions, and HVAC systems, add 15 psf as miscellaneous dead load.	 Fig. E3.2—Beams B1 and B4 framing into B2
	Dead load Slab self-weight (7 in. thick) supported by Beam B4	P_s = (7 in./12)(14 ft)(16.5 ft/2)(0.15 kip/ft³) = 10.1 kip
	Beam B4 self-weight: assume beam is 18 in. wide by 30 in. deep Note: 7 in. is the slab thickness	$P_B = \left(\frac{18\text{ in.}}{12}\right)\left(\frac{30\text{ in.}-7\text{ in.}}{12}\right)\left(\frac{16.5\text{ ft}-1\text{ ft}}{2}\right)(0.15\text{ kip/ft}^3)$ = 3.3 kip
	Superimposed dead load of 15 psf:	P_{SD} = (15 psf/1000)(14 ft)(16.5 ft)/2 = 1.7 kip
	Total dead load at B2 midspan	$\sum P$ = 10.1 kip + 3.3 kip + 1.7 kip = 15.1 kip
	Live load Concentrated load between Column Line 3 and girder transferred at midspan	P_L = (0.1 ksf)(14 ft)(16.5 ft)/2 = 11.6 kip
	Beams B1 frame into Beam B2 at 6 ft 3 in. and 21 ft 9 in. from Column Line B (Fig. E3.2). The beams' factored reactions were calculated in Example 2 and were found to be: 28.5 kip in Step 4	P_u = 28.5 kip
	The beam resists gravity load only. Lateral forces are not considered in this problem.	
	Distributed load	
5.3.1a	$w_u = 1.4D$	w_u = 1.4(0.56 kip/ft + (15 psf)(1.5 ft)/1000) = 0.82 kip/ft
5.3.1b	$w_u = 1.2D + 1.6L$	w_u = 1.2(0.82 kip/ft)/1.4 + 1.6((100 psf)(1.5 ft)/1000) = 0.94 kip/ft **Controls**
	Concentrated load	
5.3.1a	$P_u = 1.4P_D$	P_u = 1.4(15.1 kip) = 21.1 kip
5.3.1b	$P_u = 1.2P_D + 1.6P_L$	P_u = 1.2(15.1 kip) + 1.6(11.6 kip) = 36.7 kip **Controls**

Beams

Step 4. Analysis		
	Beam B2 is monolithic with supports.	
9.4.1.2	Chapter 6 permits several analysis procedures to calculate the required strengths.	
	For this example, calculate beam moment at supports using coefficients from Table B-1 Reinforced Concrete Design Handbook Design Aid Analysis Tables, which can be downloaded from https://www.concrete.org/MNL1721Download1	
	$M_u = w_u\ell^2/12 + P_u\ell/8 + P_ua^2b/\ell^2 + P_uab^2/\ell^2$	$M_u = (0.94\text{ kip/ft})(28\text{ ft})^2/12 + (36.7\text{ kip})(28\text{ ft})/8$ $+ (28.5\text{ kip})(6.25\text{ ft})^2(21.75\text{ ft})/(28\text{ ft})^2$ $+ (28.5\text{ kip})(6.25\text{ ft})(21.75\text{ ft})^2/(28\text{ ft})^2$ $= 328\text{ ft-kip}$
	and the analysis shows the beam shear is $V_u = w_u\ell/2 + P_u/2$	$V_u = (0.94\text{ kip/ft})(28\text{ ft})/2 + (36.7\text{ kip})/2 + 2(28.5\text{ kip})/2$ $= 60\text{ kip}$ Note that the (2)(28.5 kip) shear force represents the two beams framing into the girder beam.
	Using coefficients from Appendix B-1 Reinforced Concrete Design Handbook Design Aid — Analysis Tables, which can be downloaded from https://www.concrete.org/MNL1721Download1, assuming maximum moment is at midspan	$M_u = 255\text{ ft-kip}$
	This distribution assumes the girder remains uncracked. If the girder, does crack, however, its stiffness is greatly reduced and redistribution of moments occurs.	
	Assume that the moments at supports are reduced by 15 percent.	$M_u = (0.85)(328)\text{ ft-kip} = 279\text{ ft-kip}$
	Accordingly, the moment at midspan must be increased by the same amount	$M_u = (255\text{ ft-kip}) + (0.15)(328\text{ ft-kip}) = 304\text{ ft-kip}$

Note:

Alan Mattock states that, "… and it is concluded that redistribution of design bending moments by up to 25% does not result in performance inferior to that of beams designed for the distribution of bending moments predicted by the elastic theory, either at working loads or at failure." (Mattock, A. H., 1959, "Redistribution of Design Bending Moments in Reinforced Concrete Continuous Beams," *Proceedings*, Institution of Civil Engineers (London), V. 13, pp. 35-46.)

H. Scholz, however, limits the moment redistribution to 20 percent: "The cut-off point at 20 percent is imposed to avoid excessive cracking at elastic service moments." (Scholz, H., 1993, "Contribution to Redistribution of Moments in Continuous Reinforced Concrete Beams," *ACI Structural Journal*, V. 90, No. 2, Mar.-Apr., pp. 150-155.)

Beams B1 and B4 frame into Beam B2. therefore, assume that (B1) reaction of 28.5 kip and (B4) reaction of 36.7 kip are applied at the face of Beam B2, but in opposite directions. Ignoring the distributed load from the slab, Beam B2 is subjected to torsion

Refer to the torsion diagram in Fig. E3.3.

From B1

$$T_u = \frac{(28.5\text{ kip})(9\text{ in.})}{12} = 21.4\text{ ft-kip}$$

and from B4

$$T_u = \frac{(36.7\text{ kip})(9\text{ in.})}{12} = 27.6\text{ ft-kip}$$

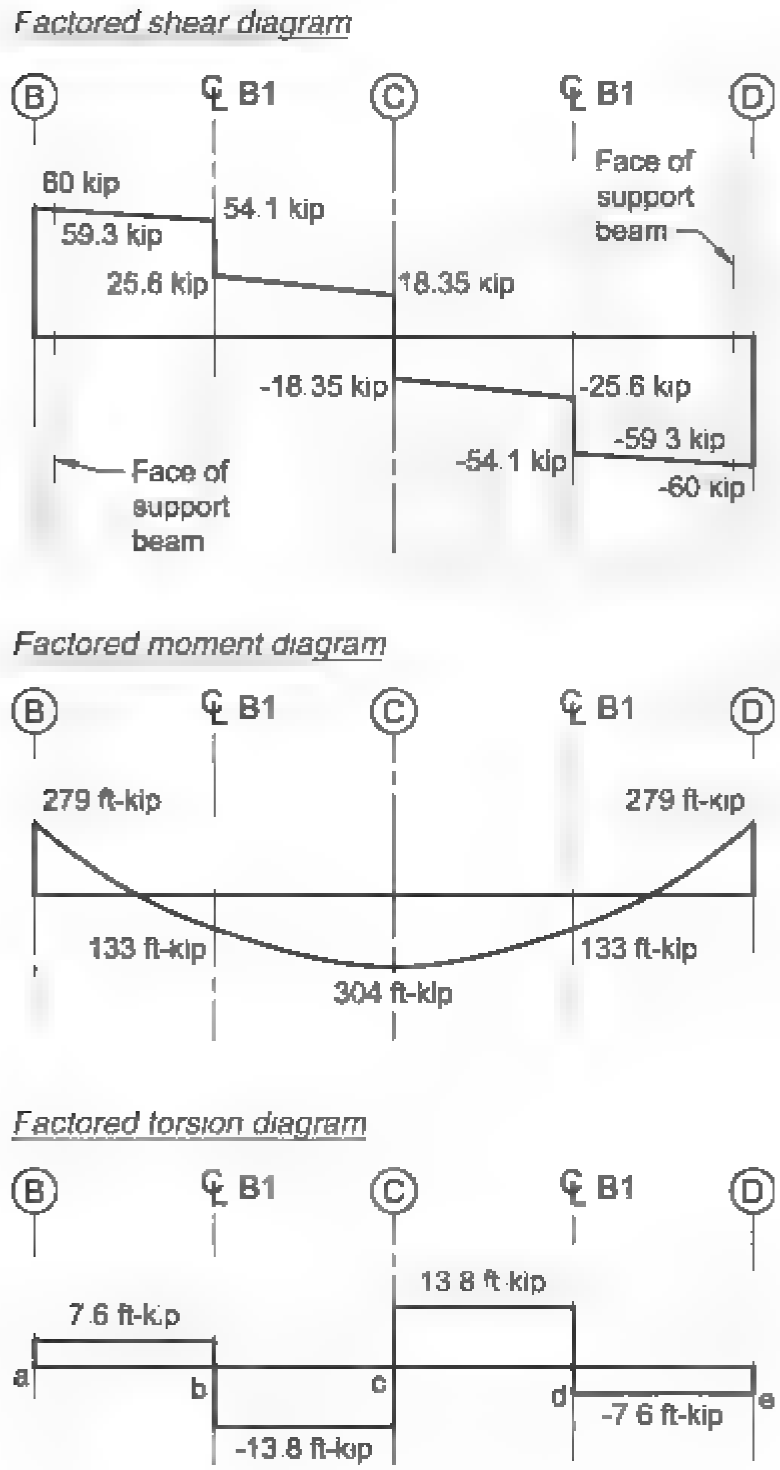
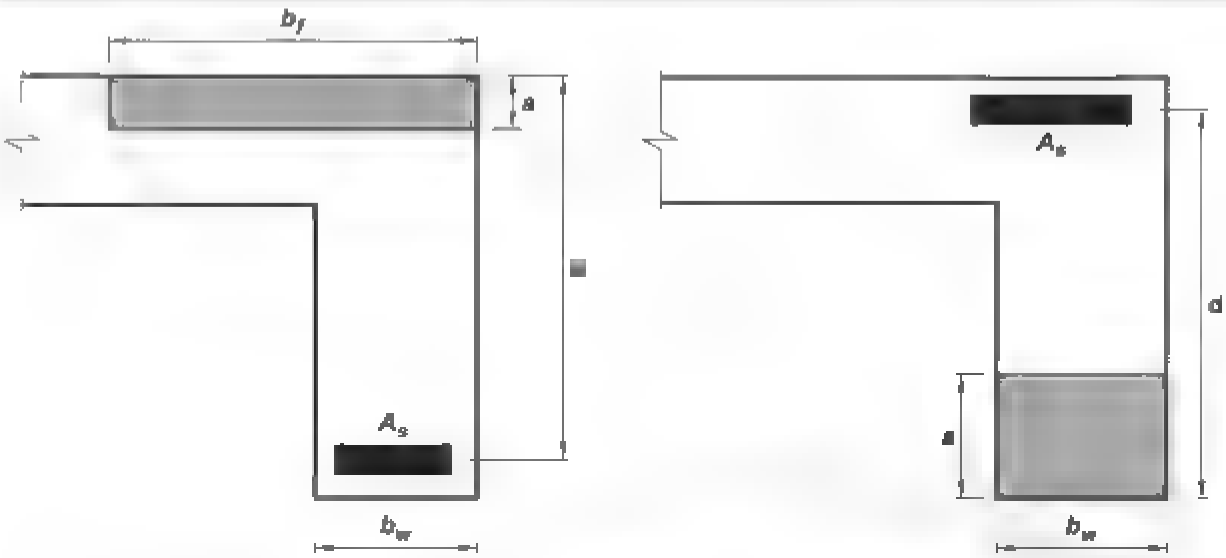


Fig. E3.3—Shear, moment, and torsion diagrams

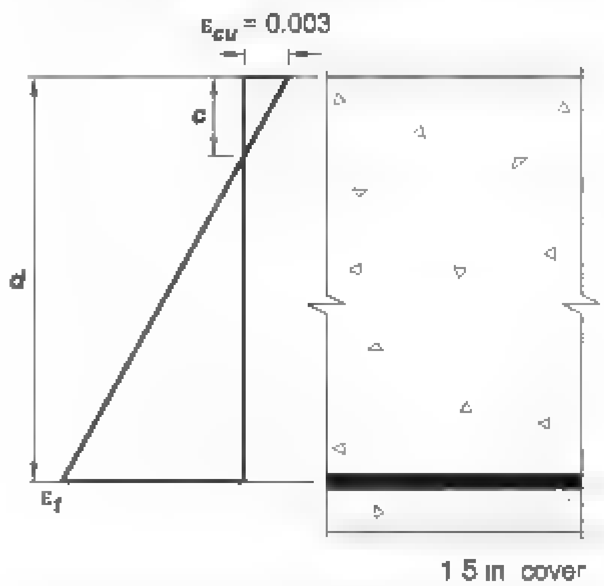
Step 5. Moment design		
9.3.3.1	Limiting steel strain restricts the amount of reinforcement to ensure warning of failure by excessive deflection and cracking. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members. Beginning with the 2019 Code, this limit is revised to require that the section be tension-controlled.	$\varepsilon_{ty} = \frac{f_y}{E_s} = \frac{60{,}000\text{ psi}}{29{,}000{,}000\text{ psi}} \cong 0.002$ $\varepsilon_t \geq \varepsilon_{ty} + 0.003 = 0.002 + 0.003 = 0.005$
21.2.2	Because the section must be tension-controlled, the strength reduction factor is 0.9	Beam must be tension-controlled in accordance with Table 21.2.2 $\phi = 0.9$
20.5.1.3.1	Calculate effective depth assuming No. 3 stirrups, No. 6 longitudinal bars, and 1.5 in. cover The effective depth of one row of longitudinal reinforcement is $d = h - \text{cover} - d_{b,stirrup} - d_{b,long}/2$	d = 28 in. − 1.5 in. − 0.375 in. − 0.75 in./2 = 25.75 in., say, d = 25.7 in.
22.2.2.1	The concrete compressive strain at which nominal moments are calculated is $\varepsilon_{cu} = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and its value is approximately 10 to 15 percent of the concrete compressive strength. For calculating nominal strength, ACI 318 neglects the concrete tensile strength. Determine the equivalent concrete compressive stress for design	
22.2.2.3	The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of nominal strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distribution of $0.85f_c'$ with a depth of $a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3	
22.2.2.4.1	For f_c' = 5000 psi	$\beta_1 = 0.85 - \frac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$
22.2.2.4.3	Find the equivalent concrete compressive depth, a, by equating the compression force in the section to the tension force (refer to Fig. E3.4): $C = T$ $0.85f_c'ba = A_sf_y$	$0.85(5000\text{ psi})(b)(a) = A_s(60{,}000\text{ psi})$

22.2.1.1	For positive moment: $b = b_f = 44.5$ in.	$a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(44.5\text{ in.})} = 0.317A_s$
	For negative moment: $b = b_w = 18$ in.	At support $0.85(5000\text{ psi})(18\text{ in.})(a) = A_s(60{,}000\text{ psi})$ $a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(18\text{ in.})} = 0.784A_s$

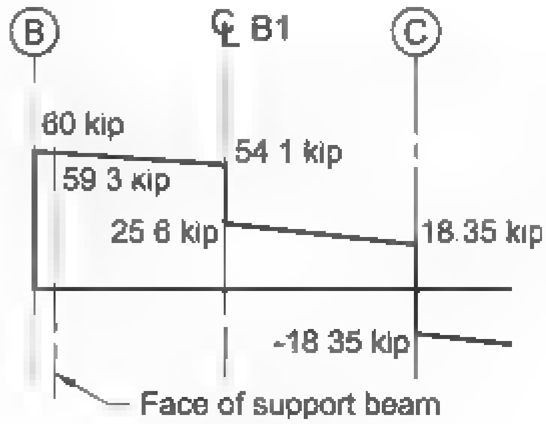


Positive moment *Negative moment*

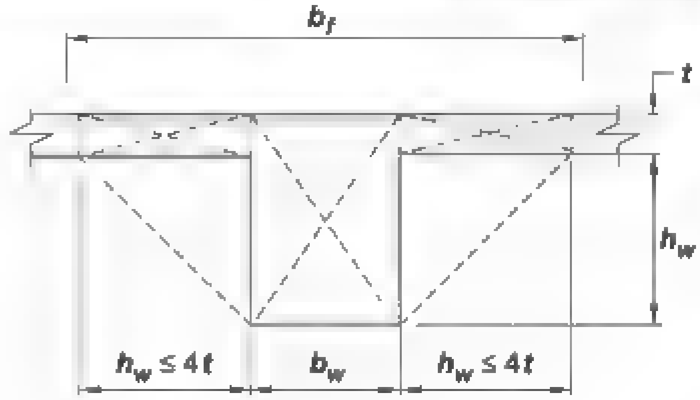
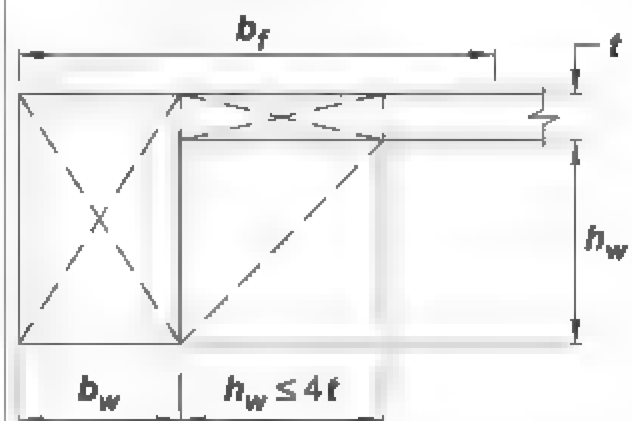
Fig. E3.4—Section reinforcement at midspan and at support.

	Design the beam for the maximum flexural moment at the midspan and the face of supports.	
9.5.1.1	The beam strength must satisfy the following inequalities at each section along its length $\phi M_n \geq M_u$ $\phi V_n \geq V_u$	
	Calculate required flexural reinforcement area using the following equation. $M_u \leq \phi M_n = \phi A_s f_y \left(d - \dfrac{a}{2}\right)$	Midspan $304\text{ ft-kip} \leq (0.9)(60\text{ ksi})A_s\left(27.7\text{ in.} - \dfrac{0.317A_s}{2}\right)$
	No. 6 bars, $d_b = 0.75$ in. and $A_s = 0.44$ in.^{2}	$A_s = 2.47$ in.^{2}; use six No. 6
	Note that $b = 18$ in. for negative moments and $b = 44.5$ in. for positive moments.	Supports $279\text{ ft-kip} \leq (0.9)(60\text{ ksi})A_s\left(27.7\text{ in.} - \dfrac{0.784A_s}{2}\right)$ $A_s = 2.31$ in.^{2} use 6-No. 6
21.2.2 9.3.3.1	Check if calculated strain is greater than 0.005 in./in. (tension-controlled). Refer to Fig. E3.5. $a = \dfrac{A_s f_y}{0.85 f_c' b}$ and $c = \dfrac{a}{\beta_1}$ where $\beta_1 = 0.8$ $\varepsilon_t = \dfrac{\varepsilon_{cu}}{c}(d - c)$	Midspan $a = 0.317A_s = (0.317)(6)(0.44\text{ in.}^2) = 0.84$ in. $c = a/0.8 = 0.84\text{ in.}/0.8 = 1.05$ in. < 7 in. slab thickness Therefore, shape assumption is correct $\varepsilon_t = \dfrac{0.003}{1.05\text{ in.}}(27.7\text{ in.} - 1.05\text{ in.}) = 0.076 > 0.005$ Supports $a = 0.784A_s = (0.784)(6)(0.44\text{ in.}^2) = 2.07$ in. $c = a/0.8 = 2.07/0.8 = 2.59$ in. $\varepsilon_t = \dfrac{0.003}{2.59\text{ in.}}(27.7\text{ in.} - 2.59\text{ in.}) = 0.029 > 0.005$ Section is tension controlled  Fig. E3.5—Strain distribution across beam section

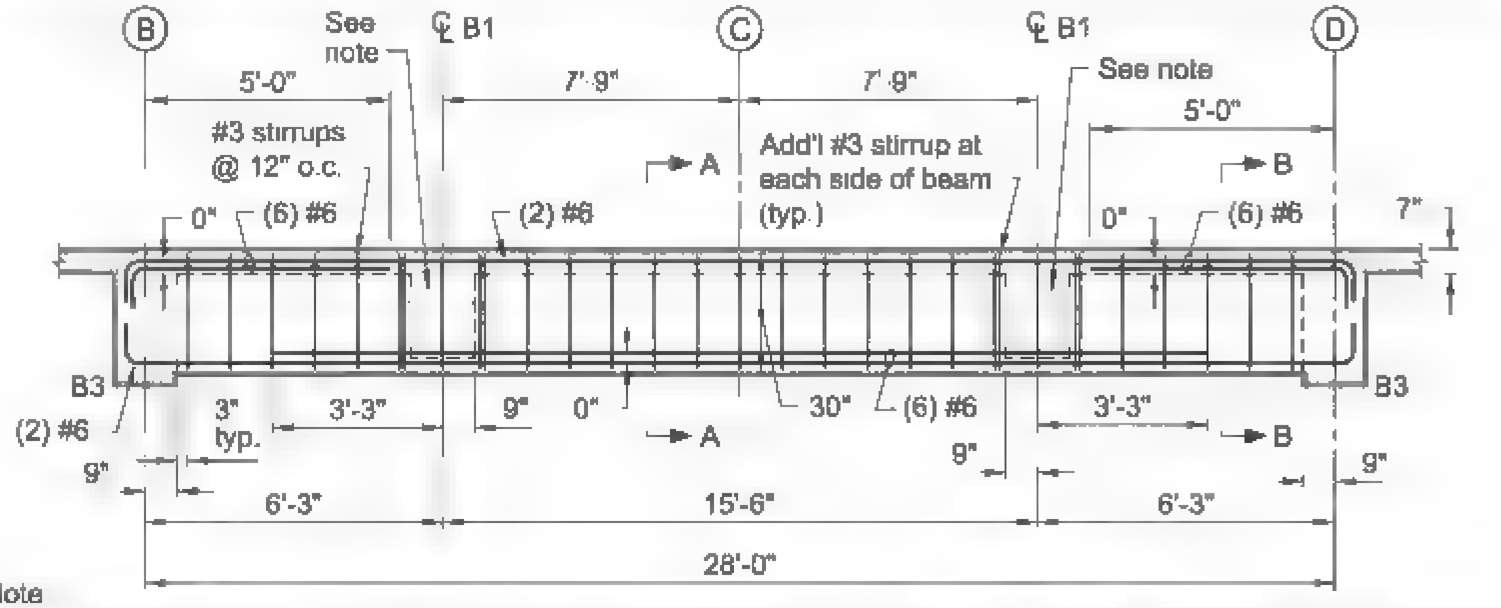
9.6.1.1 9.6.1.2	<u>Minimum reinforcement area</u> The reinforcement area must exceed the minimum required at every section along the length of the beam (a) $A_s = \dfrac{3\sqrt{f_c'}}{f_y} b_w d$ (b) $A_s = \dfrac{200}{f_y} b_w d$	$A_s = \dfrac{3\sqrt{5000\text{ psi}}}{60{,}000\text{ psi}}(18\text{ in.})(27.7\text{ in.}) = 1.76\text{ in.}^2$
	Because $f_c' > 4444$ psi, Eq. (9.6.1.2a) controls.	At midspan: $A_{s(prov)} = 2.64\text{ in.}^2 > A_{s,min} = 1.76\text{ in.}^2$ **OK** At support: $A_{s(prov)} = 2.64\text{ in.}^2 > A_{s,min} = 1.76\text{ in.}^2$ **OK**

Step 6: Shear design		
	<u>Shear strength</u>	
21.2.1b 9.5.1.1	Shear strength reduction factor: $\phi V_n \geq V_u$	$\phi_{shear} = 0.75$
9.5.3.1 22.5.1.1	$V_n = V_c + V_s$	
9.4.3.1 9.4.3.2	Design shear force is taken at the face of the support because the vertical reaction causes vertical tension rather than compression (Fig. E3.5). Condition (b), and (c) of 9.4.3.2 are satisfied. Condition (a), however, is not satisfied.	*Fig. E3.6—Shear-critical section*
22.5.5.1	2019 Code introduced size effect for shear design in which the shear strength of an element that does not contain shear reinforcement is not directly proportional to its depth. This effect is addressed by incorporating a size effect factor for λ_s into the concrete contribution equation. If shear reinforcement is not present, then the concrete contribution to shear strength must be reduced by the size effect factor. If minimum shear reinforcement is provided, then the Eq. 22.5.5.1a can be used to calculate V_c. Minimum shear reinforcement is required where $V_u > \phi\lambda\sqrt{f_c'}b_w d$ For this example, use minimum shear reinforcement over entire length of beam. The concrete contribution to shear strength is then:	$V_u = 59.3$ kip
	$V_c = 2\sqrt{f_c'}b_w d$ (22.5.5.1a)	$V_c = 2\sqrt{5000\text{ psi}}(18\text{ in.})(27.7\text{ in.})/1000 = 70.5\text{ kip}$
	Check if $\phi V_n \geq V_u$ Cross-sectional dimensions are selected to satisfy Eq. (22.5.1.2)	$\phi V_c = (0.75)(70.5\text{ kip}) = 52.9\text{ kip}$ $\phi V_n = 52.9\text{ kip} < V_u = 59.3\text{ kip}$ **NG** Therefore, shear reinforcement is required for strength
22.5.1.2	$V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d)$	$V_u \leq \phi\left(70.5\text{ kip} + 8\sqrt{5000\text{ psi}}(18\text{ in.})(27.7\text{ in.})\right)$ ≤ 212 kip Section dimensions are satisfactory
	Note: Because the girder soffit is below mid-depth, hanger reinforcement may be required as outlined in Commentary R9.7.6.2 and discussed in A. H. Mattock and J. F. Shen, "Joints between Reinforced Concrete Members of Similar Depth," *ACI Structural Journal*, V. 89, No 3, May-June 1992, pp. 290-295. See Example 11 for example calculations.	

	<u>Shear reinforcement</u>	
22.5.8.5.1	Transverse reinforcement is required at each section where $V_u > \phi V_c$ satisfying Eq. (22.5.8.5.3)	
	<u>Section AB</u>	
22.5.8.5.3	$\phi V_s \geq V_u - \phi V_c$	$\phi V_s \geq (59.3\text{ kip}) - (52.9\text{ kip}) = 6.4\text{ kip}$
22.5.8.5.5	where $V_s = \dfrac{A_v f_{yt} d}{s}$	$V_s \geq \dfrac{6.4\text{ kip}}{0.75} = 8.5\text{ kip}$
	$\dfrac{A_v}{s} \geq \dfrac{V_s}{f_{yt} d}$	$\dfrac{A_v}{s} \geq \dfrac{(8.5\text{ kip})}{(60\text{ ksi})(27.7\text{ in.})} = 0.0051\text{ in.}^2/\text{in.}$
9.7.6.2.2	Check maximum allowable stirrup spacing	$4\sqrt{f_c'}b_w d = 4\left(\sqrt{5000\text{ psi}}\right)(18\text{ in.})(27.7\text{ in.}) = 140.5\text{ kip}$ say, 141 kip
	Is $V_s \leq 4\sqrt{f_c'}b_w d$?	$V_s = 8.5\text{ kip} < 4\sqrt{f_c'}b_w d = 141\text{ kip}$ **OK** Therefore, for stirrup spacing, use the lesser of: $d/2 = 27.7\text{ in.}/2 = 13.8\text{ in.}$ or 24 in. Try No. 3 stirrups at 12 in. on center $\left(\dfrac{A_v}{s}\right)_{prov} = \dfrac{2(0.11\text{ in.}^2)}{12\text{ in.}} = 0.018\text{ in.}^2/\text{in.} > 0.005\text{ in.}^2/\text{in.}$ **OK**
9.6.3.4	Specified shear reinforcement must be at least the larger of: $A_{v,min}/s = 0.75\sqrt{f_c'}\dfrac{b_w}{f_{yt}}$ and $A_{v,min}/s = 50\dfrac{b_w}{f_{yt}}$	$\dfrac{A_{v,min}}{s} \geq 0.75\sqrt{5000\text{ psi}}\dfrac{18\text{ in.}}{60{,}000\text{ psi}} = 0.016\text{ in.}^2/\text{in.}$ **Controls** $\dfrac{A_{v,min}}{s} = 50\dfrac{18\text{ in.}}{60{,}000\text{ psi}} = 0.015\text{ in.}^2/\text{in.}$ Provided: $\dfrac{A_v}{s} \geq \dfrac{2(0.11\text{ in.}^2)}{12\text{ in.}} = 0.018\text{ in.}^2/\text{in.} > \dfrac{A_{v,min}}{s} = 0.016\text{ in.}^2/\text{in.}$ satisfies 9.6.3.4, therefore **OK**

Step 7: Torsion design		
	Determine if girder torsion can be neglected	 *Fig. E3.7—T-beam geometry to resist torsion.*
22.7.4.1(a)	Check threshold torsion T_{th}. $T_{th} = \lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	
	Determine portion of slab to be included with the beam for the torsional design T-section between *a* and *b* and between *d* and *e* (Fig. E3.2) and	
	where $A_{cp} = \sum b_i h_i$ is the area enclosed by outside perimeter of concrete	$A_{cp} = (18 \text{ in.})(30 \text{ in.}) + 2(23 \text{ in.})(7 \text{ in.}) = 862 \text{ in.}^2$
	p_{cp} = perimeter of concrete gross area	$p_{cp} = 2(18 \text{ in.} + 23 \text{ in.} + 23 \text{ in.} + 23 \text{ in.} + 7 \text{ in.}) = 188 \text{ in.}$ $T_{th} = (1.0)\sqrt{5000 \text{ psi}}\left(\frac{(862 \text{ in.}^2)^2}{188 \text{ in.}}\right)$
	Refer to Fig. E3.3 for torsional value T_u near supports.	$T_{th} = 279{,}474 \text{ in.-lb} = 23.3 \text{ ft-kip}$
21.2.1c	Torsional strength reduction factor $\phi = 0.75$	$\phi T_{th} = (0.75)(23.3 \text{ ft-kip}) = 17.5 \text{ ft-kip}$ $\phi T_{th} = 17.5 \text{ ft-kip} > T_u = 7.6 \text{ ft-kip}$ **OK** Torsion reinforcement is not required between *a* and *b* and between *d* and *e* (Fig. E3.2).
	Determine portion of the slab to be included with the beam for torsional design. L-section between *b* and *d* (Fig. E3.2) The overhanging flange dimension is equal to the smaller of the projection of the beam below the slab (23 in.) and four times the slab thickness (28 in.). Therefore, use 23 in. (refer to Fig. E3.8).	 *Fig. E3.8—L-beam geometry to resist torsion.* $A_{cp} = (18 \text{ in.})(30 \text{ in.}) + (23 \text{ in.})(7 \text{ in.}) = 701 \text{ in.}^2$ $p_{cp} = 2(18 \text{ in.} + 23 \text{ in.} + 7 \text{ in.} + 23 \text{ in.}) = 142 \text{ in.}$ $T_{th} = (1.0)\left(\sqrt{5000 \text{ psi}}\right)\left(\frac{(701 \text{ in.}^2)^2}{142 \text{ in.}}\right)$ $T_{th} = 244{,}699 \text{ in.-lb} = 20.4 \text{ ft-kip}$
	Refer to Fig. E3.3 for torsional value T_u at midspan.	$\phi T_{th} = (0.75)(20.4 \text{ ft-kip}) = 15.3 \text{ ft-kip}$ $\phi T_{th} = 15.3 \text{ ft-kip} > T_u = 13.8 \text{ ft-kip}$ **OK** Torsion reinforcement is not required between *b* and *d* (Fig. E3.2).

Step 8: Reinforcement detailing



Note:
Hanger reinforcement may be necessary in the supporting girder. See Example 11 for hanger reinforcement calculations.

Fig. E3.9—Longitudinal and transverse reinforcement.

9.7.2.1 — Minimum top bar spacing

From Appendix A of MNL-17(21) Reinforced Concrete Design Handbook Design Aid – Analysis Tables, which can be downloaded from https://www.concrete.org/MNL172/Downloads, six No. 6 bars can be placed in one layer within an 18 in. wide beam.

Bar spacing can also be calculated as shown as follows for bottom bars.

Minimum bottom bar spacing

9.7.2.1, 25.2.1 — Minimum clear spacing between the longitudinal bars is the greatest of:

$$\text{Clear spacing} \begin{cases} 1\text{ in.} \\ d_b \\ 4/3(d_{agg}) \end{cases}$$

1 in. **Controls**

0.75 in.

4/3(3/4 in.) = 1 in. **Controls**

Assume 3/4 in. maximum aggregate size.

Therefore, clear spacing between horizontal bars must not be less than 1.0 in.

Check if six No. 6 bars can be placed in the beam's web.

$$b_{w,req'd} = 2(\text{cover} + d_{stirrup} + 0.75\text{ in.}) + 5d_b + 5(1.0\text{ in.})_{min,spacing} \quad (25.2.1)$$

$$b_{w,req'd} = 2(1.5\text{ in.} + 0.375\text{ in.} + 0.75\text{ in.}) + 5(0.75\text{ in.}) + 5(1.0\text{ in.})$$

$b_{w,req'd}$ = 14 in. < 18 in. **OK**

Refer to Fig. E3.10 for reinforcement placement in Beam B2.

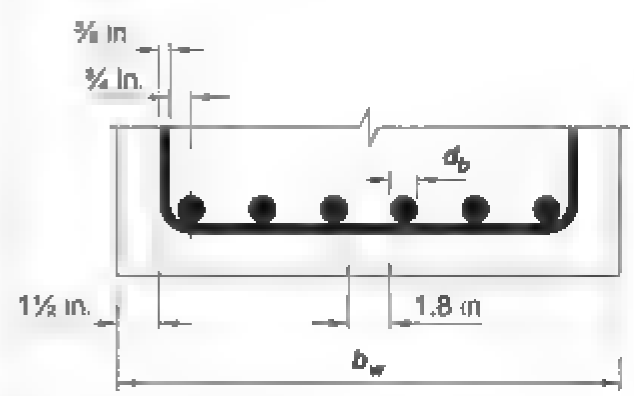


Fig. E3.10 for reinforcement placement.

9.7.2.2 24.3.1 24.3.2	Maximum bar spacing at the tension face must not exceed the lesser of $s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c$ $s = 12\left(\frac{40{,}000}{f_s}\right)$ This spacing is to limit flexural cracking widths, where c_c = 2 in. is the least distance from surface of deformed reinforcement to the tension face	$s = 15\left(\frac{40{,}000\text{ psi}}{40{,}000\text{ psi}}\right) - 2.5(2\text{ in.}) = 10\text{ in.}$ **Controls** $s = 12\left(\frac{40{,}000\text{ psi}}{40{,}000\text{ psi}}\right) = 12\text{ in.}$ 1.8 in. spacing is provided, therefore **OK**
 9.7.3 9.7.1.2 25.4.2.3 25.4.2.5	Development length of No. 6 reinforcing bar The simplified method is used to calculate the development length of No. 6 bars $\ell_d = \left(\frac{f_y\psi_t\psi_e\psi_g}{25\lambda\sqrt{f_c'}}\right)d_b$ where ψ_t is bar location; ψ_t = 1.3 for top horizontal bars, because more than 12 in. of fresh concrete is placed below them, and ψ_t = 1.0 for bottom horizontal bars, because not more than 12 in. of fresh concrete is placed below them,ψ_e is coating factor; and ψ_e = 1.0 because bars are uncoated ψ_g = reinforcement grade factor, ψ_g = 1.0 for Grade 60 reinforcement	Top bars $\ell_d = \left(\frac{(60{,}000\text{ psi})(1.3)(1.0)(1.0)}{(25)(1.0)\sqrt{5000\text{ psi}}}\right)(0.75\text{ in.}) = 33.1\text{ in.}$ say, 36 in. Bottom bars $\ell_d = \left(\frac{(60{,}000\text{ psi})(1.3)(1.0)(1.0)}{(25)(1.0)\sqrt{5000\text{ psi}}}\right)(0.75\text{ in.}) = 25.5\text{ in.}$ say, 30 in.
 9.7.1.3 25.5.2.1	Splice length of No. 6 reinforcing bar Per Table 25.5.2.1, splice length is 1.3(ℓ_d)	Top: 1.3ℓ_d = (1.3)(33.1 in.) = 43.0 in., say, 4 ft 0 in. Bottom: 1.3ℓ_d = (1.3)(25.5 in.) = 33.2 in., say, 3 ft 0 in.
9.7.3	Bar cutoff Bottom tension reinforcement Four No. 6 bottom bars are terminated beyond Beam B1 a distance equal to the development length of 30 in. Extend two No. 6 bottom bars the full length of the beam and develop into the girder beams along Column Lines B and D at each end with a hook	Four No. 6 lengths: ℓ = 14 ft + 2(1.5 ft) + 2(2.5 ft) = 22 ft

9.7.3.2	Top tension reinforcement Reinforcement must be developed at points of maximum stress and points along the span where terminated tension reinforcement is no longer required to resist flexure Six No. 6 bars are required to resist the factored moment at the support Four No. 6 bars will be terminated at the inflection point.	$-(279\text{ ft-kip}) - 0.94\text{ kip/ft}\dfrac{x^2}{2} + 60\text{ kip}(x) = 0$
9.7.3.8.4	At least one-third of the negative moment reinforcement at a support must have an embedment length beyond the point of inflection the greatest of d, $12d_b$, and $\ell_n/16$. Place four No. 6 bars within the Beam B2 web and two No. 6 bars on either side of the beam web over 36 in. (refer to Fig. E3.10, Section B)	$x = 4.8$ ft, say, 5 ft For No. 6 bars 1) $d = 27.7$ in. **Controls** 2) $12d_b = 12(1.0\text{ in.}) = 12$ in. 3) $\ell_n/16 = 21$ in. Therefore, extend four No. 6 bars a distance d beyond the inflection point 60 in. + 27.7 in. = 67.7 in. Extend the remaining two No. 6 bars over the full length of the beam as hanger bars for the stirrups
Step 9: Integrity reinforcement		
9.7.7.2	Integrity reinforcement Either one of the two conditions must be satisfied, but not both In this example, both are satisfied. At least one-fourth the maximum positive moment reinforcement, but not less than two bars must be continuous Longitudinal reinforcement must be enclosed by closed stirrups along the clear span of the beam.	This condition was satisfied above by extending two No. 6 bottom reinforcement bars into the support. 2 No. 6 > (1/4)6 No. 6 This condition is satisfied by extending stirrups over the full length of the beam. Refer to Fig. E3.9

Beams

Step 10: Deflection		
9.3.2	Calculate deflection limit.	
24.2.3.1	Immediate deflection is calculated using elastic deflection approach and considering concrete cracking and reinforcement for calculating stiffness. Modulus of elasticity:	
19.2.2.1	$E_c = 57{,}000\sqrt{f_c'}$ psi (19.2.2.1b) The beam is subjected to a factored distributed force of 0.94 kip/ft or service dead load of 0.58 kip/ft and 0.15 kip/ft service live load. The beam is also subjected to concentrated loads from Beam B1 at 6 ft 3 in. and 21 ft 9 in. from Column Line B framing into it having the following reaction; factored 28.5 kip or service dead load of 14.4 kip and 7 kip service live load. Also, Beam B2 is subjected to a concentrated load at midspan from Beam B4 of 36.7 kip factored or 15.1 kip dead service load and 11.6 kip service live load. The deflection equation for distributed load with fixity at both ends: $\Delta = w\ell^4/384EI$ For concentrated load at midspan: $\Delta = P\ell^3/192EI$ Note: w and P are unfactored loads.	$E_c = 57{,}000\sqrt{5000\text{ psi}}/1000 = 4030$ ksi
24.2.3.5	The effective moment of inertia equation was altered in the 2019 Code to more accurately reflect the deflections in members with low quantities of reinforcement. Where the applied moment (M_a) is greater than 2/3 of the cracking moment (M_{cr}), the following equation is used: $I_e = \dfrac{I_{cr}}{1 - \left(\dfrac{2/3M_{cr}}{M_a}\right)^2\left(1 - \dfrac{I_{cr}}{I_g}\right)}$ where $M_{cr} = \dfrac{f_r I_g}{y_t}$ (24.2.3.5b) M_a is the moment due to service load. For deflection calculation, use moments obtained from elastic analysis. Coefficients from Table B-1 Reinforced Concrete Design Handbook Design Aid – Analysis Tables, which can be downloaded from https://www.concrete.org/MNL1721Download1 are used to calculate the moments. The beam is assumed cracked, therefore, calculate the moment of inertia of the cracked section, I_{cr}.	For simplicity, assume that the beam is rectangular for the calculation of the moment of inertia (conservative). $I_g = \dfrac{bh^3}{12} = \dfrac{(18\text{ in.})(30\text{ in.})^3}{12} = 40{,}500\text{ in.}^4$ $M_{cr} = \dfrac{7.5(\sqrt{5000\text{ psi}})(40{,}500\text{ in.}^4)}{(15\text{ in.})(12{,}000)} = 120$ ft-kip For moment calculation, refer to table below:

	Determine neutral axis of the cracked section $nA_s(d-c) = \frac{bc^2}{2} + (n-1)A_s'(c-d')$ where c is the uncracked remaining concrete depth $n = E_s/E_c$	For c values, refer to the table below n = 29,000 ksi/4030 ksi = 7.2
	Cracking moment of inertia, I_{cr}: $I_{cr} = \frac{bc^3}{3} + (n-1)A_s'(c-d')^2 + nA_s(d-c)^2$	For I_{cr} values, refer to the table below

A_s	2.64 in.^{2}	0.88 in.^{2}	2.64 in.^{2}
A_s'	0.88 in.^{2}	2.64 in.^{2}	0.88 in.^{2}
c	3.67 in.	6.5 in.	3.67 in.
I_{cr}	4016 in.^{4}	10,314 in.^{4}	4016 in.^{4}
Distributed load $M_{a,dist}$			
Dead:	38 ft-kip	19 ft-kip	38 ft-kip
Live:	10 ft-kip	5 ft-kip	10 ft-kip
Concentrated load $M_{a,conc}$			
Dead:	123 ft-kip	101 ft-kip	123 ft-kip
Live:	75 ft-kip	64 ft-kip	75 ft-kip
Total M_a	246 ft-kip	189 ft-kip	246 ft-kip
I_e	4439 in.^{4}	11,904 in.^{4}	4439 in.^{4}
Use	4400 in.^{4}	11,000 in.^{4}	4400 in.^{4}

where $M_{a,dist} = \alpha(w, \text{kip/ft})(28\text{ ft})^2$, α = 1/12 at support and 1/24 at midspan and $M_{a,conc} = \beta P\ell$, β = 0.125 for concentrated load at midspan and $M_a = Pa^2b/\ell^2$, where a is the distance of the concentrated load to the left support

24.2.3.6	For continuous beams or beams fixed at both ends (positive and negative moments), the Code permits I_e to be taken as the average of values obtained from Eq.(24.2.3.5b) for the critical positive and negative moments. $I_{e,avg} = \frac{I_{e,left@supp} + I_{e,right@supp} + I_{e,midspan}}{3}$	$I_{e,avg} = \frac{4400\text{ in.}^4 + 11,000\text{ in.}^4 + 4400\text{ in.}^4}{3}$ $I_{e,avg}$ = 6600 in.^{4}
R24.2.3.7	If a more detailed analysis is required, the Commentary refers to ACI 435R-95 for alternate equations to calculate the average equivalent moment of inertia in a beam with two fixed or continuous ends (Eq. (2.15a) of ACI 435R-95). $I_{e,avg} = 0.7I_{e,midspan} + 0.15(I_{e,left@supp} + I_{e,right@supp})$	$I_{e,avg}$ = 0.7(11,000 in.^{4}) + 0.15(4400 in.^{4} + 4400 in.^{4}) $I_{e,avg}$ = 9020 in.^{4}

	<u>Immediate deflections</u> Deflection due to total distributed load	$\Delta_{dist} = \dfrac{(0.58\text{ kip/ft} + 0.15\text{ kip/ft})(28\text{ ft})^4(12)^4}{384(4030\text{ ksi})(6600\text{ in.}^4)} = 0.076\text{ in.}$
	Deflection due to total concentrated load $P_{D+L} = 15.1\text{ kip} + 11.6\text{ kip} = 26.7\text{ kip}$	$\Delta_{conc} = \dfrac{(26.7\text{ kip})(28\text{ ft})^3(12)^3}{192(4030\text{ ksi})(6600\text{ in.}^4)} = 0.20\text{ in.}$
	At midspan Deflection at midspan due to B1 at 6 ft 3 in. and 21 ft 9 in. from Column Line B $P_{D+L} = 14.4\text{ kip} + 7\text{ kip} = 21.4\text{ kip}$	$\Delta_{CL} = \dfrac{2(21.4\text{ kip})(6.25\text{ ft})^2(14\text{ ft})^2(12)^3}{6(4030\text{ ksi})(6600\text{ in.}^4)(28\text{ ft})^3}$ $\times\,(3(21.75\text{ ft})(28\text{ ft}) - 3(21.75\text{ ft})(14\text{ ft})$ $-(6.25\text{ ft})(14\text{ ft}) = 0.14\text{ in.}$
	Total load deflection	$\Delta_{TL} = 0.076\text{ in.} + 0.20\text{ in.} + 0.14\text{ in.} = 0.42\text{ in.}$
	Equation is obtained from Reinforced Concrete Design Handbook Design Aid – Analysis Tables, which can be downloaded from https://www.concrete.org/MNL1721Download1	

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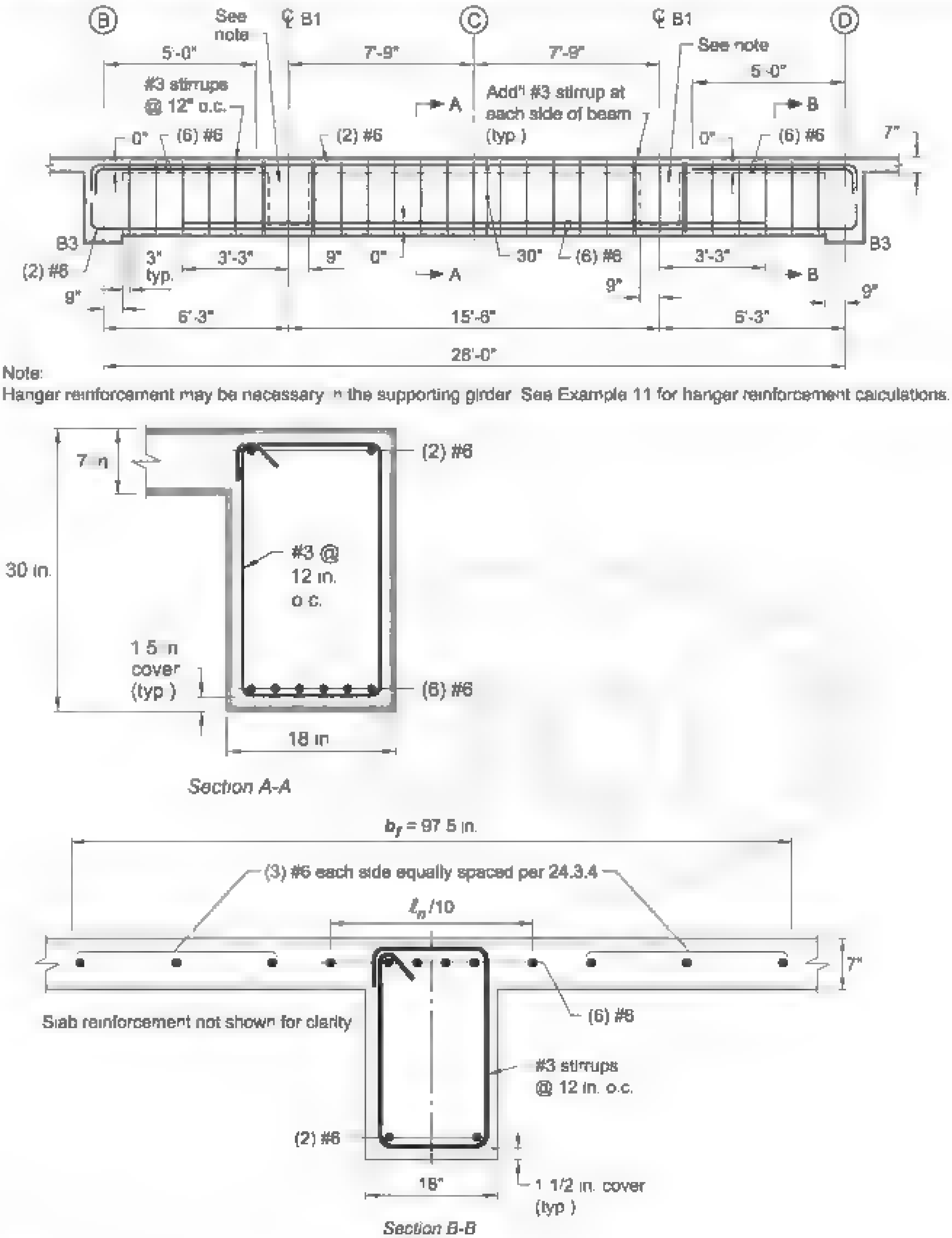
	Dead load deflections Deflections due to live load are the difference between total deflection and dead load deflection	
	Deflection due to dead load	
	Distributed load	$\Delta_{distr} = \dfrac{(0.58\text{ kip/ft})(28\text{ ft})^4(12)^3}{384(4030\text{ ksi})(6600\text{ in.}^4)} = 0.104\text{ in.}$
	Concentrated load at midspan	$\Delta_{conc} = \dfrac{(15.1\text{ kip})(28\text{ ft})^3(12)^3}{192(4030\text{ ksi})(6600\text{ in.}^4)} = 0.112\text{ in.}$
	Concentrated load at 6.25 ft and 21.75 ft, respectively	$\Delta_{c} = \dfrac{2(14.4\text{ kip})(6.25\text{ ft})^2(14\text{ ft})^2(12)^3}{6(4030\text{ ksi})(6600\text{ in.}^4)(28\text{ ft})^3}$ $\times\{3(21.75\text{ ft})(28\text{ ft}) - 3(21.75\text{ ft})(14\text{ ft})$ $-(6.25\text{ ft})(14\text{ ft})\} = 0.090\text{ in.}$
	Dead load deflection:	$\Delta_D = 0.104\text{ in.} + 0.112\text{ in.} + 0.090\text{ in.} = 0.31\text{ in.}$
	Deflection due to live load	$\Delta_L = 0.42\text{ in.} - 0.31\text{ in.} = 0.11\text{ in.}$
24.2.2	Check live load deflection limit from Table 24.2.2. Assume that the floor is not supporting or attached to nonstructural elements likely to be damaged by large deflections. Use $\ell/360$.	$\Delta_{all} = (28\text{ ft})(12\text{ in./ft})/360 = 0.93\text{ in.}$ $\Delta_{all} = 0.93\text{ in.} >> \Delta_L = 0.11\text{ in.}$ **OK**
24.2.4.1.1	Calculate long-term deflection $\lambda_\Delta = \dfrac{\xi}{1+50\rho'}$	$\lambda_\Delta = \dfrac{2.0}{1+50\dfrac{0.88\text{ in.}^2}{(18\text{ in.})(27.7\text{ in.})}} = 1.84$
24.2.4.1.3	From Table 24.2.4.1.3, the time-dependent factor for sustained load duration of more than 5 years $\xi = 2.0$	
	Assume entire dead load is sustained. Time-dependent deflection due to sustained load is then	
	$\Delta_{TD} = \Delta_D(1 + \lambda_\Delta)$	$\Delta_{TD} = 0.31\text{ in.}(1 + 1.84) = 0.88\text{ in.}$
	Check sustained load deflection limit from Table 24.2.2. Assume that the floor is supporting or attached to nonstructural elements not likely to be damaged by large deflections. Use $\ell/240$	$\Delta_{all} = (28\text{ ft})(12\text{ in./ft})/240 = 1.4\text{ in.}$ $\Delta_{all} = 1.4\text{ in.} >> \Delta_{TD} = 0.88\text{ in.}$ **OK**
	If the sustained load deflections exceeded this limit, then the sustained load deflection calculation should be refined to include only that portion of the deflection that occurs after attachment of partitions.	

Step 11 Detailing



Slab reinforcement not shown for clarity

Fig. E3.11—Beam reinforcement details.

Beam Example 4: *Continuous edge beam*

Determine the size of a continuous six-bay edge beam built integrally with a 7 in. slab on the exterior of the building. Design and detail the beam. Ignore openings at Column Lines 3 and 5.

Given:

$f_c' = 5000$ psi (normalweight concrete)

$f_y = 60{,}000$ psi

Beam width: 18 in.

Beam height: 30 in.

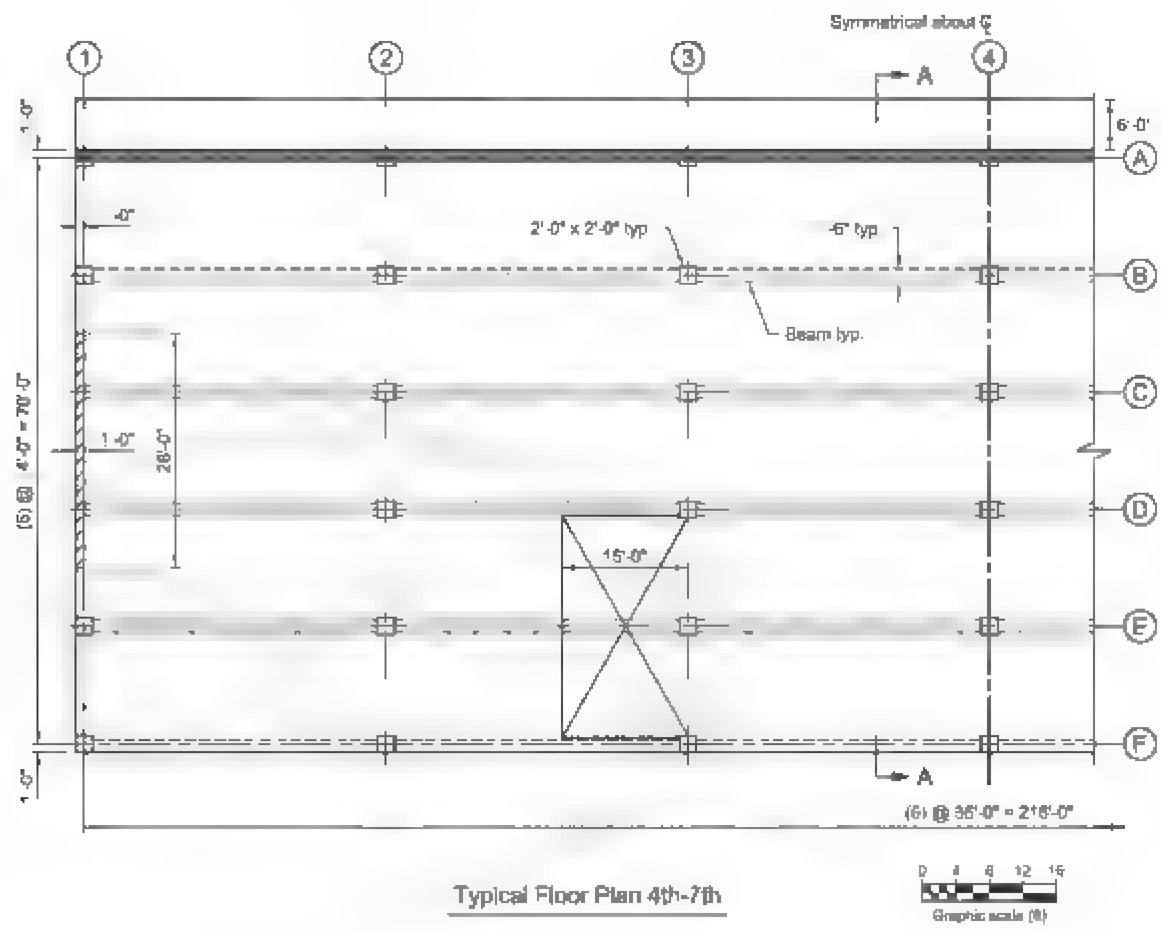


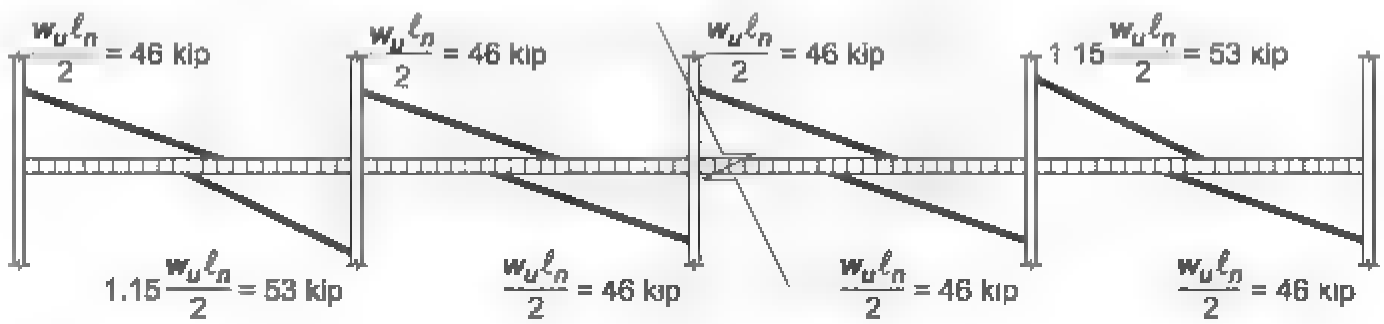
Fig. E4.1—Plan of a six-span perimeter beam.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
9.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements of ACI 318. The designer determines the durability classes. Please refer to Chapter 4 of MNL-17 for an in-depth discussion of the Categories and Classes.	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi.
	ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications.	
	There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26.

Step 2. Beam geometry		
9.3.1.1	Beam depth If the depth of a beam satisfies Table 9.3.1.1, ACI 318 permits a beam design without having to check deflections, if the beam is not supporting or attached to partitions or other construction likely to be damaged by large deflections. Otherwise, beam deflections must be calculated and the deflection limits in 9.3.2 must be satisfied.	The beam has six continuous spans. Taking the controlling condition of having one end continuous $h = \frac{\ell}{18.5} = \frac{(36\text{ ft})(12\text{ in./ft})}{18.5} = 23.4\text{ in.}$ Use 30 in. A deeper section is selected so all beams will have the same depth
	Self weight Beam: Slab: Façade: assume façade weight is 35 psf spanning 12 ft 0 in. vertically	$w_b = [(18\text{ in.})(30\text{ in.})/(144)](0.150\text{ kip/ft}^3) = 0.56\text{ kip/ft}$ $w_s = [((14\text{ ft} - 15\text{ in./12})/2)(7\text{ in./12})](0.150\text{ kip/ft}^3) = 0.56\text{ kip/ft}$ $w_{cladding} = (35\text{ psf})(12\text{ ft})/1000 = 0.42\text{ kip/ft}$
9.2.4.2	Flange width The beam is placed monolithically with the slab and will behave as an L-beam. The flange width to one side of the beam is obtained from Table 6.3.2.1	
6.3.2.1	One side of web is the least of $\begin{cases} 6h_{slab} \\ s_w/2 \\ \ell_n/12 \end{cases}$ Flange width $b_f = \ell_n/12 + b_w$	$(6)(7\text{ in.}) = 42\text{ in.}$ $[((14\text{ ft})(12) - 15\text{ in.})/2 = 76.5\text{ in.}$ $(34\text{ ft})(12)/12 = 34\text{ in.}$ **Controls** $b_f = 34\text{ in.} + 18\text{ in.} = 52\text{ in.}$
Step 3. Loads and load patterns		
	The service live load is 50 psf in offices and 80 psf in corridors per Table 4-1 in ASCE/SEI 7. This example will use 65 psf as an average as the actual layout is not provided. To account for the weight of ceilings, partitions, and mechanical (HVAC) systems, add 15 psf as miscellaneous dead load. The beam resists gravity load only and lateral forces are not considered in this problem.	
5.3.1	$U = 1.4D$	$w_u = 1.4(0.56\text{ kip/ft} + 0.56\text{ kip/ft} + 0.42\text{ kip/ft} + (15\text{ psf})((14\text{ ft})/2)/1000)) = 2.3\text{ kip/ft}$
	$U = 1.2D + 1.6L$	$w_u = 1.2(2.3\text{ kip/ft})/1.4 + 1.6(65\text{ psf}/1000)(14\text{ ft})/2 = 2.7\text{ kip/ft}$ **Controls**

Step 4. Analysis		
9.4.3.1	The beams are built integrally with supports, therefore, the factored moments and shear forces (required strengths) are calculated at the face of the supports.	Clear span. ℓ_n = 36 ft – 2 ft = 34 ft
9.4.1.2 6.5.1	Chapter 6 permits several analysis procedures to calculate the required strengths The beam required strengths can be calculated using approximations per Table 6.5.2, if the conditions in Section 6.5.1 are satisfied	
	Members are prismatic	Beams are prismatic
	Loads uniformly distributed $L < 3D$	Satisfied (no concentrated loads) 65 psf < 3(87.5 psf + 15 psf + beam self-weight) **OK**
	There are at least two spans	6 spans > 2 spans
	Difference between two spans does not exceed 20 percent.	Beams have equal clear span lengths 34 ft-0 in.
		All five conditions are satisfied, therefore, the approximate procedure is used.

6.5.4 *Shear diagram*



6.5.2 *Moment diagram*

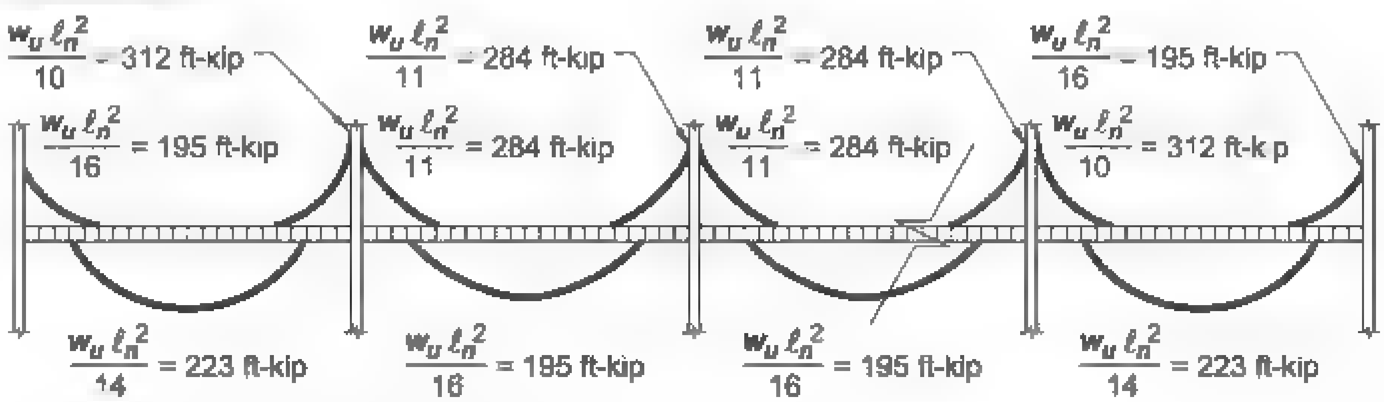


Fig. E4.2—Shear and moment diagrams.

6.5.3 Note. The moments calculated using the approximate method cannot be redistributed in accordance with Section 6.6.5.1

The slab load is eccentric with respect to the edge beam center. Therefore, the beam needs to resist a torsional moment (Fig. E4.3).

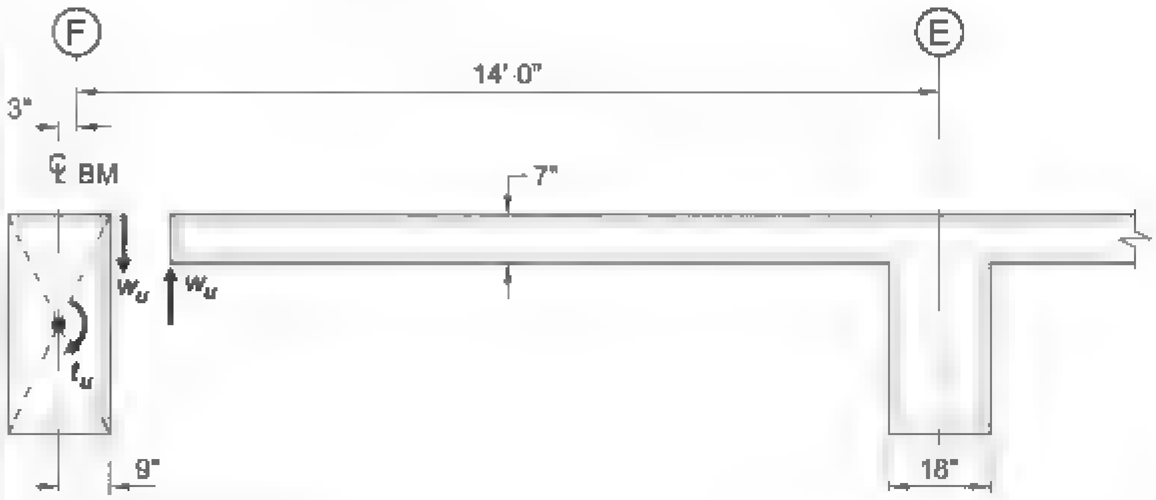


Fig. E4.3—Torsion forces.

Load at slab/beam interface

w_u = [((1.2)((7 in./12)(0.15 kip/ft^3) + (0.015 ksf)) + (1.6)(0.065 ksf))][(14 ft − 1.5 ft/2)/2 + 3 in./12]

w_u = 1.56 kip/ft

The torsional moment along the beam length is (Fig. E4.4):

$t_u = w_u(b_w/2)$ = (1.56 kip/ft)(18 in./2/12) = 1.17 ft-kip/ft

Factored torsional moment at the face of columns: T_u = (1.17 ft-kip/ft)(34 ft)/2 = 20 ft-kip

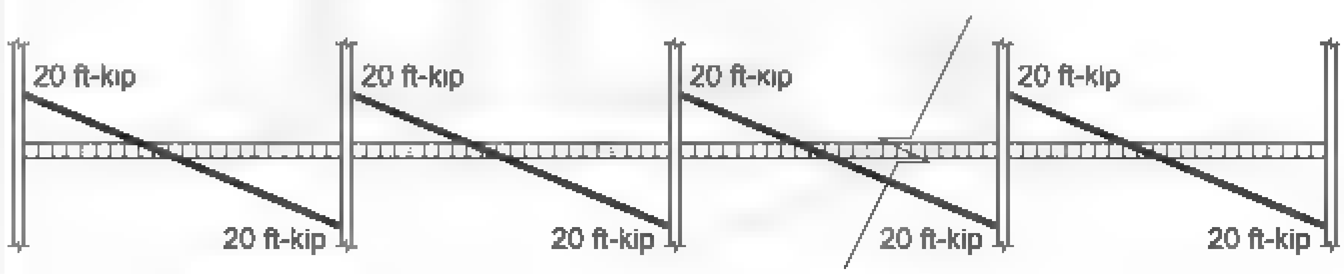


Fig. E4.4—Torsion diagram.

Step 5. Moment design		
9.3.3.1	Limiting steel strain restricts the amount of reinforcement to ensure warning of failure by excessive deflection and cracking. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members. Beginning with the 2019 Code, this limit is revised to require that the section be tension-controlled.	$\varepsilon_y = \dfrac{f_y}{E_s} = \dfrac{60{,}000\text{ psi}}{29{,}000{,}000\text{ psi}} \cong 0.002$ $\varepsilon_t \geq \varepsilon_y + 0.003 = 0.002 + 0.003 = 0.005$
21.2.2	Because section must be tension-controlled, the strength reduction factor is 0.9	Beam must be tension-controlled in accordance with Table 21.2.2 $\phi = 0.9$
	Determine the effective depth assuming No. 4 stirrups, No. 6 bars, and 1.5 in. cover	
20.5.1.3	One row of reinforcement $d = h - \text{cover} - d_{tie} - d_b/2$	
	The concrete compressive strain at nominal moment strength is:	
22.2.2.1	$\varepsilon_{cu} = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and is approximately 10 to 15 percent of the concrete compressive strength. ACI 318 neglects the concrete tensile strength to calculating nominal strength.	$d = 30\text{ in.} - 1.5\text{ in.} - 0.5\text{ in.} - 0.75\text{ in.}/2 = 27.6\text{ in.}$
	Determine the equivalent concrete compressive stress at nominal strength	
22.2.2.3	The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distri-	
22.2.2.4.1	bution of $0.85f_c'$ with a depth of	
22.2.2.4.3	$a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3. For $f_c' < 5000$ psi	$\beta_1 = 0.85 - \dfrac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$
	Find the equivalent concrete compressive depth, a, by equating the compression force to the tension force within the beam cross section (Fig. E4.5): $C = T$ $0.85f_c'ba = A_s f_y$	$0.85(5000\text{ psi})(b)(a) = A_s(60{,}000\text{ psi})$
	For positive moment: $b = b_f = 52$ in.	$a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(52\text{ in.})} = 0.271A_s$
	For negative moment: $b = b_w = 18$ in.	$a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(18\text{ in.})} = 0.784A_s$

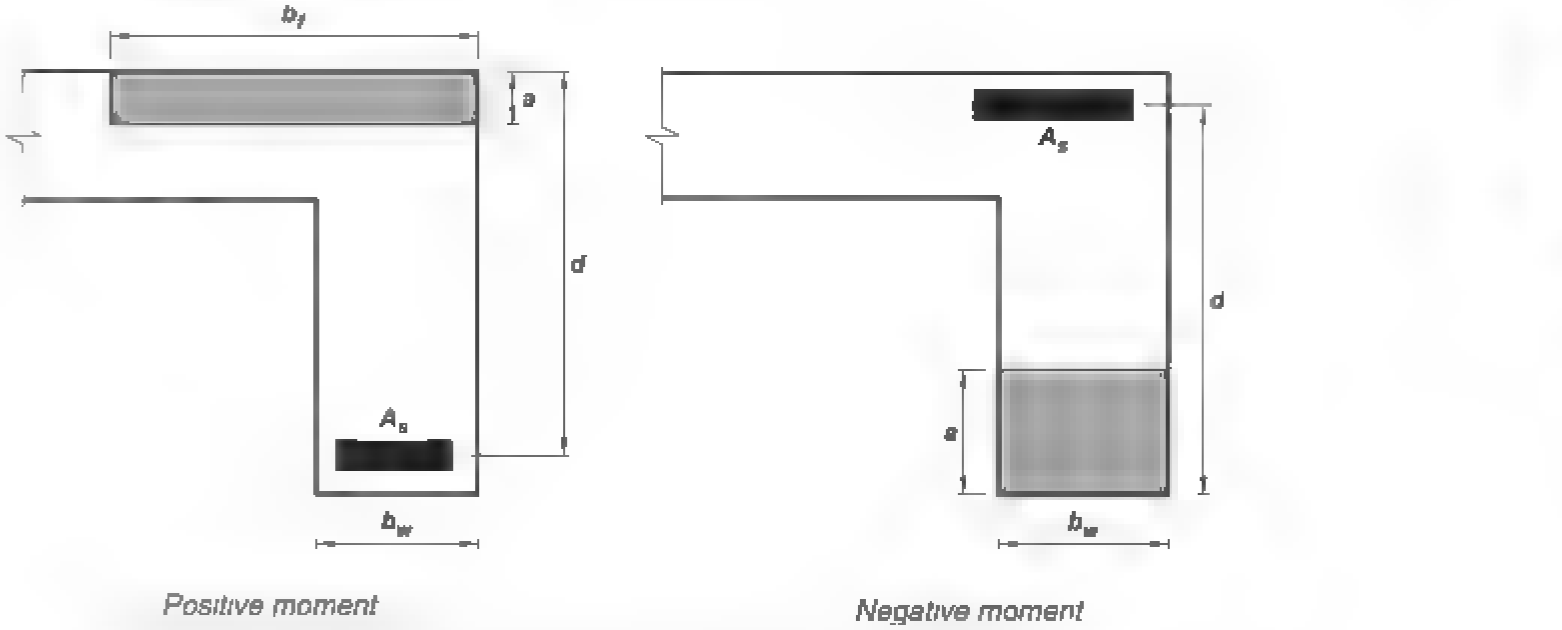


Fig. E4.5—Section compressive block and reinforcement at midspan and at support

The beam is designed for the maximum flexural moments obtained from the approximate method above

The first interior support will be designed for the larger of the moments on either side of the column.

The first interior moment $M_{max} = 306$ ft-kip

9.5.1.1 The beam design strength must be at least the required strength at each section along its length (Fig. E4.6)

$\phi M_n \geq M_u$

$\phi V_n \geq V_u$

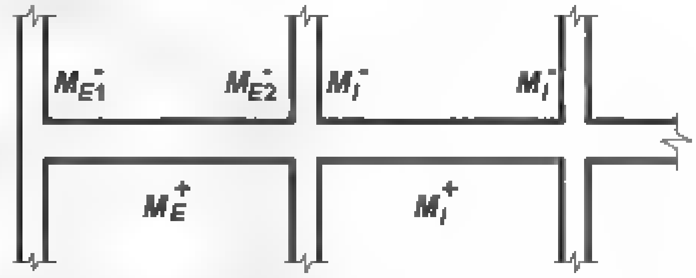


Fig. E4.6—Moment key to use with table below

Calculate required reinforcement area.

$$\phi M_n \geq M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

Each No. 6 bar has a d_b = 0.75 in. and an A_s = 0.44 in.²

Table 4.1—Required reinforcement to resist factored moment

	M_u, ft-kip	$A_{s,req'd}$, in.²	Number of No. 6 bars	
			Req'd	Prov
M_{E1}	195	59	3.6	4
M_{E2}	312	2.6	5.91	6
M_I	284	2.36	5.36	6
M_E^+	223	1.81	4.11	5
M_I^+	195	1.57	3.57	4

21.2.2
9.3.3.1 Check if the calculated strain exceeds 0.005 in./in. to ensure section is tension-controlled (Fig. E4.7).

$$a = \frac{A_s f_y}{0.85 f_c' b} \text{ and } c = \frac{a}{\beta_1}$$

where β_1 = 0.8 (calculated above)

Note that b = 18 in. for negative moments and 57.75 in. for positive moments (refer to Fig. E4.5).

$$\varepsilon_t = \frac{\varepsilon_{cu}}{c}(d - c)$$

Table 4.2—Strain in tension bars

	M_u, ft-kip	$A_{s,prov}$, in.²	a, in.	ε_t, in./in.	$\varepsilon_t >$ 0.005?
M_E	195	1.76	1.38	0.045	Y
M_{E2}	312	2.64	2.07	0.029	Y
M_I	284	2.64	2.07	0.029	Y
M_E^+	223	2.20	0.60	0.108	Y
M_I^+	195	1.76	0.48	0.136	Y

All strain values exceed 0.005. Use ϕ = 0.9

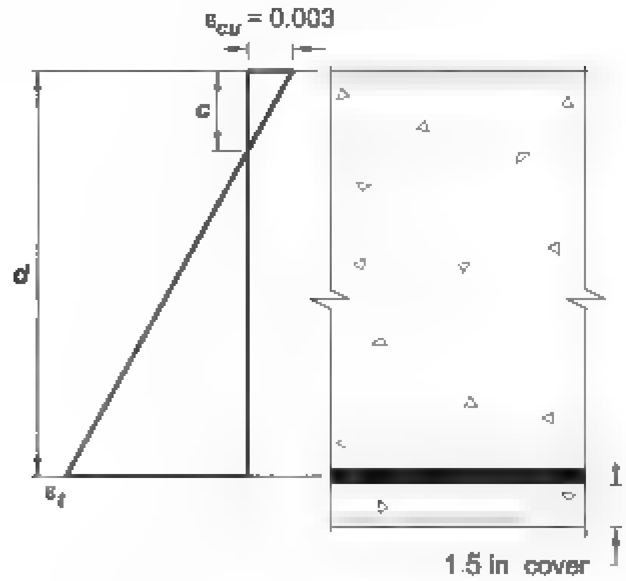


Fig. E4.7—Strain distribution across beam section.

Minimum reinforcement

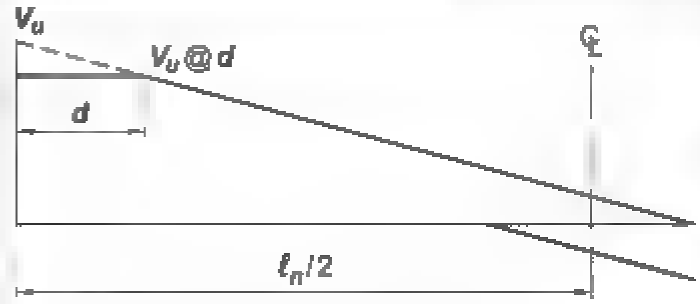
9.6.1.1 The reinforcement area must be at least the
9.6.1.2 minimum required reinforcement area at every section along the length of the beams.

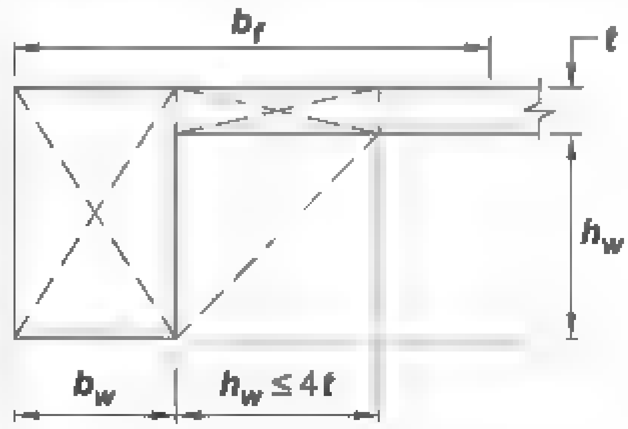
$$A_s = \frac{3\sqrt{f_c'}}{f_y} b_w d$$

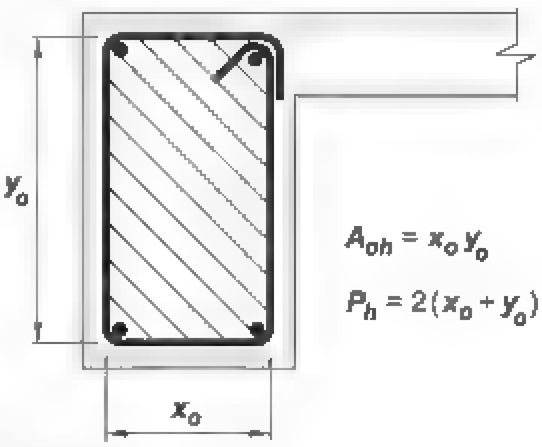
$$A_s = \frac{3\sqrt{5000\text{ psi}}}{60{,}000\text{ psi}}(18\text{ in.})(27.6\text{ in.}) = 1.76\text{ in.}^2 \quad \textbf{Controls}$$

Equation (9.6.1.2a) controls because $f_c' > 4444$ psi

All calculated reinforcement areas exceed the minimum required reinforcement area. Therefore, **OK**

	Exterior spans	
Step 6. Shear design		
	Shear strength The shear forces in the exterior and interior spans are relatively equal: 53 kip versus 46 kip, therefore, the continuous beam will be designed for 53 kip of shear force	 *Fig. E4.8—Shear critical section.*
9.4.3.2	Because conditions (a), (b), and (c) of 9.4.3.2 are satisfied, the design shear force critical section is taken at a distance d from the face of the support (Fig. E4.8)	
	The controlling factored load combination must satisfy	$V_{u@d}$ = (53 kip) − (2.7 kip/ft)(27.6 in./12) = 47 kip
9.5.1.1	$\phi V_n \geq V_u$	
9.5.3.1	$V_n = V_c + V_s$	
22.5.1.1	2019 Code introduced size effect for shear design in which the shear strength of an element that does not contain shear reinforcement is not directly proportional to its depth. This effect is addressed by incorporating a size effect factor λ_s into the concrete contribution equation. If shear reinforcement is not present, then the concrete contribution to shear strength must be reduced by the size effect factor. If minimum shear reinforcement is provided, then the Eq. 22.5.5.1a can be used to calculate V_c	
	Minimum shear reinforcement is required where $V_u > \phi\lambda\sqrt{f_c'}b_w d$	
	For this example, use minimum shear reinforcement over entire length of beam. The concrete contribution to shear strength is then	
	$V_c = 2\sqrt{f_c'}b_w d$ (22.5.5.1a)	$V_c = (2)\sqrt{5000\text{ psi}}(18\text{ in.})(27.6\text{ in.}) = 70.3\text{ kip}$
21.2.1b	Shear strength reduction factor: $\phi V_c = \phi 2\sqrt{f_c'}b_w d$	$\phi_{shear} = 0.75$ $\phi V_c = (0.75)(70.3\text{ kip}) = 52.7\text{ kip}$
9.5.1.1b	Check if $\phi V_n \geq V_u$	$\phi V_c = 52.7\text{ kip} > V_u = 47\text{ kip}$ **OK** Therefore, shear reinforcement is not required for strength
9.6.3.1	Code requires that minimum shear reinforcement must be provided over sections where $V_u > \phi\lambda\sqrt{f_c'}b_w d$	$V_u = 47\text{ kip} > \phi\lambda\sqrt{f_c'}b_w d = 1/2(52.7\text{ kip}) = 26.4\text{ kip}$
	Check if the cross-sectional dimensions satisfy Eq. (22.5.1.2)	
22.5.1.2	$V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d)$	$V_u \leq \phi\left(70.3\text{ kip} + 8\sqrt{5000\text{ psi}}(18\text{ in.})(27.6\text{ in.})\right)$ $\leq 211\text{ kip}$ Section dimensions are satisfactory.

Step 7. Torsion design		
	Threshold torsion	
9.4.4.3	Calculate the torsional moment at d from the face of the support	
	t_u = 1.17 ft-kip/ft and T_u = 20 ft-kip (Step 4, Fig. E4.3)	$T_{u@d}$ = (20 ft-kip) − (1.17 ft-kip/ft)(27.6 in.)/12 = 17.3 ft-kip
9.2.4.4	Determine the concrete section resisting torsion	
	The overhanging flange dimension is equal to the smaller of the projection of the beam below the slab (23 in.) and four times the slab thickness (28 in.). Therefore, 23 in. controls (Fig. E4.9)	
22.7.4.1	Calculate the threshold torsion value	
	$T_{th} = \lambda\sqrt{f'_c}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)$	Fig. E4.9—L-beam geometry to resist torsion
	where A_{cp} is the area enclosed by outside perimeter of concrete cross section, and	A_{cp} = (18 in.)(30 in.) + (23 in.)(7 in.) = 701 in.^{2}
	p_{cp} is the outside perimeter of concrete cross section	p_{cp} = 2(18 in. + 2(23 in.) + 7 in.) = 142 in.
		$T_{th} = (1.0)\sqrt{5000\text{ psi}}\left(\dfrac{(701\text{ in.}^2)^2}{142\text{ in.}}\right)$ = 20.4 ft-kip
21.2.1(c)	Torsion strength reduction factor ϕ = 0.75	ϕT_{th} = (0.75)(20.4 ft-kip) = 15.3 ft-kip
9.5.4.1	Check if torsion can be ignored, does $T_{u@d} < \phi T_{th}$?	$T_{u@d}$ = 17.3 ft-kip > ϕT_{th} = 15.3 ft-kip **NG**
		Torsional effects cannot be neglected and reinforcement and detailing requirements for torsion must be considered.

	<u>Torsion reinforcement</u>	
9.5.1.1 9.5.4.2 22.7.5.1	Calculate cracking torsion $T_{cr} = 4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)$	$T_{cr} = 4(1.0)\sqrt{5000\text{ psi}}\left(\frac{(701\text{ in.}^2)^2}{142\text{ in.}}\right) = 81.5\text{ ft-kip}$
22.7.3.2	Check if cross section will crack under the torsional moment.	$T_u = 17.3$ ft-kip < $T_{cr} = 81.5$ ft-kip **OK** Reducing T_u to T_{cr} is not required.  *Fig. E4.10—*A_{oh} *area* $p_h = 2[(18\text{ in.} - 2(1.5\text{ in.}) - 0.5\text{ in.}) + (30\text{ in.} - 2(1.5\text{ in.}) - 0.5\text{ in.})] = 82\text{ in.}$ $A_{oh} = (14.5\text{ in.})(26.5\text{ in.}) = 384.25\text{ in.}^2$
	Check if cross section is adequate to resist the torsional moment	
22.7.7.1	$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$ where p_h is the perimeter of centerline of outermost closed transverse torsional reinforcement; and A_{oh} is the area enclosed by centerline of the outermost closed transverse torsional reinforcement	$\sqrt{\left(\frac{46{,}000\text{ lb}}{(18\text{ in.})(27.6\text{ in.})}\right)^2 + \left(\frac{(17.3\text{ ft-kip})(12\times 10^3)(82\text{ in.})}{1.7(384.25\text{ in.}^2)^2}\right)^2}$ $\leq (0.75)\left(\frac{70{,}300\text{ lb}}{(18\text{ in.})(27.6\text{ in.})} + 8\sqrt{5000\text{ psi}}\right)$ 115 psi < 530 psi **OK** Therefore section is adequate to resist torsion.
9.7.5 9.7.6	Calculate required transverse and longitudinal torsion reinforcement	
22.7.6.1	Transverse: $T_n = \frac{2A_o A_t f_{yt}}{s}\cot\theta$ (22.7.6.1a)	$\frac{T_{u,slab}}{\phi} = \frac{(17.3\text{ ft-kip})(12)}{0.75} \leq T_n = \frac{2(327\text{ in.}^2)A_t(60\text{ ksi})}{s}\cot 45$ $A_t/s = 0.007\text{ in.}^2/\text{in.}$
	Longitudinal: $T_n = \frac{2A_o A_\ell f_y}{p_h}\tan\theta$ (22.7.6.1b) where: 30 < θ < 60; use θ = 45 degrees	$\frac{T_{u,slab}}{\phi} = \frac{(17.3\text{ ft-kip})(12)}{0.75} \leq T_n = \frac{2(327\text{ in.}^2)A_\ell(60\text{ ksi})}{82\text{ in.}}\tan 45$ $A_\ell > 0.58\text{ in.}^2$
22.7.6.1.2 22.7.6.1.1	$A_o = 0.85A_{oh}$ is the gross area enclosed by torsional shear flow path	$A_o = 0.85(384.25\text{ in.}^2) = 327\text{ in.}^2$

9.5.4.3	The required area for shear and torsional transverse reinforcement are additive $\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{s}$ $A_v = 0$ in.^{2} A_t is defined in terms of one leg. Therefore, A_t is multiplied by 2 Calculate the maximum spacing of stirrups at d from the column face.	$\frac{A_{v+t}}{s} = 0$ in.^{2}/in. + 2(0.007 in.^{2}/in.) = 0.014 in.^{2}/in.
9.7.6.3.3	Maximum spacing of transverse torsional reinforcement must not exceed the lesser of $p_h/8$ and 12 in.	Assume No. 4 stirrup $p_h = 82$ in. calculated above $p_h/8 = 82$ in./8 = 10 in. < 12 in., use 10 in.
9.6.4.2	Check maximum transverse torsional reinforcement: $(A_v + 2A_t)_{min}/s$ must be greater than $0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$ and $50\frac{b_w}{f_{yt}}$ $A_v = 0$ in.^{2} because calculations showed that shear reinforcement is not required. Minimum shear reinforcement is, however, provided. $A_t = (2)(0.20$ in.$^2) = 0.4$ in.^{2} Use two legs for torsional reinforcement.	$\frac{(A_{v+t})_{min}}{s} \geq 0.75\sqrt{5000\text{ psi}}\frac{18\text{ in.}}{60{,}000\text{ psi}} = 0.016$ in. $\frac{(A_{v+t})_{min}}{s} = 50\frac{18\text{ in.}}{60{,}000\text{ psi}} = 0.015$ in. Provided: $\frac{A_{v+t}}{s} = \frac{(2)(0.2\text{ in.}^2/\text{in.})}{10} = 0.04$ in.^{2}/in. $\frac{A_{v+t}}{s} = 0.04$ in.^{2}/in. $> \frac{(A_{v+t})_{min}}{s} = 0.016$ in. **OK**
9.6.4.3	The torsional longitudinal reinforcement $A_{\ell,min}$ must be the lesser of: $A_{\ell,min} = \frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}$ $A_{\ell,min} = \frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{25b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}$ $A_{cp} = 701$ in.^{2} calculated above	$A_{\ell,min} = \frac{5\sqrt{5000\text{ psi}}(701\text{ in.}^2)}{60{,}000\text{ psi}} - (0.02\text{ in.}^2/\text{in.})(82\text{ in.})\frac{60\text{ ksi}}{60\text{ ksi}}$ $= 2.49$ in.^{2} **Controls** $A_{\ell,min} = \frac{5\sqrt{5000\text{ psi}}(701\text{ in.}^2)}{60{,}000\text{ psi}} - \left(\frac{25(18\text{ in.})}{60{,}000\text{ psi}}\right)(82\text{ in.})\frac{60\text{ ksi}}{60\text{ ksi}}$ $= 3.5$ in.^{2} $A_{\ell,calc} = 0.58$ in.$^2 < A_{\ell,req'd} = 2.49$ in.^{2} **OK** The longitudinal reinforcement must be added to the flexural reinforcement.

9.5.4.3 Torsion longitudinal reinforcement, A_ℓ, must be distributed around the cross section and the portion of A_ℓ that needs to be placed where A_s is needed is added to A_s found in Step 5, Table 4.1
Assume that two No. 6 bars will be added at each side face and the remainder will be divided equally between top and bottom of beam with one in each corner

$\Delta A_\ell = A_\ell - 4A_{No.6}$

$\Delta A_\ell = (2.49\ \text{in.}^2 - 4(0.44\ \text{in.}^2)) = 0.7\ \text{in.}^2$

Add 0.7 in.^{2}/2 = 0.35 in.^{2} to M^- and M^+ from Step 5, Table 1.

Table 4.3—Total longitudinal reinforcement at tension side

	$A_{s,req'd}$, in.^{2}	$\Delta A_\ell/2$, in.^{2}	$A_s + \Delta A_\ell$, in.^{2}	Number of No. 6 bars	
				Req'd	Prov.
M_{E1}	1.59	0.35	1.94	4.4	5
M_{E2}	2.6	0.35	2.95	6.7	7
M_I	2.36	0.35	2.71	6.2	7
M_E^+	1.81	0.35	2.16	4.9	5
M^+	1.57	0.35	1.92	4.4	5

9.7.5.1 The spacing of longitudinal torsional reinforcement should not exceed 12 in. on center and the
9.7.5.2 minimum diameter must be greater than 0.042 times the transverse reinforcement spacing, but not less than 3/8 in.

12 in. spacing between longitudinal reinforcement is satisfied, refer to Fig. E4.12 and E4.13

$d_{b,min} = (0.042)(10\ \text{in.}) = 0.42\ \text{in.}$
$d_{b,No.6} = 0.75\ \text{in.} > d_{b,min} = 0.42\ \text{in.}$ **OK**

Typical span reinforcement due to torsion moment
The torsional moment varies from maximum at the face of the support to zero at span mid-length. Theoretically, torsional reinforcement is required over a distance equal to:

$$x = \frac{\phi T_{th}}{T_u}\ell_n/2$$

$$x = \frac{((20\ \text{ft-kip}) - (15.3\ \text{ft-kip}))(34\ \text{ft}/2)}{20\ \text{ft-kip}} = 4\ \text{ft}$$

from the face of the support

9.7.5.3 Longitudinal torsional reinforcement must be developed beyond this length a minimum of $x_o + d$ (Fig. E4.11).

$x_o + d = 14.5\ \text{in.} + 27.6\ \text{in.} = 42.1\ \text{in.}$

9.7.5.4 Develop longitudinal reinforcement at face of support.

Therefore, bars due to torsion moment must extend a minimum distance of:
(4 ft)(12) + 42.1 in. = 90.1 in., say, 7 ft 6 in. from the face of the support on both sides of the span.

Note: Bars can be discontinued per the calculations above. Practically, however, bars are extended over the full length of the beam.

Stirrup spacing.

Extend stirrups No. 4 at 10 in. on center over 7 ft from supports. Remainder of span provide No. 4 stirrups at 12 in. on center

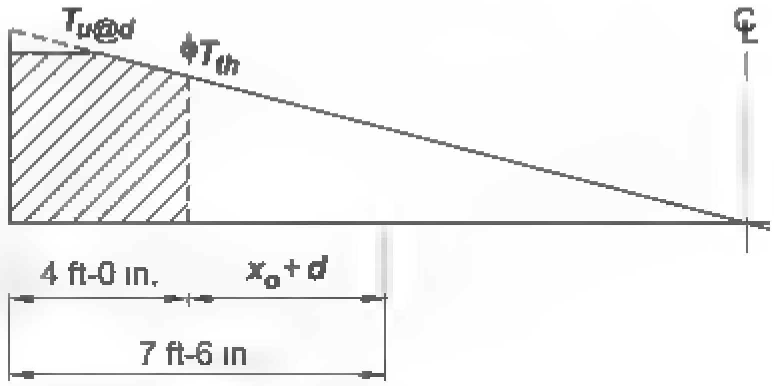
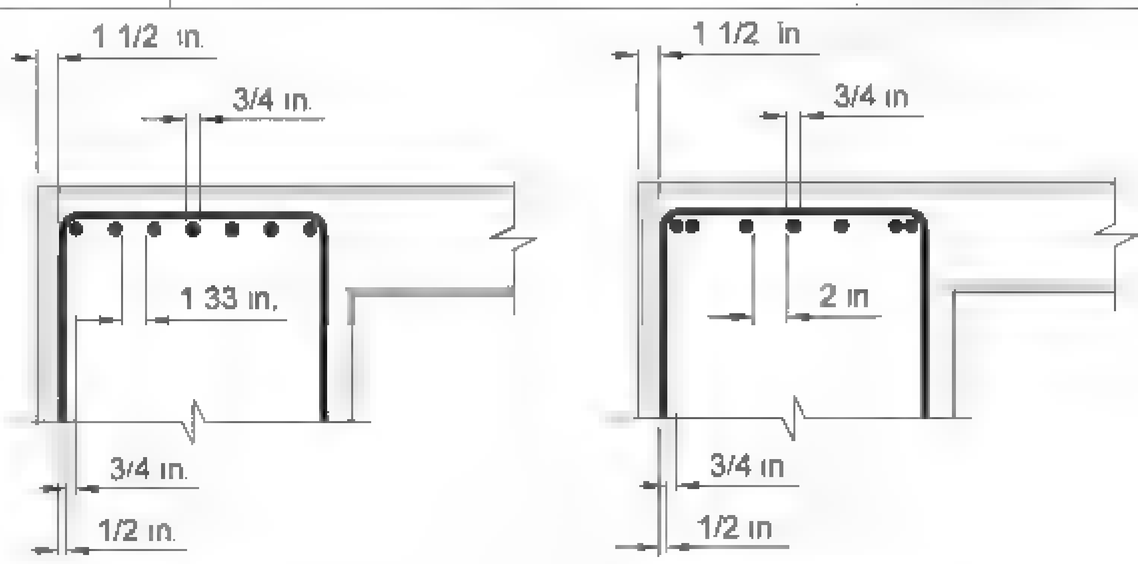


Fig. E4.11—Typical torsion reinforcement in a span applied at both ends of a span.

Step 8. Reinforcement detailing

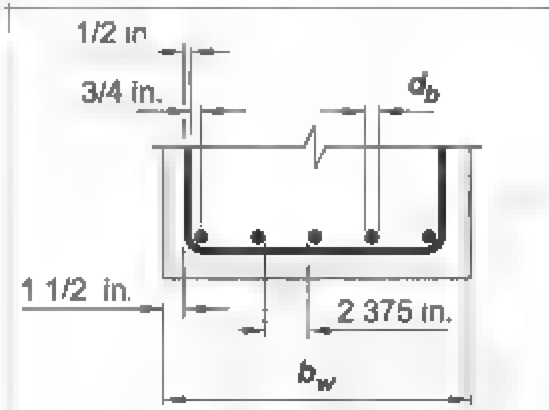
	<u>Minimum top bar spacing</u> Top bars	
9.7.2.1	At maximum and interior negative moments	
25.2.1	The clear spacing between the horizontal bars must be at least the greatest of:	
	Clear spacing greater of $\begin{cases} 1\text{ in.} \\ d_b \\ 4/3(d_{agg}) \end{cases}$	1 in. 0.75 in. 4/3(3/4 in.) = 1 in.
	Assume 3/4 in. maximum aggregate size.	Therefore, clear spacing between horizontal bars must be at least 1 in.
	Check if seven No. 6 bars can be placed in one layer in the beam's web: $b_{w,req'd} = 2(\text{cover} + d_{stirrup} + 0.75\text{ in.}) + 6d_b + 6(1\text{ in.})_{min,spacing}$ (25.2.1)	$b_{w,req'd} = 2(1.5\text{ in.} + 0.5\text{ in.} + 0.75\text{ in.}) + 4.5\text{ in.} + 6\text{ in.}$ $= 16\text{ in.} < 18\text{ in.}$ **OK** Therefore, seven No. 6 bars can be placed in one layer in the 18 in. beam web.



<u>(a) Bars evenly spaced</u> <u>(b) Bars bundled together</u>

Fig. E4.12—Top bar layout.

<u>Note</u>: A preferred solution would be to bundle a few bars together to provide larger spacing between them and to allow for improved concrete placement (Fig. E4.12b).

25.2.1	<u>Minimum bottom bar spacing</u> Bottom bars The clear spacing between the horizontal bars must be at least the greatest of	
	Clear spacing greater of $\begin{cases} 1\text{ in.} \\ d_b \\ 4/3(d_{agg}) \end{cases}$	1 in. 0.75 in. 4/3(3/4 in.) = 1 in.
	Check if five No. 6 bars can be placed in one layer in the beam's web.	Therefore, clear spacing between horizontal bars must be at least 1 in.
	$b_{w,req'd} = 2(\text{cover} + d_{stirrup} + 0.75\text{ in.}) + 4d_b + 4(1\text{ in.})_{min,spacing}$ (25.2.1)	$b_{w,req'd} = 2(1.5\text{ in.} + 0.5\text{ in.} + 0.75\text{ in.}) + 3.0\text{ in.} + 4\text{ in.} = 14.3\text{ in.} < 18\text{ in.}$ **OK**
	Refer to Fig. E4.13 for steel placement in beam web	Therefore five No. 6 bars can be placed in one layer in the 18 in. beam web
	 *Fig. E4.13—Bottom reinforcement layout*	
9.7.2.2 24.3.1 24.3.2	Maximum bar spacing at tension face must not exceed	
	$s = \text{ the lesser of } \begin{cases} 15\left(\dfrac{40,000}{f_s}\right) - 2.5c_c \\ 12\left(\dfrac{40,000}{f_s}\right) \end{cases}$	$s = 15\left(\dfrac{40,000}{40,000}\right) - 2.5(2\text{ in.}) = 10\text{ in.}$ **Controls** $s = 12\left(\dfrac{40,000}{40,000}\right) = 12\text{ in.}$
	where $f_s = 2/3f_y = 40,000$ psi This limit is intended to control flexural cracking width, where $c_c = 2$ in. is the least distance from the No. 6 bar surface to the tension face	2.3 in. spacing is provided, therefore, **OK**

Bottom reinforcing bar length along first span

9.7.3 Calculate the inflection point for positive moment

Assume the maximum moment occurs at midspan (Fig. E4.14). From equilibrium, the point of inflection is obtained from a freebody diagram

$M_{max} - w_u(x)^2/2 = 0$

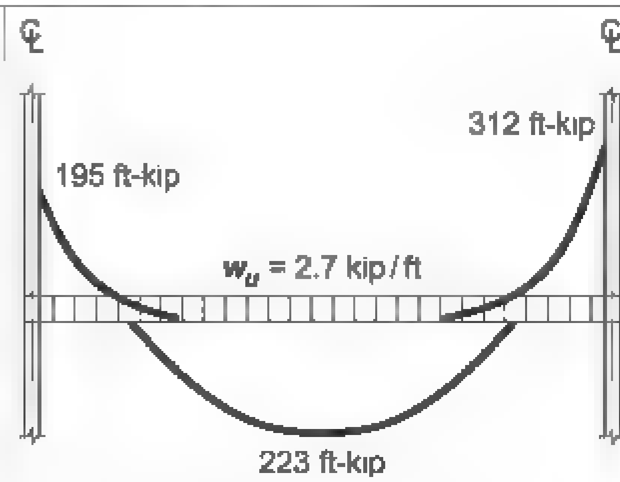


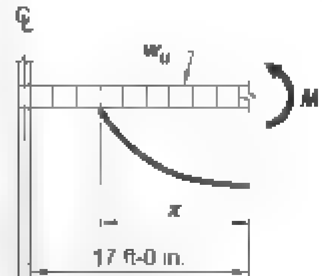
Fig. E4.14—Typical span moment diagram

Top reinforcing bar length along first span

At the exterior support

Calculate the inflection point for the negative-moment diagram

$M_{max} - w_u(x)^2/2 + V_u x = 0$



(223 ft-kip) − (2.7 kip/ft)$(x)^2/2 = 0$

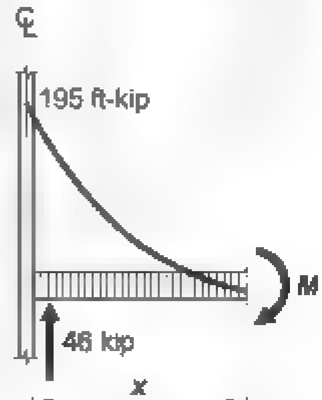
x = 12.86 ft, say, 13 ft

Inflection point of maximum positive moment

At the first interior support

Calculate the inflection point for the negative-moment diagram

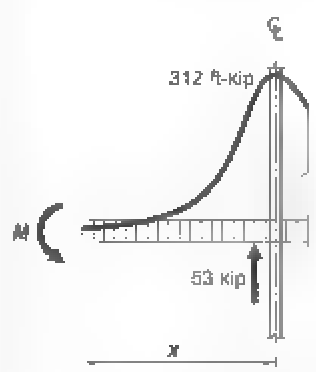
$M_{max} - w_u(x)^2/2 + V_u x = 0$



(−195 ft-kip) − (2.7 kip/ft)$(x)^2/2 + 46x = 0$

x = 4.96 ft, say, 5 ft 0 in.

Inflection point of exterior negative moment



$\sum M$ = (−312 ft-kip)−(2.7 kip/ft)$(x)^2/2 + 53x = 0$

x = 7.2 ft, say, 7 ft 3 in.

Inflection point of interior negative moment

Fig. E4.15—Inflection point locations

	Development length of No. 6 bar The simplified method is used to calculate the development length of a No. 6 bar	
25.4.2.3	$\ell_d = \left(\dfrac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f'_c}}\right) d_b$	Top bars $\ell_d = \left(\dfrac{(60{,}000\text{ psi})(1.3)(1.0)(1.0)}{(25)(1.0)\sqrt{5000\text{ psi}}}\right)(0.75\text{ in.}) = 33.1\text{ in.}$ say, 36 in.
25.4.2.5	where ψ_t is the bar location, $\psi_t = 1.3$, because more than 12 in. of fresh concrete is placed below top horizontal bars, and $\psi_t = 1.0$, because not more than 12 in. of fresh concrete is placed below bottom horizontal bars ψ_g = reinforcement grade factor, $\psi_g = 1.0$ for Grade 60 reinforcement ψ_e is coating factor, and $\psi_e = 1.0$, because bars are uncoated	Bottom bars: $\ell_d = \left(\dfrac{(60{,}000\text{ psi})(1.0)(1.0)(1.0)}{(25)(1.0)\sqrt{5000\text{ psi}}}\right)(0.75\text{ in.}) = 25.4\text{ in.}$ say, 30 in.
	First span top reinforcement	
9.7.3.2	Lengths at the exterior support Reinforcement must be developed at sections of maximum stress and at sections along the span where bent or terminated tension reinforcement is no longer required to resist flexure. Four No. 6 bars are required to resist the beam factored negative moment at the exterior column face. Calculate a distance x from the face of the column where two No. 6 bars are sufficient to resist the factored moment.	$(\ .95\text{ ft-kip}) - 2.7\text{ kip/ft}\dfrac{x^2}{2} + 46\text{ kip}(x) = 2(0.44\text{ in.}^2)$ $\times (0.9)(60\text{ ksi})\left(27.6\text{ in.} - \dfrac{2(0.44\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(\ 8\text{ in.})}\right)$ $x = 2\text{ ft }0\text{ in.}$
9.7.3.3	Reinforcement must extend beyond the section at which it is no longer required to resist flexure for a distance equal to the greater of d or $12d_b$.	For No. 6 bars. 1) $d = 27.6$ in. **Controls** 2) $12d_b = 12(0.75\text{ in.}) = 9$ in. Therefore, extend two No. 6 bars the longer of the development length (36 in.) and the sum of 24 in. + 27.6 in. = 51.6 in., say, 52 in. or 4 ft 4 in. from the face of the column. The sum of the theoretical cutoff point and d controls—extend two No. 6 bars 4 ft 6 in. from the interior face of the exterior support shown bold in Fig. E4.16.
9.7.3.8.4	At least one-third of the negative moment reinforcement at a support must have an embedment length beyond the point of inflection the greatest of d, $12d_b$, and $\ell_n/16$ The inflection point is calculated above at 5 ft 0 in. from the face of the column. The remaining two No. 6 bars are extended over the full length of the beam.	For No. 6 bars. 1) $d = 27.6$ in. **Controls** 2) $12d_b = 12(0.75\text{ in.}) = 9$ in. 3) $\ell_n/16 = (36\text{ ft} - 2\text{ ft})/16 = 2.1\text{ ft} = 25.5$ in. The remaining two No. 6 bars (1/2 of the bars > 1/3) must be extended a minimum of 5 ft 0 in. (60 in.) + 27.6 in. = 87.6 in. They are, however, spliced at midspan with the bars from the opposite support to act as hanger bars for stirrups

	First span top reinforcement	
9.7.3.2 9.7.3.3 9.7.3.8.4	Lengths at the interior support Following the same steps above, seven No. 6 bars are required to resist the factored moment at the first interior column face. Calculate a distance x from the face of the column where three No. 6 bars are sufficient to resist the factored moment. (Four No. 6 bars will be discontinued)	$-(312\text{ ft-kip}) - 2.7\text{ kip/ft}\dfrac{x^2}{2} + 53\text{ kip}(x) = 3(0.44\text{ in.}^2) \times (0.9)(60\text{ ksi})\left(27.6\text{ in.} - \dfrac{3(0.44\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(18\text{ in.})}\right)$ $x = 3.1$ ft, say, 3 ft-3 in. Therefore, extend four No. 6 bars the greater of the development length (36 in.) and the sum of theoretical cutoff point (3.25 ft) and d 39 in. + 27.6 in. = 56.6 in. The distance of 56.6 in. shown bold in Fig. E4.16 from the exterior face of the exterior support controls. Say 5 ft 0 in. as shown in Fig. E4.16. Extend the remaining three No. 6 bars the longer of the development length (36 in.) from where the four No. 6 bars are cut off and d = 27.6 in. beyond the inflection point which is 7 ft 3 in. from the interior face of the exterior support. The longer length is the distance d beyond the inflection point shown bold in Fig. E4.16. One of the three No. 6 bars will be terminated at 7 ft 3 in. + 27.6 in. ≈ 10 ft 0 in. The remaining two No. 6 top bars are extended and spliced at midspan

	<u>First span bottom reinforcement</u> <u>Lengths from midspan toward the column</u>	
9.7.3.2 9.7.3.3	Following the same steps above, five No. 6 bars are required to resist the factored moment at the midspan. Calculate a distance x from the face of the column where two No. 6 bars can resist the factored moment. (Three No. 6 bars will be cut off)	$(223\text{ ft-kip}) - 2.7\text{ kip/ft}\dfrac{x^2}{2} = 2(0.44\text{ in.}^2)(0.9)(60\text{ ksi})$ $\times\left(27.6\text{ in.} - \dfrac{2(0.44\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(52\text{ in.})}\right)$ $x = 9.21$ ft, say, 9 ft 3 in. = 111 in. Therefore, extend the three No. 6 bars the longer of the development length (30 in.) and 111 in. + 27.6 in. = 138.6 in. ≈ 11 ft 9 in. from maximum positive moment at midspan. 11 ft 9 in. is longer – shown bold in Fig. E4.16 from midspan
9.7.3.8.2	A minimum of one-fourth of the positive tension reinforcement must extend into the support minimum 6 in. The 6 in. requirement is superseded by the integrity reinforcement requirement to develop the bar at the column face.	Extend the remaining bars (two No. 6 bars > 1/4 five No. 6) the greater of the development length (30 in.) from the three No. 6 bar cutoff and d = 27.6 in. beyond the inflection point and a minimum of 6 in. into the support. The controlling bottom bar length is the distance 6 in. into the support shown in bold in Fig. E4.16
9.7.3.8.3	At point of inflection, d_b for positive moment tension reinforcement must be limited such that ℓ_d for that reinforcement satisfies $\ell_d \le \dfrac{M_n}{V_u} + \ell_a$ where M_n is calculated assuming all reinforcement at the section is stressed to f_y; V_u is calculated at the section. At the support, ℓ_a is the embedment length beyond the center of the support. At the point of inflection, ℓ_a is the embedment length beyond the point of inflection limited to the greater of d and $12d_b$.	Point of inflection occurs at 4 ft from the face of the column $V_u = 46\text{ kip} - (2.69\text{ kip/ft})(4\text{ ft}) = 35.2\text{ kip}$ At that location assume two No. 6 bars are effective $M_n = 2(0.44\text{ in.}^2)(60\text{ ksi})\left(27.6\text{ in.} - \dfrac{2(0.44\text{ in.}^2)(60\text{ ksi})}{(2)(0.85)(5\text{ ksi})(52\text{ in.})}\right)$ $M_n = 1451$ in.-kip $\ell_d \le \dfrac{1451\text{ in.-kip}}{35.2\text{ kip}} + 27.6\text{ in.} = 69\text{ in.}$ This length exceeds ℓ_d = 30 in., therefore **OK**
9.7.3.5	If bars are cut off in regions of flexural tension, then stress discontinuity in the continuing bars will occur. Therefore, the Code requires that flexural tensile reinforcement must not be terminated in a tensile zone unless (a), (b), or (c) is satisfied (a) $V_u \le (2/3)\phi V_n$ at the cutoff point (b) Continuing reinforcement provides double the area required for flexure at the cutoff point and the area required for flexure at the cutoff point and $V_u < (3/4)\phi V_n$ (c) Stirrup or hoop area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance 3/4d from the termination point. Excess stirrup or hoop area shall be at least $60b_ws/f_{yt}$. Spacing s shall not exceed $d/(8\beta_b)$.	(a) At 9 ft 3 in. $V_u = 46\text{ kip} - (2.69\text{ kip/ft})(17\text{ ft} - 9.25\text{ ft}) = 25.2\text{ kip}$ $\phi V_n = \phi(V_c + V_s)$ where V_c is calculated in Step 6 $\phi V_n = 0.75(70.3\text{ kip} + 0) = 52.7\text{ kip}$ $2/3\phi V_n = 2/3(52.7\text{ kip}) = 35\text{ kip}$ $V_u = 25.2\text{ kip} < 2/3\phi V_n = 35\text{ kip}$ **OK** Because only one of the three conditions needs to be satisfied, the other two will not be checked

	Integrity reinforcement for the perimeter beam	
9.7.7.1 9.7.7.1a	At least one-fourth the maximum positive moment reinforcement, but at least two bars must be continuous	Two No. 6 bars are extended into the column region two No. 6 > 1/4 (five No. 6)—satisfied (refer to Fig. E4.16)
9.7.7.1b	At least one-sixth the maximum negative moment reinforcement at the support, but at least two bars must be continuous	Four No. 6 bars are extended into the column region four No. 6 > 1/6 (seven No. 6)—satisfied (refer to Fig. E4.16)
9.7.7.1c	Longitudinal reinforcement must be enclosed by closed stirrups along the clear span of the beam. Longitudinal structural reinforcement must pass through the region bounded by the longitudinal reinforcement of the column.	Longitudinal reinforcement is enclosed by No. 4 stirrups at 12 in. on center along the full beam length satisfied This condition is satisfied by extending the two No. 6 top and bottom bars full length and through the column cores
9.7.7.4 25.4.3.1	Integrity reinforcement must be anchored to develop f_y at the face of the support. Therefore, development length for deformed bars in tension terminating in a standard hook must be the greater of:	At the exterior support, the No. 6 bars must be developed at the face. Calculate if a standard hook will allow a No. 6 bar to develop within the column.
25.4.3	Determine required hook development length using the following equations	
25.4.3.1	$\ell_{dh} \geq \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}}\right) d_h^{1.5}$ $\ell_{dh} \geq 8d_h$ $\ell_{dh} \geq 6$ in.	$\lambda = 1.0$ Bars are uncoated $\psi_e = 1.0$
25.4.3.2	ψ_e Coating factor ψ_r Confining reinforcement factor ψ_o Location factor ψ_c Concrete compressive strength factor	Neither confinement nor spacing meet requirement in table $\psi_r = 1.6$ Bars terminate in column core, but do not meet side cover requirements $\psi_o = 1.25$ Concrete strength less than 6000 psi $\psi_c = \dfrac{5000}{15{,}000} + 0.6 = 0.933$ Required hook development length $\dfrac{6000\text{ psi}(1.0)(1.6)(1.25)(0.933)}{55(1.0)\sqrt{5000\text{psi}}}(0.75)^{1.5} = 18.7$ in. $\ell_{avail} = 24$ in. $-$ 1.5 in. $-$ 0.5 in. $=$ 22 in. **OK**

9.7.7.5	Splices are necessary in continuous structural integrity reinforcement. The beam's longitudinal reinforcement shall be spliced in accordance with (a) and (b).	Splice length = (1.3)(development length) ℓ_{dc} = 1.3(36 in.) = 47 in., say, 4 ft 0 in.
	(a) Positive moment reinforcement shall be spliced at or near the support	Refer to Fig. E4.17
	(b) Negative moment reinforcement shall be spliced at or near midspan	Refer to Fig. E4.17
9.7.7.6	Use Class B tension lap splice	

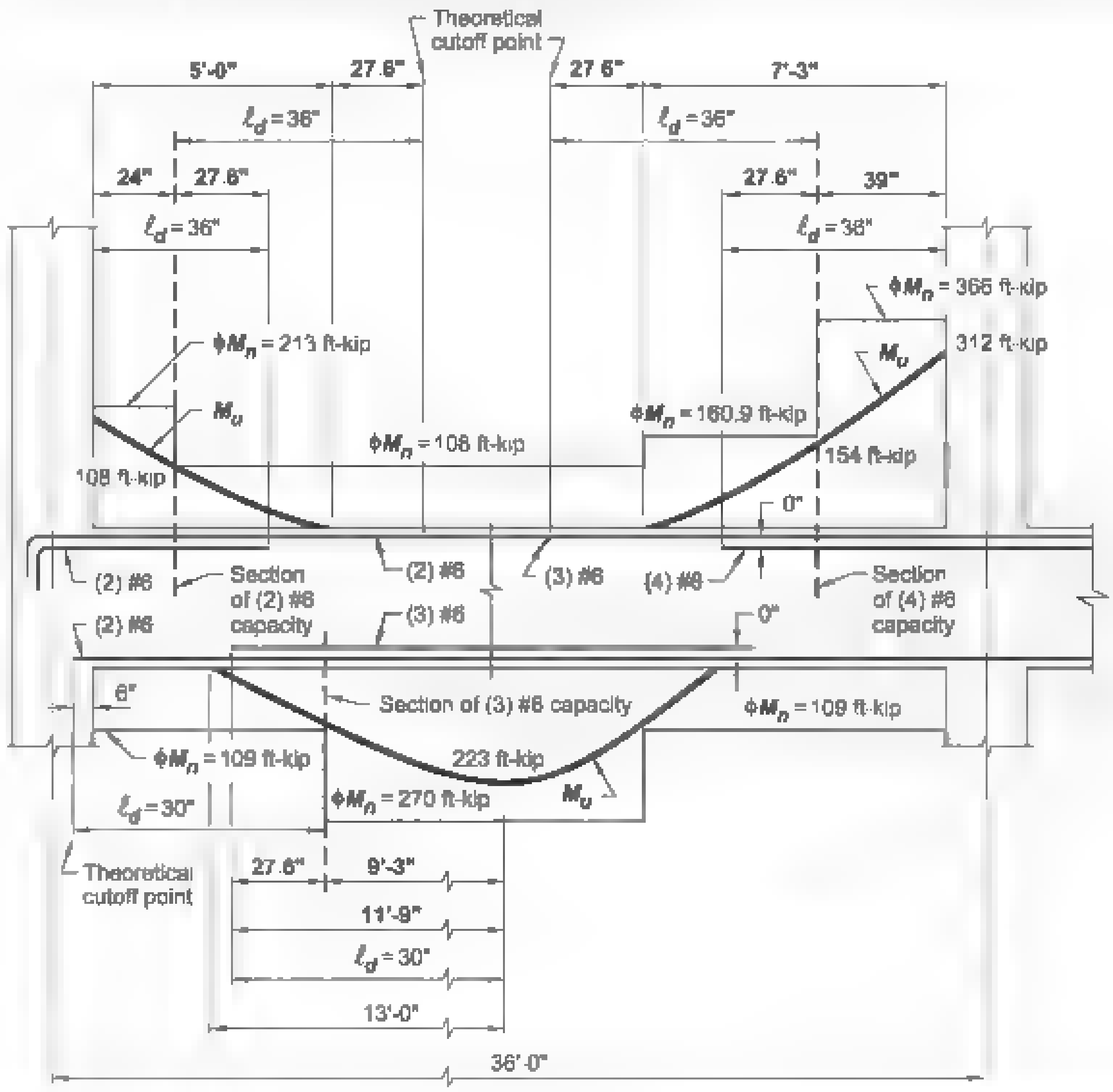


Fig. E4.16—End span reinforcement cutoff locations.

Step 9: Interior spans		
	Flexure reinforcement was calculated above in Step 5.	
	Six No. 6 top bars are required at supports.	
	Five No. 6 bottom bars are required at midspan.	
9.7.6.2.2	Shear and torsion reinforcement following the same calculation in Steps 6 and 7, No. 4 at 10 in. are required for minimum 7 ft 0 in. from face of each column. Space No. 4 stirrups at maximum 12 in. on center (d/2) for the remainder of the span.	

Step 10: Detailing

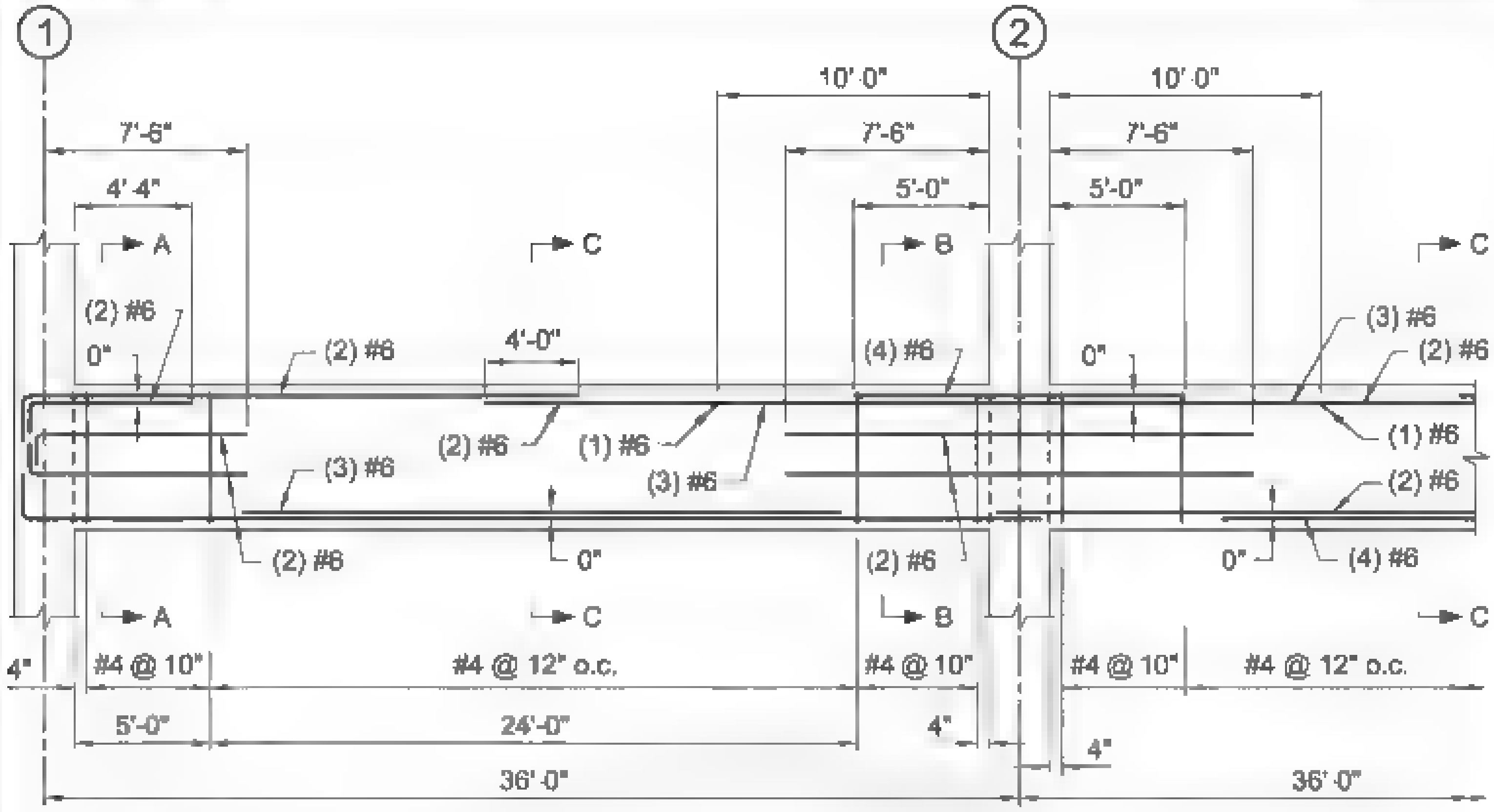


Fig. E4.17—Beam reinforcement details

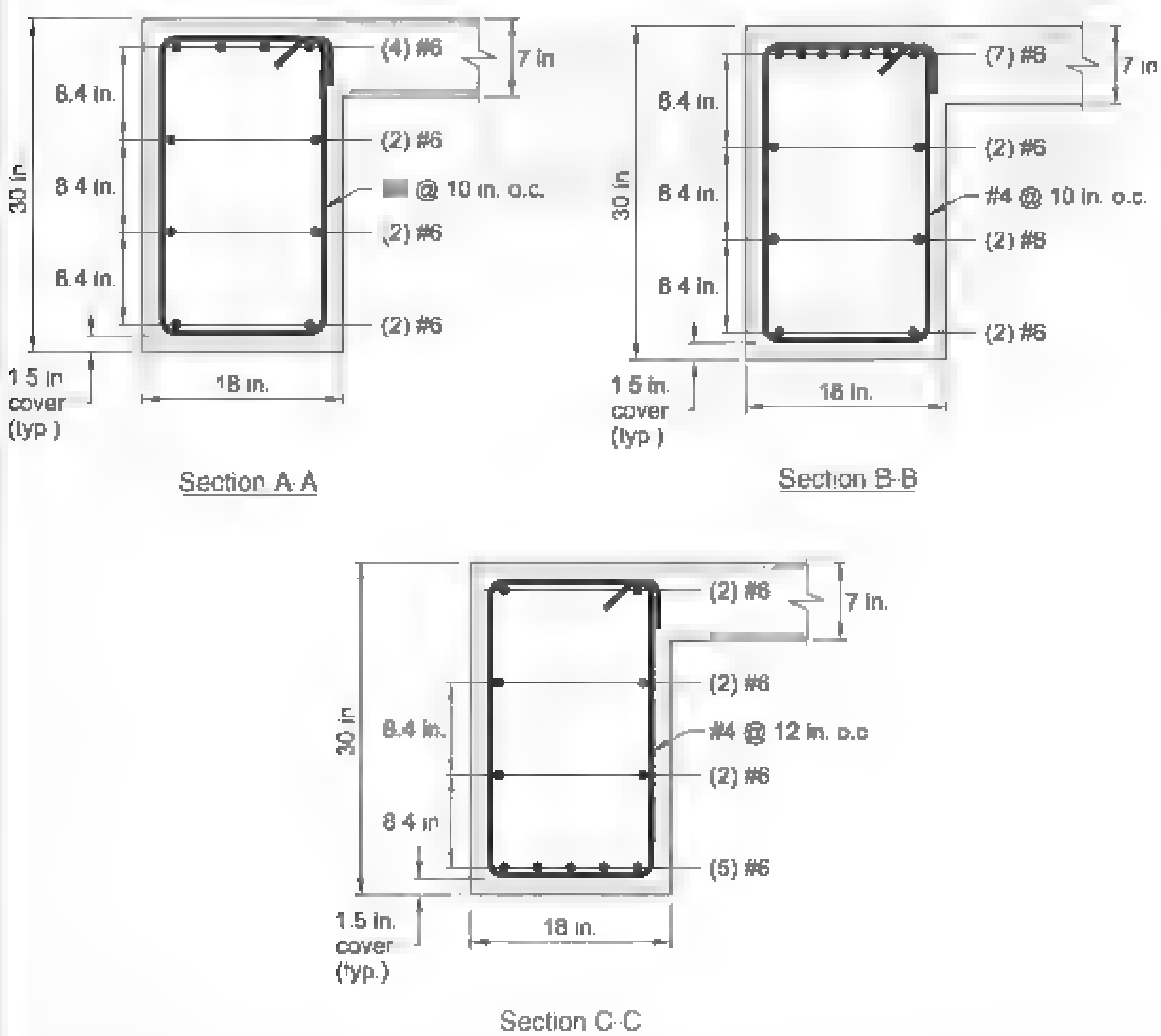


Fig. E4.18—Sections

Beam Example 5: *Continuous transfer girder*

Design and detail an interior, continuous, four bay beam, built integrally with a 7 in. slab. The span between Column Lines B and D is a transfer girder supporting five stories above.

Given:

Load—
Service additional dead load $D = 15$ psf
Service roof live load $LR = 35$ psf
Service floor live load $L = 65$ psf
Girder, beam and slab self weights are given below

Material properties—
$f_c' = 5000$ psi (normalweight concrete)
$f_y = 60{,}000$ psi
$\lambda = 1.0$ normalweight concrete

Span length—
Typical beam: 14 ft
Girder: 28 ft
Beam and girder width: 24 in.
Column dimensions: 24 in. x 24 in.

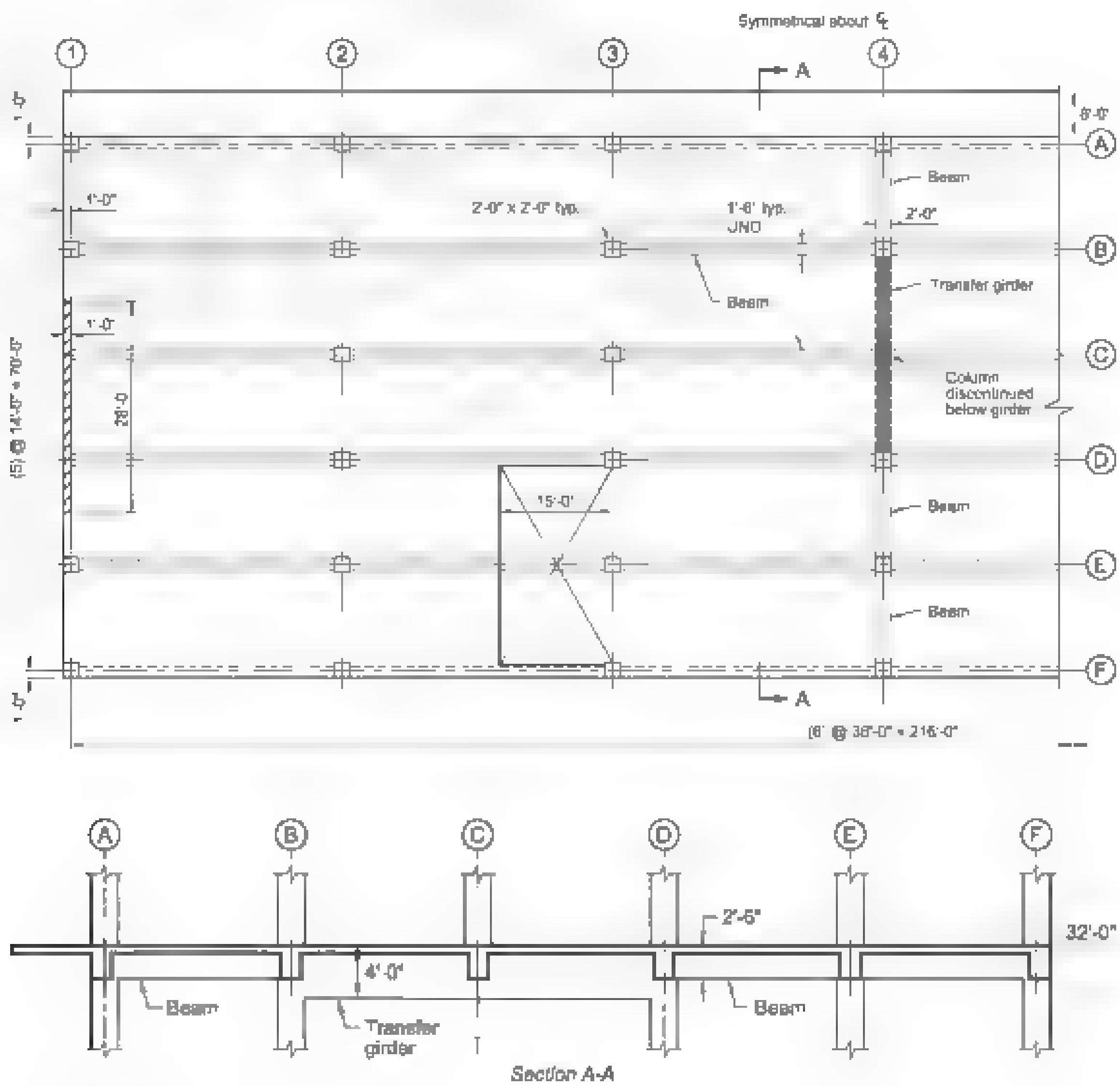
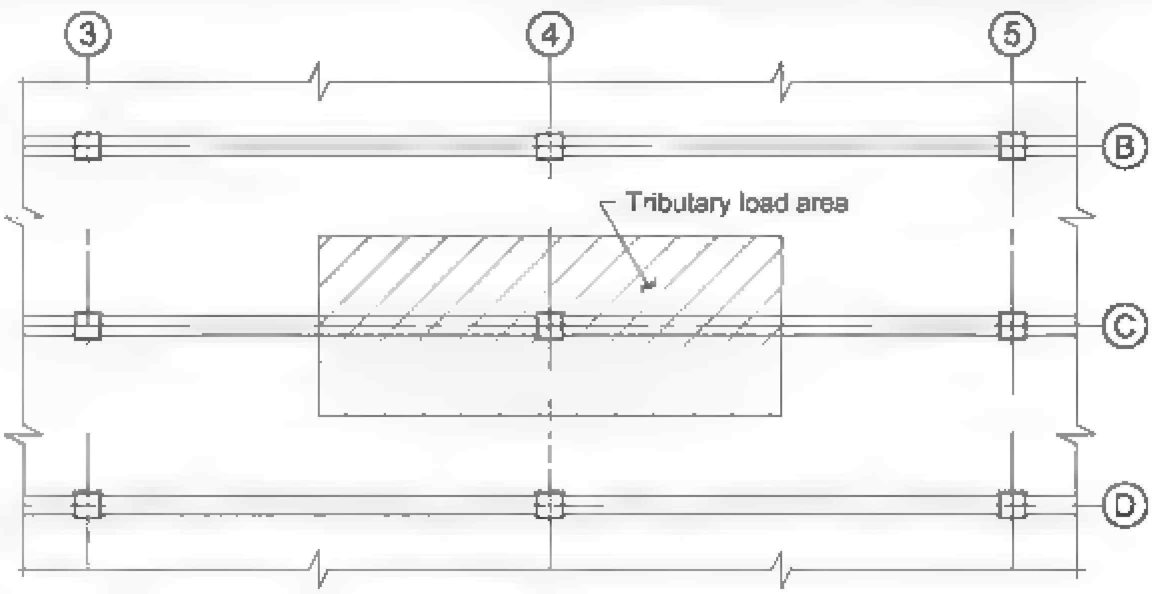


Fig. E5.1—Plan and elevation of transfer girder and beams.

ACI 318	Discussion	Calculation
Step 1. Material requirements		
9.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 (ACI 318) and structural strength requirements. The designer determines the durability classes. Please refer to Chapter 4 of MNL-17 for an in-depth discussion of the Categories and Classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi. Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26.
Step 2. Beam geometry		
9.3.1.1	Girder depth The transfer girder supports a column at midspan with tributary loads from the third level, four stories, and a roof. Therefore, the depth limits in Table 9.3.1.1 cannot be used, and calculated deflections must satisfy the deflection limits in 9.3.2. Deflections were checked using structural analysis software that was used for the moment and shear analysis. To provide continuity at the ends of the transfer girder, continue beams that will contain the top reinforcement from the transfer girder. These shallower beams extend from the ends of the girder and are subjected to uniform load. Therefore, the depth limits in Table 9.3.1.1 are used and the controlling condition for beam depth is one end continuous.	$h=\frac{\ell}{18.5}=\frac{(14\text{ ft})(12\text{ in./ft})}{18.5}=9.1\text{ in.}$ Because the beams are intended to provide continuity to strengthen the transfer girder, they must have enough stiffness for this purpose. Use a continuing beam depth of 30 in. This depth is selected to ensure consistent beam depths in floor system. Depth selection will be confirmed with the structural analysis of the continuous girder. Assume 48 in. deep continuous transfer girder for adequate strength and stiffness supporting columns above.
9.2.4.2	Flange width The transfer girder and beams are poured monolithically with the slab and will behave as a T-beam. The effective flange width on each side of the transfer girder is obtained from Table 6.3.2.1.	
6.3.2.1	Transfer girder flange width Each side of web is the least of $\begin{cases}8h_{slab}\\S_w/2\\\ell_n/8\end{cases}$	$(8)(7\text{ in.})=56\text{ in.}$ $((28\text{ ft})(12)-24\text{ in.})/8=39\text{ in.}$ **Controls**
	Flange width $b_f=\ell_n/8+b_w+\ell_n/8$	$b_f=39\text{ in.}+24\text{ in.}+39\text{ in.}=102\text{ in.}$
	Beams flange width Each side of web is the least of $\begin{cases}8h_{slab}\\S_w/2\\\ell_n/8\end{cases}$	$(8)(7\text{ in.})=56\text{ in.}$ $((14\text{ ft})(12)-24\text{ in.})/8=18\text{ in.}$ **Controls**
	Flange width $b_f=\ell_n/8+b_w+\ell_n/8$	$b_f=18\text{ in.}+24\text{ in.}+18\text{ in.}=60\text{ in.}$

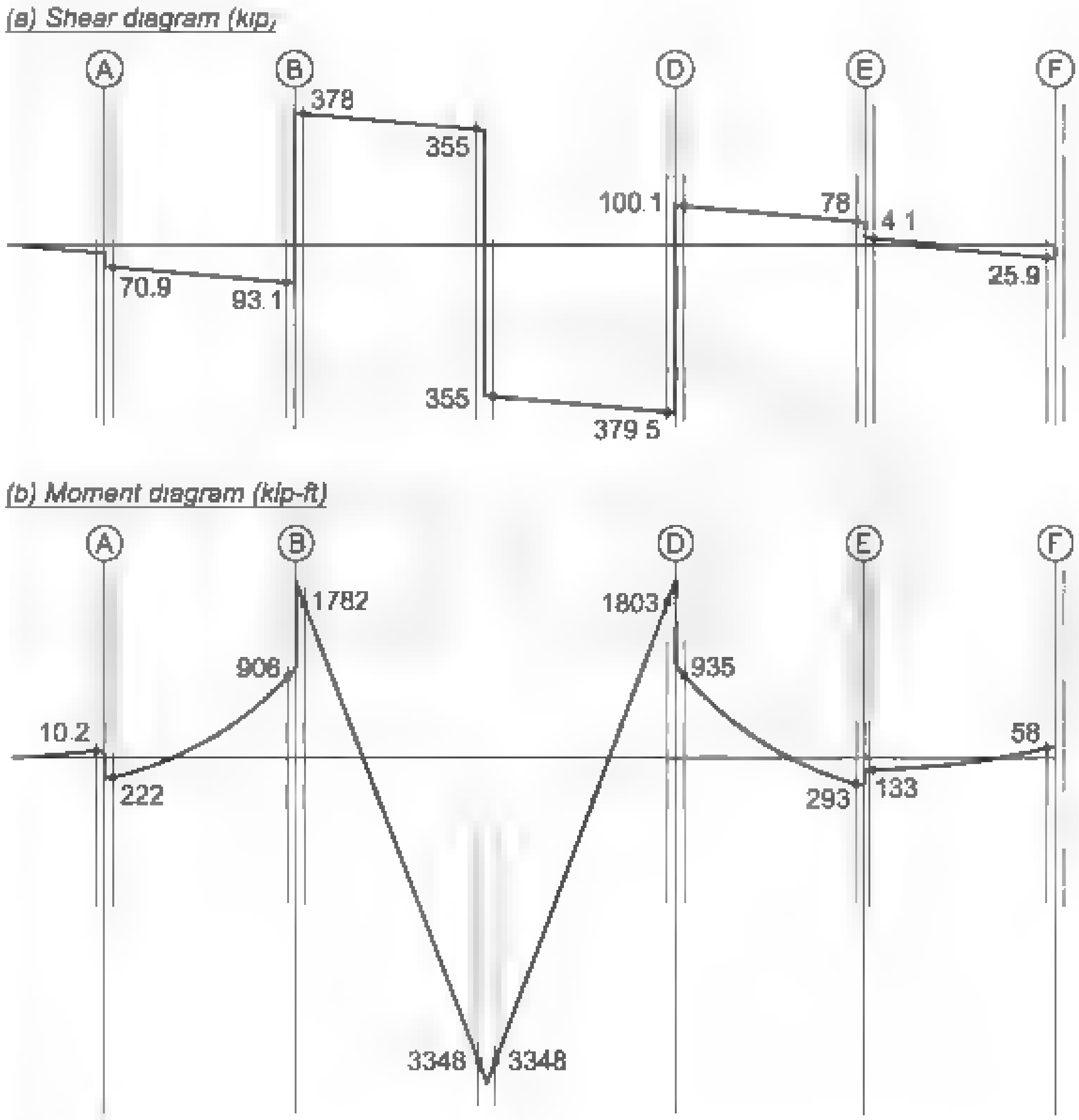
Beams

Step 3: Loads and load patterns		
	<u>Applied load on transfer girder</u> The service live load is 50 psf in offices and 80 psf in corridors per Table 4-1 in ASCE/SEI 7. This example will use 65 psf as an average live load as the actual layout is not provided. To account for the weight of ceilings, partitions, and HVAC systems, add 15 psf as miscellaneous dead load	
	<u>Dead load.</u> Transfer girder self-weight without flanges Concentrated load on girder from column above (Fig. E5.2)	$W_{gir} = [(24\text{ in.})(48\text{ in.})](0.150\text{ kip/ft}^3)/144 = 1.2\text{ kip/ft}$
	Slab weight per level	$P_{sl} = (7\text{in.}/12)(14\text{ ft})(36\text{ ft})(0.15\text{ kip/ft}^3) = 44.1\text{ kip}$
	Column weight per level	$P_{col} = (2\text{ ft})(2\text{ ft})\left(12\text{ ft} - \dfrac{7\text{ in.}}{12}\right)(0.15\text{ kip/ft}^3) = 6.85\text{ kip}$
	Typical beam weight framing into girder at midspan less slab thickness (refer to plan):	$P_{BM} = \left(\dfrac{18\text{ in.}}{12}\right)\left(\dfrac{30\text{ in.} - 7\text{ in.}}{12}\right)(34\text{ ft})(0.15\text{ kip/ft}^3)$ $= 14.7\text{ kip}$
	To account for the weight of ceilings, partitions, and mechanical (HVAC) systems, add 15 psf as miscellaneous dead load.	
	Total dead load applied on girder:	$P_{SDL} = (14\text{ ft})(36\text{ ft})(0.015\text{ kip/ft}^2) = 7.6\text{ kip}$
	Beams self-weights on both ends of the girder	$P_D = (44.1\text{ kip} + 14.7\text{ kip} + 7.6\text{ kip})(6) + (6.85\text{ kip})(5)$ $= 433\text{ kip}$
	<u>Live load.</u> The service live load is 50 psf in offices and 80 psf in corridors per Table 4-1 in ASCE/SEI 7. This example will use 65 psf as an average, as the actual layout is not provided. A 7 in. slab weighs 88 psf service dead load	$w_s = [(24\text{ in.})(30\text{ in.}) + (60\text{ in.} - 24\text{ in.})(7\text{ in.})]$ $\times [(0.150\text{ kip/ft}^3)/144] = 1.03\text{ kip/ft}$
	Roof live load 35 psf Total live load—per ASCE/SEI 7 live load with exception of roof load is permitted to be reduced by $L = L_o\left(0.25 + \dfrac{15}{\sqrt{K_{LL}A_T}}\right)$ where L is reduced live load; L_o is unreduced live load, K_{LL} is live load element factor = 4 for interior columns and 2 for interior beam (ASCE/SEI 7 Table 4-2), A_T—tributary are and $K_{LL}A_T > 400\text{ ft}^2$ $K_{LL} = 4$, $4(36\text{ ft})(14\text{ ft}) = 2016\text{ ft}^2 > 400\text{ ft}^2$	$P_{Roof} = (14\text{ ft})(36\text{ ft})(0.035\text{ kip/ft}^2) = 18\text{ kip}$ Fourth to seventh reduced live load per level $L = (0.065\text{ kip/ft}^2)\left(0.25 + \dfrac{15}{\sqrt{2016\text{ ft}^2}}\right) = 0.038\text{ ksf}$ $L = 0.038\text{ ksf} > 0.4L_o = 0.026\text{ ksf}$ **OK**

	$K_{LL} = 2$, $2(36\text{ ft})(14\text{ ft}) = 1008\text{ ft}^2 > 400\text{ ft}^2$ and $L > 0.4L_o = 26$ psf $\sum P_L = 4P_{L,col} + P_{L,3rd} + P_{Roof}$	Reduced third level live load on beam $L = (0.065\text{ kip/ft}^2)\left(0.25 + \dfrac{15}{\sqrt{1008\text{ ft}^2}}\right) = 0.047\text{ ksf}$ $L = 0.047\text{ ksf} > 0.4L_o = 0.026\text{ ksf}$ **OK** Concentrated live load to column per level $P_{L,col} = (14\text{ ft})(36\text{ ft})(0.38\text{ kip/ft}^2) = 19.2\text{ kip}$ Concentrated live load at third level $P_{L,3rd} = (14\text{ ft})(36\text{ ft})(0.047\text{ kip/ft}^2) = 23.7\text{ kip}$ Concentrated live load on girder: $\sum P_L = (23.7\text{ kip}) + (19.2\text{ kip})(4\text{ levels}) + (18\text{ kip})$ $= 119\text{ kip}$
5.3.1	The transfer girder resists gravity only and lateral forces are not considered in this problem $U = 1.4D$	Transfer girder Distributed $w_u = 1.4(1.2\text{ kip/ft}) = 1.68\text{ kip/ft}$ **Controls** Concentrated $P_u = 1.4(433\text{ kip}) = 607\text{ kip}$
	The superimposed dead load is calculated above and is included in the concentrated load. Live load is applied over the width of the girder (2 ft 0 in.) $U = 1.2D + 1.6L$	Distributed $w_u = 1.2(1.2\text{ kip/ft}) + 1.6(0.065\text{ ksf})(2\text{ ft})$ $= 1.65\text{ kip/ft}$ $P_u = 1.2(433\text{ kip}) + 1.6(119\text{ kip}) = 710\text{ kip}$ **Controls** Beams $w_u = 1.4(1.03\text{ kip/ft} + (15\text{ psf})(60\text{ in.}/12)/1000)$ $= 1.55\text{ kip/ft}$ $w_u = 1.2(1.55\text{ kip/ft})/1.4 + 1.6((65\text{ psf})(60\text{ in.}/12)/(1000)$ $= 1.85\text{ kip/ft}$ **Controls**
	 *Fig. E5.2—Column C/4 tributary area*	

Beams

Step 4: Analysis		
9.4 3.1	The beams are built integrally with supports, therefore, the factored moments and shear forces (required strengths) are calculated at the face of the supports The beams were analyzed as part of a frame The moment and shear diagram obtained from a software are presented below (Fig. E5 3) Deflections under service load were checked using the software and found to meet the limitations of Code 9 3 2	



Notes.
1 Factored moments and shear forces are shown at faces of columns.
2 Span BD is subjected to large concentrated load at midspan and relatively small distributed beam self-weight Therefore, the appearance of a straight line moment diagram.

Fig E5 3 Shear and moment diagrams

Step 5. Moment design		
9.3.3.1	Limiting steel strain restricts the amount of reinforcement to ensure warning of failure by excessive deflection and cracking. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members. Beginning with the 2019 Code, this limit is revised to require that the section be tension-controlled.	$\varepsilon_y = \dfrac{f_y}{E_s} = \dfrac{60{,}000\text{ psi}}{29{,}000{,}000\text{ psi}} \cong 0.02$ $\varepsilon_t \geq \varepsilon_{ty} + 0.003 = 0.002 + 0.03 = 0.005$
21.2.2	Because section must be tension-controlled, the strength reduction factor is 0.9	Beam must be tension-controlled in accordance with Table 21.2.2 $\phi = 0.9$
20.5.1.3.1	Determine the effective depth assuming No. 5 stirrups and No. 11 bars for the transfer girder positive moment and No. 9 bars for the transfer girder negative moment. Assume No. 4 stirrups and No. 6 and No. 9 bars for beams positive and negative moments, respectively. Girder beam and beams will have 1.5 in. cover Assume that the transfer girder requires two rows of reinforcement with one bar spacing between the two rows within the span and one layer of bars at the supports. Beams require one row only $d = h - \text{cover} - d_{tie} - 3d_b/2$	
		Transfer girder
	positive moment	$d = 48\text{ in.} - 1.5\text{ in.} - 0.625\text{ in.} - 3(1.41\text{ in.})/2 = 43.76\text{ in.}$ use $d = 43.7$ in.
	negative moment	$d = 48\text{ in.} - 1.5\text{ in.} - 0.625\text{ in.} - 1.128\text{ in.}/2 = 45.3\text{ in.}$ use $d = 45$ in.
		Beams
	positive moment	$d = 30\text{ in.} - 1.5\text{ in.} - 0.5\text{ in.} - (0.75\text{ in.})/2 = 27.625\text{ in.}$ use $d = 27.6$ in.
	negative moment	$d = 30\text{ in.} - 1.5\text{ in.} - 0.5\text{ in.} - 1.128\text{ in.}/2 = 27.4\text{ in.}$ use $d = 27.4$ in.

22.2.2.1	The concrete compressive strain at nominal moment strength is calculated at ε_{cu} = 0.003 in./in.	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and is approximately 10 to 15 percent of the concrete compressive strength. ACI 318 neglects the concrete tensile strength to calculate nominal strength.	
22.2.2.3	Determine the equivalent concrete compressive stress at nominal strength The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distri-	
22.2.2.4.1	bution of $0.85f_c'$ with a depth of $a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3	
22.2.2.4.3	For f_c' = 5000 psi:	$\beta = 0.85 - \dfrac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$
22.2.1.1	Find the equivalent concrete compressive depth a by equating the compression force to the tension force within the beam cross section (Fig. E5.4): $C = T$ $0.85f_c'ba = A_sf_y$	$0.85(5000\text{ psi})(b)(a) = A_s(60{,}000\text{ psi})$
	Transfer girder: For positive moment: $b = b_f$=102 in.	$a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(102\text{ in.})} = 0.138A_s$
	Beams For positive moment: $b = b_f$ = 60 in.	$a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(60\text{ in.})} = 0.235A_s$
	Transfer girder and beams: For negative moment: $b = b_w$ = 24 in.	$a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(24\text{ in.})} = 0.588A_s$

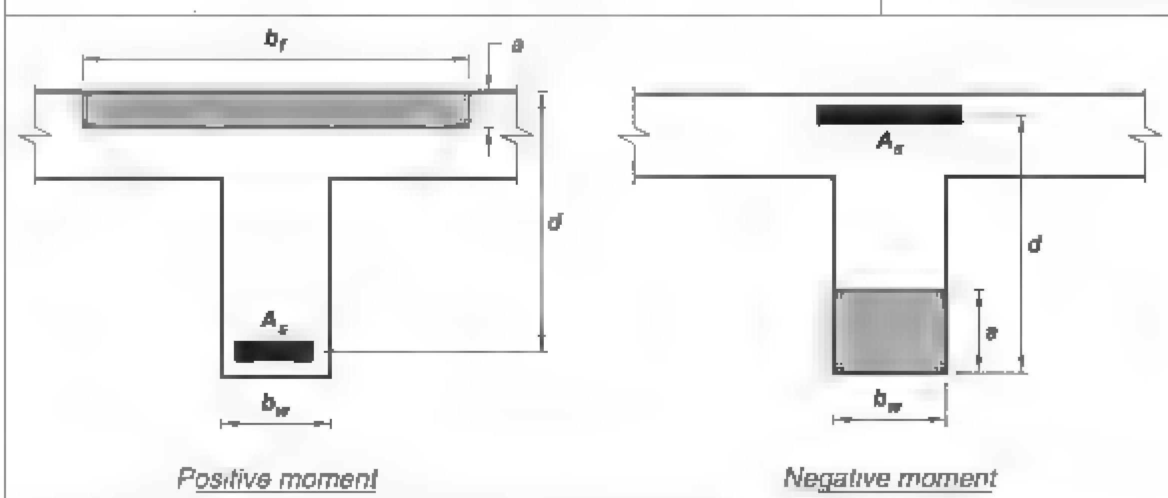


Fig. E5.4—Section compression block and reinforcement locations.

The transfer girder and beams are designed for the maximum flexural moments shown in the above moment diagram (Step 4)

The beams' design strength must be at least the required strength at each section along their lengths

9.5.1.1 $\phi M_n > M_u$

$\phi V_n > V_u$

Calculate the required reinforcement area

$$\phi M_n \geq M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

For top negative reinforcement use One layer of No 9 bar; $d_b = 1.128$ in., $A_s = 1.0$ in.^{2}, and $d = 45$ in.

For bottom positive reinforcement use: Two layers of No. 11 bar; $d_b = 1.41$ in., $A_s = 1.56$ in.^{2}, and $d = 43.7$ in.

Transfer girder:

	M_u, ft-kip	$A_{s,req'd}$, in.^{2}	Number of bars	
			Req'd	Select
Max. M^-	1782	9.4	9.4	0
Max. M^+	3348	17.51	11.22	12
Max. M^-	1803	9.5	9.5	10

Minimum reinforcement

9.6.1.1 The provided reinforcement must be at least the

9.6.1.2 minimum required reinforcement at every section along the length of the beam.

Use $d_{@support} = 45$ in. $> d_{@midspan} = 43.7$ in.

will yield higher required minimum reinforcement area

$$A_s = \frac{3\sqrt{f_c'}}{f_y} b_w d$$

$$A_s = \frac{3\sqrt{5000 \text{ psi}}}{60{,}000 \text{ psi}}(24 \text{ in.})(45 \text{ in.}) = 3.73 \text{ in.}^2$$

Because $f_c' > 4444$ psi, Eq. (9.6.1.2a) only applies.

Provided reinforcement area exceeds the minimum required. Therefore, **OK**

9.7.2.3 The girder is 48 in. deep. To control cracks within

24.3.2 the web, ACI 318 requires skin reinforcement to be placed near the vertical faces of the tension zone over a distance of:

48 in./2 = 24 in.

Spacing of skin reinforcement in girder must not exceed the lesser of:

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \text{ and}$$

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - 2.5(2.125 \text{ in.}) = 9.7 \text{ in.}$$ **Controls**

$$s = 12\left(\frac{40{,}000}{f_s}\right)$$

$$s = 12\left(\frac{40{,}000}{40{,}000}\right) = 12 \text{ in.}$$

where c_c is the clear cover from the skin reinforce-

24.3.2.1 ment to the side face.

$c_c = 1.5$ in. + 0.625 in. = 2.125 in.

and $f_s = 2/3 f_y = 40$ ksi

Place two No. 8 skin reinforcement at girder middepth, 24 in. and the second pair of No. 8 bars at 33.5 in. from the top of the girder

9.7.2.3	Skin reinforcement can be used in the strength calculation of the girder	
	Positive reinforcement at midspan	$\phi M_n = (0.9)(60\text{ ksi})(2)(0.79\text{ in.}^2)\left(\left(33.5\text{ in.} - \frac{2.58\text{ in.}}{2}\right) + \left(24\text{ in.} - \frac{2.58\text{ in.}}{2}\right)\right)$
	Using strain compatibility, try two No. 8 bars in two layers on both sides of the girder. Assume reinforcement is yielding. It will be checked later	ϕM_n = 4685.8 in.-kip = 390.5 ft-kip, say, 390 ft-kip
	Re-evaluating the positive tension reinforcement at girder midspan Calculated design moment 3348 ft-kip (Step 4)	
	Moment to be resisted by No. 11 bars	ϕM_n = (3348 ft-kip) – (390 ft-kip) = 2958 ft-kip
	Required reinforcement area by solving the following equation $\phi M_n \geq M_u = \phi A_s f_y\left(d - \frac{a}{2}\right)$	$A_{s,No.11}$ = 15.42 in.^{2} Required No. 11 bars = 15.42 in.^{2}/1.56 in.^{2} = 9.9 Choose 10 No. 11 bars.
	Negative reinforcement at the supports Provide three layers of skin reinforcement on both sides in the top half of the girder. Extend the two No. 8 middepth skin bars over the full length of the girder and use for the top half at the support two layers of No. 6 bars on both sides of the girder. Assume that skin reinforcement reach yielding (will be checked later). Calculate the provided moment from the skin reinforcement:	$\phi M_n = (0.9)(60\text{ ksi})(2)(0.79\text{ in.}^2)\left(24\text{ in.} - \frac{5.29\text{ in.}}{2}\right)$ $+ (0.9)(60\text{ ksi})(2)(0.44\text{ in.}^2)\left(31.25\text{ in.} - \frac{5.29\text{ in.}}{2}\right)$ $+ (0.9)(60\text{ ksi})(2)(0.44\text{ in.}^2)\left(38.5\text{ in.} - \frac{5.29\text{ in.}}{2}\right)$ ϕM_n = 4885.1 in.-kip = 407.1 ft-kip, say, 407 ft-kip
	Re-evaluating the positive tension reinforcement at girder midspan Calculated design moment 1803 ft-kip (Step 4)	
	Moment to be resisted by No. 9 bars.	ϕM_n = (1803 ft-kip) – (407 ft-kip) = 1396 ft-kip
	Required reinforcement area by solving the following equation $\phi M_n \geq M_u = \phi A_s f_y\left(d - \frac{a}{2}\right)$	$A_{s,No.9}$ = 7.42 in.^{2} Required No. 9 bars = 7.42 in.^{2}/1.0 in.^{2} = 7.49 Choose 8 No. 9 bars

21.2.2
9.3.3.1

Check if the calculated strain exceeds 0.005 in. in to ensure section is tension-controlled

$$a = \frac{A_s f_y}{0.85 f'_c b} \text{ and } c = \frac{a}{\beta}$$

where $\beta_1 = 0.8$

$$\varepsilon_t = \frac{0.003}{c}(d - c)$$

Note that b = 24 in. for negative moments and 102 in. for girder positive moments.

Place eight No. 9 in one layer with d = 45.3 in.

Transfer girder

	M_u, ft-kip	$A_{s,prov}$ in.^{2}	a, in.	c, in.	ε_t, in./in.	ε_t > 0.005?
M^-	1782	8	4.7	5.9	0.020	Y
M^+	3348	15.6	2 5	2.7	0.0466	Y
M^-	1803	8	4.7	5.9	0.020	Y

Section is tension-controlled. Use φ = 0.9

Check skin reinforcement strain (Fig. E5.5):

Positive half, strain at middepth

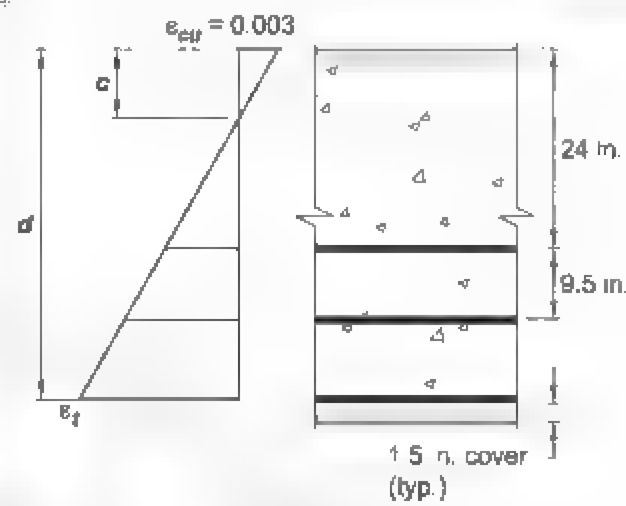


Fig. E5.5—Strain distribution over beam depth

Check strain in the upper skin layer at 24 in. from the top

$$\varepsilon_s = \frac{0.003}{2.7 \text{ in.}}(24 \text{ in.} - 2.7 \text{ in.}) = 0.024 > 0.005 \quad \textbf{OK}$$

By inspection the other skin reinforcement layer has higher strain, therefore, reinforcement in both skin reinforcement layers is yielding and the assumption of using ϕ = 0.9 is correct

Negative half, strain at supports B and D (Fig. E5.6)

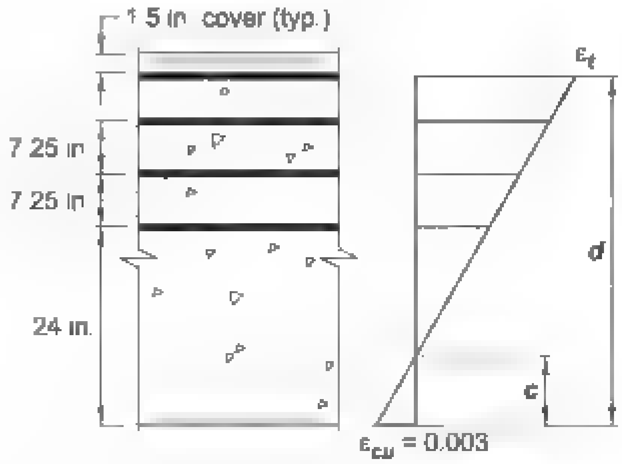


Fig. E5.6—Strain distribution

Check strain in the lower skin layer at 24 in. from the bottom

$$\varepsilon_s = \frac{0.003}{5.9\text{ in.}}(24\text{ in.} - 5.9\text{ in.}) = 0.009 > 0.005$$

By inspection the other skin reinforcement layers have higher strain, therefore, reinforcement in all skin reinforcement layers is yielding and the assumption made of using $\phi = 0.9$ is correct

Beams

Design beams for the maximum load condition Extend No. 9 top bars from Span BD to resist the 906 ft-kip and 935 ft-kip moments at Column Lines B and D in Spans AB and DE, respectively, and No. 6 bars to resist the rest of the moments.

No. 6 bar, d_b = 3/4 in., A_s = 0.44 in.^{2}, and d = 27.6 in.
No. 9 bar; d_b = 1.128 in., A_s = 1.0 in.^{2}, and d = 27.4 in. for one layer

Span Column Line AB

	M_u, ft-kip	$A_{s,req'd}$, in.^{2}	Number of bars Req'd	Select
M^- (No. 9)	906	8.0	8.0	8
M^+ (No. 6)	222	1.8	4.1	5

Span Column Line DE

	M_u, ft-kip	$A_{s,req'd}$, in.^{2}	Number of bars Req'd	Select
M^- (No. 9)	935	8.33	8.33	9
M^+ (No. 6)	293	2.38	5.42	6

Span Column Line EF

	M_u, ft-kip	$A_{s,req'd}$, in.^{2}	Number of bars Req'd	Select
M^- (No. 6)	58	0.43	0.97	2
M^+ (No. 6)	133	1.08	2.4	3

Minimum reinforcement

9.6.1.1
9.6.1.2 The provided reinforcement must be at least the minimum required reinforcement at every section along the length of the beam.

$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y} b_w d$$

Because $f_c' > 4444$ psi, Eq. (9.6.1.2a) only applies.

Check to see if the required reinforcement areas exceeds the code minimum reinforcement area at all locations.

Beams:

$$A_{s,min} = \frac{3\sqrt{5000\text{ psi}}}{60{,}000\text{ psi}}(24\text{ in.})(27.6\text{ in.}) = 2.34\text{ in.}^2$$

Provide a minimum six No. 6 bars at all beam tension locations, Except at Column Lines B and D, where transfer girder top reinforcement is extended over adjacent spans to resist the negative moment and Span DE positive moment, where six No. 6 longitudinal bars is required.

Note: Reduce the total number of No. 9 bars to eight No. 9 by extending the two top No. 6 skin bars from the girder into Beam DE to resist part of the negative moment.

Calculate strain in No. 6 bars (Fig. E5.7).

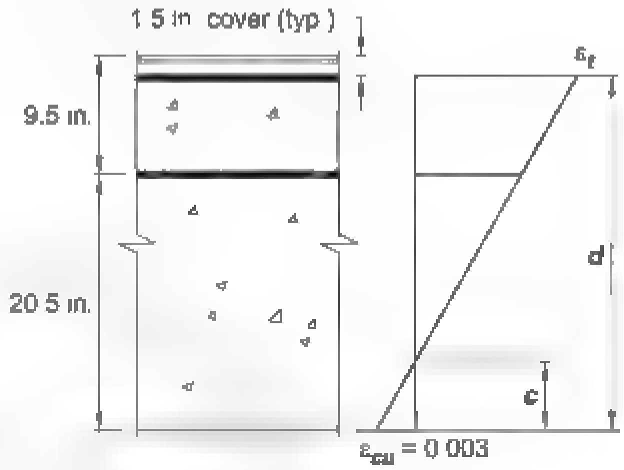


Fig. E5.7—Strain diagram.

$$\varepsilon_s = \frac{0.003}{5.9\text{ in.}}(20.5\text{ in.} - 5.9\text{ in.}) = 0.007 > 0.005$$

Therefore, reinforcement in the two No. 6 skin bars is yielding and the assumption made of using $\phi = 0.9$ is correct.

21.2.2 Check if the calculated steel strain exceeds 0.005 to ensure section is tension-controlled (refer to Fig. E5.8):

$$a = \frac{A_s f_y}{0.85 f_c' b} \text{ and } c = \frac{a}{\beta_1}$$

where $\beta_1 = 0.8$

$$\varepsilon_s = \frac{0.003}{c}(d - c)$$

Note that $b = 24$ in. for negative moments and 102 in. and 60 in. for girder and beam positive moments, respectively.

$c < h_f$ for both transfer girder and beams, therefore, the T-section members assumption for positive moments is correct.

Beams (only maximum moments are checked)

	M_u ft-kip	$A_{s,prov}$ in.^{2}	a, in.	c, in.	ε_t, in./in.	$\varepsilon_t >$ 0.005?
M^-	935	8	4.7	5.9	0.0 1	Y
M^+	293	2.64	0.62	0.78	0.103	Y

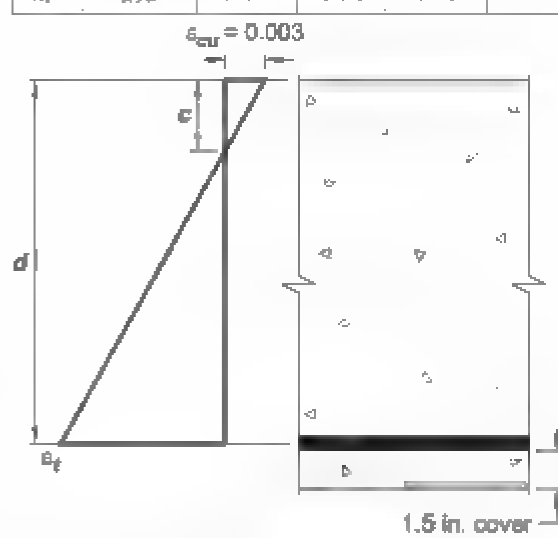
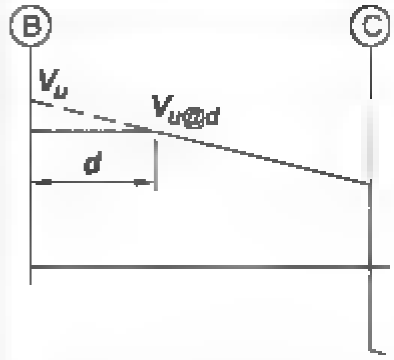


Fig. E5.8—Strain distribution across beam section.

Step 6. Shear design		
	Transfer girder	
	Shear strength	
9.4.3.2	Because conditions a), b), and c) of 9.4.3.2 are satisfied, the design shear force is taken at critical section at distance d from the face of the support (Fig. E5.9); use $d = 45.3$ in. at support. The controlling factored load combination must satisfy:	*Fig. E5.9—Shear at the critical section.*
9.5.1.1 9.5.3.1 22.5.1.1	$\phi V_n \geq V_u$ $V_n = V_s + V_c$ 2019 Code introduced size effect for shear design in which the shear strength of an element that does not contain shear reinforcement is not directly proportional to its depth. This effect is addressed by incorporating a size effect factor λ_s into the concrete contribution equation. If shear reinforcement is not present, then the concrete contribution to shear strength must be reduced by the size effect factor. If minimum shear reinforcement is provided, then the Eq. 22.5.5.1a can be used to calculate V_c. Minimum shear reinforcement is required where $V_u > \phi\lambda\sqrt{f_c'}b_w d$ For this example, use minimum shear reinforcement over entire length of beam. The concrete contribution to shear strength is then: $V_c = 2\sqrt{f_c'}b_w d$ (22.5.5.1a)	$V_{u@d} = (379.5\text{ kip}) - (1.68\text{ kip/ft})(45\text{ in.}/12) = 373.2\text{ kip}$ $V_c = 2\sqrt{5000\text{ psi}}(24\text{ in.})(45\text{ in.}) = 152.7\text{ kip}$
21.2.1b	Shear strength reduction factor: $\phi V_c = \phi 2\sqrt{f_c'}b_w d$	$\phi_{shear} = 0.75$ $\phi V_c = 0.75(152.7\text{ kip}) = 114.6\text{ kip}$
9.5.1.1b	Check if $\phi V_c > V_u$	$\phi V_c = 114.6\text{ kip} < V_{u@d} = 373.7\text{ kip}$ **NG** Therefore, shear reinforcement is required for strength.
	Prior to calculating shear reinforcement, check if the cross-sectional dimensions satisfy Eq. (22.5.1.2).	
22.5.1.2	$V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d)$	$V_u \leq \phi\left(152.7\text{ kip} + \dfrac{8\sqrt{5000\text{ psi}}(24\text{ in.})(45\text{ in.})}{1000\text{ lb/kip}}\right)$ $\leq 576.6\text{ kip}$ **OK**, therefore, section dimensions are satisfactory.

22.5.8.5.1	<u>Shear reinforcement</u> Transverse reinforcement satisfying equation 22.5.8.5.3 is required at each section where $V_u > \phi V_c$ $V_s \ge \frac{V_u}{\phi} - V_c$	$V_s \ge \frac{373.2\text{ kip}}{0.75} - 152.7\text{ kip} = 344.9\text{ kip}$ Try a No. 5 bar, two legged stirrup
22.5.8.5.3 22.5.8.5.5	Spacing required for No. 5 stirrups where $V_s = \frac{A_v f_{yt} d}{s}$	$344.9\text{ kip} = \frac{(2)(0.31\text{ in.}^2)(60{,}000\text{ psi})(45\text{ in.})}{s(1000\text{ lb/kip})}$ $s = 4.85$ in. This is a relatively tight spacing. Use two No. 5 double stirrups side by side. This will yield a spacing of 9.7 in.; say, 8 in. spacing.
9.7.6.2.2	Calculate maximum allowable stirrup spacing First, does the beam transverse reinforcement value need to exceed the threshold value? $V_s \le 4\sqrt{f_c'}b_w d$?	$4\sqrt{f_c'}b_w d = \frac{4\left(\sqrt{5000\text{ psi}}\right)(24\text{ in.})(45\text{ in.})}{1000\text{ lb/kip}} = 305.5\text{ kip}$ $V_s = 344.9\text{ kip} > 4\sqrt{f_c'}b_w d = 305.5\text{ kip}$ **OK**
9.7.6.2.2	Because the required shear strength is higher than the threshold value, the maximum stirrup spacing is the lesser of $d/4$ and 12 in. Because shear force does not vary significantly over the length of the transfer girder (Fig. E5.3a), use two No. 5 stirrups at 8 in. spacing over the full length of the girder.	$d/4 = 45\text{ in.}/4 = 11\text{ in.} < 12\text{ in.}$ Use $s = 8$ in. = $d/4 = 11$ in. < 12 in. ∴ **OK**
9.6.3.4	Specified shear reinforcement must be at least $0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}$ and $50\frac{b_w}{f_{yt}}$ Because f_c' > 4444 psi, Eq. (1) controls	$\frac{A_{v,min}}{s} \ge 0.75\sqrt{5000\text{ psi}}\frac{18\text{ in.}}{60{,}000\text{ psi}} = 0.016\text{ in.}^2\text{/in.}$ Provided 8 in. spacing: $\frac{A_{v,min}}{s} = \frac{4(0.31\text{ in.}^2)}{8\text{ in.}} = 0.155\text{ in.}^2\text{/in.}$ Spacing satisfies 9.6.3.4, therefore, **OK**

Beams

Shear strength

21.2.1b Shear strength reduction factor

$\phi_{shear} = 0.75$

9.5.1.1 $\phi V_n \geq V_u$

9.5.3.1, 22.5.1.1 $V_n = V_c + V_s$

9.4.3.2 Because conditions a), b), and c) of 9.4.3.2 are satisfied, the design shear force is taken at critical section at distance d from the face of the support (Fig. E5.10). Provide minimum shear reinforcement over the full length of each beam. This allows the use of the following equation for V_c.

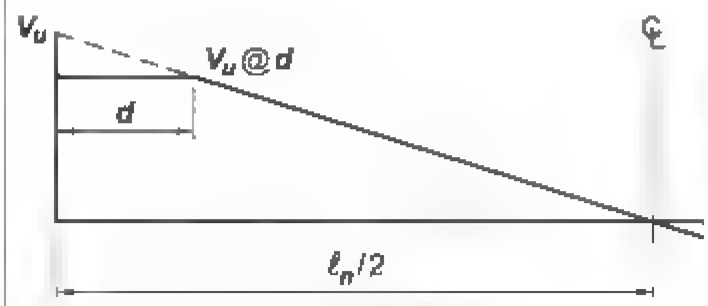


Fig. E5.10—Shear at the critical section.

$V_{u@d} = V_u - (1.85\text{ kip/ft})(27.4\text{ in./12}) = V_u - 4.2\text{ kip}$

$$V_c = 2\sqrt{f_c'}b_w d \qquad (22.5.5.1a)$$

$V_c = (2)\left(\sqrt{5000\text{ psi}}\right)(24\text{ in.})(27.4\text{ in.}) = 93\text{ kip}$

$\phi V_c = (0.75)(93\text{ kip}) = 69.7\text{ kip}$

Check if shear reinforcement is required for strength. Provide minimum shear reinforcement over the full length of each beam.

		$V_{u@d}$, kip	Is $\phi V_c > V_u$?
Beam [illegible]	Left	66.7	Y
	Right	88.9	N
Beam 3	Left	95.9	N
	Right	73.8	N
Beam 4	Left	0	Y
	Right	21.7	Y

Shear reinforcement

$V_s \geq \dfrac{V_u}{\phi} - V_c$

$V_s \geq \dfrac{95.9\text{ kip}}{0.75} - 93\text{ kip} = 34.9\text{ kip}$

where $V_s = \dfrac{A_v f_{yt} d}{s}$ and $V_c = 93$ kip

Using No. 3 stirrups

$s = \dfrac{(2)(0.2\text{ in.}^2)(60{,}000\text{ psi})(27.4\text{ in.})}{34{,}900\text{ lb}} = 18.8\text{ in.}$

The spacing exceeds the maximum allowed $d/2 = 13.7$ in., therefore, use 12 in. spacing over the full length of beam.

By inspection provisions, 9.7.6.2.2 and 9.6.3.3 are satisfied.

Step 7. Reinforcement detailing		
	Minimum bar spacing Bottom reinforcement—girder.	
25.2.1	The clear spacing between the horizontal No.11 bars must be at least the greatest of	
	Clear spacing the greater of: $\begin{cases} 1 \text{ in.} \\ d_b \\ 4/3(d_{agg}) \end{cases}$	1 in. 1.41 in. **Controls** 4/3(3/4 in.) = 1 in.
	Assume 3/4 in. maximum aggregate size Check if five No.11 bars (resisting positive moment) can be placed in the beam's web, refer to Fig. E5.11	Therefore, clear spacing between horizontal bars must be at least 1.41 in., say, 1.5 in.
	$b_{w,req'd} = 2(\text{cover} + d_{stirrup} + 1.0 \text{ in.}) + 4d_b + 4(1.5 \text{ in.})_{min,spacing}$ (25.2.1)	$b_{w,req'd} = 2(1.5 \text{ in.} + 0.625 \text{ in.} + 1.0 \text{ in.}) + 5.64 \text{ in.} + 6.0 \text{ in.}$ $= 17.9 \text{ in.} < 24 \text{ in.}$ **OK** Therefore, five No. 11 bars can be placed in one layer in the 24 in. transfer girder web
	Spacing between longitudinal bars.	$sp = \dfrac{24 \text{ in.} - [2(1.5 \text{ in.} + 0.625 \text{ in.} + 1.0 \text{ in.}) + 4(1.41 \text{ in.})]}{4}$ $= 3.0 \text{ in.}$

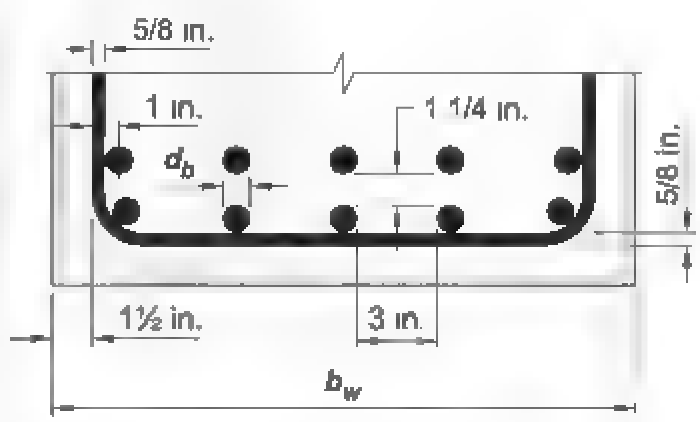


Fig. E5.11—Bottom reinforcement layout one layer is shown.

	Bottom reinforcement—beams. Beams AB, DE, and EF are reinforced with six No. 6 bottom bars uniformly spaced. The calculated spacing is 3 in. Therefore, **OK**	

Top reinforcement

24.3.4 Tension reinforcement in flanges must be distributed within the effective flange width, $b_f = 102$ in. (Step 2), but not wider than $\ell_n/10$

Girder
$\ell_n/10 = (26\ \text{ft})(12)/10 = 31.2\ \text{in.} < 102\ \text{in.}$

Because effective flange width exceeds $\ell_n/10$, additional bonded reinforcement is required in the outer portion of the flange

Beam
$\ell_n/10 = (12\ \text{ft})(12)/10 = 14.4\ \text{in.} < 60\ \text{in.}$

Use No. 6 placed in slab over b_f for additional bonded reinforcement, refer to Fig. E5.12

This requirement is to control cracking in the slab due to wide spacing of bars across the full effective flange width and to protect flange if reinforcement is concentrated within the web width

$$\text{Bar spacing} = \frac{24\ \text{in.} - [2(3.125\ \text{in.}) + 5(1.128\ \text{in.})]}{5}$$

$$= 2.4\ \text{in.}$$

Span		Prov. No. 9	No. 9 in web	No. 10 in $\ell_n/10$*	No. 6 in outer portion†
AB	L	4	4		4
	R	8	6	2	4
BD	L	8	6	2	6
	R	8	6	2	6
DE	L	8	6	2	4
	R	4	4		4
EF	L	5†	5†	—	4
	R	5†	5†		4

*Bars to be divided equally on both sides of the web (refer to Fig. E5.12-sections)

†Section reinforced with No. 6 bars

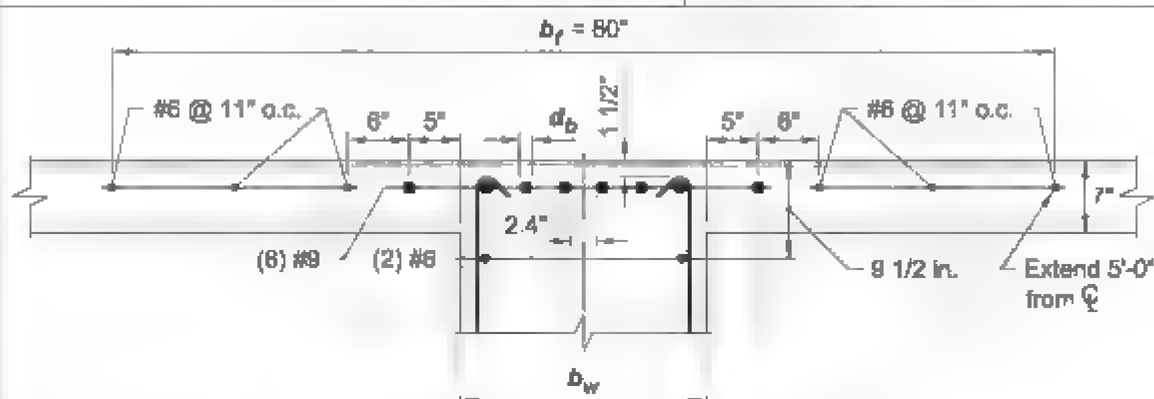


Fig. E5.12—Longitudinal bars distributed with flange

Note: Slab shrinkage and temperature reinforcement can be used as the additional bonded reinforcement in the outer portion of flange to satisfy ACI 318, Section 24.3.4

Step 8: Development length

Development length of No. 6, No. 9, and No. 11 bar

The simplified method is used to calculate the development length of No. 8, No. 9, and No. 11 bars:

25.4.2.3

$$\ell_d = \left(\frac{f_y\psi_t\psi_e\psi_g}{20\lambda\sqrt{f_c'}}\right)d_b$$

$$\ell_d = \left(\frac{(60{,}000\ \text{psi})(1.0)(1.0)(1.0)}{(20)(1.0)\sqrt{5000\ \text{psi}}}\right)(d_b) = 42.43d_b$$

25.4.2.5 where ψ_t is the cast position, $\psi_t = 1.3$, if more than 12 in. of fresh concrete is placed below top horizontal bars, and $\psi_t = 1.0$, if not more than 12 in. of fresh concrete is placed below bottom horizontal bars.

Top: $1.3d_b = 1.3(42.43\ d_b) = 55\ d_b$

Conservatively use the simplified equation for No. 7 and larger bars for the No. 6 development length

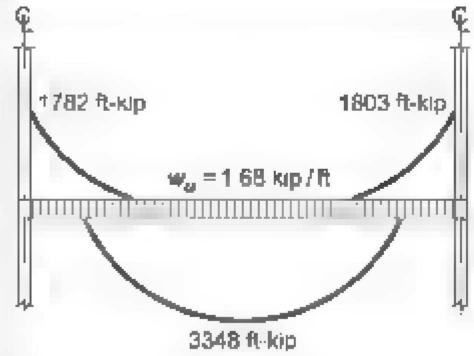
ψ_e is the coating factor; and $\psi_e = 1.0$, because bars are uncoated

ψ_g = reinforcement grade factor; $\psi_g = 1.0$ for Grade 60 reinforcement

		No. 6	No. 9	No. 8	No. 11
Top	ℓ_d, in.	41.3	62	55	
	Use ℓ_d, in.	42	63	57	
Bottom	ℓ_d, in.	31.8			59.8
	Use ℓ_d, in.	36			60

Step 9: Inflection points

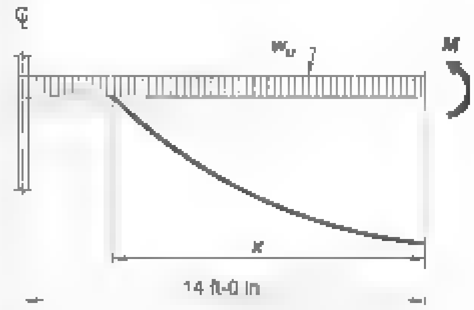
The moment diagram inflection points are calculated at both supports and at midspan (Fig. E5.13a).



(a) Girder moment diagram

Bottom bar length along girder

Calculate the inflection point for positive moment (Fig. E5.13b).



Maximum moment at midspan

3348 ft-kip

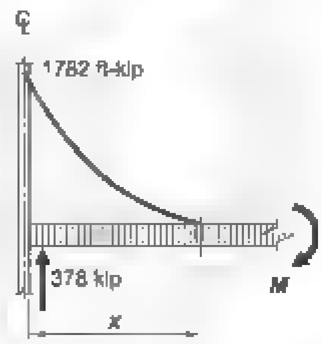
(b) Inflection point of positive moment

3348 ft-kip − (1.68 kip/ft)$(x)^2/2$ − 355x = 0

x = 9.2 ft, say, 9 ft 3 in.

Assume the maximum positive moment occurs at midspan. From equilibrium, the point of inflection is obtained from the following free body diagram:

$M_{max} - w_u(x)^2/2 - P/2x = 0$



Top bar length along transfer span

Left support

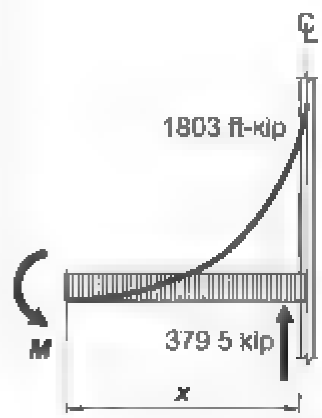
Calculate the inflection point for negative moment diagram (Fig. E5.13(c)).

$M_{max} - w_u(x)^2/2 + V_u x = 0$

(c) Inflection point at Support B

(−1782 ft-kip) − (1.68 kip/ft)$(x)^2/2$ + (378 kip)x = 0

x = 4.76 ft, say, 4 ft 10 in.



Right support

Calculate inflection point for the negative moment diagram (Fig. E5.13(d)).

$M_{max} - w_u(x)^2/2 + V_u x = 0$

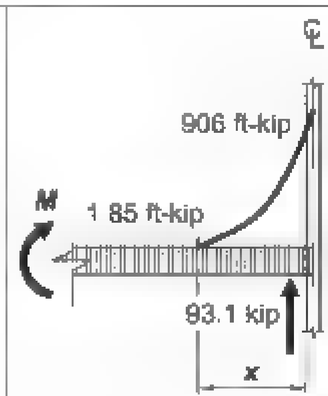
(d) Inflection point at Support D

(−1803 ft-kip) − (1.68 kip/ft)$(x)^2/2$ + (379.5 kip)x = 0

x = 4.8 ft, say, 4 ft 10 in.

Fig. E5.13—Girder inflection point locations.

Check the inflection point for Spans AB, DE, and EF



The moment diagram envelop shows that all three spans (AB, DE, and EF) do not have a defined maximum moment at midspan. The moment varies from maximum negative moment at one support and increases to maximum positive moment at the other support.

Span AB

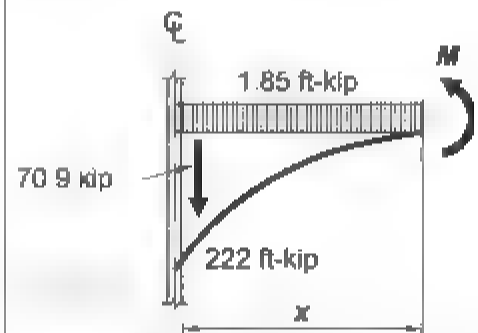
Calculate inflection point at Support B (Fig. E5.14a)

$M_{max} - w_u(x)^2/2 + V_u x = 0$

(a) Inflection point location at Support B

(−906 ft-kip) − (1.85 kip/ft)$(x)^2/2$ + (93.1 kip)$x = 0$

$x = 10.9$ ft, say, 11 ft 0 in.



Calculate inflection point at Support A (Fig. E5.14b)

$M_{max} - w_u(x)^2/2 + V_u x = 0$

(b) Inflection point location at Support A

(222 ft-kip) − 1.85 kip/ft$(x)^2/2$ − (70.9 kip)$x = 0$

$x = 3.0$ ft

Fig. E5.14—Beam AB inflection point locations.

Following the same concept for spans three and four the inflection points are calculated at:

Span	*x*-left	*x*-right
DE	10.3 ft, say, 10 ft 6 in.	3.6 ft, say, 3 ft 9 in.
EF	12.2 ft, say, 12 ft 3 in.	2.44 ft, say, 2 ft 6 in.

Step 10: Cutoff locations

Transfer girder

Support

9.7.3.2
9.7.3.3 Bars must be developed at locations of maximum stress and locations along the span where bent or terminated tension bars are no longer required to resist flexure.

Eight No. 9 bars and four No. 6 and two No. 8 skin bars are required to resist the factored moment at Column Lines B and D.

The two end moments at the supports are close, so the calculation will be applied at one end only. Calculate a distance x from the column face where four No. 9 bars can be discontinued and the contribution from the skin reinforcement is ignored.

Note: Skin reinforcement are extended over the full girder length and are properly developed or extended into the adjacent spans at the supports.

Cutoff point:

$$-1803\text{ ft-kip} - 1.69\text{ kip/ft}\frac{x^2}{2} + 379.5\text{ kip}(x) = -4(1.0\text{ in.}^2)$$

$$(0.9)(60\text{ ksi})\left(45\text{ in.} - \frac{4(1.0\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(24\text{ in.})}\right)$$

x = 2.69 ft, say, 2 ft 9 in. from column face

For No. 9 bars:
1) d = 45 in. **Controls**
2) $12d_b$ = 12(1.128 in.) = 13.5 in.

Therefore, extend the four No. 9 bars the greater of the development length (63 in., Step 8) from the column face and d from theoretical cutoff point (33 in.):
33 in. + 45 in. = 78.3 in. > ℓ_d = 63 in.

Therefore, 78.3 in. **Controls**

Four No. 9 bars can be terminated 80 in. from the face of the column, shown bold in Fig. E5.15.

9.7.3.8.4 At least one-third of the bars resisting negative moment at a support must have an embedment length beyond the inflection point the greatest of d, $12d_b$, and $\ell_n/16$.

For No. 9 bars:
1) d = 45 in. **Controls**
2) $12d_b$ = 12(1.128 in.) = 13.5 in.
3) $\ell_n/16$ = (28 ft – 2 ft)/16 = 1.625 ft = 19.5 in.

Extend the remaining four No. 9 bars the larger of the development length (63 in.) beyond the theoretical cutoff point (33 in.) and d = 45 in. beyond the inflection point (4 ft 10 in.) (Step 9) shown bold in Fig. E5.15).

63 in. + 33 in. = 96 in.
58 in. + 45 in. = 103 in. **Controls**
Extend the remaining four No. 9 bars 8 ft 8 in. from the face of the column.

The four No. 9 bars will be, however, extended over the full length of the girder.

	Transfer girder bottom bars	
9.7.3.2 9.7.3.3	Following the same steps above, ten No. 11 bars and four No. 8 skin bars are required to resist the factored moment at midspan.	
	Calculate a distance x from the midspan where four No. 11 bars can resist the factored moment Note: Skin bars are extended over full girder length.	$(3348\text{ ft-kip}) - (1.68\text{ kip/ft})\dfrac{x^2}{2} - (355\text{ kip})x = 4(1.56\text{ in.}^2) \times (0.9)(60\text{ ksi})\left(43.7\text{ in.} - \dfrac{4(1.56\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(102\text{ in.})}\right)$ $x = 5.92$ ft, say, 6 ft 0 in. from midspan Therefore, extend six No. 11 bars the larger of the development length (60 in. – Step 8) and a distance d beyond the theoretical cutoff point (6 ft 0 in. **Controls**) 72 in. + 43.7 in. = 115.7 in., say, 9.75 ft from maximum positive moment at midspan (Fig. E5.15). Extend the remaining four No. 11 bars at least the longer of 6 in. into the column or ℓ_d = 60 in. past the theoretical cutoff point (Fig. E5.15). 60 in. + 72 in. = 132 in. < (13 ft)(12) + 6 in. = 162 in. The 6 in. into the column controls, however, it is recommended to extend bars to the far face of the column and develop them.
9.7.3.8.2	At least one-fourth of the positive tension bars must extend into the column at least 6 in.	4 bars > 1/4(10 bars) = 2.5 bars **OK**

Beams

9.7.3.8.3	At the point of inflection, d_b for positive moment tension bars must be limited such that ℓ_d for that bar size satisfies.	Point of inflection occurs at 26 ft/2 − 9.25 ft = 3.75 ft from the column face V_u = 379.5 kip − (1.68 kip/ft)(3.75 ft) = 373.2 kip At that location, four No. 11 bars are effective $M_n = 4(1.56\text{ in.}^2)(60\text{ ksi})\left(43.7\text{ in.} - \dfrac{4(1.56\text{ in.}^2)(60\text{ ksi})}{(2)(0.85)(5\text{ ksi})(24\text{ in.})}\right)$ M_n = 15,674 in.-kip
	$\ell_d \le \dfrac{M_n}{V_u} + \ell_a$	$\ell_d \le \dfrac{15{,}674\text{ in.-kip}}{373.2\text{ kip}} + 43.7\text{ in.} = 85.7\text{ in.}$
	where M_n is calculated assuming all bars at the section are stressed to f_y. V_u is calculated at the section. The term l_a is the embedment length beyond the point of inflection, limited to the greater of d and $12d_b$.	This length exceeds ℓ_d = 60 in., therefore, **OK**
9.7.3.5	If bars are cut off in regions of flexural tension, then a bar stress discontinuity occurs. Therefore, the code requires that flexural tensile bars must not be terminated in a tensile zone unless (a), (b), or (c) is satisfied (a) $V_u \le (2/3)\phi V_n$ at the cutoff point Continuing bars provides double the area required for flexure at the cutoff point and (b) $V_u \le (3/4)\phi V_n$ (c) Stirrup or hoop area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance 3/4d from the termination point. Excess stirrup or hoop area shall be at least $60b_ws/f_{yt}$. Spacing s shall not exceed $d/(8\beta_b)$.	$V_n = V_c + V_s$ = 153.8 + 344.5 = 498.3 kip 2/3 ϕV_n = 249.2 kip Conditions (a) and (b) are not satisfied (c) Therefore, over a distance of 3/4(45.3 in.) = 34 in. from the end of the terminated bars space stirrups at 45.3 in./(8(8/12)) = 8.5 in. on center and excess stirrup area must be at least $A_{v,excess}$ = 60(24 in.)(8 in.)/60,000 psi = 0.192 in.^{2} Calculated required stirrup area at this location.
	$V_{u@7ft}$ = 379.5 kip − (1.68 kip/ft)(7 ft) = 367.7 kip	$367.7\text{ kip} - 115.3\text{ kip} = \dfrac{(0.75)A_{v,req'd}(60\text{ ksi})(45.3\text{ in.})}{8\text{ in.}}$
	$\Delta A_{v,excess} = A_{v,prov} - A_{v,req'd}$	$A_{v,req'd}$ = 1.0 in.^{2} $\Delta A_{v,excess}$ = 4(0.31 in.^{2}) − 1.0 in.^{2} = 0.24 in.^{2} 0.24 in.^{2} > 0.192 in.^{2}, therefore, **OK** Because only one of the three conditions needs to be satisfied, this requirement is satisfied

	<u>Integrity reinforcement</u>	
9.7.7.2	At least one-fourth of the maximum positive moment bars, but at least two bars, must be continuous and developed at the face of the column.	This condition was satisfied above by extending four No. 11 bars into the support 4 bars/10 bars = 2.5 > 1/4 **OK**
9.7.7.3	Beam longitudinal bars must be enclosed by closed stirrups along the clear span.	This condition is satisfied by extending stirrups at 8 in. on center over the full length of the beam.
	Beam structural integrity bars shall pass through the region bounded by the longitudinal column bars.	Four No. 11 bars are extended through the column longitudinal reinforcement, therefore satisfying this condition. Note: The girder has the same width as the columns' dimensions (24 in.). Therefore, beam longitudinal reinforcement must be offset to clear column reinforcement.

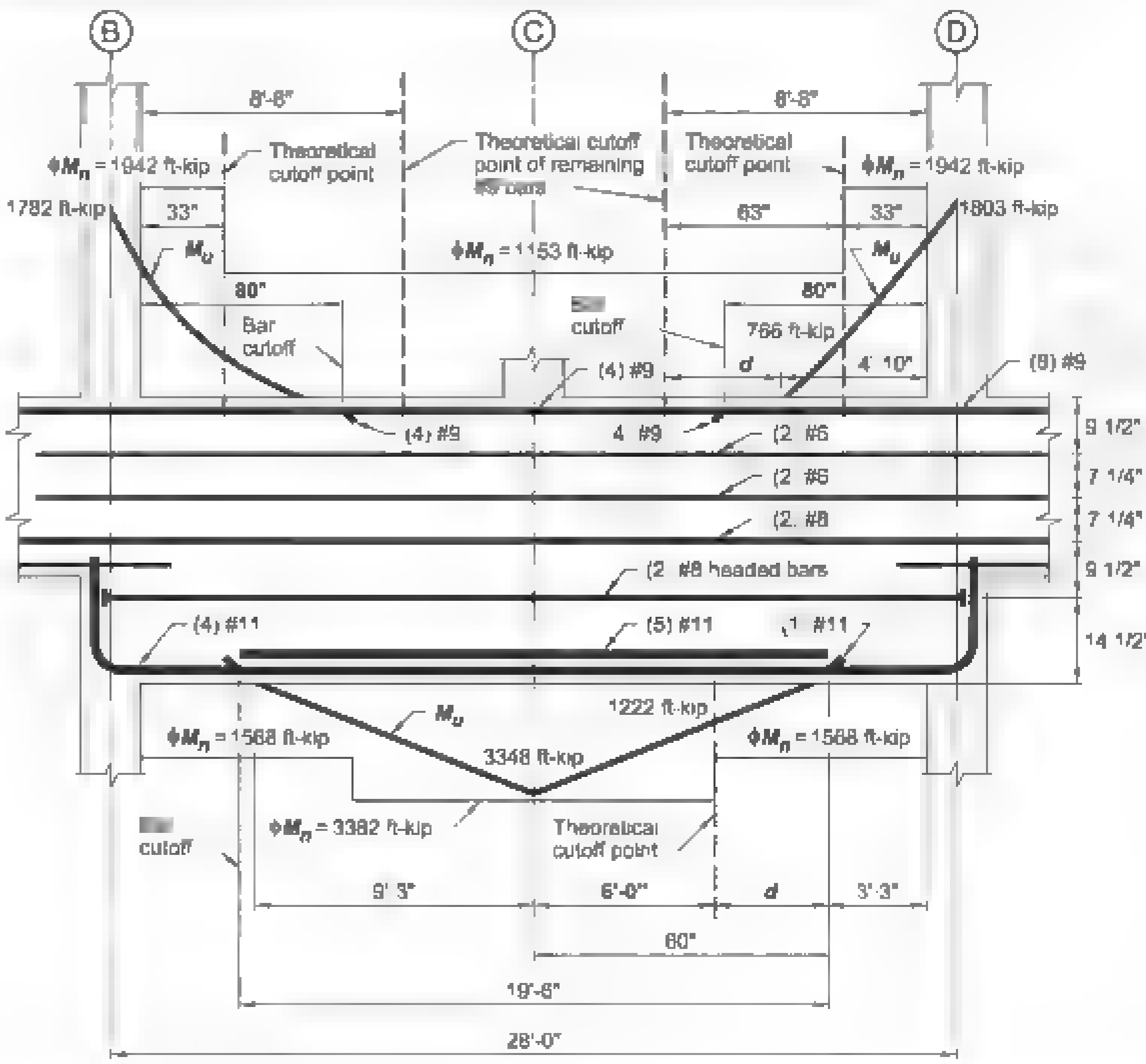


Fig. E5.15—Girder bar cutoff locations.

Step 11. Beams		
	Spans AB and DE	
9.7.3.2	Beams spanning between column A and B and between D and E are subjected to comparable factored flexure and shear forces. Therefore, the two beams will be designed for the same loads (larger of the two beams) To simplify detailing, simply extend eight No. 9 bars and two No. 6 skin bars (span DE only) from the transfer girders to resist the negative moment in the adjacent beams (Step 5). Calculate a distance x from the column face where four No. 9 bars can resist the factored moment	$-(906\text{ ft-kip}) - (1.85\text{ kip/ft})\dfrac{x^2}{2} + 93.1\text{ kip}(x) = -4(1.0\text{ in.}^2)\dfrac{(0.9)(60\text{ ksi})}{12\text{ in./ft}}\left(27.4\text{in.} - \dfrac{4(1.0\text{ in.}^2)(60\text{ ksi})}{2(0.85)(5\text{ ksi})(24\text{ in.})}\right)$ $x = 4.9$ ft, say, 5 ft At 5 ft 0 in. in. from the column face, four No. 9 can be cut off and the remainder four No. 9 bars can resist the factored moment
9.7.3.3	The four No. 9 cutoff bars must extend beyond the location where they are no longer required to resist flexure for a distance equal to the greater of d or $12d_b$.	For No. 9 bars: 1) $d = 27.4$ in. **Controls** 2) $12d_b = 12(1.128\text{ in.}) = 13.54$ in.
9.7.3.8.4	At least one-third of the bars resisting negative moment at a support must have an embedment length beyond the inflection point the greatest of d, $12d_b$, and $\ell_n/16$	Therefore, extend four No. 9 bars the greater of the development length (63 in. – Step 8) from the column face and the sum of theoretical cutoff point and d: 60 in. + 27.4 in. = 87.4 in. > 63 in. **Controls** Say, 90 in. or 7 ft 6 in. For No. 9 bars: (a) $d = 27.4$ in. **Controls** (b) $12d_b = 12(1.128\text{ in.}) = 13.54$ in. (c) $\ell_n/16 = (28\text{ ft} - 2\text{ ft})/16 = 1.625\text{ ft} = 19.5$ in. Extend the remainder four No. 9 bars, the greater of the development length (63 in.) beyond the theoretical cutoff point (5 ft) and $d = 27.4$ in. beyond the inflection point (11 ft 0 in.), $\ell = 63\text{ in.}/12 + 5\text{ ft} = 10.25$ ft $\ell = 11.0\text{ ft} + 27.4\text{ in.}/12 = 13.3$ ft **Controls** Therefore, extend the remainder bars over the full length of the beam, refer to Fig. E5.16 Spans AB and DE.
	At the Support E, where there is no negative moment, provide minimum reinforcement of six No. 6 bars extending minimum the development length (42 in.) on both sides of the column. Of the six No. 6 bars, extend two bars over the full length to support the stirrups (hanger bars), refer to Fig. E5.16	

<table>
<tr><td></td><td>For the positive moment region, five No. 6 bottom bars are extended over the full span length for Beams 1, 3, and 4.</td><td></td></tr>
<tr><td colspan="3">Step 12. Splicing and bar spacing</td></tr>
<tr><td>9.7.7.5</td><td>Splices are necessary for continuous bars. The bars shall be spliced in accordance with (a) and (b).
(a) Positive moment bars shall be spliced at or near the support
(b) Negative moment bars shall be spliced at or near midspan</td><td>Splice length = (1.3)(development length)
No. 11: ℓ_{dc} = 1.3(60 in.) = 78 in. = 6 ft 6 in.
No. 6: ℓ_{dc} = 1.3(36 in.) = 46.8 in., say, 48 in. = 4 ft 0 in.
No. 9: ℓ_{dc} = 1.3(63 in.) = 81.9 in., say, 84 in. = 7 ft 0 in.
No. 6: ℓ_{dc} = 1.3(42 in.) = 54.6 in., say, 57 in. = 4 ft 9 in.</td></tr>
<tr><td>9.7.2.2
24.3.1
24.3.2

24.3.2.1</td><td>Maximum bar spacing at the tension face must not exceed the lesser of
$$s = 15\left(\frac{40,000}{f_s}\right) - 2.5c_c$$
and
$s = 12(40,000/f_s)$
where $f_s = 2/3\,f_y = 40,000$ psi

This limit is intended to control flexural cracking width. Note that c_c is the cover to the longitudinal bars, not to the tie.</td><td>$$s = 15\left(\frac{40,000\text{ psi}}{40,000\text{ psi}}\right) - 2.5(2\text{ in.}) = 10\text{ in.}$$ **Controls**
$$s = 12\left(\frac{40,000\text{ psi}}{40,000\text{ psi}}\right) = 12\text{ in.}$$
<table><tr><th></th><th>No. 6</th><th>No. 9</th><th>No. 11</th></tr><tr><td>Spacing, in.</td><td>3.75</td><td>2.4</td><td>3</td></tr></table>
All longitudinal bar spacing satisfy the maximum bar spacing requirement, therefore, **OK**</td></tr>
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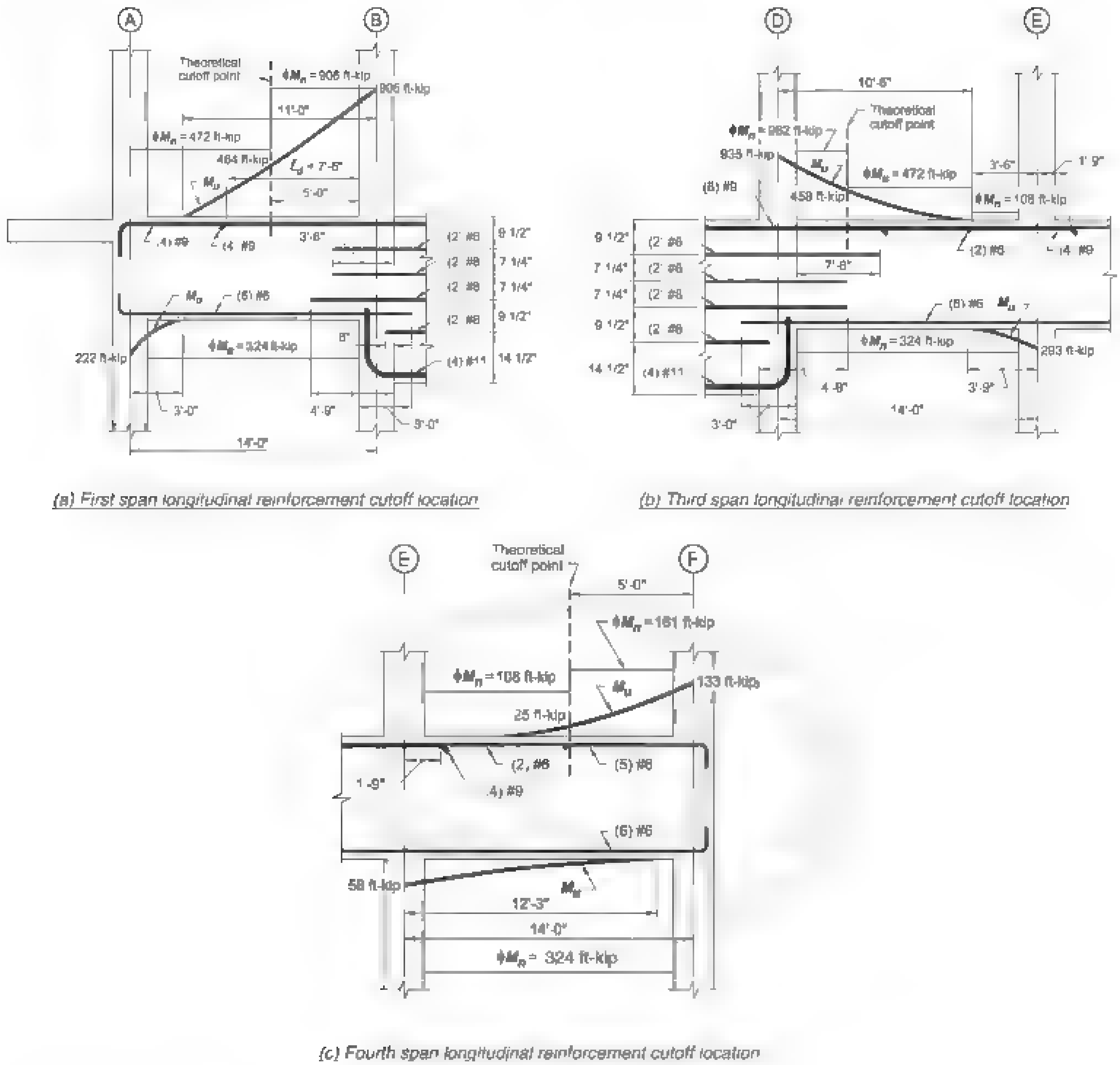


(a) First span longitudinal reinforcement cutoff location

(b) Third span longitudinal reinforcement cutoff location

(c) Fourth span longitudinal reinforcement cutoff location

Fig. E5.16—Longitudinal reinforcement cutoff locations

Step 13: Detailing

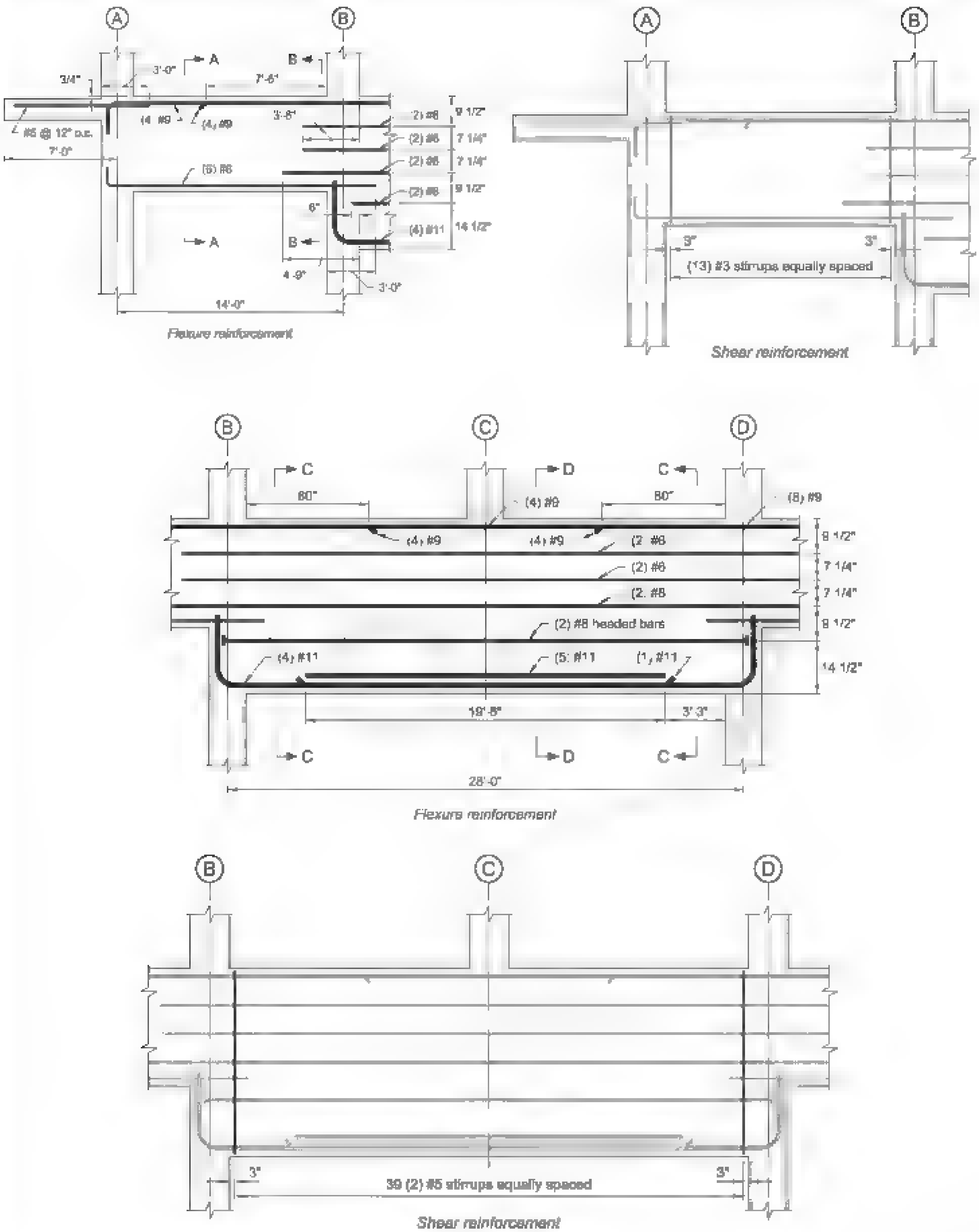
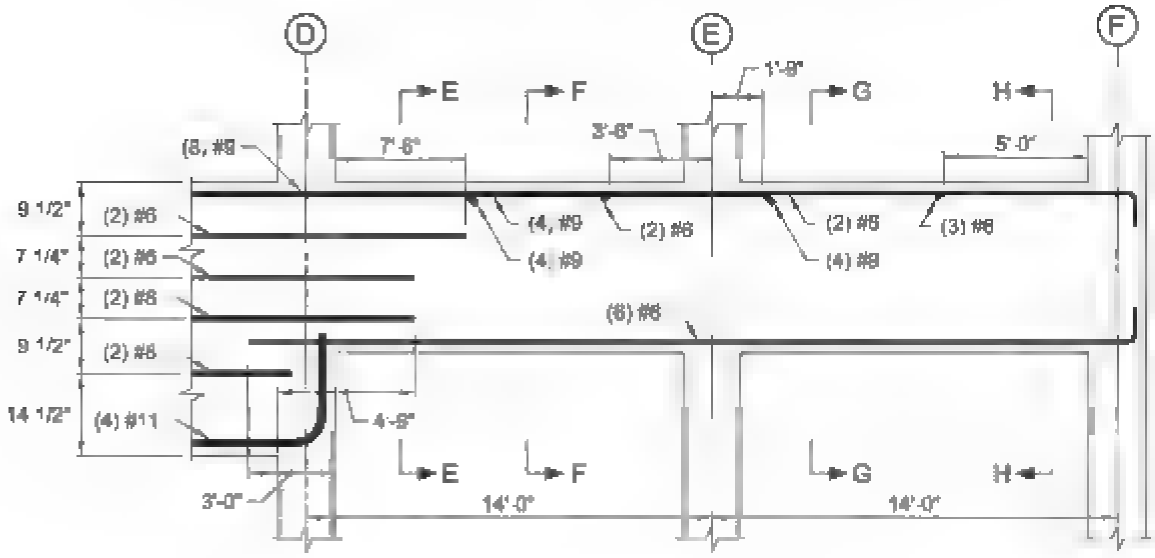
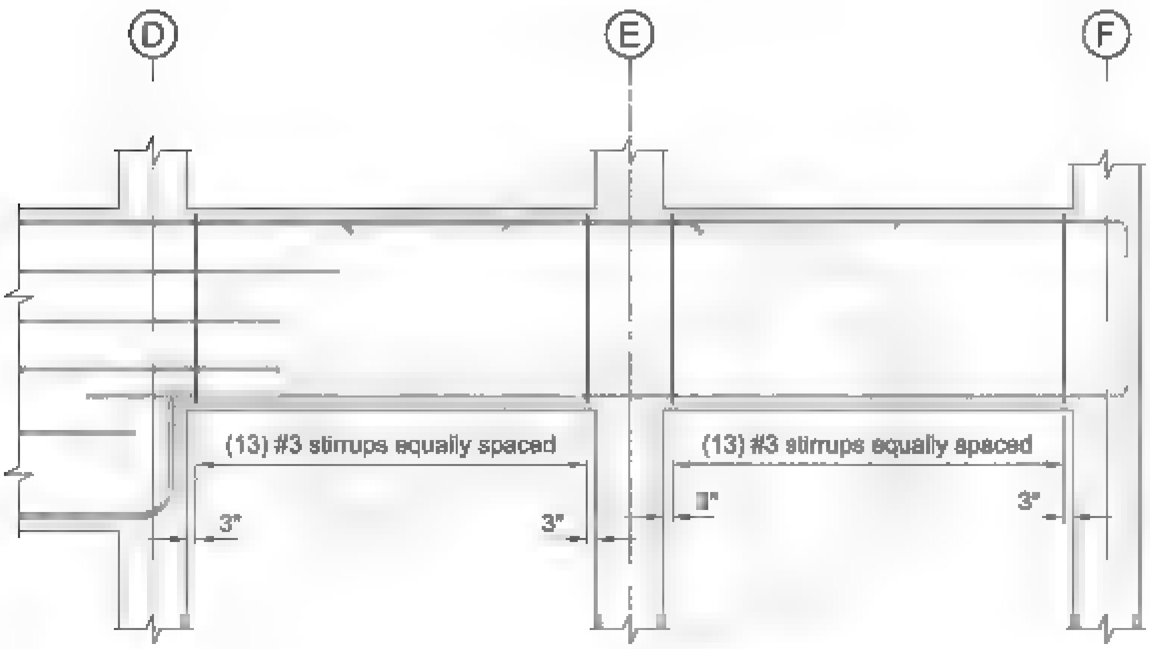


Fig. E5.17—Beam bar details. (Note: Figure continued on next page.)



Flexure reinforcement



Shear reinforcement

Fig. E5.17(cont.)—Beam bar details

Notes.
1. Place first stirrup at 3 in. from the column face.
2. The contractor may prefer to extend two No. 7 top reinforcement over the full beam length to replace the two No. 5 hanger beams. Bars should be spliced at mid-length.

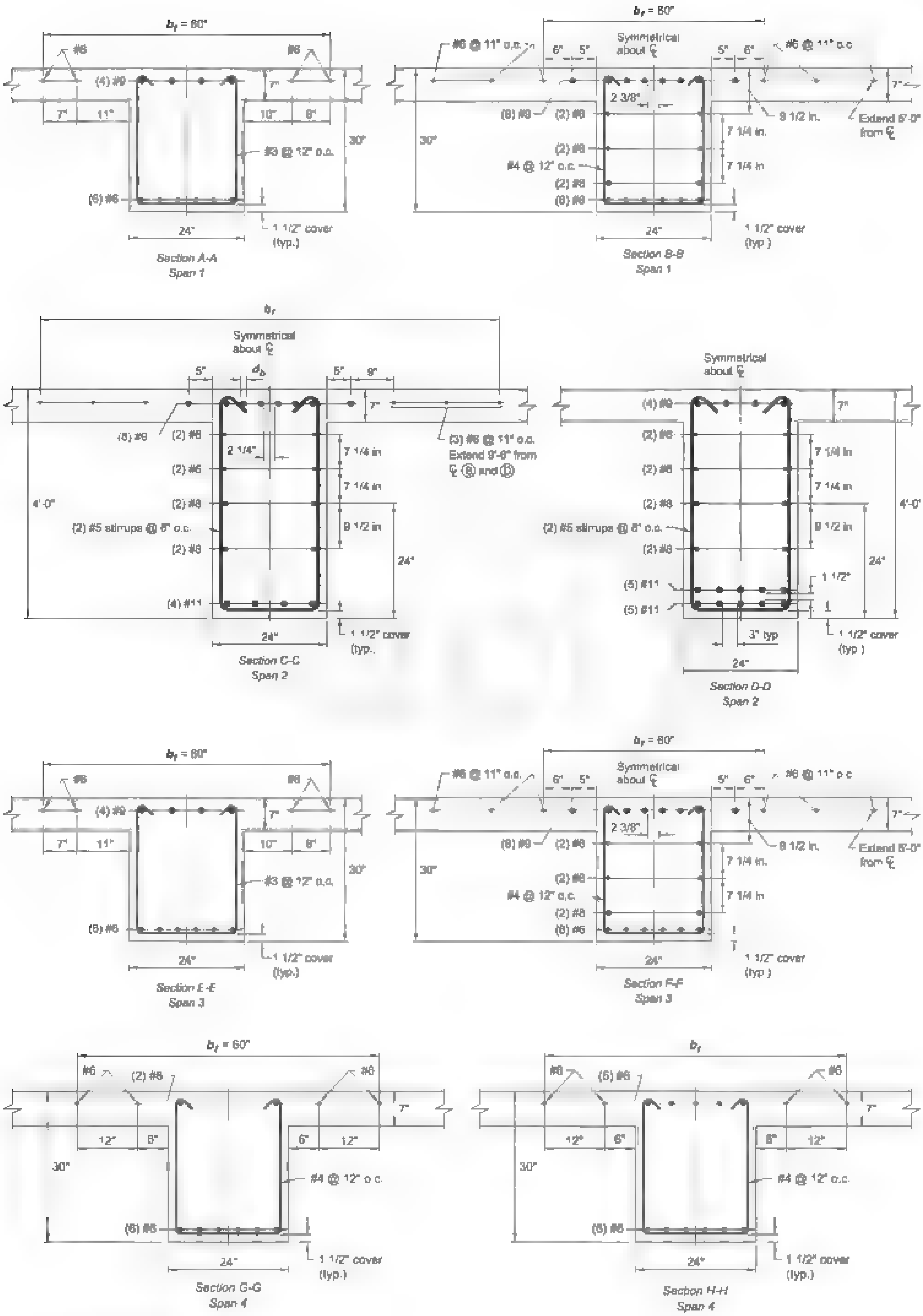


Fig. E5.18—Sections

Beam Example 6: *Post-tensioned transfer girder*

Design and detail an interior, post-tensioned, transfer girder supporting five stories and built integrally with a 7 in. slab. Girder tendons will be stressed when concrete compressive strength reaches the specified f_{ci}' = 4000 psi. Assume the tendon center of gravity at beam midspan is 4 in. from the bottom and at the beam's center of gravity at the column. Tendon is composed of 1/2 in. diameter individually coated and sheathed seven-wire prestressing strands.

Given:

Load—
Service additional dead load D = 15 psf
Service roof live load LR = 35 psf
Service floor live load L = 65 psf
Girder, beam and slab self weights are given below

Material properties—
f_c' = 5000 psi (normalweight concrete)
f_{ci} = 4000 psi
f_y = 60,000 psi
f_{pu} = 270,000 psi
λ = 1.0 (normalweight concrete)

Span length—
Girder: 28 ft
Beam and girder width: 24 in.
Column dimensions: 24 in. x 24 in.

Area of 1/2 in. diameter strand = 0.153 in.^{2}

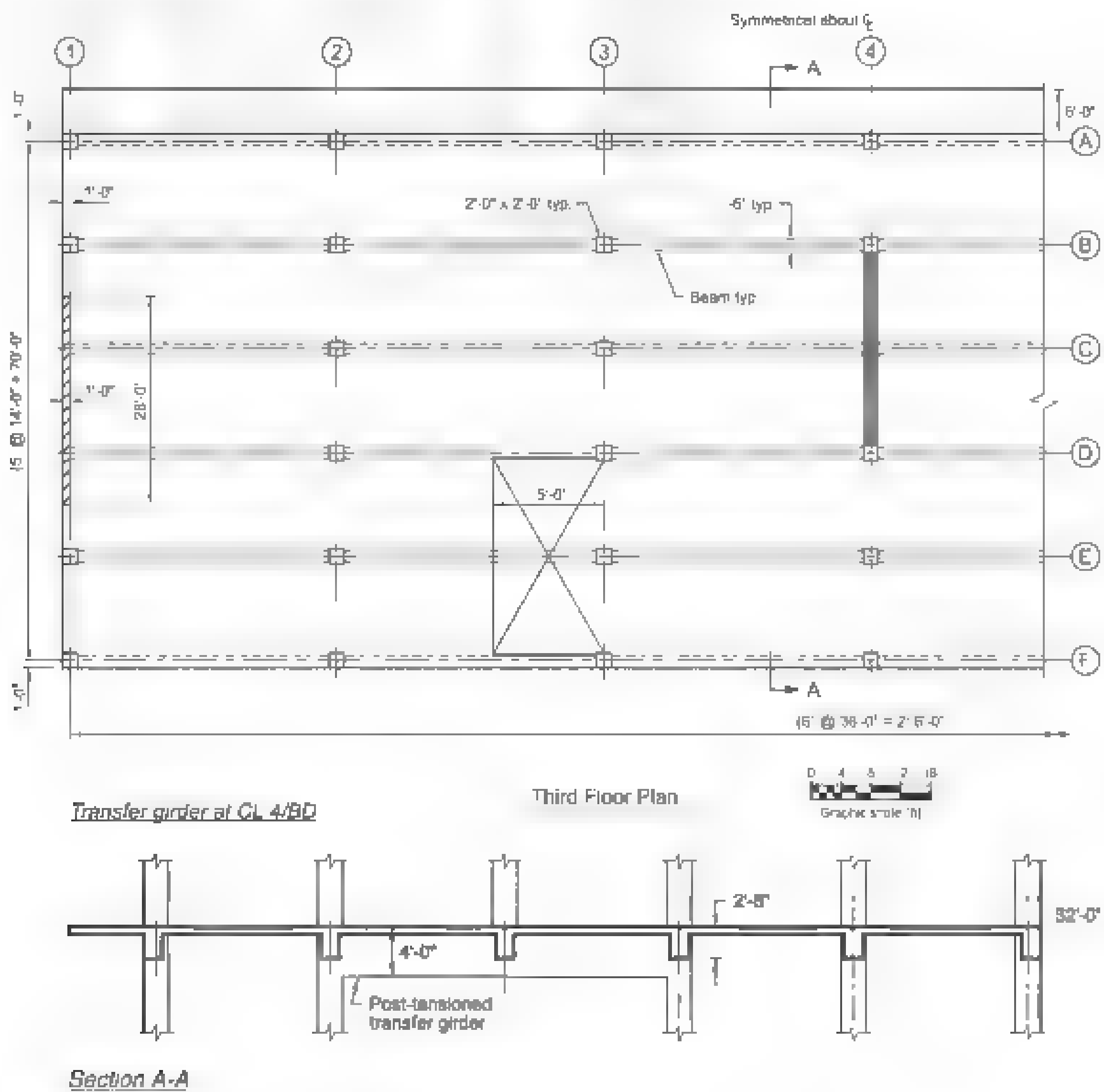


Fig. E6.1—Plan and partial elevation of third level transfer girder and beams.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
9.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 (ACI 318) and structural strength requirements. The designer determines the durability classes. Please refer to Chapter 2 of MNL-17 for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi. The engineer must specify the transfer stress—in this case, 4000 psi is selected. Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26.
Step 2: Beam geometry		
9.3.1.1	Girder depth The transfer girder supports a column at midspan. The column load includes self-weight and its tributary loads from the third level, the four stories above it, and the roof. Because of this large concentrated load, the depth limits in Table 9.3.1.1 cannot be used, and calculated deflections must satisfy the deflection limits in 9.3.2. Deflections were checked using software.	Assume 48 in. deep transfer girder.
9.2.4.2 6.2.3.1	Flange width The transfer girder is poured monolithically with the slab and will behave as a T-beam.	
R6.3.2.3	It is allowed per ACI 318 comment to ignore the flange width requirements based on experience and past performances. Determination of an effective flange width for prestressed T-beams is therefore left to the experience and judgment of the licensed design professional.	
6.3.2.1	Use $b_f = 8h + b_w + 8h$	b_f = (8)(7 in.) + 24 in. + (8)(7 in.) = 136 in.

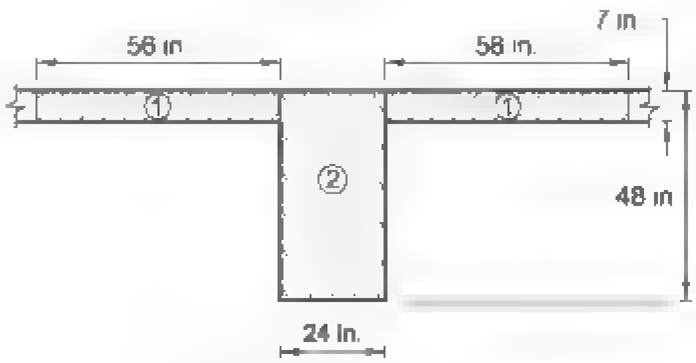


Fig. E6.2—Transfer girder geometry.

	A, in.^{2}	y, in.	Ay, in.^{3}	$A(y - y_t)^2$, in.^{4}	I_o, in.^{4}	$\sum I_o$, in.^{4}
1	(2)(56 in.)(7 in.) = 784	3.5	2744	116,690	3201	119,891
2	(48 in.)(24 in.) = 1152	24	27,648	79,361	221,184	300,545
$\sum$	1936		30,392	196,051	224,385	420,436

Center of gravity: $y_t = \dfrac{30{,}392\text{ in.}^3}{1936\text{ in.}^2} = 15.7\text{ in.}$ from top of girder

$y_b = 48\text{ in.} - 15.7\text{ in.} = 32.3\text{ in.}$ from bottom of girder

Section modulus: $S_{Top} = \dfrac{420{,}436\text{ in.}^4}{15.7\text{ in.}} = 26{,}779\text{ in.}^3$

$$S_{Bottom} = \frac{420{,}436\text{ in.}^4}{32.3\text{ in.}} = 13{,}017\text{ in.}^3$$

Beams

Step 3: Loads	
<u>Applied load on transfer girder</u> The service live load is 50 psf in offices and 80 psf in corridors per Table 4-1 in ASCE/SEI 7. This example will use 65 psf as an average live load, as the actual layout is not provided. To account for the weight of ceilings, partitions, and HVAC systems, add 15 psf as miscellaneous dead load	
<u>Dead load:</u> Transfer girder self-weight without flanges	$w_b = [(24\text{ in.})(48\text{ in.})][(0.150\text{ kip/ft}^3)/144]$ $= 1.2\text{ kip/ft}$
Slab weight per level:	$P_{sl} = (7\text{ in.}/12)(14\text{ ft})(36\text{ ft})(0.15\text{ kip/ft}^3) = 44.1\text{ kip}$
Column weight per level	$P_{col} = (2\text{ ft})(2\text{ ft})\left(12\text{ ft} - \dfrac{7\text{ in.}}{12}\right)(0.15\text{ kip/ft}^3) = 6.85\text{ kip}$
Typical beam weight (refer to plan) 18 in. × (30 in. − 7 in.) × (36 ft − 2 ft)	$P_{bm} = \left(\dfrac{18\text{ in.}}{12}\right)\left(\dfrac{23\text{ in.}}{12}\right)(34\text{ ft})(0.15\text{ kip/ft}^3) = 14.7\text{ kip}$
Miscellaneous dead load per level	$P_{SDL} = (14\text{ ft})(36\text{ ft})(0.015\text{ kip/ft}^2) = 7.6\text{ kip}$
Total dead load applied on girder:	$P_D = (44.1\text{ kip} + 14.7\text{ kip} + 7.6\text{ kip})(6) + (6.85\text{ kip})(5)$ $= 433\text{ kip}$
<u>Live load:</u> Roof live load: 35 psf	$P_{L,Roof} = (14\text{ ft})(36\text{ ft})(0.035\text{ kip/ft}^2) = 18\text{ kip}$
Total live load—per ASCE 7 live load with exception of roof load is permitted to be reduced by:	
$L = L_o\left(0.25 + \dfrac{15}{\sqrt{K_{LL}A_T}}\right)$	Fourth to seventh reduced live load per level: $L = (0.065\text{ kip/ft}^2)\left(0.25 + \dfrac{15}{\sqrt{2016\text{ ft}^2}}\right) = 0.038\text{ ksf}$ $L = 0.038\text{ ksf} > 0.4L_o = 0.026\text{ ksf}$ **OK**
where L is reduced live load, L_o is unreduced live load, K_{LL} is live load element factor = 4 for interior columns and 2 for interior beam (ASCE 7 Table 4-2) A_T is tributary area and $K_{LL}A_T > 400\text{ ft}^2$ $4(36\text{ ft})(14\text{ ft}) = 2016\text{ ft}^2 > 400\text{ ft}^2$ $2(36\text{ ft})(14\text{ ft}) = 1008\text{ ft}^2 > 400\text{ ft}^2$ and $L > 0.4L_o = 26\text{ psf}$	Reduced third level live load on beam $L = (0.065\text{ kip/ft}^2)\left(0.25 + \dfrac{15}{\sqrt{1008\text{ ft}^2}}\right) = 0.047\text{ ksf}$ $L = 0.047\text{ ksf} > 0.4L_o = 0.026\text{ ksf}$ **OK** Concentrated live load to column per level $P_L = (14\text{ ft})(36\text{ ft})(0.038\text{ kip/ft}^2) = 19.2\text{ kip}$ Concentrated live load at third level $P_{L@3rdLevel} = (14\text{ ft})(36\text{ ft})(0.047\text{ kip/ft}^2) = 23.7\text{ kip}$
Total live load applied on girder (four levels)	$\sum L = (19.2\text{ kip})(4) + (23.7\text{ kip}) + (18\text{ kip}) = 119\text{ kip}$

Step 4. Material properties

20.3.1.1 20.3.2.2	Post tensioned strands: ASTM A416	f_{pu} 270,000 psi
20.3.2.5.1	Stress in tendon immediately after force transfer:	$0.7f_{pu}$ = 189,000 psi
20.3.2.6.	Considering lump sum losses as an estimate (ACI 423.10R-16) use an effective prestress of	$0.65f_{pu}$ = 175,000 psi
R20.3.2.1	Modulus of elasticity is assumed for design and checked against test results after a supplier is contracted	
	For design use Concrete compressive strength Concrete strength at initial stressing	E_p = 28,500,000 psi f_c' = 5000 psi f_{ci}' = 4000 psi
24.5.2.1	Assume a maximum concrete flexure stress as $12\sqrt{f_c'}$, Class T, at service.	$7.5\sqrt{f_c'}$ = 530 psi < $f_t \leq 12\sqrt{f_c'}$ = 848 psi
	Concrete compressive and tensile stresses immediately after transfer	
24.5.3.1 24.5.3.2	The transfer girder will be reinforced with nonprestressed bonded reinforcement. Therefore, the $3\sqrt{f_{ci}'}$ can be exceeded. Use $7.5\sqrt{f_{ci}'}$	See table below
24.5.4.1	Concrete compressive stress limits at service loads	See table below

Location	Type	Stress limit, psi
A	Compression	0.60 = 2400
	Tension	$7.5\sqrt{f_{ci}'}$ = 474 psi

Load condition	Concrete compressive stress limits
Prestress plus sustained load	$0.45f_c'$ 2250 psi
Prestress plus total load	$0.60f_c'$ = 3000 psi

Step 5. Design assumptions

The girder is designed using unbonded single strand tendons, as it is the typical type of tendon used in building construction in the United States. Bonded tendons may be preferred in other parts of the world. The tendons center of gravity profile is shown in Fig. E6.3.

9.4.3 The beam is built integrally with end supports, therefore, the beam is analyzed as part of a frame. The factored moments and shear forces (required strengths) are calculated at the face of the supports. The moment and shear diagrams at different stages are obtained from PTData software.

The girder is stressed in two stages. At the first stressing stage, few tendons are stressed after concrete reaches a concrete compressive strength of minimum 4000 psi and subjected only to its self-weight. Before stressing the remaining tendons, the girder will be supporting three levels dead load only. At the second stage, the remaining tendons are stressed and construction of the upper floors continues.

Step 6. Post-tensioning design		
	<u>Find the total number of strands required to resist the total load</u> The required prestress force to support the total dead and live load is calculated at service load condition to satisfy the limit in the selected Class in Table 24.5.2.1 The service load is 433 kip concentrated dead load and 119 kip concentrated live load from all levels above the girder—refer to Step 3. Assume that approximately 75 percent of the dead load is balanced by the harped post-tensioned tendons	$0.75(433\text{ kip}) = 324.8\text{ kip}$, say, 325 kip $325\text{ kip} = 2F\sin\theta$
	This gives a tendon eccentricity of 28.3 in., refer to Fig. E6.3	$\theta = \tan^{-1}\left(\dfrac{28.3\text{ in.}}{(13\text{ ft})(12\text{ in./ft})}\right) = 10.3\text{ degrees}$ $F = \dfrac{325\text{ kip}}{2\sin 10.3} = 908\text{ kip}$
	Calculate number of strands in the tendon. $f_{pe} = 0.65f_{pu} = 175\text{ ksi}$ (Step 4)	Required tendon area = (908 kip)/(175 ksi) = 5.19 in.^{2} Number of strands = 5.19 in.^{2}/(0.153 in.^{2}) = 33.9 Say, 34 strands. Therefore, the design force is $F = (34)(0.153\text{ in.}^2)(175\text{ ksi}) = 910\text{ kip} > 908\text{ kip}$ **OK**
R20.3.2.6.2	Note: ACI 318 requires a detailed check of losses. Estimation of friction losses in post-tensioned tendons is addressed in PTI TAB 1-06. The lump-sum losses were used herein to determine effective prestress force. ACI 423.3R can be used to calculate refined time-dependent losses	

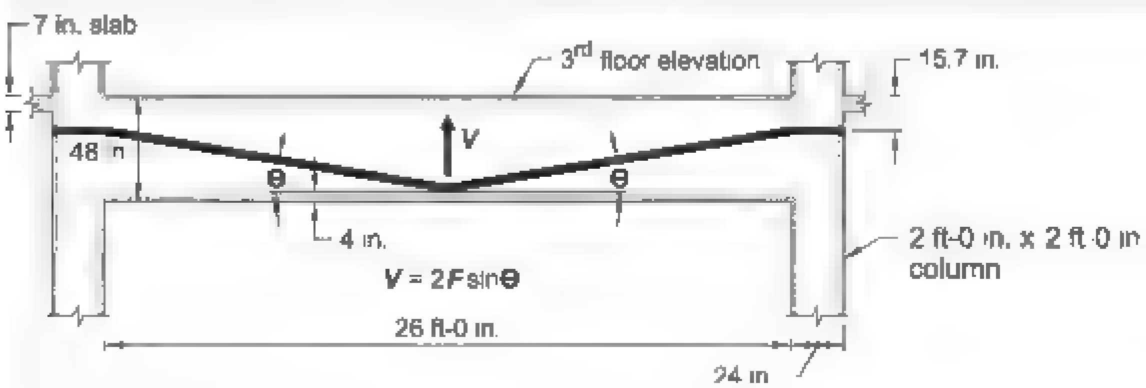


Fig. E6.3—Tendon profile.

	Moments at support and midspan for dead load, live load, and post-tensioning are obtained from PTData, Fig. E6.4, 6.5, and 6.6, respectively	

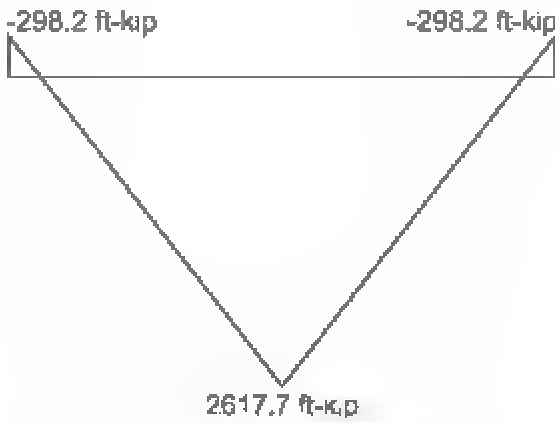


Fig. E6.4—Service dead load moment diagram

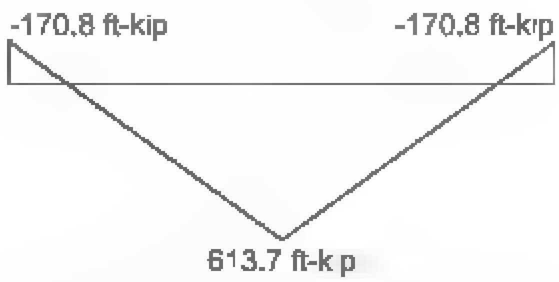


Fig. E6.5—Service live load moment diagram.

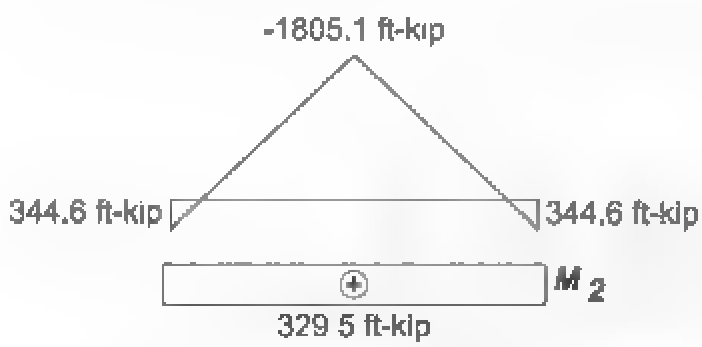


Fig. E6.6—Post-tensioning moment diagram

Stress calculations.
Note: for sign convention
Compression (−)
Tension (+)

Note: When comparing compression stresses, absolute values are used.

At transfer.
Assume zero dead load present and all 34 diameter strands are stressed. Stresses immediately following prestress transfer are checked here. To conservatively account for time-dependent losses that have not yet occurred, apply a factor of 7/6 to the effective prestress force determined previously.

Moment due to balance load at midspan

$$M_{bal} = \frac{7}{6}(1805 \text{ ft-kip}) = 2106 \text{ ft-kip}$$

$$f_{top} = -\frac{(7/6)(910 \text{ kip})}{1936 \text{ in.}^2} + \frac{(2106 \text{ ft-kip})(12)}{26,779 \text{ in.}^3} = 0.395 \text{ ksi}$$

Midspan stresses.

Top: $f_{top} = -\frac{P}{A} + \frac{M}{S_{top}}$

$f_{top} = 395$ psi < $f_{all} = 474$ psi **OK**

Check if actual stress is less than allowable stress (Step 4)

$$f_{bot} = -\frac{(7/6)(910 \text{ kip})}{1936 \text{ in.}^2} + \frac{(2106 \text{ ft-kip})(12)}{13,017 \text{ in.}^3} = 2.49 \text{ ksi}$$

Bottom: $f_{bot} = -\frac{P}{A} - \frac{M}{S_{bot}}$

$f_{bot} = 2490$ psi ≈ $f_{all} = 2400$ psi **OK**

Check if actual stress is less than allowable stress (Step 4)

This compressive stress will decrease as time-dependent losses occur

	Support stresses.	Moment due to balance at support $M_{bal} = \frac{7}{6}(-345 \text{ ft-kip}) = -403 \text{ ft-kip}$
	Top $f_{top} = -\frac{P}{A} - \frac{M}{S_{top}}$	$f_{top} = -\frac{(7/6)(910 \text{ kip})}{1936 \text{ in.}^2} - \frac{(-403 \text{ ft-kip})(12)}{26,779 \text{ in.}^3} = -0.729 \text{ ksi}$
	Check if actual stress is less than allowable stress (Step 4)	$f_{top} = 729 \text{ psi} < f_{all} = 2400 \text{ psi}$ **OK**
	Bottom: $f_{bot} = -\frac{P}{A} + \frac{M}{S_{bot}}$	$f_{bot} = -\frac{(7/6)(910 \text{ kip})}{1936 \text{ in.}^2} + \frac{(-403 \text{ ft-kip})(12)}{13,017 \text{ in.}^3} = -0.177 \text{ ksi}$
	Check if actual stress is less than allowable stress (Step 4)	$f_{bot} = 177 \text{ psi} < f_{all} = 474 \text{ psi}$ **OK**
	Conclusion. No stage stressing is required. Stress all tendons after girder beam concrete attained 4000 psi compressive strength.	
	At service:	
	Midspan $M_{TL} = M_D + M_L$	Service moment at midspan $M_{TL} = (2618 \text{ ft-kip}) + (614 \text{ ft-kip}) = 3232 \text{ ft-kip}$ $M_{Bal} = 1805 \text{ ft-kip}$
	$\Delta M = M_{TL} - M_{Bal}$	$\Delta M = (3232 \text{ ft-kip}) - (1805 \text{ ft-kip}) = 1427 \text{ ft-kip}$
	Top $f_{top} = -\frac{P}{A} - \frac{M}{S_{top}}$	$f_{top} = -\frac{910 \text{ kip}}{1936 \text{ in.}^2} - \frac{(1427 \text{ ft-kip})(12)}{26,779 \text{ in.}^3} = -1.109 \text{ ksi}$
	Check if actual stress is less than allowable stress (Step 4)	$f_{top} = 1109 \text{ psi} < f_{all} = 2250 \text{ psi}$ **OK**
	Bottom: $f_{bot} = -\frac{P}{A} + \frac{M}{S_{bot}}$	$f_{bot} = -\frac{910 \text{ kip}}{1936 \text{ in.}^2} + \frac{(1427 \text{ ft-kip})(12)}{13,017 \text{ in.}^3} = 0.845 \text{ ksi}$
	Check if actual stress is less than allowable stress Class T (Step 4).	$f_{bot} = 845 \text{ psi} < f_{all} = 848 \text{ psi}$ **OK**
	Support.	Service moment at support $M_{TL} = (-298 \text{ ft-kip}) - (171 \text{ ft-kip}) = -469 \text{ ft-kip}$ $M_{Bal} = +345 \text{ ft-kip}$
	$\Delta M = M_{TL} - M_{Bal}$	$\Delta M = (-469 \text{ ft-kip}) + (345 \text{ ft-kip}) = 124 \text{ ft-kip}$
	Top: $f_{top} = -\frac{P}{A} + \frac{M}{S_{top}}$	$f_{top} = -\frac{910 \text{ kip}}{1936 \text{ in.}^2} + \frac{(124 \text{ ft-kip})(12)}{26,779 \text{ in.}^3} = -0.414 \text{ ksi}$
	Check if actual stress is less than allowable stress (Step 4)	$f_{top} = 414 \text{ psi} < f_{all} = 2250 \text{ psi}$ **OK**
	Bottom: $f_{bot} = -\frac{P}{A} - \frac{M}{S_{bot}}$	$f_{bot} = -\frac{910 \text{ kip}}{1936 \text{ in.}^2} - \frac{(124 \text{ ft-kip})(12)}{13,017 \text{ in.}^3} = -0.548 \text{ ksi}$
	Check if actual stress is less than allowable stress Class T (Step 4).	$f_{bot} = 548 \text{ psi} < f_{all} = 2250 \text{ psi}$ **OK**

Conclusions and summary.

All service load stresses are acceptable.

Initial stressing

24.5.2.1
24.5.3.1
24.5.3.2
24.5.3.2.1
25.5.4.1

	Location	Stress, psi	Allowable stress, psi	Status
Support	Top	−729	−2400	OK
	Bottom	177	474	OK
Midspan	Top	395	474	OK
	Bottom	2490	2400	~OK

At service.

	Location	Stress, psi	Allowable stress, psi	Status
Support	Top	414	2250	OK
	Bottom	584	2250	OK
Midspan	Top	1109	2250	OK
	Bottom	845	848	OK

Step 7: Design strength

(a) Flexure

Factored loads

Shear and moment diagrams are obtained from PTData Fig. E6.7.

Moment at midspan

The beam resists gravity only and lateral forces are not considered in this problem.

From Moment diagrams, Fig. E6.6

5.3.1 $U = 1.4D$ — $M_u = 1.4(2618\text{ ft-kip}) = 3665\text{ ft-kip}$

$U = 1.2D + 1.6L$ — $M_u = 1.2(2618\text{ ft-kip}) + 1.6(614\text{ ft-kip}) = 4123\text{ ft-kip}$

Controls

From PTData, secondary moments are: $M_2 = 329.5$ ft-kip, say, 330 ft-kip

Add secondary moments

$M_u = 4124\text{ ft-kip} + 330\text{ ft-kip} = 4454\text{ ft-kip}$

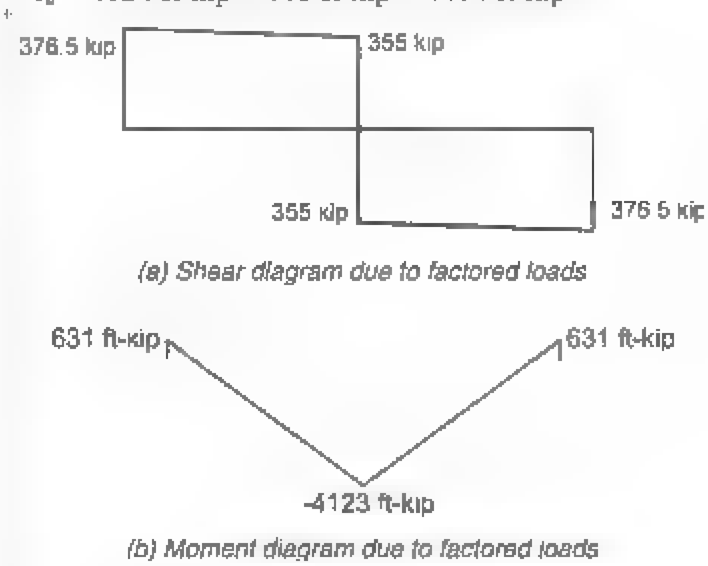
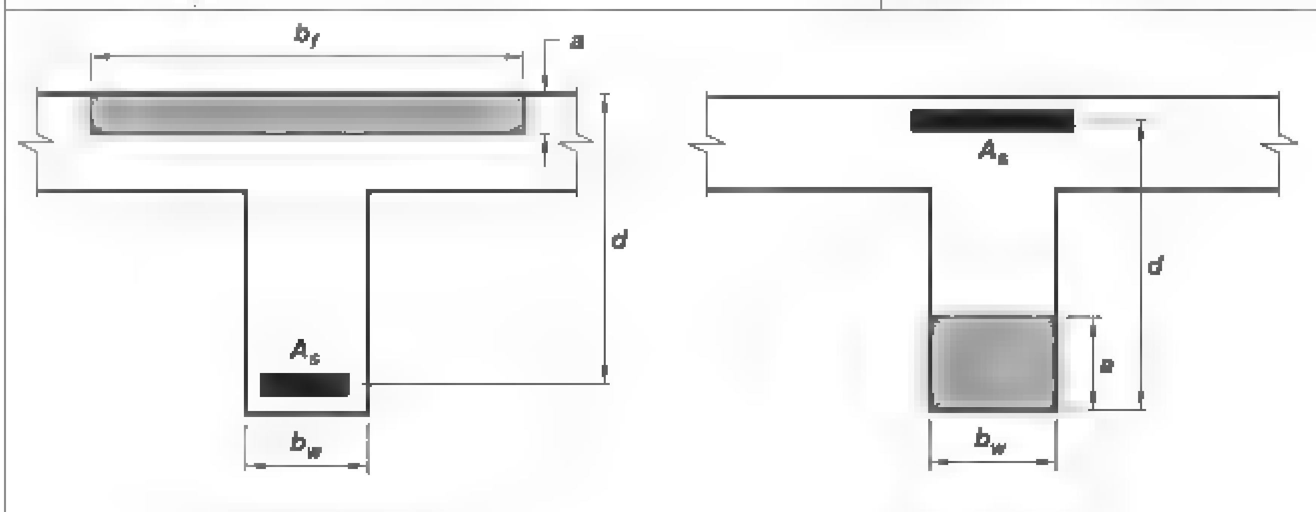


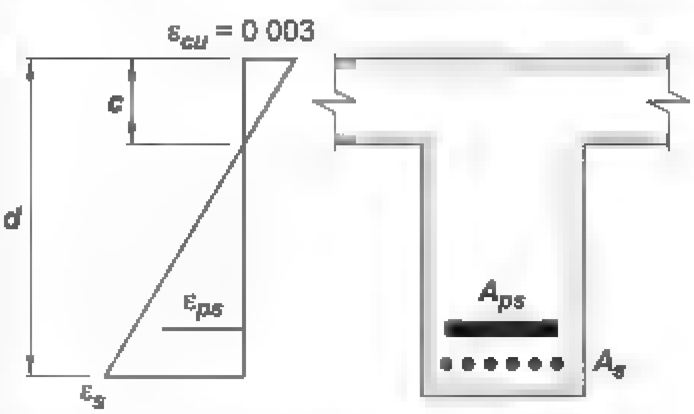
Fig. E6.7—Shear and moment diagrams.

21.2.1a	Because section must be tension-controlled, the strength reduction factor is 0.9. Determine the effective depth assuming No. 10 bars for the girder with 1.5 in. cover:	
20.5.1.3.1	$d = h - cover - d_{tie} - d_b/2$	Transfer girder $d = 48\text{ in.} - 1.5\text{ in.} - 0.5\text{in.} - (1.128\text{ in.})/2 = 45.4\text{ in.}$
22.2.2.1	The concrete compressive strain at nominal moment strength is: $\varepsilon_c = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and is approximately 10 to 15 percent of the concrete compressive strength. ACI 318 neglects the concrete tensile strength to calculate nominal strength.	
22.2.2.3	Determine the equivalent concrete compressive stress at nominal strength: The concrete compressive stress distribution is inelastic at high stress. The code permits any stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distribution of $0.85f_c'$ with a depth of	
22.2.2.4.1	$a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3.	
22.2.2.4.3	For $f_c' = 5000$ psi	$\beta_1 = 0.85 - \dfrac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$

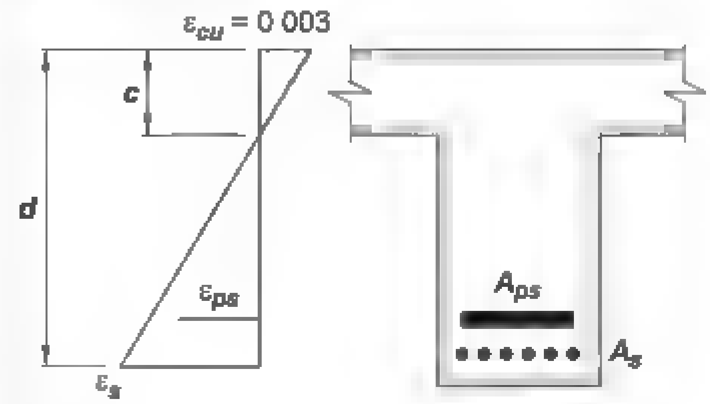


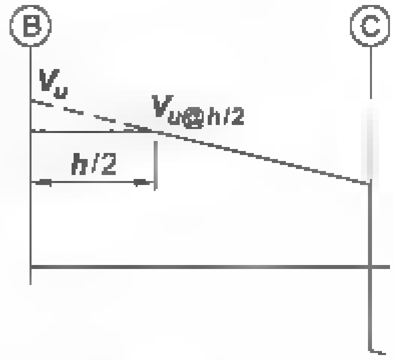
Positive moment Negative moment

Fig. E6.8—Section compression block and reinforcement locations.

20.3.2.4.1	For unbonded tendons and as an alternative to a more accurate calculation, the stress in post-tensioned tendons at nominal flexural strength is the least of: (a) $f_{se} + 10{,}000 + f_c'/(100\rho_p)$ (b) $f_{se} + 60{,}000$ (c) f_{py} if $f_{se} = 175$ ksi $> 0.5f_{pu} = 135$ ksi $\ell_n/h = (26\text{ ft})(12)/(48\text{ in.}) = 6.5 < 35$ and where $\rho_p = \dfrac{A_{ps}}{bd_p} = \dfrac{(34)(0.153\text{ in.}^2)}{(136\text{ in.})(44\text{ in.})} = 0.00087$	(a) 175 ksi + 10 ksi + 5 ksi/(100(0.00087)) = 242.5 ksi (b) 175 ksi + 60 ksi = 235 ksi **Controls** (c) $f_{py} = 0.9f_{pu} = (0.9)(270\text{ ksi}) = 243$ ksi Therefore, use $f_{ps} = 235$ ksi
9.6.2.3	The strands are unbonded. A minimum area of deformed reinforcement is required to ensure flexural behavior at nominal girder strength, rather than tied arch behavior. In addition, the reinforcing bar should limit crack width and spacing. To calculate the minimum area: $A_{s,min} = 0.004A_{ct}$ where A_{ct} is the area of that part of the cross section between the flexural tension face and the centroid of the gross section	At midspan: $A_{ct} = (24\text{ in.})(48\text{ in.} - 15.7\text{ in.}) = 775.2\text{ in.}^2$ $A_{s,min} = (0.004)(775.2\text{ in.}^2) = 3.1\text{ in.}^2$ Try four No. 8 $A_{s,prov} = (4)(0.79\text{ in.}^2) = 3.16\text{ in.}^2$ **OK**
	Calculate design moment strength of section at midspan with only PT tendons $C = T$ $0.85f_c'ba = A_{ps}f_{ps} + A_sf_y$	$a = \dfrac{(34)(0.153\text{ in.}^2)(235\text{ ksi}) + (4)(0.79\text{ in.}^2)(60\text{ ksi})}{(0.85)(5\text{ ksi})(136\text{ in.})}$ $a = 2.44\text{ in.} < h_f = 7\text{ in.}$ Therefore, compression block in flange $c = 2.44/0.8 = 3.05$
	Check that section is tension-controlled Deformed bars: $\varepsilon_t = \left(\dfrac{d}{c} - 1\right)\varepsilon_{cu}$ Alternatively $c/d = (30.5\text{ in.})/(44\text{ in.}) = 0.069 < 3/8$	$\varepsilon_t = \left(\dfrac{(45.4\text{ in.})}{3.05\text{ in.}} - 1\right)(0.003) = 0.042\text{ in./in.} > 0.005$ Section is tension-controlled  Fig. E6.9—Strain distribution over girder depth.

	Calculate flexural strength of section $M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right)$	$M_n = (5.20\text{ in.}^2)(235\text{ ksi})\left(44\text{ in.} - \frac{2.44\text{ in.}}{2}\right)$ $+ (4)(0.79\text{ in.}^2)(60\text{ ksi})\left(45.5\text{ in.} - \frac{2.44\text{ in.}}{2}\right)$ $\phi M_n = (0.9)(52{,}277\text{ in.-kip} + 8395\text{ in.-kip})$ $= 54{,}605\text{ in.-kip}$ $\phi M_n = 4550\text{ ft-kip} > M_u = 4454\text{ ft-kip}$ **OK**
	Moment at support The beam resists gravity only and lateral forces are not considered in this problem	
5.3.1	$U = 1.4D$ $U = 1.2D + 1.6L$	From moment diagrams, Fig. E6.4 $M_u = 1.4(298\text{ ft-kip}) = 417\text{ ft-kip}$ $M_u = 1.2(298\text{ ft-kip}) + 1.6(171\text{ ft-kip}) = 631\text{ ft-kip}$ **Controls**
	From PTData, secondary moments are:	$M_2 = 329.5\text{ ft-kip, say, } 330\text{ ft-kip}$ Add secondary moments $M_u = -631\text{ ft-kip} + 330\text{ ft-kip} = -301\text{ ft-kip}$
20.3.2.4.1	Stress in post-tensioned tendons at nominal flexural strength is the least of: (a) $f_{se} + 10{,}000 + f_c'/(100\rho_p)$	(a) $175\text{ ksi} + 10\text{ ksi} + 5\text{ ksi}/(100(0.0067)) = 192.0\text{ ksi}$ **Controls**
	(b) $f_{se} + 60{,}000$ (c) f_{py}	(b) $175\text{ ksi} + 60\text{ ksi} = 235\text{ ksi}$ (c) $f_{py} = 0.9 f_{pu} = (0.9)(270\text{ ksi}) = 243\text{ ksi}$
	if $f_{se} = 175\text{ ksi} > 0.5 f_{pu} = 135\text{ ksi}$ $\ell/h = (26\text{ ft})(12)/48\text{ in.}) = 6.5 < 35$ and where $\rho_p = \frac{A_{ps}}{bd_p} = \frac{(34)(0.153\text{ in.}^2)}{(24\text{ in.})(32.3\text{ in.})} = 0.0067$	Therefore, use $f_{ps} = 192\text{ ksi}$
9.6.2.3	Minimum area of deformed reinforcement at support $A_{s,min} = 0.004A_{ct}$ where A_{ct} is the area of that part of the cross section between the flexural tension face and the centroid of the gross section	At supports $A_{ct} = (136\text{ in.})(7\text{ in.}) + (24\text{ in.})(8.7\text{ in.}) = 1161\text{ in.}^2$ $A_{s,min} = (0.004)(1161\text{ in.}^2) = 4.64\text{ in.}^2$ Try four No. 10 $A_{s,prov} = (4)(1.27\text{ in.}^2) = 5.08\text{ in.}^2$ **OK**
	Calculate design moment strength of section at midspan with only PT tendons. $C = T$ $0.85 f_c'ba = A_{ps}f_{ps} + A_sf_y$	$a = \frac{(34)(0.153\text{ in.}^2)(192\text{ ksi}) + (4)(1.27\text{ in.}^2)(60\text{ ksi})}{(0.85)(5\text{ ksi})(24\text{ in.})}$ $a = 12.78\text{ in.}$ Therefore, compression block in flange $c = 12.78/0.8 = 15.98\text{ in.}$

	Check that section is tension-controlled Is the strain in bars closest to the tension face are greater than 0.005? Deformed bars $\varepsilon_s = \left(\frac{d}{c} - 1\right)\varepsilon_{cu}$	$\varepsilon_s = \left(\frac{(45.4\text{ in.})}{15.98\text{ in.}} - 1\right)(0.003) = 0.0055\text{ in./in.} > 0.005$ Therefore, use $\phi = 0.9$  *Fig. E6.10—Strain distribution over girder depth*
	Calculate flexural strength of section $M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_sf_y\left(d - \frac{a}{2}\right)$	$M_n = (5.20\text{ in.}^2)(192\text{ ksi})\left(32.3\text{ in.} - \frac{12.78\text{ in.}}{2}\right)$ $+ (4)(1.27\text{ in.}^2)(60\text{ ksi})\left(45.5\text{ in.} - \frac{12.78\text{ in.}}{2}\right)$ $\phi M_n = (0.9)(25{,}868\text{ in.-kip} + 11{,}921\text{ in.-kip})$ $= 34{,}010\text{ in.-kip}$ $\phi M_n = 2834\text{ ft-kip} > M_u = 301\text{ ft-kip}$ **OK**

(b) Shear design		
	<u>Shear strength</u> $V_u = w_u\ell/2 + P_u/2$	$V_u = (1.2)\left((1.2\text{ kip/ft})(13\text{ ft}) + \dfrac{433\text{ kip}}{2}\right)$ $+ (1.6)\left((0.065\text{ kip/ft}^2)(2\text{ ft})(13\text{ ft}) + \dfrac{119\text{ kip}}{2}\right)$ $V_u = 376.4$ kip
9.4.3.2	Because conditions a), b), and c) of 9.4.3.2 are satisfied, the design shear force is taken at critical section at distance $h/2$ from the face of the support (Fig. E6.11)	*Fig. E6.11—Factored shear at the critical section.*
		$V_{u@h/2} = (376.4\text{ kip}) - (1.65\text{ kip/ft})(24\text{ in./12}) = 373$ kip
21.2.1b	Shear strength reduction factor	$\phi_{shear} = 0.75$
9.5.1.1	$\phi V_n \geq V_u$	
9.5.3.1 22.5.1.1	$V_n = V_c + V_s$	
22.5.6.2 22.5.2.1	$\phi V_c = \phi 2\sqrt{f_c'}b_w d$ $d = d_p = 0.8h = 0.8(48\text{ in.}) = 38.4$ in.	$\phi V_c = 0.75(2)(\sqrt{5000\text{ psi}})(24\text{ in.})(38.4\text{ in.}) = 97.7$ kip
9.5.1.1	Check if $\phi V_c > V_u$	$\phi V_c = 97.7\text{ kip} < V_{u@h/2} = 373$ kip **NG** Therefore, shear reinforcement is required.
22.5.1.2	Before calculating shear reinforcement, check if the cross-sectional dimensions satisfy Eq. (22.5.1.2) $V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d)$	$V_u \leq \phi\left(97.7\text{ kip} + \dfrac{8(\sqrt{5000\text{ psi}})(24\text{ in.})(38.4\text{ in.})}{1000\text{ lb/kip}}\right)$ $= 489$ kip $V_u = 373\text{ kip} \leq \phi(V_c + 8\sqrt{f_c'}b_w d) = 489$ kip **OK**, therefore, section dimensions are satisfactory.

22 5 6.2	Check if $A_{ps}f_{se} > 0.4(A_{ps}f_{pu} + A_s f_y)$	$(0\ 153\ \text{in.}^2)(34)(175\ \text{ksi}) = 910\ \text{kip}\ 0.4((0.153\ \text{in.}^2)(34)(270\ \text{ksi}) + (5\ 08\ \text{in}^2)(60\ \text{ksi})) = 684\ \text{kip}$ 910 kip > 684 kip
22 5 6.2	Therefore, using the simplified approach, V_c is the least of the following three equations (a) $\left(0.6\lambda\sqrt{f_c'} + 700\dfrac{V_u d_p}{M_u}\right)b_w d$ (b) $\left(0\ 6\lambda\sqrt{f_c'} + 700\right)b_w d$ (c) $5\lambda\sqrt{f_c'}b_w d$	
22 5.6.2	and $V_w \geq 2\lambda\sqrt{f_c'}b_w d$	$V_w \geq 2(1\ 0)\sqrt{5000\ \text{psi}}(24\ \text{in})(38\ 4\ \text{in.}) = 130.3\ \text{kip}$ For the calculation of the three equations, refer to the table below

x, ft	V_u, kip	M_u, ft-kip	d_p, in.	$V_u d_p/M_u$	V_c (a), kip	V_c (b), kip	V_c (c), kip
1	376.5	631	38.4	1.909	1072 7	684	326
2	374 8	–255	38 4	–4.694	–2353.6	684	326
3	373 2	118	38 4	10 078	15 262 9	684	326
4	371.5	491	38 4	2.422	1852 5	684	326
5	369 9	861	38.4	[illegible].374	1001.2	684	326
6	368 2	123[illegible]	38.4	0.958	692 9	684	326
7	366 6	1598	38 4	0.734	533 7	684	326
8	364.9	1964	38 4	0 595	436.5	684	326
9	363 3	2328	38 4	0.499	370 9	684	326
10	361.6	2690	38.4	0.430	323.8	684	326
11	360 0	305[illegible]	38.4	0 378	288.2	684	326
12	358 3	3410	39 6	0 347	276 3	706	336
13	356 7	3767	4[illegible]8	0 330	278 7	745	355
14	355.0	4123	44.0	0.316	282.1	784	373

Equation (22.5 6.2a) controls the middle 8 ft of the girder; the rest of the span is controlled by Eq (22 5.6.2c), shown shaded in table above. Both equations are greater than Eq. 22.5 6.2

$V_u > \phi V_c$ at all sections along the girder length. Therefore, shear reinforcement is required. Try No. 4 stirrups, $A_v = 2(0.2\ \text{in.}^2) = 0.4\ \text{in.}^2$ and $d_{p,min} = 38.4$ in.

x, ft	V_u, kip	ϕV_c Eq. (22.5.8.2a) and (22.5.8.2c), kip	$\phi V_s = V_u - \phi V_c$, kip	$s = \dfrac{\phi A_v f_{yt} d_p}{V_u - \phi V_c}$, in.	$s_{prov'd}$, in.
1	377	244	132	5.23	5
2	375	244	130	5.30	5
3	373	244	129	5.37	5
4	372	244	127	5.44	5
5	370	244	125	5.5[illegible]	5
6	368	244	124	5.58	5
7	367	244	122	5.66	5
8	365	244	121	5.73	5
9	363	244	19	5.8	5
10	362	243	119	5.82	5
11	360	216	144	4.81	4
12	358	207	151	4.72	4
13	357	209	148	5.10	4
14	355	212	143	5.23	4

<u>Shear reinforcement</u>

22.5.8.5.1 Transverse reinforcement satisfying Eq. (22.5.8.5.3) is required at each section where $V_u > \phi V_c$

$$V_s \geq \frac{V_u}{\phi} - V_c$$

22.5.8.5.3
22.5.8.5.5 where $V_s = A_v f_{yt} d/s$

$V_s = 212\ \text{kip}/0.75 = 282\ \text{kip}$

9.7.6.2.2 Calculate maximum allowable stirrup spacing. First, does the beam transverse reinforcement value need to exceed the threshold value?

$V_s < 4\sqrt{f_c'}b_w d$?

$4\sqrt{f_c'}b_w d = 4(\sqrt{5000\ \text{psi}})(24\ \text{in.})(38.4\ \text{in.}) = 260\ \text{kip}$

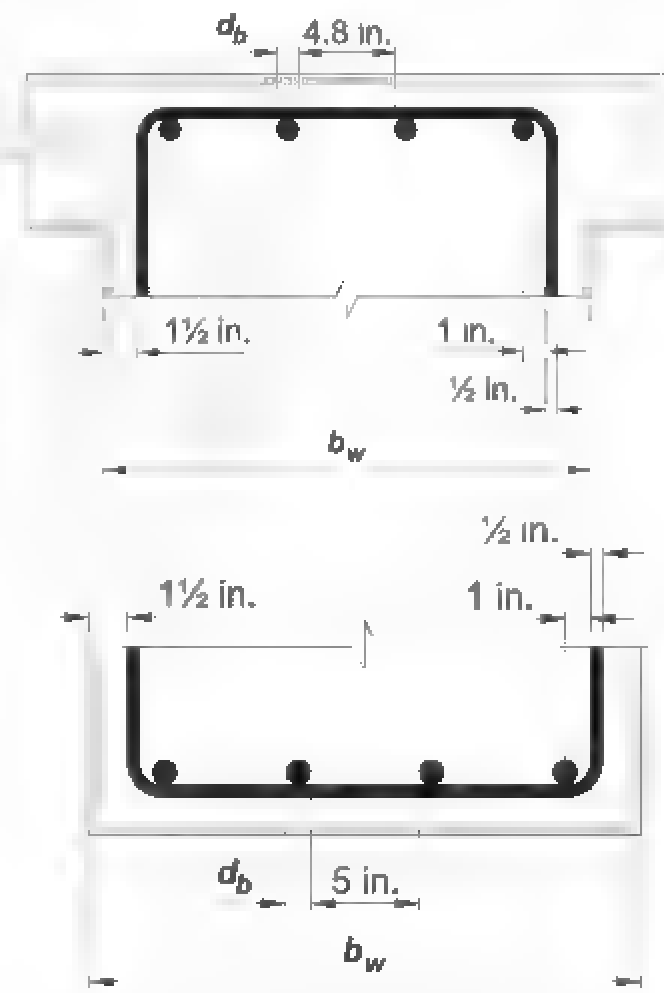
The required shear strength is less than the threshold value, therefore, provide maximum stirrup spacing as the lesser of $3h/8$ and 12 in.

$V_s = 282\ \text{kip} > 4\sqrt{f_c'}b_w d = 260\ \text{kip}$ **OK**

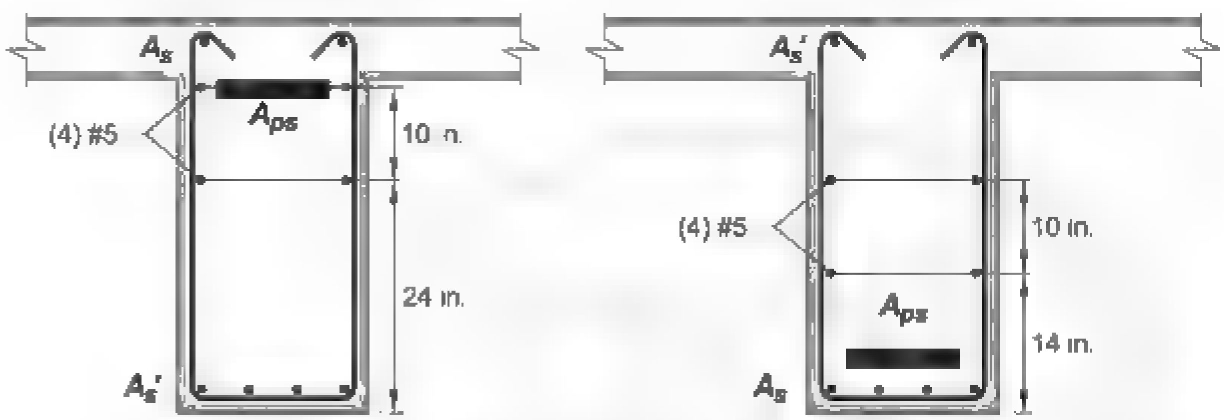
$3h/8 = (3)(48\ \text{in.})/8 = 18\ \text{in.} > d/4 = 12\ \text{in.}$ **Controls**

Place first No. 4 stirrup at 3 in. from the face of support. Place No. 4 stirrups at 4 in. on center in the middle 6 ft of the girder. Place No. 4 stirrups at 5 in. on center on both sides of the midsection.

9.6.3.4	Specified shear reinforcement must be the least of the greater of (c) and (d) and (e)	
	(c) $0.75\sqrt{f'_c}\dfrac{b_w}{f_{yt}}$, and	$\dfrac{A_{v,min}}{s} \geq 0.75\sqrt{5000\text{ psi}}\dfrac{24\text{ in.}}{60{,}000\text{ psi}} = 0.021\text{ in.}^2/\text{in.}$
	(d) $50\dfrac{b_w}{f_{yt}}$	$\dfrac{A_{v,min}}{s} = 50\dfrac{24\text{ in.}}{60{,}000\text{ psi}} = 0.02\text{ in.}^2/\text{in.}$
	(e) $\dfrac{A_{ps}f_{pu}}{80f_{yt}d}\sqrt{\dfrac{d}{b_w}}$	$\dfrac{(0.153\text{ in.}^2)(34)(270\text{ ksi})}{80(60\text{ ksi})(38.4\text{ in.})}\sqrt{\dfrac{38.4\text{ in.}}{24\text{ in.}}} = 0.01\text{ in.}^2/\text{in.}$ **Controls**
	to develop ductile behavior	Provided 5 in. spacing: $\dfrac{A_{v,min}}{s} = \dfrac{2(0.2\text{ in.}^2)}{5\text{ in.}} = 0.08\text{ in.}^2/\text{in.}$ spacing satisfies 9.6.3.4 ∴ **OK**

Step 8. Reinforcement detailing		
9.7.2.1 25.2.1	<u>Minimum longitudinal bar spacing</u> The clear spacing between longitudinal No. 10 bars Clearing spacing greater of $\begin{cases} 1 \text{ in.} \\ d_b \\ 4/3(d_{agg}) \end{cases}$	1 in. 1.27 in. **Controls** 4/3(3/4 in.) = 1 in. assuming a 3/4 in. maximum aggregate size
	Check if four No. 10 bars (resisting positive moment) can be placed in the beam's web (Fig. E6.12).	Therefore, clear spacing between horizontal bars must be at least 1.27 in., say, 1.5 in.
	Top bar layout $b_{w,req'd} = 2(\text{cover} + d_{stirrup} + 1.0 \text{ in.}) + 3d_b + 3(1.5 \text{ in.})_{min,spacing}$ (25.2.1)	$b_{w,req'd} = 2(1.5 \text{ in.} + 0.5 \text{ in.} + 1.0 \text{ in.}) + 3.81 \text{ in.} + 4.5 \text{ in.} = 14.3 \text{ in.} < 24 \text{ in.}$ **OK** Therefore, four No. 10 bars can be placed in one layer in the 24 in. transfer girder web
9.7.2.2 24.3.4	Tension reinforcement in flanges must be distributed within the effective flange width, $b_f = 136$ in. (Step 2), but not wider than $\ell_n/10$. Because effective flange width exceeds $\ell_n/10$, additional bonded reinforcement is required in the outer portion of the flange. Use No. 5 placed at slab middepth for additional bonded reinforcement. This requirement is to control cracking in the slab due to wide spacing of bars across the full effective flange width and to protect flange if reinforcement is concentrated within the web width.	 *Fig. E6.12—Bottom reinforcement layout.*
	Bottom bar layout $b_{w,req'd} = 2(\text{cover} + d_{stir} + 1.0 \text{ in.}) + 3d_b + 3(1.5 \text{ in.})$	$b_{w,req'd} = 2(1.5 \text{ in.} + 0.5 \text{ in.} + 1.0 \text{ in.}) + 3 \text{ in.} + 4.5 \text{ in.} = 13.5 \text{ in.} < 24 \text{ in.}$ **OK**

9.7.2.2 24.3.1 24.3.2	Maximum bar spacing at the tension face must not exceed the lesser of $s = 15\left(\dfrac{40,000}{f_s}\right) - 2.5c_c$ and $s = 12(40,000/f_s)$ This limit is intended to limit flexural cracking width. Note that c_c is the cover to the longitudinal bars, not to the tie	$s = 15\left(\dfrac{40,000\text{ psi}}{40,000\text{ psi}}\right) - 2.5(2\text{ in.}) = 10\text{ in.}$ **Controls** $s = 12\left(\dfrac{40,000\text{ psi}}{40,000\text{ psi}}\right) = 12\text{ in.}$ Longitudinal bar spacing satisfy the maximum bar spacing requirement, therefore, **OK**
9.7.2.3 24.3.2	Skin reinforcement The transfer girder is 48 in. deep > 36 in. Although the Code does not require skin reinforcement for Class T prestressed beams, many engineers provide it. Skin reinforcement is placed a distance $h/2$ from the tension face	Use two No. 5 bars each face side as shown in Fig. E6.13



(a) Skin reinforcement at support *(b) Skin reinforcement at midspan*

Fig. E6.13—Skin reinforcement in girder.

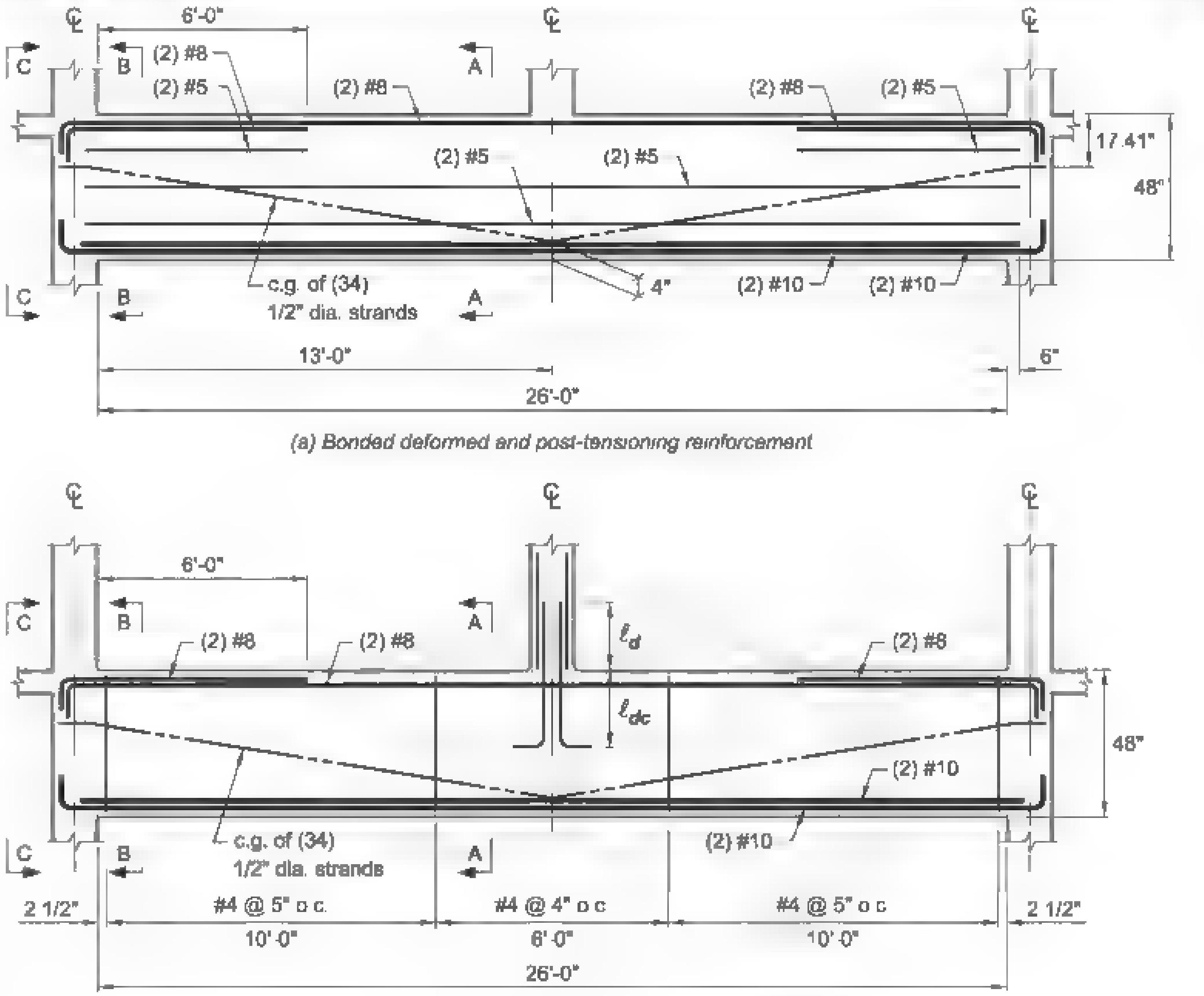
Step 9: Bar cutoff		
	Development length Extend top and bottom deformed bars over the full length of the beam. This will ensure better resistance to creep stresses and control of cracks in the girder. Therefore, development length calculation is not required into the girder. Bars, however, must be developed within the support at each end. The simplified method is used to calculate the development length of No. 8 bars	
25.4.2.3	$\ell_d = \left(\dfrac{f_y\psi_t\psi_e\psi_g}{20\lambda\sqrt{f_c'}}\right)d_b$	$\ell_d = \left(\dfrac{(60,000\text{ psi})(1.0)(1.0)(1.0)}{(20)(1.0)\sqrt{5000\text{ psi}}}\right)(d_b) = 42.43d_b$
25.4.2.5	where ψ_t = 1.0 top bar location with not more than 12 in. of fresh concrete below horizontal reinforcement. Otherwise, use 1.3. ψ_g = reinforcement grade factor; ψ_g = 1.0 for Grade 60 reinforcement ψ_e = 1.0, bars are uncoated	For top bars, ψ_t = 1.3, ℓ_d = (1.3)(42.43d_b) No. 8 bars ℓ_d = 42.43(1.0 in.) = 42.43 in., say, 48 in. because No. 8 bars are top bars, development length is 1.3ℓ_d = (1.3)(42.43 in.)(1.27 in.) = 70 in., say, 6 ft 0 in.
9.7.7.5	Girder is a single 26 ft long span, therefore, reinforcement splicing is not required	

Beams

	Integrity reinforcement	
9.7.7.2	The girder is an interior member; therefore, either a) or b) of 9.7.7.2 must be satisfied. In this example, condition a) is satisfied by having at least one-quarter of the positive moment bars, but not less than two bars continuous.	This condition is satisfied by extending two No. 10 bars into the support with standard hooks. **OK**
9.7.7.3	Beam structural integrity bars must pass through the region bounded by the longitudinal column bars	Place hooks on two interior No. 10 bottom bars to ensure that they are inside the longitudinal column reinforcement

	Post-tensioning detailing	
25.8.1	Anchorages for tendons must develop 95 percent of f_{pu} when tested in an unbonded condition.	
25.9.1.1 25.9.3	The Code defines two zones that require attention when considering the design of anchorages for post-tensioned tendons. The first is the local zone, which is the region immediately surrounding the anchorage device along with confining reinforcement placed nearby. Concrete in this area is typically stressed beyond its unconfined compressive strength. Consequently, the strength is highly dependent on the anchorage device geometry and the quantity and configuration of the confining reinforcement. This zone is typically addressed by the tendon supplier using either analysis, or testing, or both.	
25.9.4 25.9.4.3	The second region is the general zone, which is the area where the concentrated PT force is distributed to a more uniform stress state across the section. This zone is typically designed by the engineer of record or PT specialty engineer. Within the general zone, bursting, spalling, and edge tension stresses can occur. The Code indicates that the general zone can be designed using strut-and-tie method. If the anchorage zone meets the criterion listed in 25.9.4.3.2, however, then simplified equations may be used to design the bursting reinforcement. This example meets these criterion and the general zone can be designed by the simplified method. For this design, however, the columns and their reinforcement provided confinement for the bursting stresses caused by the anchorage. See beam details in Fig. E6.14 and Fig. E6.15	

Step 10: Detailing



(b) Shear reinforcement

Fig. E6.14—Beam bar details. (Note: Place first stirrup at 3 in. from the column face.)

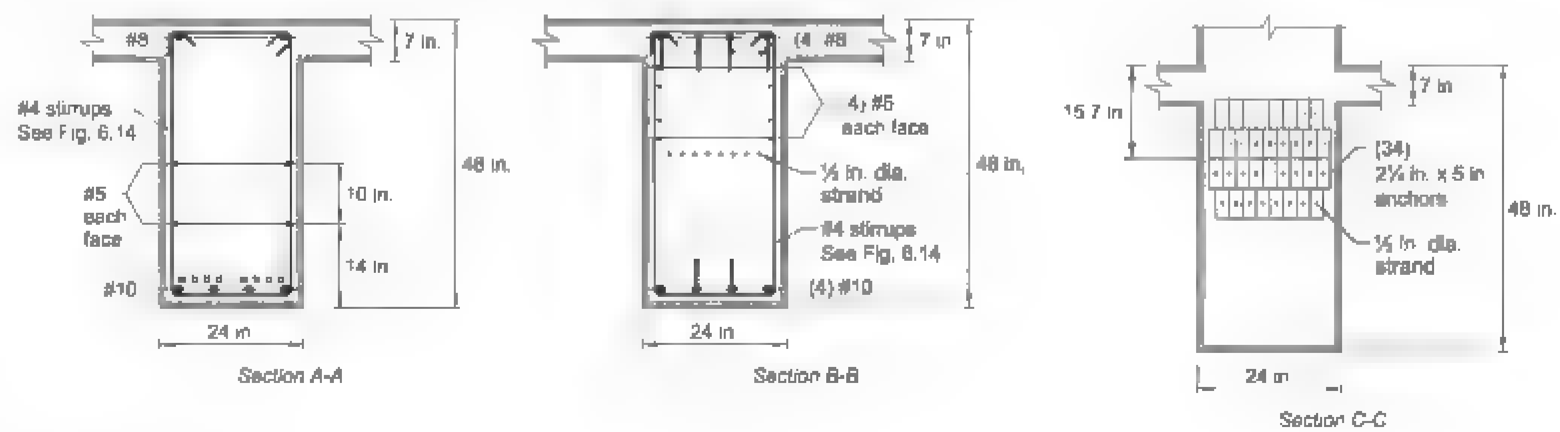


Fig. E6.15—Sections

Beam Example 7: *Precast concrete beam*

Design and detail an interior, simply supported precast beam supporting factored concentrated forces of 15 kip located at 4 ft 6 in. from each end and a continuously distributed factored force of 4.6 kip/ft. The beam is supported on a 6 in. ledge (Fig. E7.1).

Given:

Material properties—
$f_c' = 4000$ psi (normalweight concrete)
$\lambda = 1.0$
$f_y = 60{,}000$ psi

Load—
$P_{u1} = 15.0$ kip at 4 ft 6 in. from each support
$w_u = 4.6$ kip/ft
Span length: 18 ft
Beam width: 14 in.
Bearing at support: 6 in.
Bearing at concentrated load: 10 in.

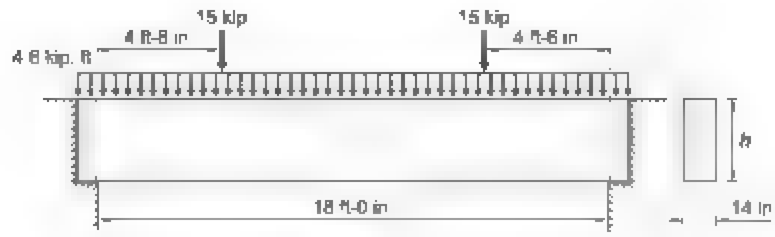
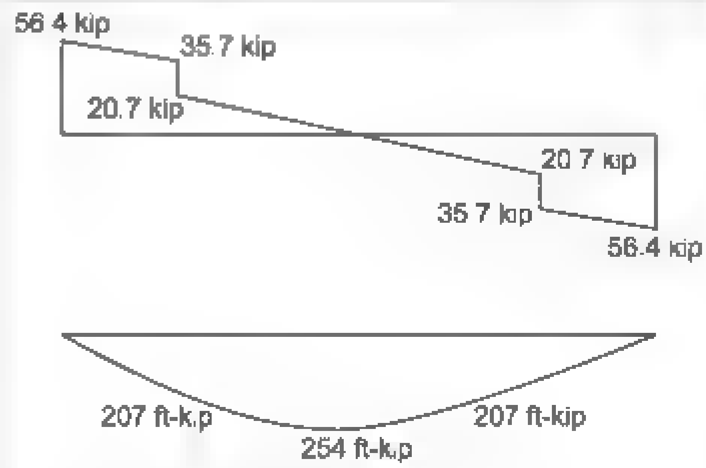
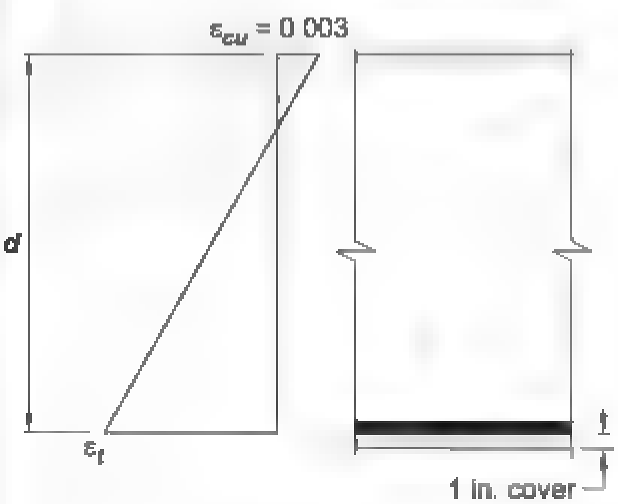


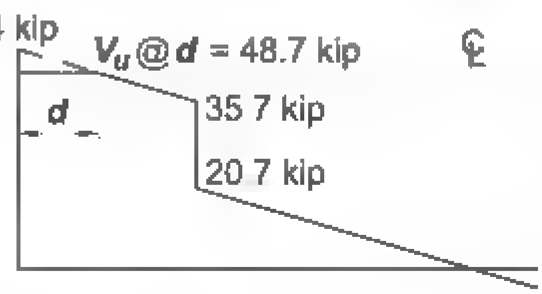
Fig. E7.1—Simply supported precast concrete beam.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
9.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 (ACI 318) and structural strength requirements. The designer determines the durability classes. Please refer to Chapter 3 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 (ACI 318) requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 4000 psi. Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26.
Step 2: Beam geometry		
9.3.1.1	Beam depth Since beam resists concentrated loads, the beam depth limits in Table 9.3.1.1 of ACI 318 cannot be used.	Assume 22 in. deep beam

Step 3: Analysis		
	The beam is simply supported and the loads are symmetrical. Therefore, the maximum shear and moment are located at supports and midspan, respectively.	 *Fig. E7.2—Shear and moment diagrams*
	$V_{u,max} = \frac{w_u(\ell)}{2} + (P_u)$	$V_{u,max} = \frac{(4.6 \text{ kip/ft})(18 \text{ ft})}{2} + (15 \text{ kip}) = 56.4 \text{ kip}$
	$M_{u,max} = \frac{w_u(\ell)^2}{8} + (P_u)(x)$	$M_{u,max} = \frac{(4.6 \text{ kip/ft})(18 \text{ ft})^2}{8} + (15 \text{ kip})(4.5 \text{ ft}) = 254 \text{ ft-kip}$
Step 4: Bearing		
16.2.6.2	The minimum seating length of the precast beam on the wall ledge is the greater of: $\ell_n/180$ and 3 in.	$\ell_n/180 = (18 \text{ ft})(12 \text{ in./ft})/180 = 1.2 \text{ in.} < 3 \text{ in.}$ Provided 6 in., therefore **OK**
22.8.3.2	<u>Bearing strength</u> Check bearing strength at seat and concentrated load. The supporting surface (ledge) is wider on three of the four sides. Therefore, condition (c) applies.	
	$0.85f_c'A_1$	$0.85(4000 \text{ psi})(14 \text{ in.})(6 \text{ in.})/1000 = 285.6 \text{ kip}$ $285.6 \text{ kip} >> 56.4 \text{ kip}$ **OK**
	A 10 in. wide beam rests on the precast beam	$0.85(4000 \text{ psi})(14 \text{ in.})(10 \text{ in.})/1000 = 476 \text{ kip} >> 15 \text{ kip}$ **OK**

Step 5. Moment design		
9.3.3.1	Limiting steel strain restricts the amount of reinforcement to ensure warning of failure by excessive deflection and cracking. Before the 2019 Code, a minimum strain limit of 0.004 was specified for nonprestressed flexural members. Beginning with the 2019 Code, this limit is revised to require that the section be tension-controlled.	$\varepsilon_{ty} = \dfrac{f_y}{E_s} = \dfrac{60{,}000\text{ psi}}{29{,}000{,}000\text{ psi}} \cong 0.002$ $\varepsilon_t \geq \varepsilon_{ty} + 0.003 = 0.002 + 0.003 = 0.005$
21.2.2	Because section must be tension-controlled, the strength reduction factor is 0.9	Beam must be tension-controlled in accordance with Table 21.2.2 $\phi = 0.9$
20.6.1.3.3	Determine the effective depth assuming No. 8 bars and 1.0 in. cover. For precast concrete beam, the minimum cover is the greater of 5/8 in. and d_b and need not exceed 1.5 in.	Use $d_b = 1$ in. cover
	One row of reinforcement $d = h - \text{cover} - d_{tie} - d_b/2$	$d = 22\text{ in.} - 1.0\text{ in.} - 0.375\text{ in.} - 1.0\text{ in.}/2 = 20.1\text{ in.}$, say, 20 in.
22.2.2.1	The concrete compressive strain at nominal moment strength is calculated at: $\varepsilon_{cu} = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and is about 10 to 15 percent of the concrete compressive strength. ACI 318 neglects the concrete tensile strength to calculate nominal strength.	
	Determine the equivalent concrete compressive stress at nominal strength	
22.2.2.3	The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distri-	
22.2.2.4.1 22.2.2.4.3	bution of $0.85f_c'$ with a depth of: $a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3	
22.2.1.1	For $f_c' \leq 4000$ psi	$\beta_1 = 0.85$
	Find the equivalent concrete compressive depth a by equating the compression force to the tension force within the beam cross section $C = T$ $0.85f_c'ba = A_sf_y$	$0.85(4000\text{ psi})(b)(a) = A_s(60{,}000\text{ psi})$
	For moment at midspan $b = 14$ in.	$a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(4000\text{ psi})(14\text{ in.})} = 1.26A_s$

9.5.1.1	The beam's design strength must be at least the required strength at each section along its length $\phi M_n \ge M_u$ $\phi V_n \ge V_u$ Calculate the required reinforcement area $\phi M_n \ge M_u = \phi A_s f_y\left(d - \frac{a}{2}\right)$ A No. 8 bar has a $d_b = 1.0$ in. and an $A_s = 0.79$ in.^{2}	$(254 \text{ ft-kip})(12) = (0.9)(60\text{ ksi})A_s\left(20\text{ in.} - \frac{1.26A_s}{2}\right)$ $A_{s,req'd} = 3.1$ in.^{2} Use four No. 8 $A_{s,prov} = 3.16$ in.^{2} > $A_{s,req'd} = 3.1$ in.^{2} **OK** Per Reinforced Concrete Design Handbook Design Aid – Analysis Tables, which can be downloaded from: https://www.concrete.org/MNL1721Download1, four No. 8 bars require a minimum of 11.5 in. wide beam. Therefore, 14 in. width is adequate
9.3.3.1	Check to ensure section is tension-controlled (Fig. E7.3) $a = \frac{A_s f_y}{0.85 f'_c b}$ and $c = \frac{a}{\beta_1}$ where $\beta_1 = 0.85$ $\varepsilon_t = \frac{\varepsilon_{cu}}{c}(d - c)$	$a = 1.26A_s = (1.26)(4)(0.79 \text{ in.}^2) = 3.98$ in. $c = a/0.85 = 3.98 \text{ in.}/0.85 = 4.68$ in. $\varepsilon_t = \frac{0.003}{4.68\text{ in.}}(20\text{ in.} - 4.68\text{ in.}) = 0.01 > 0.005$ **OK**  *Fig. E7.3—Strain distribution across beam section*
9.6.1.1 9.6.1.2	<u>Minimum reinforcement</u> The provided reinforcement must be at least the minimum required reinforcement at every section along the length of the beam. $A_{s,min} = \frac{3\sqrt{f'_c}}{f_y}b_w d$ $A_{s,min} = \frac{200}{f_y}b_w d$	$A_{s,min} = \frac{3\sqrt{4000\text{ psi}}}{60,000\text{ psi}}(14\text{ in.})(20\text{ in.}) = 0.89$ in.^{2} $A_{s,min} = \frac{200}{60,000\text{ psi}}(14\text{ in.})(20\text{ in.}) = 0.93$ in.^{2} **Controls** $A_{s,prov} = 3.16$ in.^{2} > $A_{s,min} = 0.93$ in.^{2} **OK** Required positive moment reinforcement areas exceed the minimum required reinforcement area at all positive moment locations

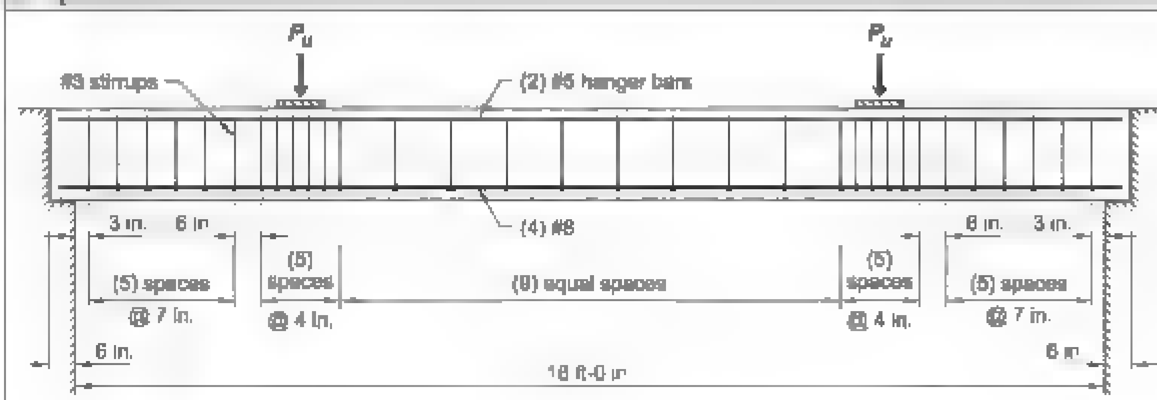
	Top reinforcement While not required by Code, top bars are needed to stabilize the beam's stirrups. Use two No. 5 continuous bars.	
Step 6: Shear design		
	Shear strength	
21.2.1b	Shear strength reduction factor:	$\phi_{shear} = 0.75$
9.5.1.1	$\phi V_n \geq V_u$	
9.5.3.1 22.5.1.1	$V_n = V_c + V_s$	
9.4.3.2	Because conditions (a), (b), and (c) of 9.4.3.2 are satisfied, the design shear force is taken at distance d from the face of the support (Fig. E7.4).	Fig. E7.4—Shear at the critical section.
22.5.5.1	2019 Code introduced size effect for shear design in which the shear strength of an element that does not contain shear reinforcement is not directly proportional to its depth. This effect is addressed by incorporating a size effect factor λ_s into the concrete contribution equation. If shear reinforcement is not present, then the concrete contribution to shear strength must be reduced by the size effect factor. If minimum shear reinforcement is provided, then Eq. 22.5.5.1a can be used to calculate V_c.	
9.6.3.1	Minimum shear reinforcement is required where $V_u > \phi\lambda\sqrt{f_c'}b_w d$ For this example, use minimum shear reinforcement over entire length of beam. The concrete contribution to shear strength is then	$V_u = (56.4\text{ kip}) - (4.6\text{ kip/ft})(20\text{ in.}/12) = 48.7\text{ kip}$
	$V_c = 2\sqrt{f_c'}b_w d$ (22.5.5.1a)	$V_c = (2)(\sqrt{4000\text{ psi}})(14\text{ in.})(20\text{ in.}) = 35.4\text{ kip}$
	Check if $\phi V_c > V_u$	$\phi V_c = (0.75)(35.4\text{ kip}) = 26.6\text{ kip} < V_u = 48.7\text{ kip}$ **NG** Therefore, shear reinforcement is required
	Determine required V_u on each side of P_u Left of P_u: $V_{u,\ell} = V_u - w_u x_1$ Right of P_u: $V_{u,r} = V_u - w_u x_1 - P_u$	 $V_{u,\ell} = 56.4\text{ kip} - (4.6\text{ kip/ft})(4.5\text{ ft}) = 35.7\text{ kip}$ $V_{u,r} = 56.4\text{ kip} - (4.6\text{ kip/ft})(4.5\text{ ft}) - 15\text{ kip} = 20.7\text{ kip}$
22.5.1.2	Prior to calculating shear reinforcement, check if the cross-sectional dimensions satisfy Eq. (22.5.1.2) $V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d)$	 $V_u \leq \phi(35.4\text{ kip} + 8(\sqrt{4000\text{ psi}})(14\text{ in.})(20\text{ in.})) = 132.8\text{ kip}$ **OK** Section dimensions are satisfactory

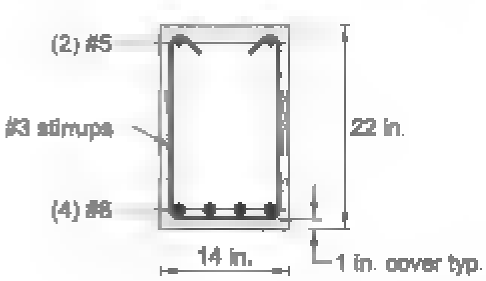
	Shear reinforcement	
9.5.3 22.5.8.5.1	Transverse reinforcement satisfying equation 22.5.8.5.3 is required at each section where $V_u > \phi V_c$	
22.5.8.5.3 22.5.8.5.5	$\phi V_s > V_u - \phi V_c$ where $\phi V_s = \frac{\phi A_v f_{yt} d}{s}$	$\phi V_s > (48.7 \text{ kip}) - (26.6 \text{ kip}) = 22.1 \text{ kip}$ Assume a No. 3 bar, two legged stirrup $22.1 \text{ kip} = \frac{(0.75)(2)(0.11 \text{ in.}^2)(60{,}000 \text{ psi})(20 \text{ in.})}{s}$ $s = 8.9 \text{ in.}$
	Calculate maximum allowable stirrup spacing First, does the required transverse reinforcement value exceed the threshold value?	$V_s = \frac{22.1 \text{ kip}}{0.75} = 29.5 \text{ kip}$
	$V_s \leq 4\sqrt{f_c'}b_w d$?	$4\sqrt{f_c'}b_w d = 4(\sqrt{4000 \text{ psi}})(14 \text{ in.})(20 \text{ in.}) = 71 \text{ kip}$ $V_s = 29.5 \text{ kip} < 4\sqrt{f_c'}b_w d = 71 \text{ kip}$ **OK**
	Because the required shear strength is below the threshold value, the maximum stirrup spacing is the lesser of $d/2$ and 24 in.	$d/2 = 20 \text{ in.}/2 = 10 \text{ in.}$ Use $s = 7$ in. $< d/2 = 10$ in., therefore, **OK**
9.7.6.2.2	It is unnecessary to use No. 3 stirrups at 7 in. on center over the full length of the beam	
	Since the maximum spacing is 10 in., determine the value of: $\phi V_n = \phi V_c + \phi V_s$ with $s = 10$ in.	$\phi V_n = 26.6 \text{ kip} + \frac{(0.75)(2)(0.11 \text{ in.}^2)(60 \text{ ksi})(20 \text{ in.})}{10 \text{ in.}}$ $\underline{\phi V_n = 46.4 \text{ kip}}$
	Determine distance x from face of support to point at which $V_u = 46.4 \text{ kip}$	$x = \frac{56.4 \text{ kip} - 46.4 \text{ kip}}{4.6 \text{ kip/ft}} = 2.2 \text{ ft}$
	Conclude use $s = 7$ in. until $\phi V_u < 44.5$ kip and use $s = 10$ in. over the remainder of the beam	From face of support use 3 in. space then Five spaces at 7 in. on center (35 in.) and the remainder at 10 in. on center. (3 in.+ 35 in. + 50 in.) = 88 in. > 78 in. **OK** The beam middle section of length (18 ft)(12) – 2(88 in.) = 40 in. does not require shear reinforcement However, extend No. 3 stirrups over the remaining length of 40 in. at 10 in. on center as good practice It is good practice to add stirrups near a concentrated load. Place six No. 3 stirrups at 4 in. centered on each concentrated load.

Step 7: Deflection		
9.3.2.1 24.2.3.1	Immediate deflection is calculated using elastic deflection approach and considering concrete cracking and reinforcement for calculating stiffness.	
24.2.3.4	Modulus of elasticity	
19.2.2.1	$E_c = 57{,}000\sqrt{f_c'}$ psi (19.2.2.1b)	$E_c = 57{,}000\sqrt{4000\text{ psi}} = 3600$ ksi
	The beam resists a factored distributed force of 4.6 kip/ft or service dead load of 2.23 kip/ft and 1.2 kip/ft service live load	
	The beam also resists a concentrated factored load of 15 kip or service dead load of 7.2 kip and 3.9 kip service live load	
	The deflection equation for distributed load with free rotation at both ends $\Delta = \dfrac{5w\ell^4}{384EI}$ For concentrated load at thirdspan $\Delta = \dfrac{P\ell^3}{28.23EI}$	
24.2.3.5	The effective moment of inertia equation was altered in the 2019 Code to more accurately reflect the deflections in members with low quantities of reinforcement. Where the applied moment (M_a) is greater than 2/3 of the cracking moment (M_{cr}), the following equation is used. $I_e = \dfrac{I_{cr}}{1-\left(\dfrac{2/3M_{cr}}{M_a}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)}$	$I_g = \dfrac{bh^3}{12} = \dfrac{(14\text{ in.})(22\text{ in.})^3}{12} = 12{,}423\text{ in.}^4$
	where: $M_{cr} = \dfrac{f_r I_g}{y_t}$ (24.2.3.5b)	$M_{cr} = \dfrac{7.5(\sqrt{4000\text{ psi}})(12{,}423\text{ in.}^4)}{11\text{ in.}} = 535{,}704$ in.-lb $M_{cr} = 44.6$ ft-kip
	M_a is the moment due to service load. The beam is assumed cracked, therefore, calculate the moment of inertia of the cracked section, I_{cr}. Calculate the neutral axis depth, c:	$n = \dfrac{29{,}000\text{ ksi}}{3600\text{ ksi}} = 8$
	$nA_s(d-c) = \dfrac{bc^2}{2} + (n-1)A_s'(c-d')$ where $n = \dfrac{E_s}{E_c}$ and $A_s' = 0$ in.^{2}	$(8)(3.16\text{ in.}^2)(20\text{ in.}-c) = \dfrac{(14\text{ in.})c^2}{2}$
	Solving for c:	$c = 6.9$ in.
	Conservatively check deflections with cracked moment of inertia. Refine calculations to consider effective moment of inertia if warranted $I_{cr} = \dfrac{bc^3}{3} + (n-1)A_s'(c-d')^2 + nA_s(d-c)^2$	$I_{cr} = 5975$ in.^{4}

	Live load deflection due to distributed load.	$\Delta_{distr} = \dfrac{5(1.2\ \text{kip/ft})(18\ \text{ft})^4(12)^3}{384(3600\ \text{ksi})(5975\ \text{in.}^4)} = 0.13\ \text{in.}$
	Live load deflection due to concentrated load	$\Delta_{conc} = \dfrac{(3.9\ \text{kip})(18\ \text{ft})^3(12)^3}{28.3(3600\ \text{ksi})(5975\ \text{in.}^4)} = 0.07\ \text{in.}$
	Live load deflection.	$\Delta = 0.13\ \text{in.} + 0.07\ \text{in.} = 0.20\ \text{in.}$
24.2.2	Check live load deflection limit from Table 24.2.2. Assume that the floor is not supporting or attached to nonstructural elements likely to be damaged by large deflections. Use $\ell/360$.	$\Delta_{all} = (18\ \text{ft})(12\ \text{in./ft})/360 = 0.6\ \text{in.}$ $\Delta_{all} = 0.6\ \text{in.} > \Delta_L = 0.20\ \text{in.}$ **OK**
24.2.4.1.1	Calculate long-term deflection $\lambda_\Delta = \dfrac{\xi}{1+50\rho'}$	$\lambda_\Delta = \dfrac{2.0}{1+50\dfrac{0.62\ \text{in.}^2}{(14\ \text{in.})(20\ \text{in.})}} = 1.8$
24.2.4.1.3	From Table 24.2.4.1.3, the time dependent factor for sustained load duration of more than 5 years $\xi = 2.0$ Therefore, long-term deflection is: $\Delta_T = (1+\lambda_\Delta)\Delta_i$	$\Delta_T = (1+1.8)(0.56\ \text{in.} - 0.20\ \text{in.}) = 1.0\ \text{in.}$
24.2.2	Check sustained load deflection limit from Table 24.2.2. Assume that the floor is supporting or attached to nonstructural elements not likely to be damaged by large deflections. Use $\ell/240$	$\Delta_{all} = (18\ \text{ft})(12\ \text{in./ft})/240 = 0.9\ \text{in.}$ $\Delta_{all} = 0.9\ \text{in.} < \Delta_{TD} = 1.0\ \text{in.}$ **NG** Sustained load deflections exceeds limit. Refine the sustained load deflection calculation to use effective moment of inertia, provide camber of 1 in. to offset time-dependent deflection, or increase compressive steel reinforcement area

Step 8. Details





Beam Example 8 *Determination of closed ties required for the beam shown to resist shear and torque*

Design and detail a simply supported precast edge beam spanning 29 ft 6 in. (Fig. E8.1). The beam is subjected to a factored load of 4.72 kip/ft. Structural analysis provided a factored shear and torsion of 61 kip and 53 ft-kip, respectively. Assume that the boundary conditions are such that torsion is required for equilibrium and cannot be distributed internally.

Given:

f_c' = 5000 psi (normalweight concrete)
λ = 1.0
f_y = 60,000 psi
d = 21.5 in.
V_u = 61 kip
T_u = 53 ft-kip
w_u = 4.72 kip/ft

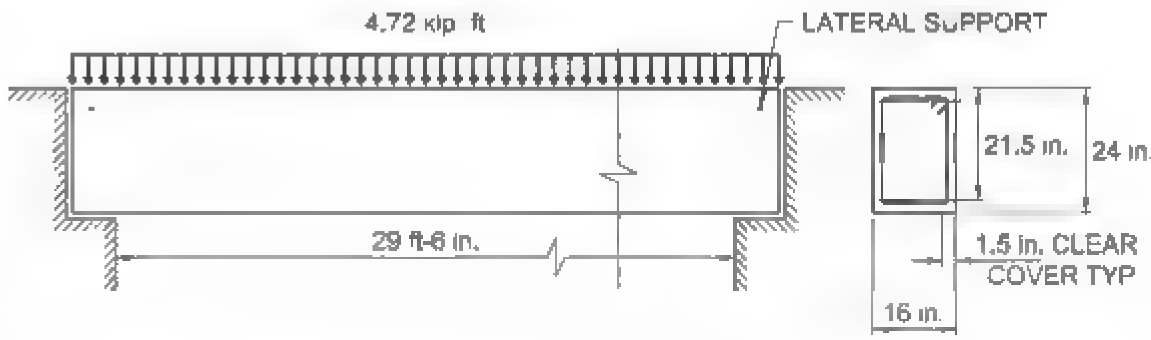
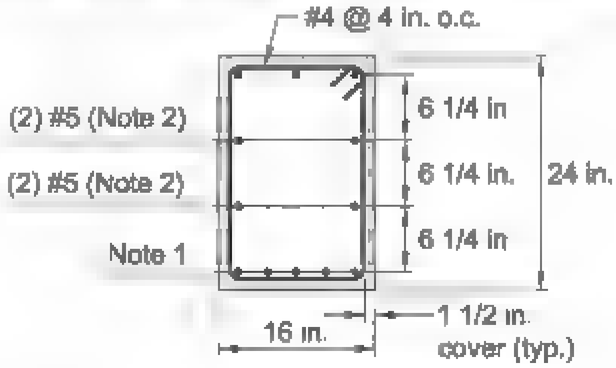


Fig. E8.1—Beam subjected to determinate torque.

ACI 318	Discussion	Calculation
Step 1. Section properties		
	Determine section properties for torsion	
9.2.4.4	$A_{cp} = b_w h$	A_{cp} = 16 in.(24 in.) = 384 in.^{2}
22.7.6.1	$A_{oh} = (b_w - 3.5\text{ in.})(h - 3.5\text{ in.})$	A_{oh} = (16 in. – 3.5 in.)(24 in. – 3.5 in.) = 256 in.^{2}
22.7.6.1.1	$A_o = 0.85A_{oh}$	A_o = 0.85(256 in.^{2}) = 218 in.^{2}
	$p_{cp} = 2(b_w + h)$	p_{cp} = 2(16 in. + 24 in.) = 80 in.
	$p_h = 2(b_w - 3.5\text{ in.} + h - 3.5\text{ in.})$	p_h = 2(16 in. – 3.5 in. + 24 in. – 3.5 in.) = 66 in.
Step 2: Cracking torsion		
22.7.5.1	Calculate cracking torsion T_{cr} $$\phi T_{cr} = 4\phi\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$	$$\phi T_{cr} = 4(0.75)(1.0)(\sqrt{5000\text{ psi}})\left(\frac{(384\text{ in.}^2)^2}{80\text{ in.}}\right)$$ = 391,000 in.-lb
21.2.1c	where torsion strength reduction factor ϕ = 0.75	ϕT_{cr} = (391,000 in.-lb)/(12,000 in.-lb/ft-kip) = 32.6 ft-kip
9.5.4.1 22.7.4.1a	Calculate threshold torsion $T_{th} = 0.25T_{cr}$	Threshold torsion = 0.25(32.6 ft-kip) = 8.2 ft-kip Because T_u = 53 ft-kip > 8.2 ft-kip, ties for torsion are therefore required
22.7.7.1	Is section large enough?	
	Calculate $f_v = V_u/(b_w d)$ Calculate $f_{vt} = T_u p_h/(1.7A_{oh}^2)$	f_v = 61 kip/(16 in. × 21.5 in.) = 0.177 ksi f_{vt} = (53 ft-kip)(12 in./ft)(66 in.)/[1.7(256 in.^{2})^2] = 0.377 ksi
	Calculate limit = $\phi(2\sqrt{f_c'} + 8\sqrt{f_c'})$	Limit = 0.75(2 + 8)($\sqrt{5000\text{ psi}}$) = 0.53 ksi
	Is $\sqrt{f_v^2 + f_{vt}^2}$ < limit?	$\sqrt{(0.177\text{ ksi})^2 + (0.377\text{ ksi})^2}$ = 0.416 ksi 0.416 ksi < limit 0.53 ksi Therefore, section is large enough.

Step 3: Calculate shear and torsion reinforcement		
	Assume that minimum shear reinforcement will be provided	
9.5.1.1 22.5.1.1 22.5.5.1 22.5.8.5.3 21.2.1b	Required shear tie area/spacing $\frac{A_v}{s} = \frac{(V_u - 2\phi\lambda\sqrt{f_c'}b_w d)}{\phi f_y d}$	$\frac{A_v}{s} = \frac{(61\text{ kip} - 2(0.75)(1.0)(\sqrt{5000\text{ psi}})(16\text{ in.})(21.5\text{ in.})}{(0.75)(60{,}000\text{ psi})(21.5\text{ in.})}$ $\frac{A_v}{s} = 0.0253\text{ in.}^2/\text{in.}$
22.7.6.1a 22.7.6.1.2	Required torsional tie area/spacing $\frac{A_t}{s} = \frac{T_u}{2\phi A_o f_y \cot\theta}$	$\frac{A_t}{s} = \frac{(53\text{ ft-kip})(12\text{ in./ft})}{2(0.75)(218\text{ in.}^2)(60\text{ ksi})\cot 45°} = 0.0324\text{ in.}^2/\text{in.}$
9.6.4.2	Calculate total tie area/spacing $(A_v/s + 2A_t/s)$	$\frac{A_v}{s} + 2\frac{A_t}{s} = 0.0253\text{ in.}^2/\text{in.} + 2(0.0324\text{ in.}^2/\text{in.})$ $= 0.09\text{ in.}^2/\text{in.}$ $s = 0.40\text{ in.}^2/(0.09\text{ in.}^2/\text{in.}) = 4.44\text{ in.}$
	Use No. 4 ties for which $(A_v + 2A_t) = 0.40$ in. Calculate $s = 0.40/(A_v/s + 2A_t/s)$.	Use 4 in.
	Check minimum transverse reinforcement. Is $\frac{0.75\sqrt{f_c'}b_w}{f_{yt}} < \left(\frac{A_v}{s} + \frac{2A_t}{s}\right)$?	$\frac{0.75(\sqrt{5000\text{ psi}})(16\text{ in.})}{60{,}000\text{ psi}} < \left(\frac{0.4\text{ in.}^2}{4\text{ in.}} + \frac{2(0.2\text{ in.}^2)}{4\text{ in.}}\right)$ $0.0141\text{ in.} < 0.2\text{ in.}$ **OK**

Step 4. Longitudinal reinforcement		
22.7.6.1 22.7.6.1.2	Calculate torsional longitudinal reinforcement from Eq. (22.7.6.1b)	
	$T_n = \frac{2A_oA_\ell f_y}{p_h}\tan\theta$	$\frac{(53\text{ ft-kip})(12\text{ in./ft})}{0.75} = \frac{2(218\text{ in.}^2)A_\ell(60\text{ ksi})}{66\text{ in.}}\tan 45°$
	Set $T_n = \frac{T_u}{\phi}$	$A_\ell = 2.14\text{ in.}^2$
9.6.4.3	The torsional longitudinal reinforcement $A_{\ell,min}$ must be the lesser of:	
	(a) $\frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}$	$\frac{5(\sqrt{5000\text{ psi}})(384\text{ in.}^2)}{60{,}000\text{ psi}} - (0.0324\text{ in.}^2/\text{in.})(66\text{ in.})\frac{60\text{ ksi}}{60\text{ ksi}}$ $= 0.12\text{ in.}^2$ **Controls**
	(b) $\frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{25b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}$	$\frac{5(\sqrt{5000\text{ psi}})(384\text{ in.}^2)}{60{,}000\text{ psi}} - \left(\frac{25(16\text{ in.})}{60{,}000\text{ psi}}\right)(66\text{ in.})\frac{60\text{ ksi}}{60\text{ ksi}}$ $= 1.82\text{ in.}^2$ $A_{\ell,prov} = 2.14\text{ in.}^2 > A_{\ell,req'd} = 0.12\text{ in.}^2$ **OK**
9.7.5.1	Distribute torsional longitudinal reinforcement around the perimeter of closed stirrups that satisfy Section 25.7.1.6 (ACI 318) (ends of stirrups are terminated with 135-degree standard hooks around a longitudinal bar)	
9.7.6.3.3	Tie spacing must not exceed the lesser of $p_h/8$ and 12 in.	$p_h/8 = 66\text{ in.}/8 = 8.25\text{ in.} < 12\text{ in.}$
	Use ten No. 5 longitudinal bars are required, three top and bottom, and two in each vertical face.	Try ten No. 5 bars. Longitudinal bars must satisfy spacing and diameter limits and provide sufficient design moment strength $A_{\ell,prov} = (10)(0.31\text{ in.}^2) = 3.1\text{ in.}^2$ **OK**
	At a distance d from support, M_u decreases by the amount of: $\Delta M_u = V_ud - w_ud^2/2$	$\Delta M_u = (61\text{ kip})(21.5\text{ in.}/12) - (4.72\text{ kip/ft})(21.5\text{ in.}/12)^2/2$
	The amount of flexural reinforcement required to resist	$\Delta M_u = 102\text{ ft-kip}$
	ΔM_u: $\Delta M_u = \phi f_yA_s(d - a/2) \approx \phi f_yA_s(0.9d)$	$\Delta M_u = (0.9)(60\text{ ksi})A_s(21.5\text{ in.})(0.9) = (102\text{ ft-kip})(12)$ $A_s = 1.17\text{ in.}^2$
9.7.5.2	No. 5 bar diameter must be at least equal to 0.042 times the transverse reinforcement spacing, but not less than 3/8 in.	$(0.042)(4\text{ in.}) = 0.168\text{ in.} < 0.625\text{ in.} = \text{No. 5}$ **OK** No. 5 > 3/8 in. **OK**
	Use No. 5 longitudinal bars. Place five No. 5 in bottom, two No. 5 in each side face, and three No. 5 in top.	

Step 5. Beam detailing

Notes 1 Bottom reinforcing bars summation of flexure and torsion reinforcement requirements. 2 Side reinforcing bar due to torsional moment

Beam Example 9 *Determine closed ties required for the beam of Example 8 to resist shear and torque*

Use the same data as that for Example 8 except that the factored torsion of 53 ft-kip is not an equilibrium requirement, but because the structure is indeterminate, can be redistributed if the beam cracks

Given:

f_c' = 5000 psi (normalweight concrete)
λ = 1.0
$f_y = f_{yt}$ = 60,000 psi
b_w = 16 in.
h = 24 in.
V_u = 61 kip
T_u = 53 ft-kip

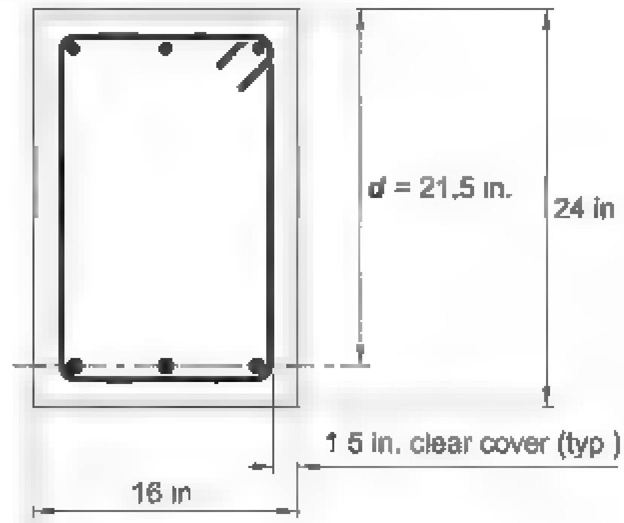


Fig. E9.1—Beam subjected to torque.

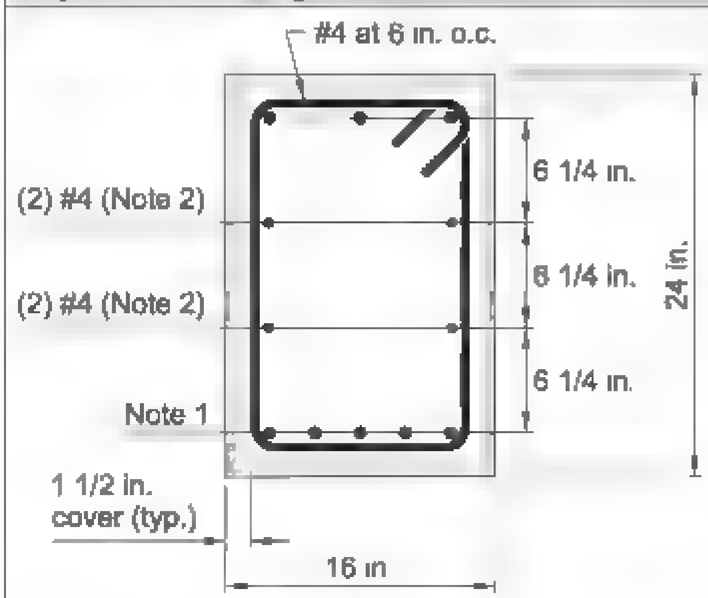
ACI 318	Discussion	Calculation
Step 1: Section properties		
	Determine section properties for torsion.	
9.2.4.4	$A_{cp} = b_w h$	A_{cp} = (16 in.)(24 in.) = 384 in.^{2}
22.7.6.1	$A_{oh} = (b_w - 3.5\text{ in.})(h - 3.5\text{ in.})$	A_{oh} = (16 in. − 3.5 in.)(24 in. − 3.5 in.) = 256 in.^{2}
22.7.6.1.1	$A_o = 0.85A_{oh}$	A_o = 0.85(256 in.^{2}) = 218 in.^{2}
	$p_{cp} = 2(b_w + h)$	p_{cp} = 2(16 in. + 24 in.) = 80 in.
	$p_h = 2(b_w - 3.5\text{ in.} + h - 3.5\text{ in.})$	p_h = 2(16 in. − 3.5 in. + 24 in. − 3.5 in.) = 66 in.
Step 2: Threshold torsion		
9.5.4.1 22.7.4.1a	Calculate threshold torsion. $\phi T_{th} = \phi\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	$\phi T_{th} = (0.75)(1.0)(\sqrt{5000\text{ psi}})\left(\frac{(384\text{ in.}^2)^2}{80\text{ in.}}\right) = 97{,}750$ in.-lb
9.5.1.2 21.2.1c	Torsion strength reduction factor ϕ = 0.75	
9.5.1.1	Check if $T_u > \phi T_{th}$	ϕT_{th} = 8.15 ft-kip T_u = 53 ft-kip > ϕT_{th} = 8.15 ft-kip **OK** Design section to resist torsional moment
22.7.5.1	Calculate cracking torsion $\phi T_{cr} = \phi 4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	ϕT_{cr} = 32.6 ft-kip
9.5.1.1	Check if $T_u > \phi T_{cr}$? In statically indeterminate structures where $T_u > \phi T_{cr}$, a reduction of T_u in the beam can occur due to redistribution of internal forces after torsion cracking. Therefore, reduce T_u to ϕT_{cr}.	T_u = 53 ft-kip > ϕT_{cr} = 32.6 ft-kip Use T_u = 32.6 ft-kip and design for torsional reinforcement

Beams

22.7.7.1	Check if cross-sectional dimensions are large enough $\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$	$\sqrt{\left(\frac{61{,}000\text{ lb}}{(16\text{ in.})(21.5\text{ in.})}\right)^2 + \left(\frac{(32.6\text{ ft-kip})(12{,}000)(66\text{ in.})}{1.7(256\text{ in.}^2)^2}\right)^2}$ $\le (0.75)(2\sqrt{5000\text{ psi}} + 8\sqrt{5000\text{ psi}})$ $\sqrt{(177.3\text{ psi})^2 + (231.7\text{ psi})^2} = 292\text{ psi} \le 530\text{ psi}$ **OK** Therefore, section is large enough
Step 3. Torsional reinforcement		
9.5.4.1 22.7.6.1a 22.7.6.1.2	Find area/spacing of ties due to shear and torsional moment Calculate torsional tie area/spacing $\phi T_n = \phi\frac{2A_o A_t f_{yt}}{s}\cot\theta \ge T_u = 32.6$ ft-kip	$\frac{A_t}{s} = \frac{(32.6\text{ ft-kip})(12{,}000)}{2(0.75)(218\text{ in.}^2)(60{,}000\text{ psi})(\cot 45°)}$ $= 0.0199\text{ in.}^2/\text{in.}$
9.5.1.1 22.5.1.1 22.5.5.1 22.5.8.5.3 21.2.1b	Calculate shear tie area/spacing $\phi V_n = \phi V_c + \phi V_s \ge V_u = 61$ kip and $\phi V_s = \phi\frac{A_v f_{yt} d}{s}$	
	Calculate total tie area/spacing ($A_v/s + 2A_t/s$)	$\frac{A_v}{s} = \frac{\frac{61{,}000\text{ lb}}{0.75} - 2\sqrt{5000\text{ psi}}(16\text{ in.})(21.5\text{ in.})}{(60{,}000\text{ psi})(21.5\text{ in.})}$ $\frac{A_v}{s} = 0.0253\text{ in.}^2/\text{in.}$
9.6.4.2	Try No. 4 ties and calculate s	$\frac{A_v}{s} + 2\frac{A_t}{s} = 0.0253\text{ in.}^2/\text{in.} + 2(0.0199\text{ in.}^2/\text{in.})$ $= 0.065\text{ in.}^2/\text{in.}$ $s = \frac{0.4\text{ in.}^2}{0.065\text{ in.}^2/\text{in.}} = 6.1\text{ in.}$ Use s = 6 in.
22.7.6.1b	Torsional longitudinal reinforcement $T_n = \frac{2A_o A_\ell f_y}{p_h}\tan\theta$	$\frac{(32.6\text{ ft-kip})(12\text{ in./ft})}{0.75} = \frac{2(218\text{ in.}^2)A_\ell(60\text{ ksi})}{66\text{ in.}}\tan 45°$ A_ℓ [illegible] 32 in.^{2}

9.6.4.3	The minimum torsional longitudinal reinforcement ℓ_{min} must be at least the lesser of:	
	(a) $\frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}$	$\frac{5(\sqrt{5000\text{ psi}})(384\text{ in.}^2)}{60{,}000\text{ psi}} - (0.0324\text{ in.}^2/\text{in.})(66\text{ in.})\frac{60\text{ ksi}}{60\text{ ksi}}$ $= 0.12\text{ in.}^2$ **Controls**
	(b) $\frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{25b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}$	$\frac{5(\sqrt{5000\text{ psi}})(384\text{ in.}^2)}{60{,}000\text{ psi}} - \left(\frac{25(16\text{ in.})}{60{,}000\text{ psi}}\right)(66\text{ in.})\frac{60\text{ ksi}}{60\text{ ksi}}$ $= 1.82\text{ in.}^2$ $A_{\ell,prov} = 1.32\text{ in.}^2 > A_{\ell,req'd} = 0.12\text{ in.}^2$ **OK**
9.7.5.1	Distribute torsional longitudinal reinforcement around the perimeter of closed stirrups that satisfy Section 25.7.1.6 (ends of stirrups are terminated with 135-degree standard hooks around a longitudinal bar)	
9.7.6.3.3	Transverse torsional reinforcement spacing must not exceed the lesser of $p_h/8$ and 12 in.	$p_h/8 = 66\text{ in.}/8 = 8.25\text{ in.} < 12\text{ in.}$
	Use No. 4 longitudinal bars. Place five No. 4 in bottom and two No. 4 in each side face. Excess flexural capacity in top at d from support can serve in place of three No. 4 in top.	Refer to Beam Example 8

Step 4. Beam detailing



Notes:

1. Bottom bars summation of moment and torsion reinforcement requirements.
2. Side bar due to torsional moments

Beam Example 10: *Precast prestressed double-tee beam*
Design and detail precast double-tee beam for parking garage

Given:

Loads—
Superimposed dead load D = 10 lb/ft^2
Live load L = 40 lb/ft^2 (passenger cars) with concentrated wheel load
Self weight calculated as follows

Material properties—
f_c' = 5000 psi
f_{ci}' = 3500 psi
f_y = 60,000 psi
f_{pu} = 270,000 psi

Length = 60 ft (assume span is equal to length)
Double-tee section = Pretopped 10DT26 PCI Design Manual 8th Edition (Fig. E10.1)

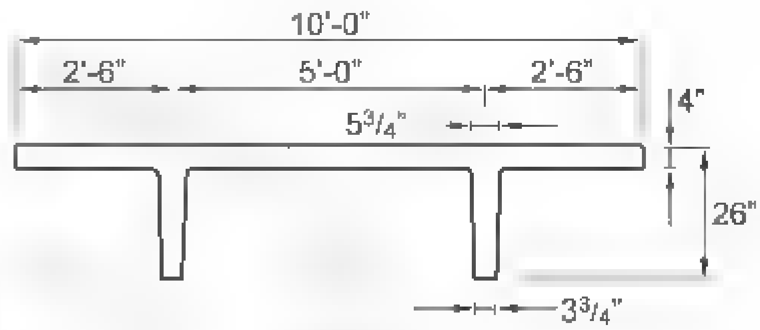


Fig. E10.1—Double-tee geometry and section properties used for design.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
9.2.1.1	The mixture proportions must satisfy the durability requirements of Chapter 19 (ACI 318) and structural strength requirements. The designer determines the durability classes. Please refer to Chapter 2 of MNL-17 for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor. Concrete properties, design information, compliance requirements, and other construction information for the contractor must be included in the construction documents in accordance with Chapter 26.	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied. Because parking garages (particularly in northern climates) are subjected to the harsh conditions of freezing-and-thawing in combination with chloride exposure due to road salts, the use of concrete is paramount. ACI 362.1R-12 provides detailed discussion and guidance on the design of durable parking structures and should be consulted as part of the double-tee design. If this double tee is produced in a precast plant, then the concrete strength will likely be higher than necessary for design to ensure that release strength is reached in as short a time as practicable. While purely for production purposes, this high early strength and plant production will simultaneously enhance the durability. Based on durability and strength requirements, and experience of the local precaster, the compressive strength of concrete is specified at 28 days to be at least 5000 psi. The minimum concrete strength at which prestress transfer can occur is typically determined by the precaster based on the strength gain of their mixtures and their production schedule among others. For this problem, use 3500 psi.

Beams

Step 2. Beam geometry

9.3.1.1	Precast prestressed double-tee sections are commonly used in parking structures. For this example, the preliminary design tables of the PCI Design Manual are used to select 10DT26. This example illustrates the design checks necessary for double-tee design. Use the pretopped section properties for this example. Depth of the member is 26 in. Standard practice is to assume that the flange of double-tee beams is fully effective.	$h = 26$ in. $A = 689$ in.^{2} $I = 30{,}716$ in.^{4} $y_b = 20.29$ in. $y_t = 5.71$ in.
	Availability and section details of double-tee beams should be verified locally for individual projects. In many cases, the engineer of record specifies the project and load requirements of the floor system and the specialty engineer provides the detailed design.	$S_b = \frac{I}{y_b} = 1514$ in.^{3} $S_t = \frac{I}{y_t} = 5379$ in.^{3} $A_{ps} = 12(0.153$ in.$^2) = 1.836$ in.^{2}
	Preliminary tendon size is a single-point harped tendon with six 0.5 in. diameter seven-wire prestressing strands placed in each web. Service and strength calculations will include the entire section and use a single 12-strand tendon. Tendon centroid is specified by its distance from the bottom of the section of 11.67 in. at the end of the member and 3.25 in. at the midspan. Tendon eccentricity is then calculated from these covers. Tendon profile is shown in Fig. E10.2.	$e_e = 20.29$ in. $-$ 11.67 in. $= 8.62$ in. $e_c = 20.29$ in. $-$ 3.25 in. $= 17.04$ in.

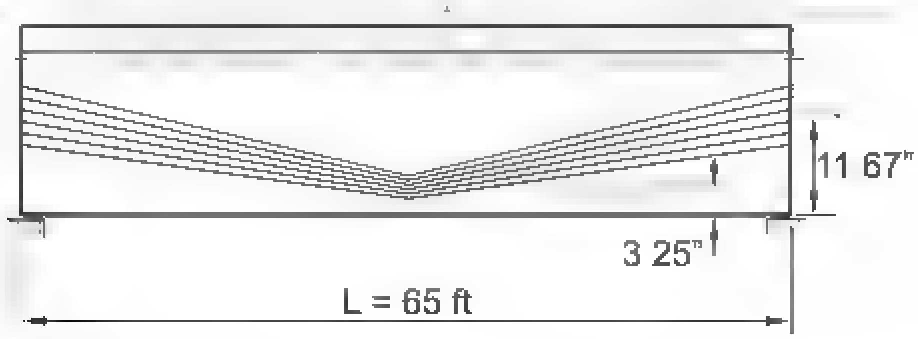


Fig. E10.2—Selected tendon profile.

Step 3. Loads		
	ASCE/SEI 7 service load requirement for passenger vehicles is 40 lb/ft^2 with no live load reduction allowed. The garage must also be checked for concentrated load of 3 kip over a 4.5 x 4.5 in. area	$w_L = 10\text{ ft}(40\text{ lb/ft}^2) = 0.4\text{ kip/ft}$
	Prestressed members must be checked for service stresses in flexure, and strength in shear and flexure. This requires the use of unfactored and factored loads and actions.	
		Compute self-weight from section area.
	Dead load is a combination of self-weight and superimposed dead load	$w_{sw} = \dfrac{689\text{ in.}^2(150\text{ lb/ft}^3)}{(12\text{ in./ft})^2} = 0.718\text{ kip/ft}$ $w_D = 0.718\text{ kip/ft} + 10\text{ ft}(10\text{ lb/ft}^2) = 0.818\text{ kip/ft}$
5.3.1		$U = 1.4(0.818\text{ kip/ft}) = 1.15\text{ kip/ft}$
		$U = 1.2(0.818\text{ kip/ft}) + 1.6(0.4\text{ kip/ft}) = 1.62\text{ kip/ft}$
Step 4. Prestressing tendon size and profile		
	The prestressing tendon size and profile has been selected based on load tables	
20.3.1.1 20.3.2.2	Prestressing strands ASTM A416	$f_{pu} = 270{,}000\text{ psi}$
20.3.2.5.1	Prestress losses must be estimated to determine effective prestress levels that will be used in both service and strength calculations. For precast prestressed concrete, strands are typically stressed against an abutment in the precast yard and located at a stress of approximately 75% of the ultimate strand strength	$f_{pj} = 0.75f_{pu} = 202{,}500\text{ psi}$
20.3.2.5.1	Concrete is then cast around the strands. Cutting the strands results in a slight shortening of the tendon as the prestress force is imposed on the section. This is the elastic loss. For the purposes of this example, use an elastic loss of 10% of the initial stress in the strands	$f_{pi} = 0.9 \cdot 0.75f_{pu} = 182{,}250\text{ psi}$
20.3.2.6	To account for both the short-term and long-term prestress losses, use a lump sum value of 30 ksi deducted from the initial stress in the strands	$f_{se} = 0.75f_{pu} - 30\text{ ksi} = 172{,}500\text{ psi}$
R20.3.2.6.1	For detailed methods to compute prestress losses, refer to ACI 423.10	
	Calculate the effective prestress force considering all 12 strands	$A_{ps} = 12(0.153\text{ in.}^2) = 1.836\text{ in.}^2$ $P = 182{,}250\text{ psi}(1.836\text{ in.}^2) = 335\text{ kip}$ $P_e = 172{,}500\text{ psi}(1.836\text{ in.}^2) = 317\text{ kip}$
25.4.8.1 21.2.3	Transfer length is the first term of the development length equation.	$\ell_{tr} = \dfrac{172{,}500\text{ psi}}{3000\text{ psi}}0.5\text{ in.} = 28.75\text{ in.}$

24.5.2.1	Classify flexural member to determine the appropriate section properties to use for service stresses and deflection.	$f_t < 7.5\sqrt{f_c'} = 530$ psi Uncracked $f_t < 12\sqrt{f_c'} = 848$ psi Transition $f_t > 12\sqrt{f_c'} = 848$ psi Cracked
24.5.3.1	Limit compressive stresses at prestress transfer based on the initial concrete strength and location	

Location	Concrete compressive stress limits
End of simply supported members	$0.70f_{ci}' = 2450$ psi
All other locations	$0.60f_{ci}' = 2100$ psi

Limit tensile stresses at prestress transfer based on the initial concrete strength and location. Assume that no additional bonded reinforcement is provided to control cracking.

Location	Concrete tensile stress limits
End of simply supported members	$6\sqrt{f_{ci}'} = 355$ psi
All other locations	$3\sqrt{f_{ci}'} = 177$ psi

Concrete compressive stress limits at service loads

Location	Concrete compressive stress limits
Prestress plus sustained load	$0.45f_c' = 2250$ psi
Prestress plus total load	$0.60f_c' = 3000$ psi

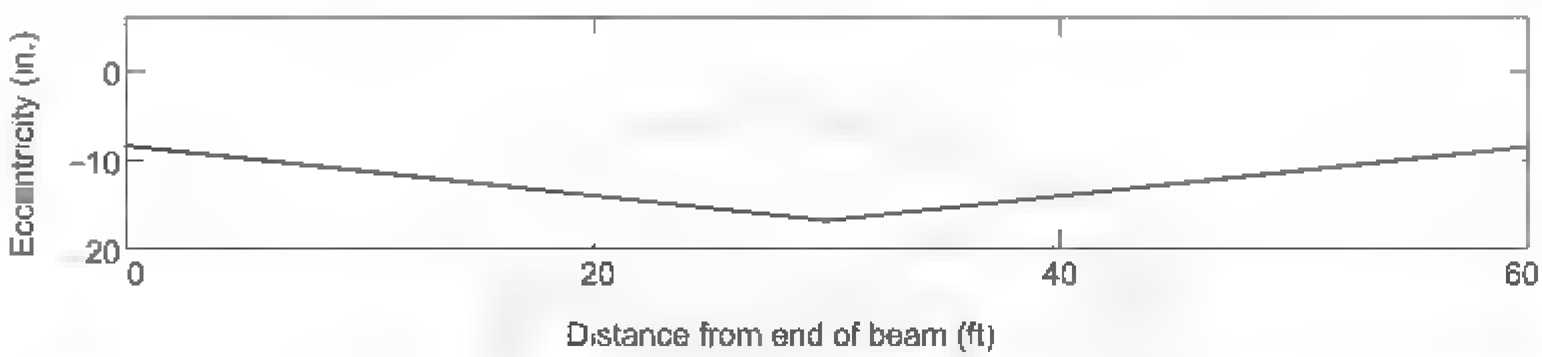


Fig. E10.3—Tendon profile.

Step 5. Analysis

	To determine moment and shear, assume that the supports are on knife edges.	
	Maximum service and factored moment occur at midspan	$M = \frac{1}{8}(0.818\text{ kip/ft} + 0.4\text{ kip/ft})(60\text{ ft})^2 = 548\text{ ft-kip}$
		$M_u = \frac{1}{8}(1.62\text{ kip/ft})(60\text{ ft})^2 = 729\text{ ft-kip}$
	Maximum factored shear is at the support	$V_u = 1/2(1.62\text{ kip/ft})(60\text{ ft}) = 48.6\text{ kip}$

Step 6. Concrete stress at transfer		
24.5.3	For precast elements, prestress transfer occurs in the casting bed after the concrete has reached f_{ci}'. When the strands are cut, the prestress force is applied to the section, resulting in camber. This action causes the supporting element to be supported at each end, effectively creating a loading condition of self-weight simultaneously with the application of prestress. This stress state must be investigated to ensure that the concrete is not overstressed at this stage.	$M_{sw} = \frac{1}{8}(0.718\text{ kip/ft})(60\text{ ft})^2 = 323\text{ ft-kip}$
	Calculate the net compressive stresses in the bottom of the section. Maximum value will occur at the bottom of the section near the end of the member at the end of the transfer length (Fig. E10.4). Conservatively check the end of the member ignoring self-weight moment.	
	$f_{bot} = \frac{P_i}{A} + \frac{P_i \cdot e_e}{S_b}$	$f_{bot} = \frac{335\text{ kip}}{689\text{ in.}^2} + \frac{335\text{ kip}(8.62\text{ in.})}{1514\text{ in.}^3} = 2394\text{ psi}$
24.5.3.1	Compare to allowable stresses at transfer of prestress at the end of the simply supported member.	$0.7(3500\text{ psi}) = 2450\text{ psi}$ **OK**
	All other locations.	$0.6(3500\text{ psi}) = 2100\text{ psi}$ **OK**
	Calculate the net tensile stresses in the top of the section ignoring the reduction of prestress force over the transfer length. Most of the rest of the member will have net compression at the top of the section. (negative indicates tension)	
	$f_{top} = \frac{P_i}{A} - \frac{P_i e_e}{S_t}$	$f_{top} = \frac{335\text{ kip}}{689\text{ in.}^2} - \frac{335\text{ kip}(8.62\text{ in.})}{5379\text{ in.}^3} = -51\text{ psi}$
24.5.3.2	Compare to the allowable tensile stresses at transfer of prestress at the end of simply supported member.	$6\sqrt{3500}\text{ psi} = 354\text{ psi}$ **OK**
	All other locations.	$3\sqrt{3500}\text{ psi} = 177\text{ psi}$ **OK**
	Figure E10.4 shows the top and bottom net stresses after transfer plotted over the full length of the member. Plots such as these can be generated in spreadsheets or other software to allow full visualization of the stress state of the member in each stage. This is particularly useful when prestress force or eccentricity or both vary with member length. Spot checking of stresses may inadvertently miss a point of maximum or minimum stress.	

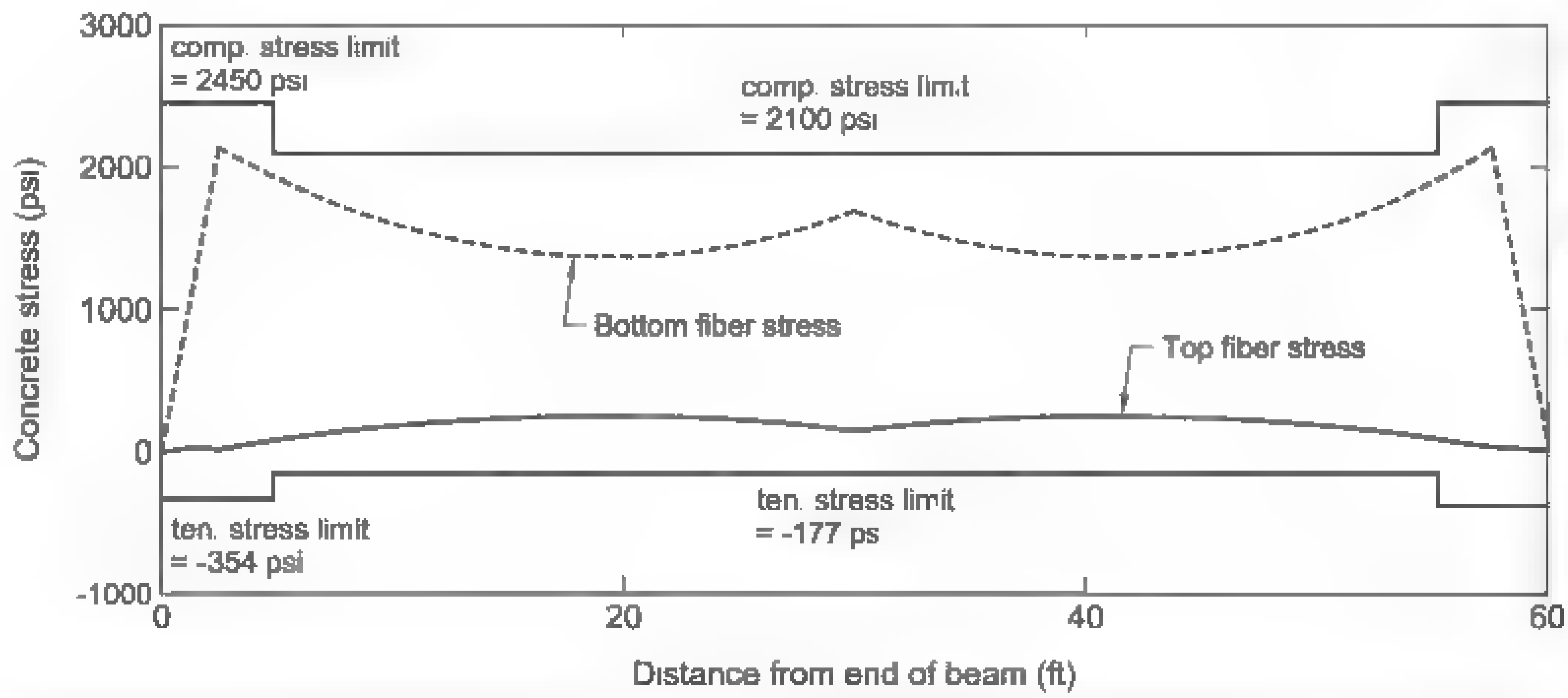


Fig. E10.4—Net concrete stresses at the top and bottom fiber of the double tee immediately following prestress transfer.

Step 7, Concrete stress under full service loads

Other intermediate stages should be checked depending on the construction sequence and timing—for instance, transportation and erection of the members or installation of composite toppings. These stress checks are similar to those illustrated here.

In this example, maximum compressive stresses remain at the bottom of the section near the support. Check compression in the top of the section near midspan. Because the prestress eccentricity varies linearly and the moment is second order, the peak compressive stress occurs away from the support. For this situation, the stresses are typically checked at ~0.4L.

Calculate moment at 0.4L

$M_{0.4L} = 0.5(w_D + w_L)[0.4L^2 - (0.4L)^2]$

$M_{0.4L} =$
$0.5(0.818\text{ kip/ft} + 0.4\text{ kip/ft})[0.4(60\text{ ft})^2 - (0.4 \times 60\text{ ft})^2]$
$M_{0.4L} = 526\text{ ft-kip}$

Calculate eccentricity at 0.4L

$$e_{0.4L} = \frac{0.4}{0.5}(e_m - e_e) + e_e$$

$$\frac{0.4}{0.5}(17.04\text{ in.} - 8.62\text{ in.}) + 8.62\text{ in.} = 15.4\text{ in.}$$

$$f_{top} = \frac{317\text{ kip}}{689\text{ in.}^2} - \frac{317\text{ kip}(15.4\text{ in.})}{5379\text{ in.}^3} + \frac{526\text{ ft-kip}}{5379\text{ in.}^3} = 726\text{ psi}$$

25.5.4.1 Check against compressive stress limits for full service load

0.6(5000 psi) = 3000 psi

Check against compressive stress limits for sustained load

0.45(5000 psi) = 2250 psi

Compressive stresses are less than the limits over the full member length

25.5.2.1 Net tension stresses in the precompressed tension zone are used to classify the member as Uncracked, Transition, or Cracked. This designation is then used to determine the appropriate section properties for use in stress and deflection calculations

$7.5\sqrt{f_c'}$ psi = 530.3 psi
$12\sqrt{f_c'}$ psi = 848.5 psi

Calculate the tension stress under full service load at 0.4L.

$$f_{bott} = -\frac{317\text{ kip}}{689\text{ in.}^2} + \frac{317\text{ kip}(15.4\text{ in.})}{1514\text{ in.}^3} - \frac{526\text{ ft-kip}}{1514\text{ in.}^3} = -485\text{ psi}$$

In general, double tee sections are designed to be in transition between cracked and uncracked (Class T), which allows the use of uncracked section properties for determining stresses, but requires the consideration of cracking when calculating deflections. For this example uncracked section properties may be used for both

Class U Use uncracked section properties

Stress state of entire member is shown in Fig. E10.5

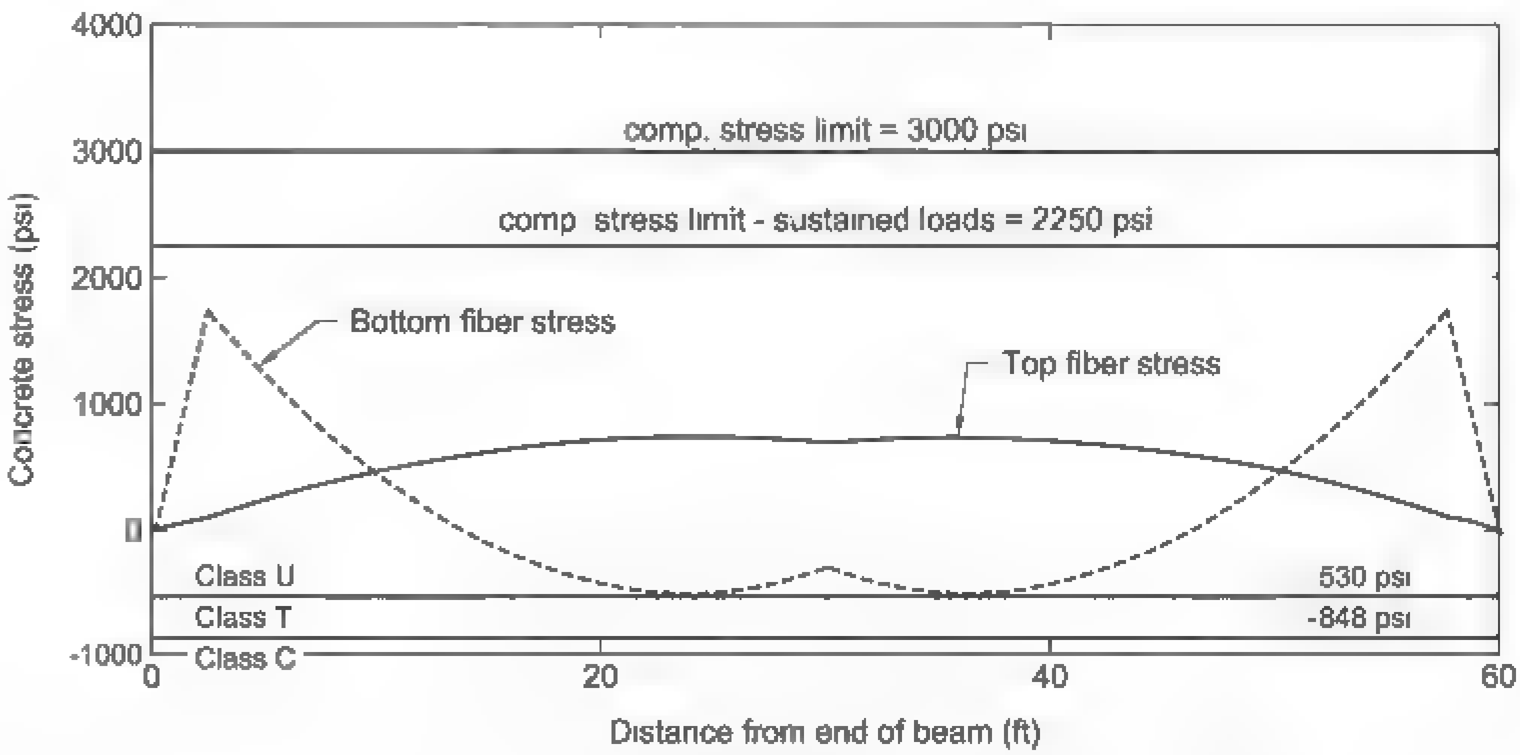


Fig. E10.5—Net concrete stresses at the top and bottom fiber of the double tee under full service loads.

Step 8. Design moment strength		
25.4.8.1	Calculate develop length to check against slip at the end of the member	$L_d = 28.75\text{ in.} + \dfrac{(f_{ps} - f_{se})}{1000\text{ psi}} 0.5\text{ in.} = 77.5\text{ in.}$
20.3.2.3.1	The stress-strain curve of prestressing strand does not have a well-defined yield point or associated yield plateau. Consequently, strain compatibility must be used to determine the stress in the prestressing strand. Alternatively, the Code allows the use of an empirical formula to determine the stress in the prestressing steel at nominal flexural strength (f_{ps}) which is $f_{ps} = f_{pu}\left\{1 - \dfrac{\gamma_p}{\beta_1}\left[\rho_p \dfrac{f_{pu}}{f'_c} + \dfrac{d}{d_p}\dfrac{f_y}{f'_c}(\rho - \rho')\right]\right\}$ (20.3.2.3.1) Most current software programs calculate the flexural strength using strain compatibility To demonstrate the process, use the empirical approach	$\dfrac{f_{py}}{f_{pu}} > 0.90$ for seven-wire prestressing strand. $\gamma_p = 0.28$ No mild steel in compression or tension $\rho = 0$ $\rho' = 0$ Prestressing steel reinforcement ratio at midspan $\rho_p = \dfrac{12(0.153\text{ in.}^2)}{120\text{ in.}(22.75\text{ in.})} = 0.000673$ $f_{ps} = 270{,}000\text{ psi}\left[1 - \dfrac{0.28}{0.8}\left(0.000673\dfrac{270{,}000}{5000}\right)\right] = 266{,}000\text{ psi}$ Prestressing steel reinforcement ratio at 0.4L: $\rho_p = \dfrac{12(0.153\text{ in.}^2)}{120\text{ in.}(15\text{ in.} + 5.71\text{ in.})} = 0.000739$ $f_{ps} = 270{,}000\text{ psi}\left[1 - \dfrac{0.28}{0.8}\left(0.000739\dfrac{270{,}000}{5000}\right)\right] = 266{,}000\text{ psi}$ Use $f_{ps} = 266$ ksi for moment strength at both midspan and 0.4L $a = \dfrac{12(0.153\text{ in.}^2)(266\text{ ksi})}{0.85(5000\text{ psi})(120\text{ in.})} = 0.958\text{ in.}$ Depth of stress block is less than the flange thickness.

20.3.2.3.1	Maximum moment occurs away from the end of the member. Transfer and development length will not control the required strength.	
21.2.2.2	Check if section is tension-controlled. For prestressed reinforcement, ε_{ty} should be assumed equal to 0.002 to determine the section classification according to the following limit: $\varepsilon_t \geq \varepsilon_{ty} + 0.003$	$\varepsilon_t = 0.003\left(\frac{\beta_1 22.75\text{ in.}}{0.958\text{ in.}} - 1\right) = 0.054$ $>0.002 + 0.003 = 0.005$ **OK** Section is tension-controlled. $\phi = 0.9$
21.2.2.1		Design moment strength at midspan: $\phi M_n = 0.9(266\text{ ksi})(12 \times 0.153\text{ in.}^2)\left(22.75\text{ in.} - \frac{0.958\text{ in.}}{2}\right)$ $\phi M_n = 816\text{ kip-ft}$ $M_u = \frac{1}{8}(1.62\text{ klf})(60\text{ ft})^2 = 729\text{ kip-ft}$ **OK** Design moment strength at $0.4L$: $\phi M_n = 0.9(266\text{ ksi})(12 \times 0.153\text{ in.}^2)\left(20.71\text{ in.} - \frac{0.958\text{ in.}}{2}\right)$ $\phi M_n = 741\text{ kip-ft}$ $M_u = 0.5(1.62\text{ klf})\left[0.4 \times 60\text{ ft}(60\text{ ft}) - (0.4 \times 60\text{ ft})^2\right] = 700\text{ kip-ft}$ **OK**
9.6.2.1	For beams with bonded prestressed reinforcement, the total quantity of reinforcement must be sufficient to develop at least $1.2M_{cr}$, where M_{cr} can be calculated using the following equation, which considers both prestressing and concrete tensile strength: $M_{cr} = S_t(7.5\lambda\sqrt{f_c'} + f_{pe})$ Determine if the prestressed flexural reinforcement satisfies the minimum flexural reinforcement requirements.	$f_{pe} = \frac{317\text{ kip}}{689\text{ in.}^2} + \frac{317\text{ kip}(11.4\text{ in.})}{1514\text{ in.}^3} = 2847\text{ psi}$ $1.2M_{cr} = 1.2(1514\text{ in.}^3)(7.5\sqrt{5000}\text{ psi} + 2847\text{ psi}) = 511\text{ kip-ft}$ $\phi M_n = 741\text{ kip-ft} > 511\text{ kip-ft}$ **OK** No additional reinforcement required.
	As with service moments, the flexural strength varies along the length of the member in proportion to the tendon eccentricity. In this example the flexural strength at approximately $0.4L$ from the end of the member controls flexural strength design. Figure E10.6 illustrates this variation and the need to evaluate the flexural strength over the entire member length. The figure also illustrates the decrease in design moment strength near the end of the member due to the potential for strand slip within the transfer and development length.	

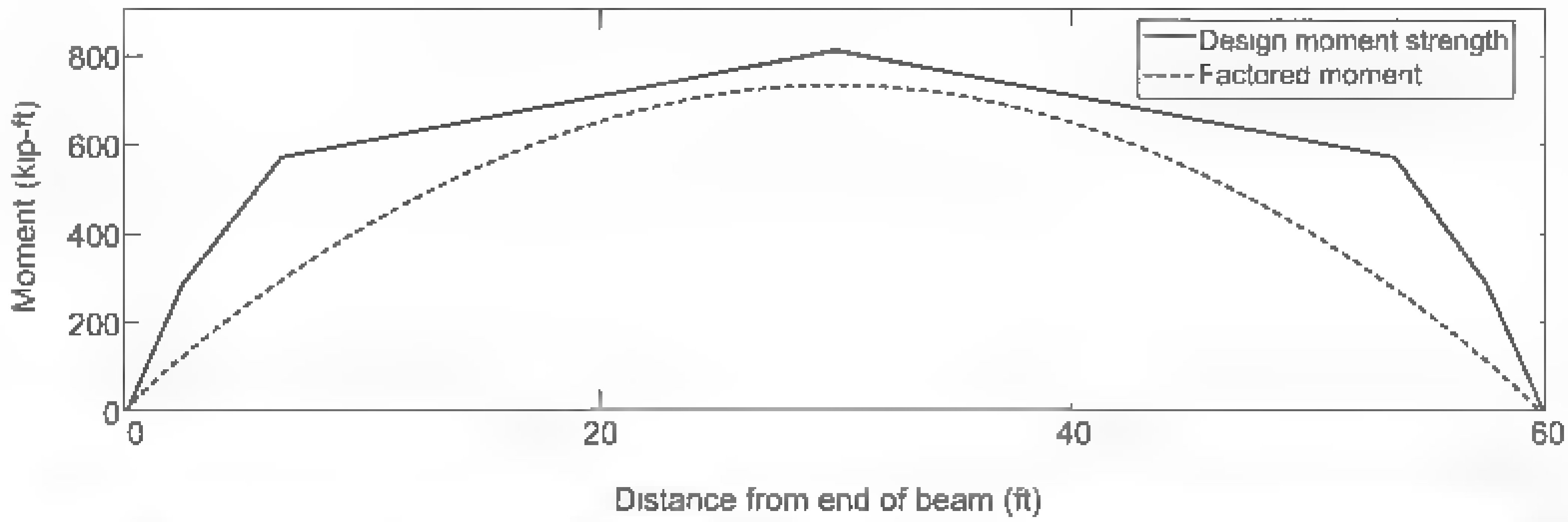


Fig. E10.6—Design moment strength envelope compared to factored moment diagram.

Step 9: Beam length and bearing

16.2.6
16.2.6.3

To design the beam for shear, use a bearing length of 5 in. at the supports, which will give a clear span of 59 ft 2 in. (Fig. E10.7). Check minimum distance from end of precast member to face of support:

$\ell_n/180$

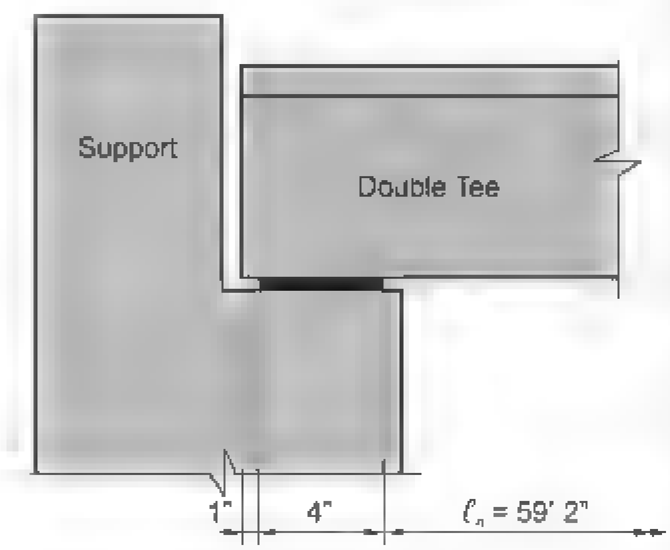


Fig. E10.7—Support conditions for double tee.

$$60\text{ ft} - \frac{2(5\text{ in.})}{12\text{ in./ft}} = 59.17\text{ ft}$$

$$\frac{59.17\text{ ft}}{180} = 3.9\text{ in.}$$

> 3 in., **OK**. Use 4 in. bearing pad with 1 in. space at end of member because double-tee stem will be unarmored.

Step 10: Shear design approach

22.5.6	For prestressed concrete members, the Code allows the use of either a simplified method for computing the concrete contribution to shear strength V_c (22.5.6.2), or a detailed method (22.5.6.3). This example will use the detailed method to calculate V_c. Unlike V_c for nonprestressed sections, V_c for prestressed sections varies with applied shear, moment, tendon eccentricity, and effective prestress, which can result in an unintuitive shape to the V_c diagram. Consequently, it desirable to plot the concrete contribution to shear (V_c) along with the factored shear diagram to determine the locations where transverse reinforcement is needed for strength and detailing. These plots can be developed in spreadsheet or calculation software or in software programs created for prestressed concrete design. Sample calculations follow at key points in the span to demonstrate the shear design approach.	

Step 11. Factored shear envelope

	The factored shear envelope is presented in Fig. E10.8. This envelope was develop using single-span live load pattern loading to determine the maximum possible shear at every location along the member length. Shear at the midspan, is due to the partial span loading in which half of the span is loaded and the other half is not.	
9.4.3.2	Calculate the maximum factored shear at the face of the support. The Code allows the use of the factored shear at $h/2$ from the face of the support to be considered the critical section for shear design. Also, calculate shear at midspan for the pattern live load where half of the span is covered with live load.	$V_{u,max} = 1.62\text{ klf}\left(0.5(59.2\text{ ft}) - 0.5\dfrac{26\text{ in.}}{12\text{ in./ft}}\right) = 46.2\text{ kip}$ $V_{u,mid} = 1.6\times0.4\text{ klf}\dfrac{59.2\text{ ft}}{8} = 4.8\text{ kip}$

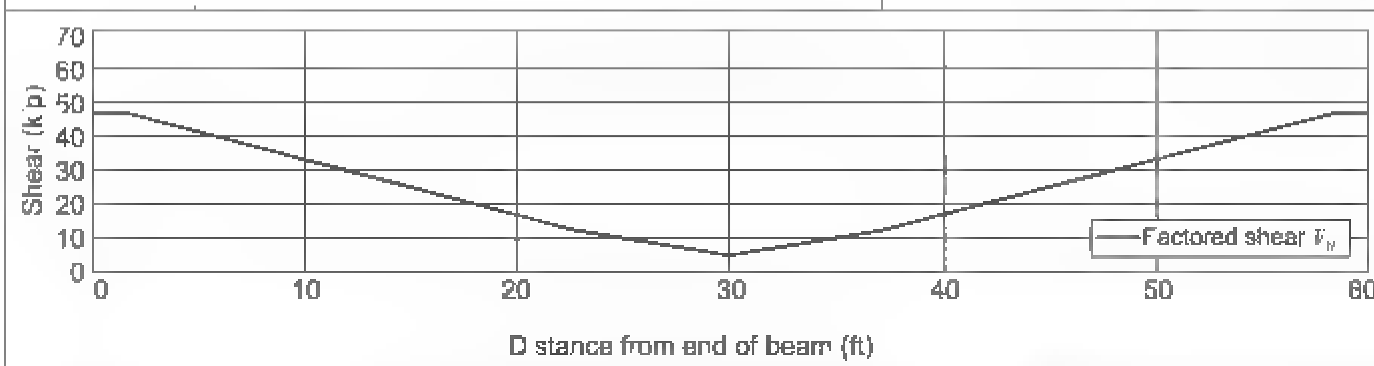


Fig. E10.8—Factored shear envelope including pattern live loading within span.

Step 12: Determine concrete contribution considering flexure-shear cracking V_{ci}

22.5.2.1	For beams containing harped or draped tendons, the Code allows the effective depth d used in shear calculations to be no less than $0.8h$ for prestressed members.	$0.8(26\text{ in.}) = 20.8\text{ in.}$

22.5.6.3	Two types of inclined cracking occur in concrete beams: web-shear cracking and flexure-shear cracking. Web shear cracking typically initiates in regions of high shear and moderate moment and begins with cracks forming in the web. Flexure-shear cracks typically form in regions of moderate shear and moment and begin with the formation of flexural cracks that extend into the web as load is increased. The nominal shear strength provided by the concrete is assumed to be the lesser of the shear required to form the two mechanisms V_{ci} and V_{cw}	
22.5.6.3.1	V_{ci} is calculated with the following equations: $V_{ci} = 0.6\lambda\sqrt{f'_c}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ but need not be less than For members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_s f_y)$, $V_{ci} = 1.7\lambda\sqrt{f'_c}b_w d$ For members with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_s f_y)$, $V_{ci} = 2\lambda\sqrt{f'_c}b_w d$	
R22.5.6.3.1	For noncomposite uniformly loaded beams, the Commentary provides the following simplification to the V_{ci} equation: $V_{ci} = 0.6\lambda\sqrt{f'_c}b_w d + \frac{V_u M_{ct}}{M_u}$ (R22.5.6.3.1d) where the cracking moment M_{ct} can be calculated using the following equation: $M_{ct} = (I / y_t)(6\lambda\sqrt{f'_c} + f_{pe})$ (R22.5.6.3.1e)	
	Figure E10.9 shows a plot of V_{ci} for this double tee. Note that V_{ci} is at its lowest value away from the support where the shear and moment are moderate. Compute V_{ci} at 10 ft from the support.	f_{pe} is the compressive stress in concrete due only to effective prestress forces, after allowance for all prestress losses, at extreme fiber of section if tensile stress is caused by externally applied loads $f_{pe} = \frac{317\text{ kip}}{689\text{ in.}^2} + \frac{317\text{ kip}(11.4\text{ in.})}{1514\text{ in.}^3} = 2847\text{ psi}$ $M_{ct} = 1514\text{ in.}^3\left(6\sqrt{5000}\text{ psi} + 2847\text{ psi}\right) = 413\text{ kip-ft}$ For web width use the average width of both webs: $b_w = 2(0.5)(3.75\text{ in.} + 5.75\text{ in.}) = 9.5\text{ in.}$ $d_p = \frac{10\text{ ft}}{30\text{ ft}}(17.04\text{ in.} - 8.62\text{ in.}) + 8.62\text{ in.} = 11.4\text{ in.}$ $0.8h = 20\text{ in.}$ **Controls** $V_u = 1.621\text{ klf}(20\text{ ft}) = 32.42\text{ kip}$ $M_u = 0.5 \times (1.621\text{ klf})[60\text{ ft}(10\text{ ft}) - (10\text{ ft})^2] =$ 405 kip-ft $V_{ci} = 0.6\sqrt{5000}\text{ psi}(9.5\text{ in.})(20.8\text{ in.}) + \frac{32.42\text{ kip}(413\text{ kip-ft})}{405\text{ kip-ft}} = 41.4\text{ kip-ft}$ **OK.** Agrees with plot.

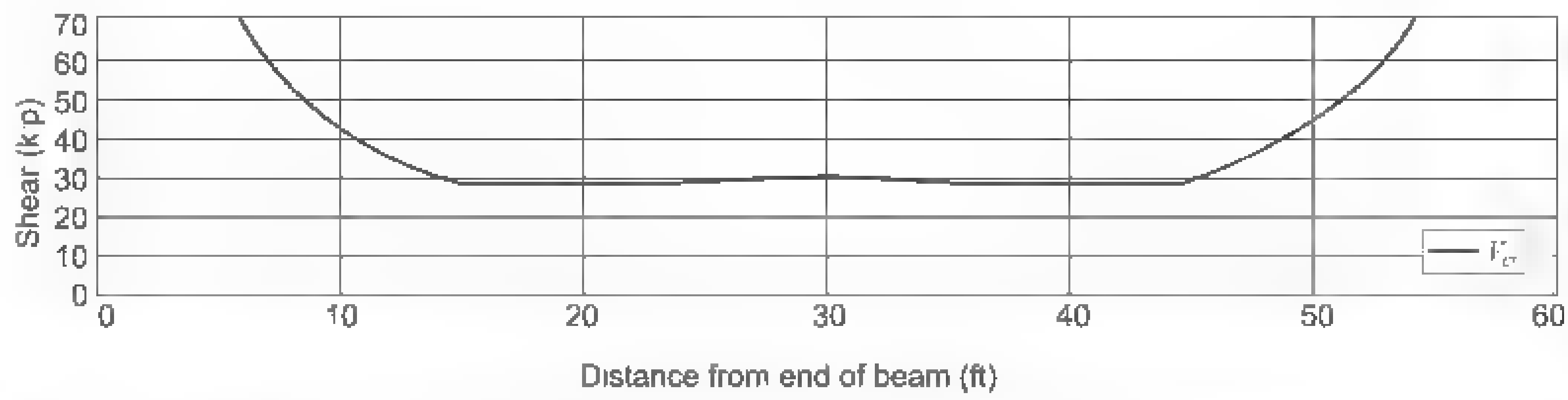


Fig. E10.9—Contribution of concrete to shear strength considering flexure-shear cracking.

Step 13: Determine concrete contribution considering web-shear cracking V_{cw}

22.5.6.3.2 V_{cw} is calculated with the following equation:

$$V_{cw} = (3.5\lambda\sqrt{f_c'} + 0.3f_{pc})b_w d_p + V_p$$

Figure E10.10 shows a plot of V_{cw} for this double tee. Note that V_{cw} is relatively constant. Near the support, the shear strength decreases due to the decrease in contribution from the prestressing force over the transfer length. Compute V_{cw} at 10 ft from the support and at the support where factored shear is maximum.

For noncomposite members, f_{pc} is the compressive stress in concrete, after allowance for all prestress losses, at the centroid of cross section.

$$f_{pc} = \frac{317\text{ kip}}{689\text{ in.}^2} = 460\text{ psi}$$

Vertical component of the effective prestress force:

$$V_p = 317\text{ kip}\frac{(17.04\text{ in.} - 8.62\text{ in.})}{30\text{ ft}} = 7.4\text{ kip}$$

$$V_{cw@10ft} = \left[3.5(\sqrt{5000}\text{ psi}) + 0.3(460\text{ psi})\right](9.5\text{ in.})(20.8\text{ in.}) + 7.4\text{ kip}$$

$$V_{cw@10ft} = 83.6\text{ kip}$$

$$V_{cw@support} = \left[3.5(\sqrt{5000}\text{ psi})\right](9.5\text{ in.})(20.8\text{ in.}) = 48.9\text{ kip}$$

OK Calculated values agree with plot

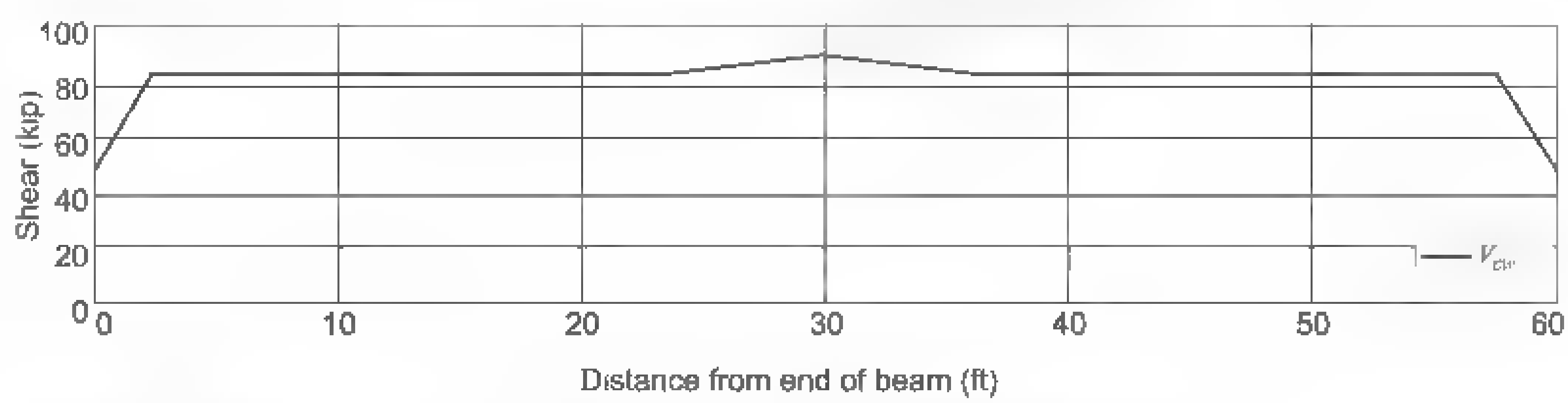


Fig. E10.10—Contribution of concrete to shear strength considering web-shear cracking.

The lower value of ϕV_{ci} and ϕV_{cw} are selected and plotted in Fig. E10.11. Factored shear exceeds the design strength provided by the concrete in two locations. One is near the support due to the reduction in prestress along the transfer length. The other is between 10 and 15 ft from the end of the double tee. Size shear reinforcement to provide adequate design shear strength.

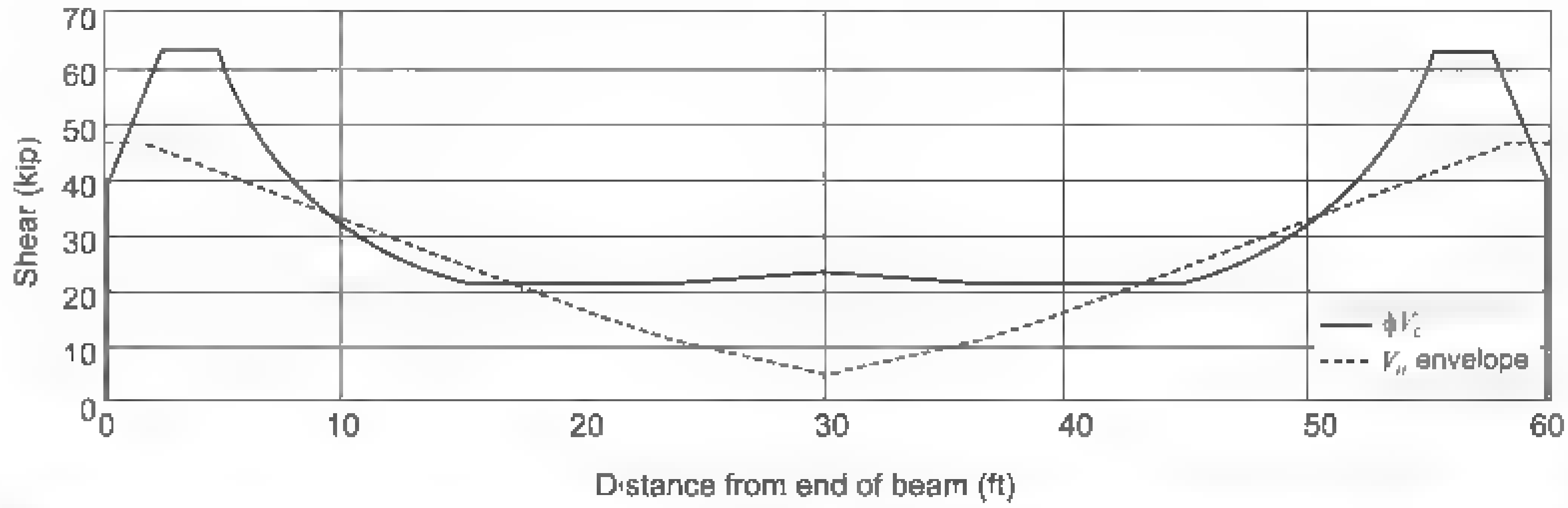


Fig. E10.11—Comparison of concrete contribution to shear strength and factored shear.

Step 14: Section size and minimum shear reinforcement

22.5.1.2 Check to ensure web size is adequate to avoid compression failure:

$$V_u \le \phi\left(V_c + 8\sqrt{f_c'}b_w d\right)$$

Ignoring V_c calculate the maximum contribution of transverse reinforcement:

$$8\left(\sqrt{5000}\text{ psi}\right)(9.5\text{ in.})(20.8\text{ in.}) = 111.8\text{ kip}$$

$V_{u,max} = 46.2$ kip **OK**. Web size is adequate

Rather than calculating the required area of steel as formulated in the Code, assume a stirrup size and configuration and then vary spacing. In the case of double tees, use WWR reinforcement for placement convenience. Use PCI Design Handbook Design Aid 15.5.4 for WWR commonly used in the stems of double tees.

From Code WRI Standard Wire Reinforcement. Use W2.5 wire size. Provide one mat in each web (A_v = 2*wire area).

$A_v = 2(0.025\text{ in.}^2) = 0.05\text{ in.}^2$

20.2.2.4 ASTM A1064 Plain WWR (Code Table 20.2.2.4b) $f_{yt} = 60{,}000$ psi

9.6.3.2 Minimum shear reinforcement must be provided where $V_u > 0.5\phi V_c$. This requirement covers nearly the entire beam. Provide minimum shear reinforcement over the full span.

9.6.3.4 Determine minimum shear reinforcement requirement:

Lesser of	Greater of	$0.75\sqrt{f_c'}\dfrac{b_w}{f_{yt}}$
		$50\dfrac{b_w}{f_{yt}}$
	$\dfrac{A_{ps}f_{pu}}{80f_{yt}d}\sqrt{\dfrac{d}{b_w}}$	

$$0.75\left(\sqrt{5000}\text{ psi}\right)\frac{9.5\text{ in.}}{60{,}000\text{ psi}} = 0.0084\text{ in.}^2/\text{in.}$$

$$50\text{ psi}\frac{9.5\text{ in.}}{60{,}000\text{ psi}} = 0.0079\text{ in.}^2/\text{in.}$$

$$\frac{1.836\text{ in.}^2(270\text{ ksi})}{80(60{,}000\text{ psi})(0.8)26\text{ in.}} = 0.0050\text{ in.}^2/\text{in.}$$

$$\frac{0.05\text{ in.}^2}{0.0050\text{ in.}^2/\text{in.}} = 10\text{ in.}$$

Use mesh with spacing of 10 in. for vertical wires. Use for entire length.

9.7.6.2.2	Check spacing limits.	$\frac{3(26\text{ in.})}{4} = 19.5\text{ in.}$
Step 15: Shear reinforcement for strength		
22.5.8.1	Determine if minimum shear reinforcement will satisfy required design strength $V_s \geq \frac{V_u}{\phi} - V_c$	Check spacing required at support $V_{s\_req\_support} = \frac{46.20\text{ kip}}{0.75} - 48.9\text{ kip} = 12.7\text{ kip}$
22.5.8.3	$V_s = \frac{A_v f_{yt} d}{s}$	$s_{support} = \frac{0.05\text{ in.}^2(60{,}000\text{ psi})(0.8)(26\text{ in.})}{12.7\text{ kip}} = 4.9\text{ in.}$ Check spacing required at approximately 15 ft from end of member $V_{s\_req\_15ft} = \frac{24.3\text{ kip}}{0.75} - 28.7\text{ kip} = 3.7\text{ kip}$ $s_{15ft} = \frac{0.05\text{ in.}^2(60{,}000\text{ psi})(0.8)(26\text{ in.})}{3.7\text{ kip}} = 16.9\text{ in.}$ Use W2.5 wire at 10 in. spacing. Double mesh at support for ~36 in. to account for reduced prestress
Step 16: Transverse bending of double tee flange under wheel load		
Ch. 7 7.5.2.3	Double-tee flange should be checked for local flange bending due to both the distributed load and concentrated wheel load required by the Building Code. The flange can be checked as a one-way nonprestressed slab in accordance with Code Chapter 7 and is typically reinforced with welded wire mesh. ASCE/SEI 7 specifies a concentrated load of 3 kip applied over a 4.5 in. square patch. The analysis of the concentrated load on the end of the double-tee flange is complex and requires either a highly indeterminate elastic solution or yield-line analysis. The concentrated load is typically considered to be resisted by moment in the flange at the outside face of the stem. The effective width has been estimated to be at a 45- to 60-degree projection from the concentrated load. This is further complicated by the use of connectors between adjacent double-tee flanges. These flange connections, depending on their spacing, provide some load sharing of the concentrated load between adjacent double tees. PCI MNL-129-15-1 indicates that experimental studies have shown that the actual flange strength is greater than predicted by elastic or yield-line models. Crack patterns suggest that the effective width is more accurately reflected by a dispersion rate of 1-to-3. For this example assume that flange connectors are widely spaced and that the concentrated load is resisted by a single double tee (Fig. E10.12). Use the dispersion rate of 60 degrees to compute the unit moment at the face of the stem.	distance from center of concentrated load to face of 3 in. chamfer 30 in. – 0.5(5.75 in.) – 3 in. – 0.5(4.5 in.) = 1.82 ft distance from edge of flange to face of 3-in. chamfer 30 in. – 0.5(5.75 in.) – 3 in. = 2.01 ft

<table>
<tr>
<td>7.6.1.1</td>
<td>Determine minimum flexural reinforcement requirements for transverse bending in double-tee flange, flange is nonprestressed in this direction, so Chapter 7 One-way slab requirements will be used.</td>
<td>$A_{s,min} = 0.0018(12\text{ in.})(4\text{ in.}) = 0.087\text{ in.}^2$</td>
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3 kip load applied over 4.5" square patch

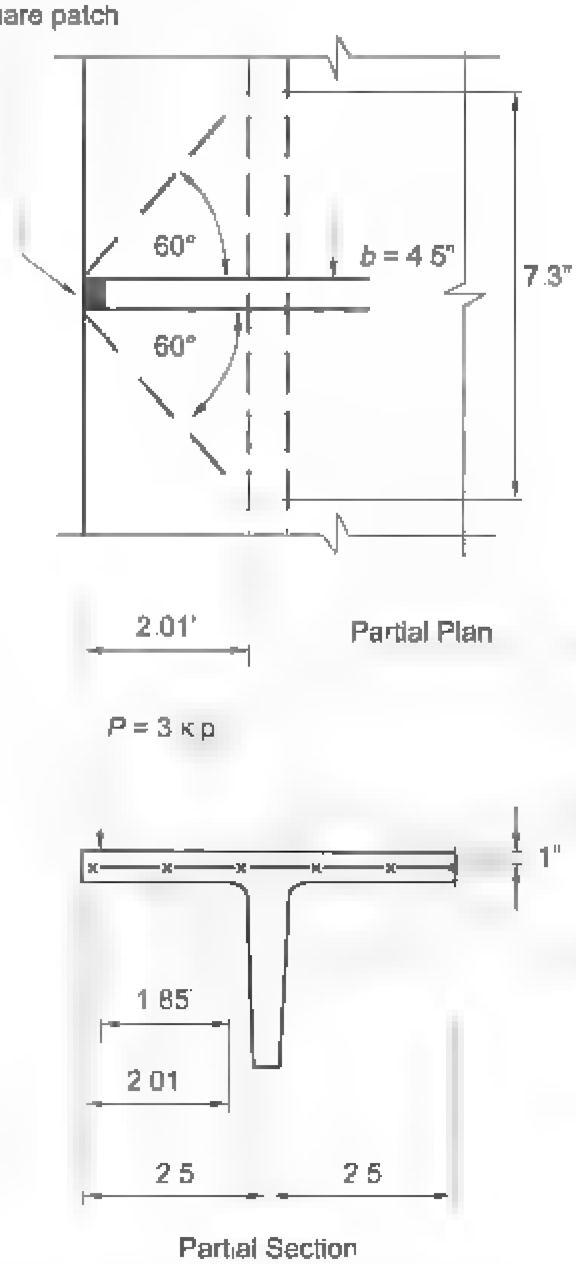

Fig. E10.12—Dispersion of moment from concentrated load at tip of double-tee flange

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Unit moment at face of stem

$m_{uDL} = 1.2(150\text{ lb/ft}^3)(4\text{ in.})(12\text{ in.})(2.01\text{ ft}) = 1.4\text{ kip-in./ft}$

$m_{uLL} = 1.6(3\text{ kip})(1.82\text{ ft})\dfrac{1}{4.5\text{ in.} + 2\tan(60\text{ deg})(2.01\text{ ft})}$

$= 14.3\text{ kip-in./ft}$

$m_u = 1.4\text{ kip-in./ft} + 14.3\text{ kip-in./ft} = 15.7\text{ kip-in./ft}$

Try WWR 4 x 4,W4.0 x W4.0 placed at 1 in. from top of deck.

$A_{s,prov} = \dfrac{0.04\text{ in.}^2}{4\text{ in.}} = 0.12\text{ in.}^2/\text{ft} > A_{s,min} = 0.087\text{ in.}^2/\text{ft}$

$a = \dfrac{(0.12\text{ in.}^2)(60\text{ ksi})}{0.85(5000\text{ psi})(12\text{ in.})} = 0.141\text{ in.}$

$\varepsilon_t = 0.003\left(\dfrac{\beta_1(3\text{ in.})}{0.141\text{ in.}} - 1\right) = 0.048$

$> 0.002 + 0.003 = 0.005$ **OK** Section is tension-controlled $\phi = 0.9$

Design moment strength on unit basis

$\phi M_n = 0.9(60\text{ ksi})(0.12\text{ in.}^2/\text{ft})\left(3\text{ in.} - \dfrac{0.141\text{ in.}}{2}\right) = 19.0\text{ kip-in./ft}$

$19.0\text{ kip-in./ft} > m_u = 15.7\text{ kip-in./ft}$ **OK**

Care must be taken to ensure that the deck reinforcement is placed at the proper depth. Small errors in placement can result in a significant reduction in flexural strength.

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Step 17: Camber and deflections

24.2
24.5.2.1
24.2.3.8

Camber is typically measured in the prestressing bed immediately following prestress transfer. It is composed of the downward deflection due to self-weight and upward deflection due to the eccentric prestressing force. These are calculated using the estimated modulus at the time of release based on the specified compressive strength of concrete at the time of prestress transfer, which is typically no more than a few days. Because this section is classified as uncracked (Class U), gross section properties are permitted to be used to calculate deflections (Code Table R24.5.2.1). Calculate the self-weight component using

$$\Delta_s = \frac{5 w_{sw} \ell^4}{384 E_{ci} I}$$

$$\Delta_s = \frac{5(0.718\text{ klf})(60\text{ ft})^4(1728\text{ in.}^3/\text{ft}^3)}{384(3372\text{ ksi})(30{,}716\text{ in.}^4)} = 2.02\text{ in.}$$

Calculate camber using equations from PCI Design Handbook

$$\Delta_{ps} = \frac{P\ell^2}{E_{ci} I}\left[\frac{4(e_c - e_e)}{48} + \frac{e_e}{8}\right]$$

$$\Delta_{ps} = \frac{335\text{ kip}(60\text{ ft})^2(144\text{ in.}^2/\text{ft}^2)}{3372\text{ ksi}(30{,}716\text{ in.}^4)} \times\left[\frac{4(17.04\text{ in.} - 8.62\text{ in.})}{48} + \frac{8.62\text{ in.}}{8}\right] = 2.98\text{ in.}$$

$$\Delta_o = 2.98\text{ in.} - 2.02\text{ in.} = 0.96\text{ in.}$$

24.2	Calculate immediate live load deflection $\Delta_{LL} = \dfrac{5w_{LL}L^4}{384E_cI}$	$\Delta_{LL} = \dfrac{5(0.4\text{ klf})(60\text{ ft})^4 1728\text{ in.}^3/\text{ft}^3}{384(4031\text{ ksi})(30{,}716\text{ in.}^4)} = 0.94\text{ in.}$
24.2.2	Compare to Code limitation assuming that there are no nonstructural elements likely to be damaged by large deflections. $L/360$ To ensure occupant comfort, vibrations should also be checked. Guidance on the vibration response of floors is given in the PCI Design Handbook Precast and Prestressed Concrete, 8th Edition, Chicago, IL (MNL-120)	$\dfrac{60\text{ ft}(12\text{ in./ft})}{360} = 2\text{ in.}$
24.2.4.2.1	Consider time-dependent deflections. Code Commentary indicates that any suitable method may be used. Use multiplier approach given in PCI Design Handbook, which is based on Martin, L.D. (1977) "A Rational Method for Estimating Camber and Deflections of Precast Prestressed Members," *PCI Journal*, V. 22, No. 1. From PCI Design Manual Table 5.8.2 apply given multipliers to the elastic deflection calculated at release. Use multipliers for final condition and conservatively assume that partitions are installed at the same time as prestress is released. When pretopped double tees are used, care must be taken to eliminate differential camber offsets at the adjoining edges of members that may trap water	Time-dependent camber (up) 2.98 in.(2.45) = 7.30 in. Time-dependent self-weight (down) 2.02 in.(2.70) = 5.45 in. Time-dependent superimposed dead load (down) $\Delta_{SD} = \dfrac{5(0.1\text{ klf})(60\text{ ft})^4 1728\text{ in.}^3/\text{ft}^3}{384(4031\text{ ksi})(30{,}716\text{ in.}^4)} = 0.24\text{ in.}$ 0.24 in.(3.00) = 0.72 in.
24.2.4.2	Compare to permissible deflections from Code Table 24.2.2. Check "That part of the total deflection occurring after attachment of nonstructural elements." This will be the sum of the live load deflection and the time-dependent deflection that occurs after the installation of partitions. In this example, partitions are assumed to be installed at the same time as release $L/480$	7.30 in. − 5.45 in. − 0.72 in. = 1.13 in. 1.13 in. − 0.96 in. − 0.94 in. = −0.77 in. $\dfrac{60\text{ ft}(12\text{ in./ft})}{480} = 1.5\text{ in.}$ **OK** The time-dependent deflections are less than the most stringent provision in the table.

Step 18: Strand spacing

25.2.4 For 0.5 in. diameter strand, center-to-center spacing must be greater than 1.75 in. Precast plants typically have permanently fixed anchors that have holes drilled at 2 in. spacing, which will satisfy this requirement. Clear spacing must also be larger than 4/3 of maximum aggregate size.

25.2.4 No specific code provisions are provided for splitting stresses in pretensioned strands. PCI Design Manual provides the following equation for computing the transverse reinforcement to resist these stresses:

$$A_{vt} = \frac{0.021 P_o h}{f_s \ell_t}$$

where

A_{vt} = required area of stirrups at the end of a component uniformly distributed over a length $h/5$ from the end

P_o = prestress force at transfer

h = depth of the component

f_s = design stress in the stirrups, usually assumed to be 30 ksi

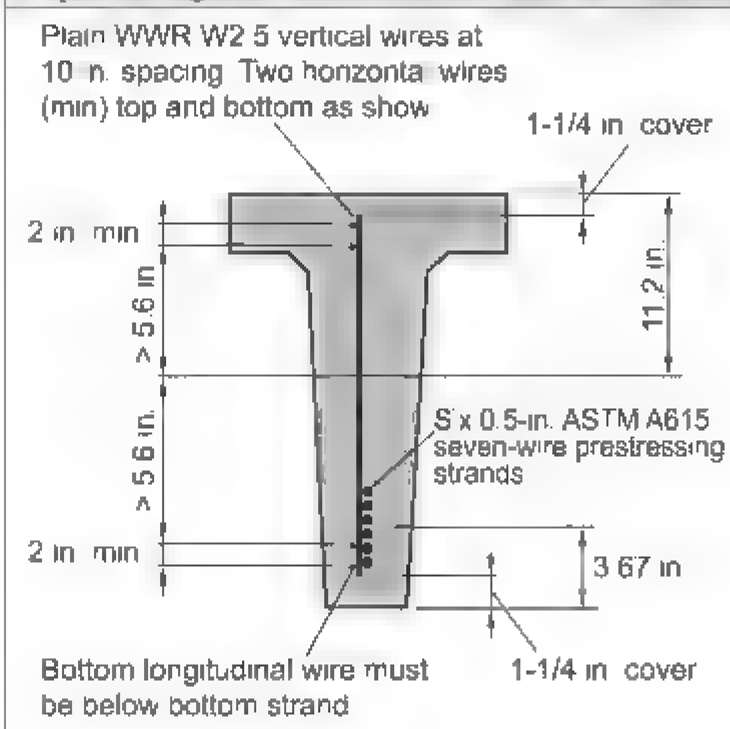
ℓ_t = strand transfer length

$$A_{vt} = \frac{0.021(335\ \text{kip})(26\ \text{in.})}{30\ \text{ksi}(28.75\ \text{in.})} = 0.212\ \text{in.}^2$$

$h/5 = 5.2$ in.

This area is for both webs of the double tee. Provide one No. 3 single leg stirrup at the end of each web to control splitting cracking. Shear reinforcement that is already present in the web may be used to satisfy this requirement.

Step 19: Design sketch



Partial Section thru Web

Fig. E10.13—Detail of web reinforcement.

Beam Example 11: *Hanger reinforcement*

Using the information from Beam Example 2 and the loads given herein, design hanger reinforcement for the connection between Beam B1 and Beam B2. Beam B1 frames a slab opening and is supported by Beam B2.

Given:

Beam B1 reaction—

Maximum factored reaction from Beam B1, from Example 2 use V_u = 28.5 kip

To demonstrate design of hanger reinforcement use V_u = 80 kip

Material properties—

f_c' = 5000 psi (normalweight concrete)

f_y = 60,000 psi

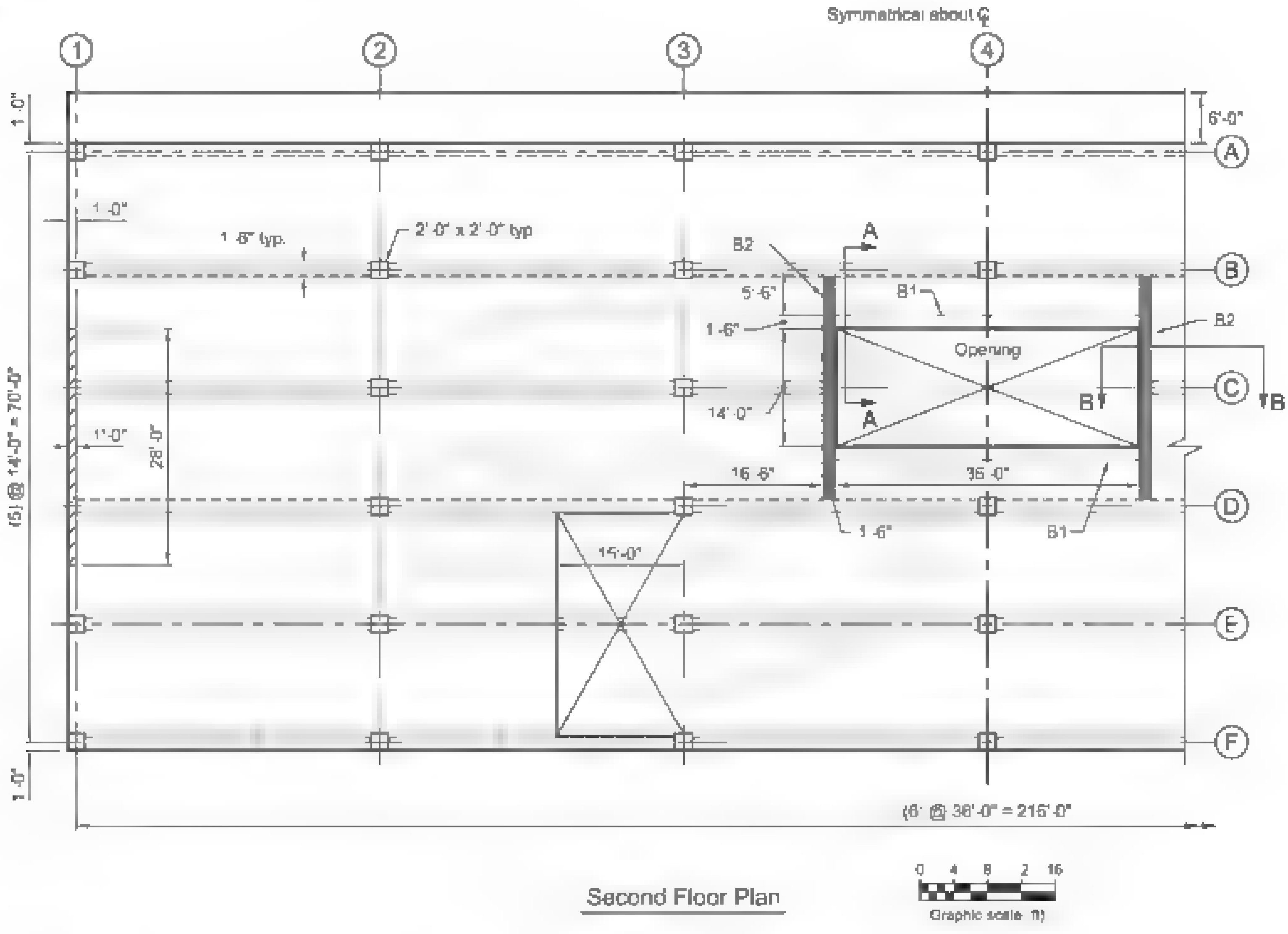


Fig. E11.1—Framing plan showing slab opening and beams framing slab opening.

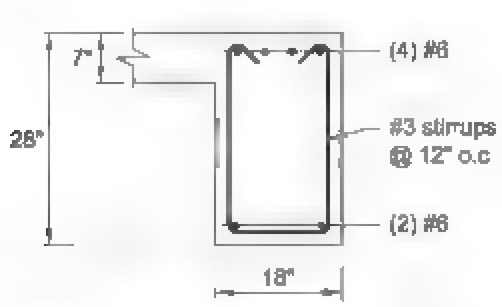


Fig. E11.2—Section A-A through Beam B1 near connection with Beam B2.

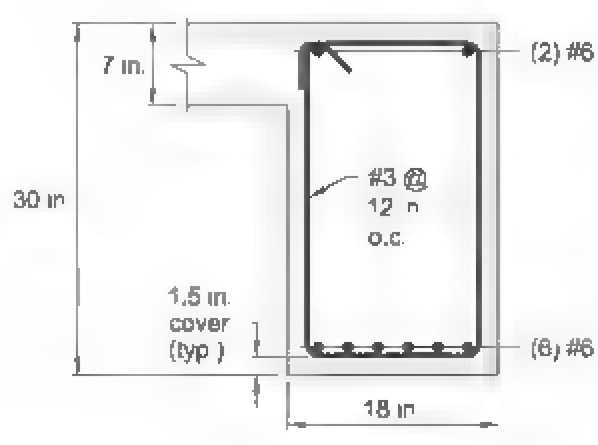


Fig. E11.3—Section B-B through Beam B2 near connection with Beam B1.

Step 1: Design approach		
R9.7.6.2.1	The Code alerts the designer to the potential reduction in strength at the intersection of monolithically cast reinforced concrete beams. The strength reduction occurs in conditions where a beam is supported from the side face of the supporting girder rather than by direct bearing. This is a typical arrangement in floor systems to minimize the space occupied by the floor framing. In this type of connection, the reaction is transferred to the supporting girder by means of internal forces. If sufficient hanger reinforcement is not detailed in the joint region to transfer these forces, then the joint strength may be less than the calculated nominal shear strength. This problem has been known for some time. The Code does not provide prescriptive detailing or analysis specifications, but rather describes the problem and refers the reader to research that evaluated the issues. ACI 314R, "Guide to Simplified Design for Reinforced Concrete Buildings," provides an approach for designing the hanger reinforcement, which will be used to solve this problem.	

Step 2. Design and detail hanger reinforcement requirements

R9.7.6.2 Where the following conditions occur, then hanger reinforcement is required

1. If the depth of the supported beam is greater than half the depth of the supporting girder

1. Depth of supported beam (h_{B1}) is 28 in. and depth of supporting girder (h_{B2}) is 30 in. Check requirement

$h_{B1} = 28 \text{ in.} > 0.5\,h_{B2} = 15 \text{ in.}$ Yes.

2. If the factored shear transferred across the interface between the two beams is greater than $\phi 3\sqrt{f_c'}\,b_w d$ in the supported beam.

2. Check magnitude of factored shear

$d = 28 \text{ in.} - 1.5 \text{ in.} - 0.375 \text{ in.} - 0.5(0.625 \text{ in.}) = 25.8 \text{ in.}$

$V_{u,limit} = 0.75\left(3\sqrt{5000} \text{ psi}\right)(18 \text{ in.})(25.8 \text{ in.}) = 73.9 \text{ kip}$

$V_u = 25.4 \text{ kip} < V_{u,limit} = 73.9 \text{ kip}$

Hanger reinforcement is not required for beam B2 in Example 3.

If hanger reinforcement is required, then the following equation from ACI 314 can be used to determine the area of steel required

$$A_s = \frac{1 - \dfrac{h_b}{h_g}}{\phi f_y} V_u$$

To demonstrate the design of hanger reinforcement use a factored reaction of $V_u = 80$ kip, which exceeds the limit.

$$A_s = \frac{1 - \dfrac{2 \text{ in.}}{30 \text{ in.}}}{0.75(60 \text{ ksi})} 80 \text{ kip} = 1.66 \text{ in.}^2$$

ACI 314 suggests distributing 1/3 of the shear reinforcement to the supported beam and 2/3 of the shear reinforcement to the supporting girder. The added stirrups must be in addition to the stirrups required for shear. Stirrups in the supported beam should be spaced over the distance $d/4$ from the interface. Stirrups in the supporting girder should be spaced over the supported beam width plus twice the distance from the bottom of the supported beam to the bottom of the supporting girder.

Use No. 3 bars to match stirrups designed for shear strength.

$$\frac{\frac{2}{3}\left(1.66 \text{ in.}^2\right)}{0.11 \text{ in.}^2} = 10.06$$

Since only one leg of the stirrup is effective as hanger reinforcement in the supporting girder, place 10 stirrups in supporting girder over 18 in. + 2(2 in.) = 22 in. width, centered on the supported beam (Fig. E11.4). This is in addition to the shear reinforcement requirement of No. 3 at 12 in. Add one stirrup to those placed over the 22 in. width. Add two more just outside this region. This will result in a center-to-center spacing of ~2 in. Check maximum aggregate size to ensure this provides sufficient clear spacing. Use closed stirrups to ensure that the hanger force is transferred to the top of the girder.

$$\frac{\frac{1}{3}\left(1.66 \text{ in.}^2\right)}{2\left(0.11 \text{ in.}^2\right)} = 2.5$$

Since both legs of the stirrups in the supported beam are effective, place three No. 3 stirrups over $d/4 = 25.8/4 = 6.5$ in. Add one stirrup to account for shear reinforcement requirement. Place the first stirrup at 0.5 in. from the face of the girder and then three spaces at 2 in.

Step 3: Hanger reinforcing details

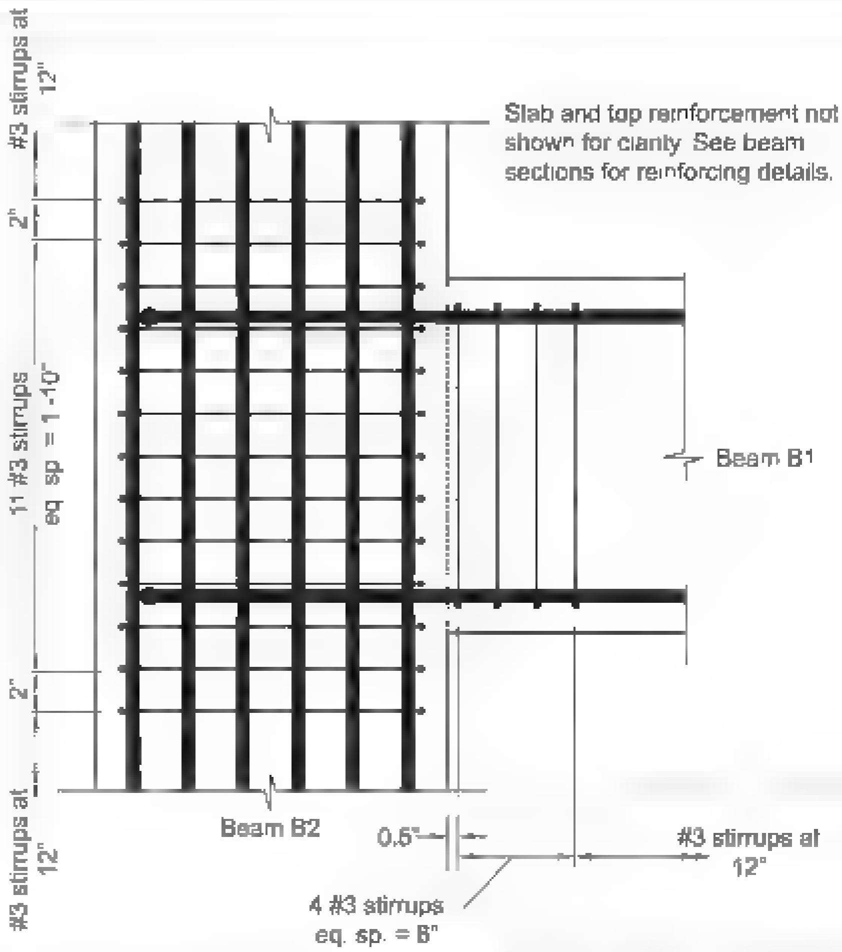


Fig. E11.4—Plan section of intersection of beams B1 and B2

CHAPTER 8—DIAPHRAGMS

8.1—Introduction

Building diaphragms are usually horizontal, reinforced concrete one-way or two-way slabs spanning between columns or walls, or both columns and walls. They can be built out of cast-in-place (CIP) concrete, precast elements with CIP topping, interconnected precast elements without CIP topping, or precast elements with end strips formed of CIP topping slab or edge beams (Moehle et al. 2016).

Building slabs are designed to resist gravity loads and also to transfer wind, earthquake, fluid, or lateral earth pressure forces to the lateral-force-resisting system, such as moment frames, shear walls, or both (Code Section 12.2). For dual-system structures such as shear walls and special moment frames, special moment frames deform in a shear mode, as shown in Fig. 8.1(a), while shear walls deform in a bending mode (cantilever), also as shown in Fig. 8.1(a). Diaphragms maintain compatible deformations between the two systems, thus tying the entire structure together (Fig. 8.1(a) and (b)). Diaphragms also provide lateral support to shear walls and columns. As a rule of thumb, approximately 2 to 5 percent of a column axial force must be resisted by the diaphragm to provide adequate lateral support to the walls and columns. This force is easily achieved in low-rise building diaphragms but must be checked for columns with high axial force, such as those in high-rise buildings (Moehle et al. 2016). Checks can include

(a) Slab-bearing force at face of columns

(b) Adequacy of diaphragm slab reinforcement anchored into columns at edge connections

(c) Adequate diaphragm buckling strength to resist the bracing forces

8.2—Material

Specified concrete compressive strength for diaphragms and collectors resisting lateral forces must be at least 3000 psi (Code Section 19.2.1.1). Where nonprestressed, bonded prestressing reinforcement is used to resist diaphragm forces, the value of steel stress used to calculate resistance should be the lesser of the specified yield strength and 60,000 psi (Code Section 12.5.1.5).

8.3—Service limits

The minimum diaphragm slab thickness must satisfy the requirements of Code Section 7.3.1 for one-way slabs or Code Section 8.3.1 for two-way slabs. The diaphragm thickness must also be sufficient to resist in-plane moment, shear, and axial forces (Code Section 12.5.2.3).

8.4—Analysis

Diaphragm slabs must resist gravity loads and lateral in-plane force combinations simultaneously. Diaphragm slabs are commonly designed as a deep beam that spans horizontally in the plane of the floor system and is supported by moment-resisting frames, or shear walls, or both (Fig. 8.4a and 8.4b). For concrete slabs, ASCE/SEI 7 Section 12.3.1.2 permits the assumption of a rigid diaphragm if the diaphragm aspect ratio, which is the span-to-depth ratio in the direction of loading, is 3 or less for seismic design and 2 or less for wind loading (ASCE/SEI 7, Section 27.4.5) if the struc-

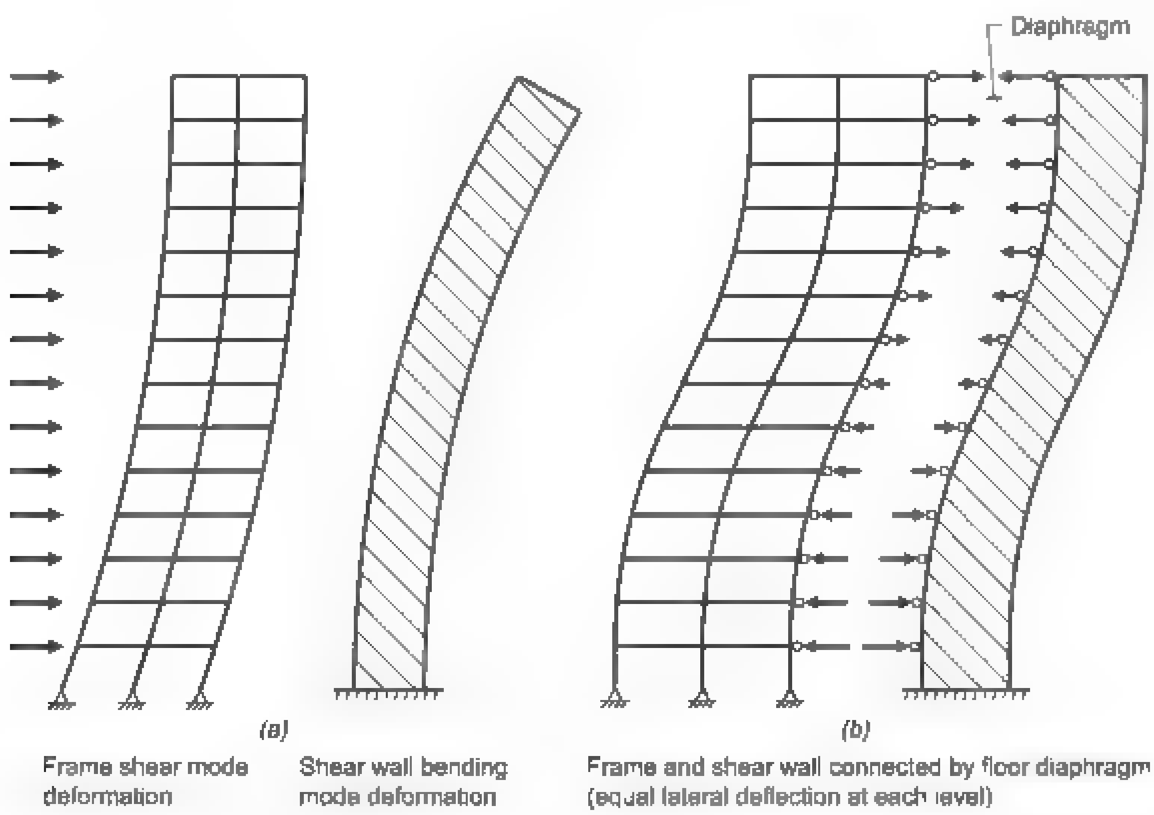


Fig. 8.1—Shear wall and moment frame dual system deformation.

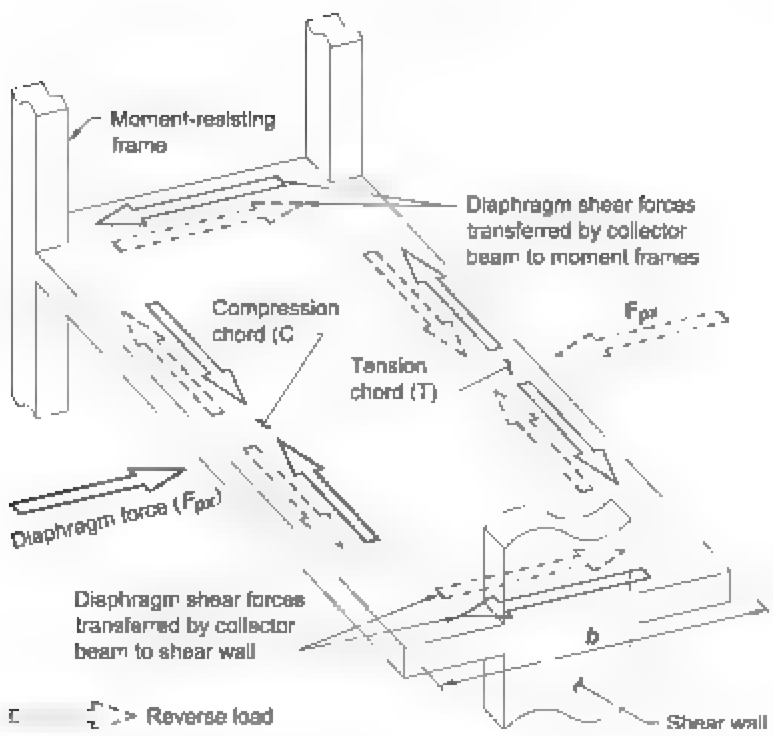


Fig. 8.4a—Diaphragm tension-compression and shear forces due to lateral forces

ture has no significant horizontal irregularities (Fig. 8.4b). Selected parts of the lateral force-resisting system, such as the base of shear walls or the beam ends in a moment frame, are designed to behave inelastically during the design seismic event; diaphragms, however, are generally designed to remain elastic during such an event.

The diaphragm reinforcement resisting tension due to flexure is placed at the tension edge perpendicular to the applied force (Fig. 8.4a). Tension and compression edges are identified as chords. Because earthquake and wind forces are reversible, equal reinforcement should be provided at both chords (Fig. 8.4a). Building edge beams, if provided, are often designed as the diaphragm's chord (Section 8.4.1 of this Manual). Chords are assumed to resist all the flexural tension from the diaphragm in-plane bending moment resulting from the lateral load. If edge beams are not provided, the slab acts as a deep rectangular beam resisting bending in the plane of the slab, with the chord tension reinforcement placed within $h/4$ of the tension face, where h is the diaphragm width in the direction of analysis (Code Section 12.5.2.3).

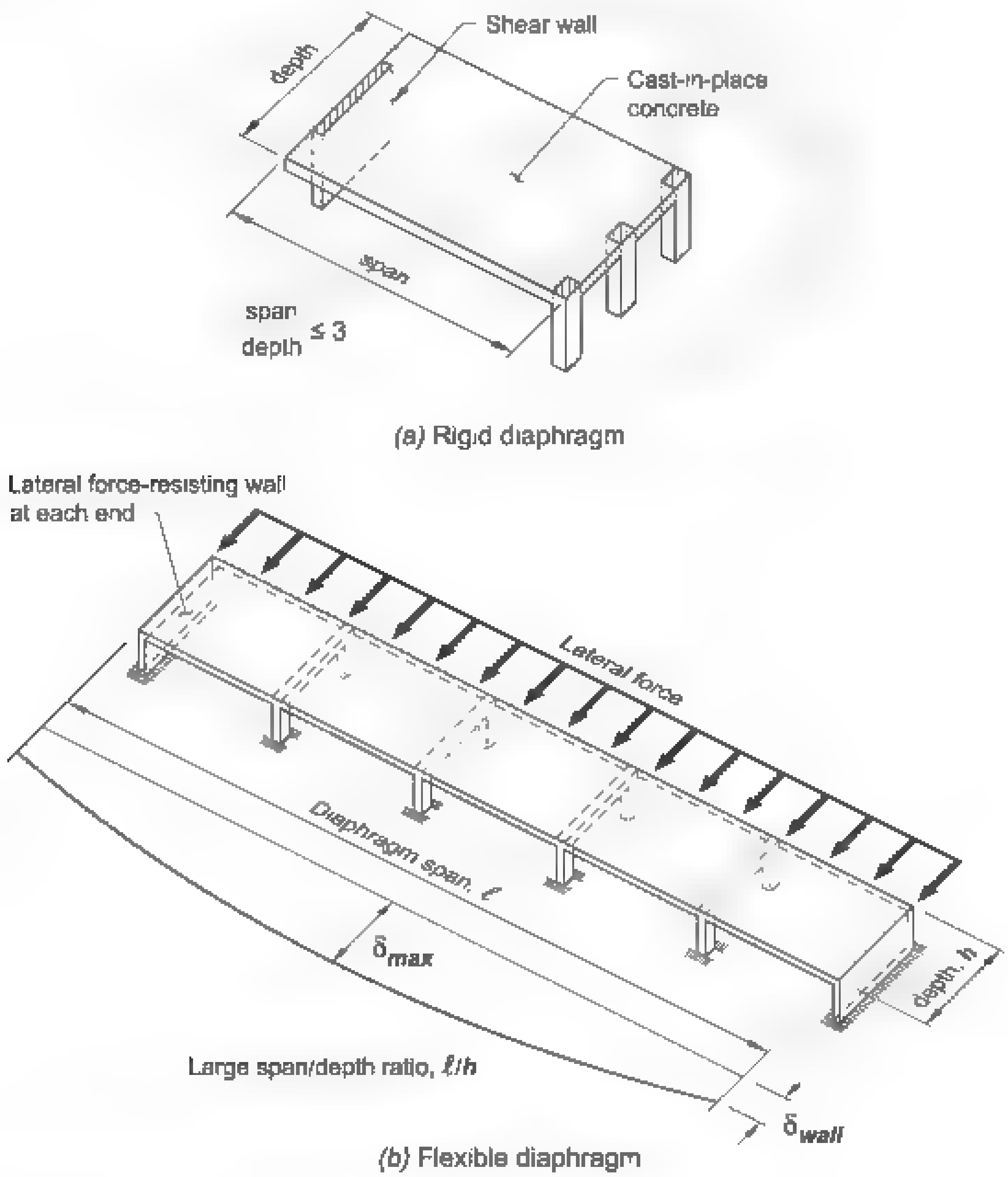
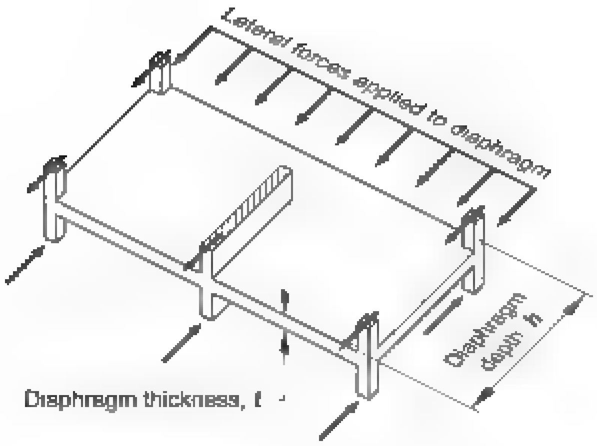


Fig. 8.4b—Rigid and flexible diaphragm



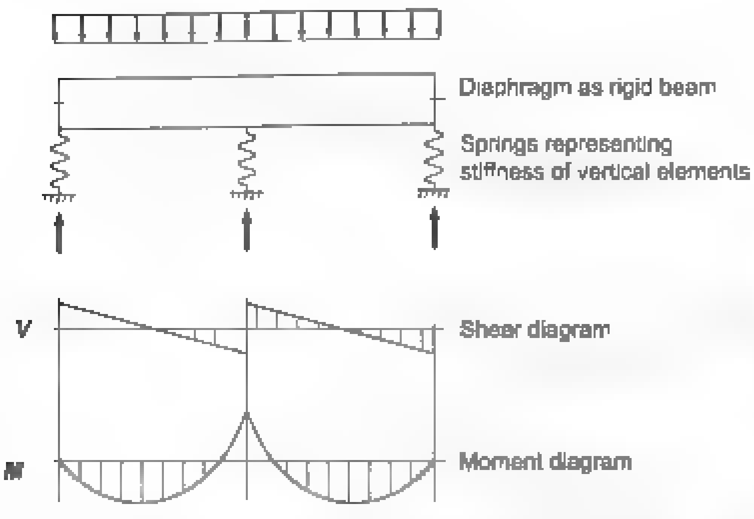


Fig. 8.4c—Rigid diaphragm lateral force distribution.

The diaphragm shear forces are resisted by the moment-resisting frames and shear walls (Fig. 8.4a). The beams or slab sections that transfer shear are identified as collectors. The collector slab or beam connection to the columns and walls must be appropriately designed and detailed to achieve shear transfer.

Rigid diaphragms (Fig. 8.4b(a)) are often modeled as deep beams with spring supports (Fig. 8.4c). Lateral force is distributed to the columns and walls according to their relative lateral stiffness. Flexible diaphragms are modeled with rigid supports (Fig. 8.4b(b)). If all supports have equal lateral resistance, the lateral force can be distributed to the columns and walls according to their tributary areas.

Also, finite element and strut-and-tie method can be used to analyze diaphragms. The finite element method should consider diaphragm flexibility (Code Section 12.4.2.4).

8.4.1 *Collectors*—Shear walls do not usually extend the full length of a building. Collectors, also called drag members or distributers, are designed to collect lateral forces from the diaphragm and transfer them to the seismic-force-resisting system, or to transfer lateral loads from a shear wall into the diaphragm. Collectors can be the full length of the diaphragm, but not necessarily (Code Section 12.5.4.1). Collectors can be defined as a section within the depth of the diaphragm or as a beam as part of the diaphragm. Collectors, as part of a rigid diaphragm, are expected to perform elastically during an earthquake event. Collectors parallel to a shear wall can have the same width as a shear wall (Fig. 8.4.1a) or be wider. Collectors eccentric to the wall

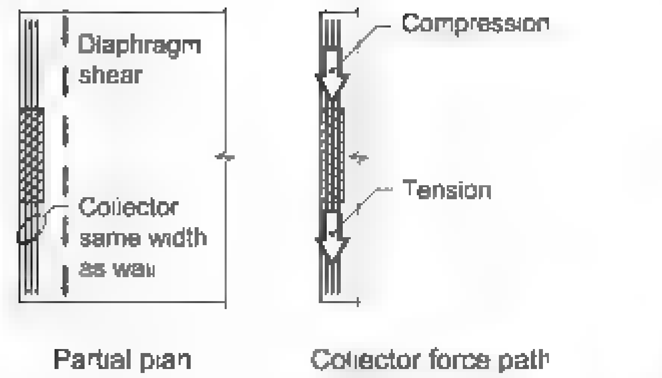


Fig. 8.4.1a—Collector having same width as shear wall, forces are reversible.

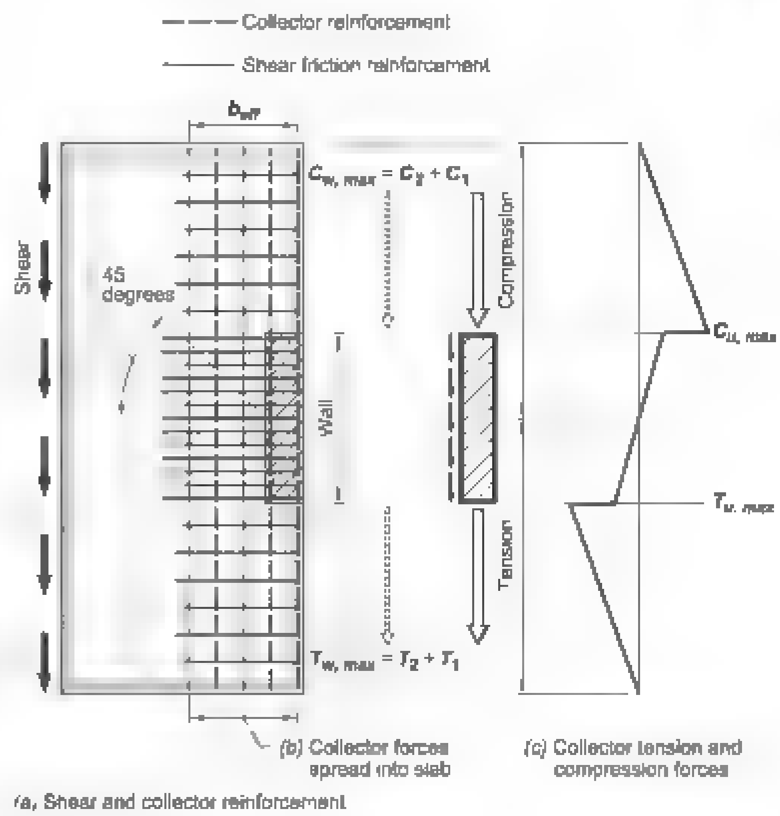


Fig. 8.4.1b—Collector wider than the shear wall, forces are reversible.

have an effective width b_{eff}, defined as not wider than the thickness of the wall plus one-half the length of the shear wall (Seismology and Structural Standards Committee [SEAOC] 2005, Code Section R12.5.4, Fig. 8.4.1b of this Manual). Collectors with the same width as the wall will simply transfer the slab lateral forces by axial compression or tension to the shear wall. Collectors having a width wider than the shear wall will transfer part of the diaphragm lateral force by axial compression or tension and the balance will be transferred along the wall length through shear friction. An eccentricity results between the resultant force in the collector and the shear wall reaction (Fig. 8.4.1b). This eccentricity creates secondary stresses in the slab transfer region adjacent to the wall. Adequate reinforcement must be provided to resist these stresses (SEAOC 2005).

Collectors, like rigid diaphragms, are expected to behave elastically under axial and compression forces. Reinforcement is usually placed at middepth in collectors. Shear reinforcement perpendicular to the walls is needed as shear

friction reinforcement for eccentric collectors, and is placed within the slab thickness (SEAOC 2005).

8.5—Design strength

Diaphragms in Seismic Design Categories (SDCs) D through F are designed in accordance with Code Chapter 18.

Slabs used as diaphragms must have sufficient thickness as is required for stability, strength, and stiffness under the required factored load combinations (Code Section 12.3.1). The shear forces and bending moments resulting from the effects of lateral loads are considered simultaneously.

Diaphragms are designed to resist the design seismic force calculated from the structural analysis, F_{px}, which must be at least (ASCE/SEI 7 Section 12.10.1.1)

$$F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px}$$

where F_{px} is the diaphragm design force at level x.

The design force applied to level x is F_i, w_i is the weight tributary to level x_i, and w_{px} is the weight tributary to the diaphragm at level x. The force calculated from this equation need not exceed $0.4S_{DS}Iw_{px}$, but needs to be at least $0.2S_{DS}Iw_{px}$ (ASCE/SEI 7 Section 12.10.1.1).

Collectors in SDCs C through F are designed for the largest of (a) through (c):

(a) F_x obtained from structural analysis using load combinations with overstrength factor Ω_o of ASCE/SEI 7 Section 12.4.3.2

(b) F_{px} using load combinations with overstrength factor Ω_o of ASCE/SEI 7 Section 12.4.3.2

(c) $F_{px,min} = 0.25S_{DS}Iw_{px}$ using load combinations of ASCE/SEI 7 Section 12.4.2.3 forces F_x are applied to all floor levels concurrently

Forces F_{px} and $F_{px,min}$ "are applied one level at a time to the diaphragm under consideration (Moehle et al. 2016)." The nominal shear strength of a diaphragm is $V_n = A_{cv}(2\lambda\sqrt{f_c'} + \rho_t f_y)$ (Code Section 12.5.3.3) and the cross-sectional dimensions must satisfy $V_n \leq \phi 8\sqrt{f'_c}A_{cv}$ (Code Section 12.5.3.4).

In Code Sections 12.5.3.3 and 12.5.3.4, A_{cv} is the gross area of concrete section bounded by web thickness and section length in the direction of shear force considered, and ρ_t is the ratio of area of distributed transverse reinforcement to gross concrete area positioned perpendicular to the diaphragm flexural reinforcement. The reduction factor ϕ for diaphragms is 0.75 (Code Section 12.5.3.2).

Diaphragms in lateral-force-resisting systems where the primary vertical elements are short structural walls may not behave as intended. Earthquake damage has been observed in the diaphragms of such structures where the walls remained essentially linear elastic, but the diaphragms responded inelastically. To provide increased relative diaphragm strength, ϕ for shear in the diaphragm should not exceed the least value of ϕ for shear used for the vertical components of the primary seismic-force-resisting system,

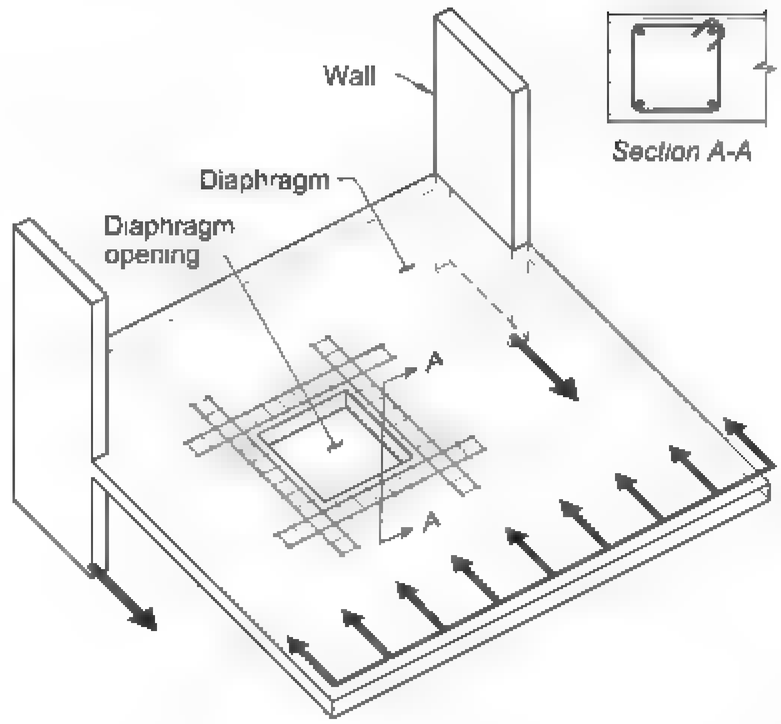


Fig. 8.6a—Reinforcement detail around opening within diaphragm.

Table 8.6a—Collector and chord reinforcement requirements in SDCs D through F for splice and anchorage zones

Reinforcement	Requirement		Code reference
Longitudinal	Spacing	$3d_b \geq 1.5$ in.	18.12.7.7(a)
	Cover	$2.5d_b \geq 2$ in.	
Transverse	Greater of	$0.75\sqrt{f'_c}\frac{b_w s}{f_{yt}}$ $\frac{50b_w s}{f_{yt}}$	18.12.7.7(b)

this is applicable for special moment frames, special structural walls, or intermediate precast structural walls in SDC D, E, or F. For low-rise stiff walls that are shear controlled, Code Section 21.2.4.1 requires the use of a shear strength reduction of 0.60, as those type structures tend to have relatively high overstrength. For this condition, ϕ for shear in the diaphragm would also be limited to 0.6.

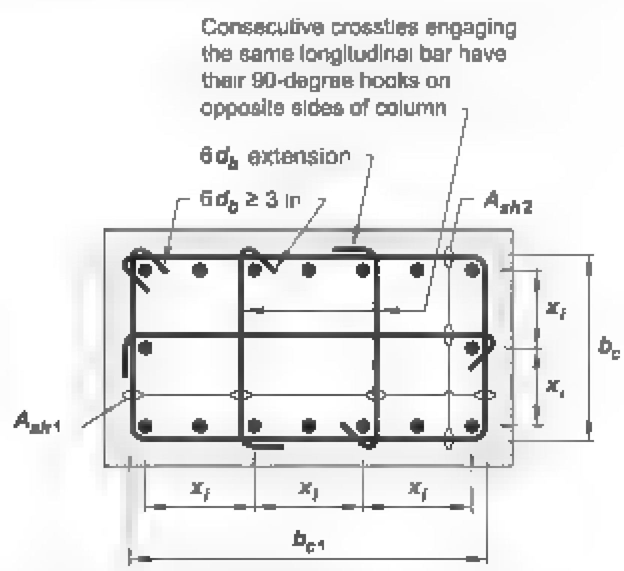
At diaphragm discontinuities, such as openings and reentrant corners, the design needs to consider the dissipation or transfer of edge (chord) forces. When combined with other forces in the diaphragm, the local design strengths should be within the shear and torsion strength of the diaphragm.

8.6—Reinforcement detailing

Generally, chord and collector reinforcement is placed around diaphragm middepth. It is common practice (Moehle et al. 2016) to reinforce diaphragm openings smaller than approximately twice the slab thickness with only the displaced reinforcement, but at least one bar on any side. Larger openings require a more rigorous analysis.

Around large openings or other discontinuities, confinement reinforcement (ties) should be placed around the chord bars surrounding the opening (Fig. 8.6a). To properly

Table 8.6b—Transverse reinforcement requirements for tension and compression collectors and chords in SDCs D through F reinforced with transverse confinement reinforcement (Code Section 18.12.7.6)

Compressive stress	Transverse reinforcement requirements	Details			Code provision
$> 0.2f_c'$ ($> 0.5f_c'$ if forces are amplified to account for over-strength)	Yes	Single or overlapping spirals per Code Sections 25.7.3.5 and 25.7.3.6. Circular hoops or rectangular hoops with or without crossties spaced not more than 14 in.  The dimension x_i from centerline to centerline of laterally supported longitudinal bars is not to exceed 14 inches. The term h_x used in Eq. (18.7.5.3) is taken as the largest value of x_i			18.12.7.6
		Transverse reinforcement spacing along length of the diaphragm is the smallest of (a) One-fourth the minimum member dimension (b) For Grade 60, $6d_b$ of smallest longitudinal reinforcement (b) For Grade 80, $5d_b$ of smallest longitudinal reinforcement (c) $s_o = 4 + \frac{14 - h_x}{3}$ and 4 in. $\le s_o \le 6$ in.			18.12.7.6 and 18.7.5.3
		Rectilinear hoop	$A_{sh} = 0.09 \frac{s b_c f_c'}{f_{yt}}$		18.12.7.6
		Spiral or circular hoop	Greater of	$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$	
				$\rho_s = 0.12\frac{f_c'}{f_{yt}}$	
$< 0.15f_c'$ ($0.4f_c'$ if forces are amplified to account for overstrength)	No	None			18.12.7.6

transfer forces between the diaphragm and columns or walls, chord bar splices should be Type 2 and chord bar spacing should satisfy the requirements of Table 8.6a. Chord bars in higher seismic zones must be confined with closed hoops or spirals per Table 8.6b.

Chords at openings need to be proportioned to resist the sum of the factored axial forces acting in the plane of the diaphragm and the force obtained by dividing the factored moment at the section by the distance between the chords at the section.

A collector parallel to a shear wall has its critical connection at the face of the shear wall. Collector longitudinal bars must extend deep enough into the shear wall to develop and transfer the lateral force to wall reinforcement (Fig. 8.6b).

The collector reinforcement is in addition to the horizontal diaphragm reinforcement required to resist the shear force (Moehle et al. 2016). Collector reinforcement must comply with Code Section 20.2.1 with the following two exceptions.

(a) Collector or chord reinforcement placed within beams of special moment frames must satisfy ASTM A706/706M, Grade 60. Reinforcement complying with ASTM A615/A615M is permitted if the conditions of Code Section 20.2.2.5(b) are satisfied.

(b) If bonded tendons are used to resist collector forces, diaphragm shear, or flexural tension, then the design yield stress for longitudinal and transverse reinforcement is limited to the smaller of the specified yield strength or 60,000 psi (Code Section 12.5.1.5).

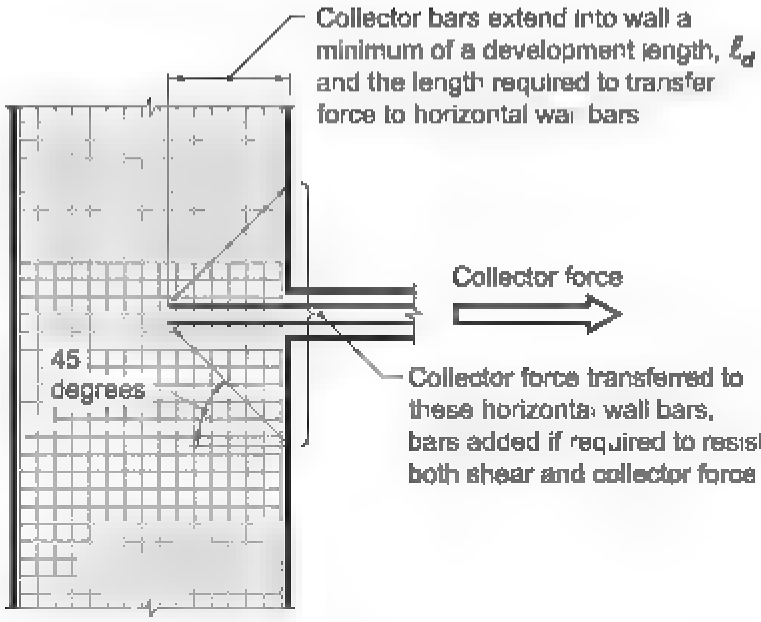


Fig. 8.6b—Collector reinforcement extended into shear wall

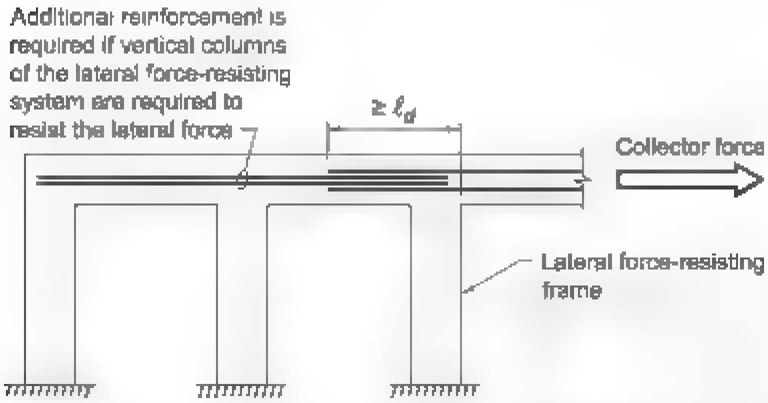


Fig. 8.6c—Collector reinforcement extended into moment frame.

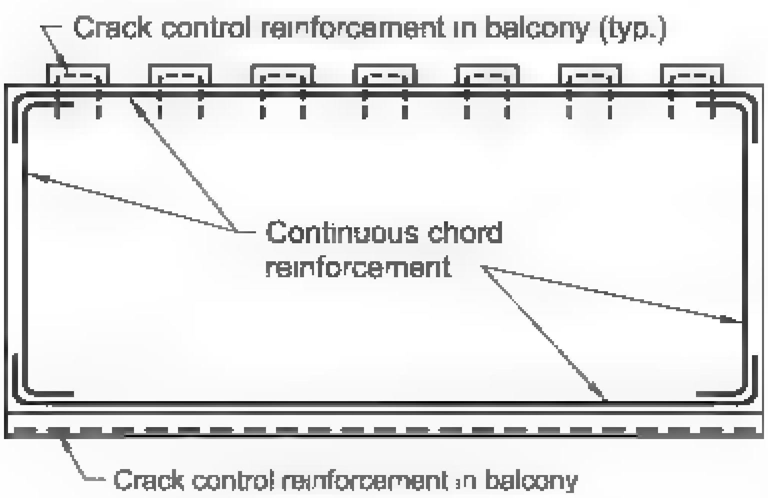
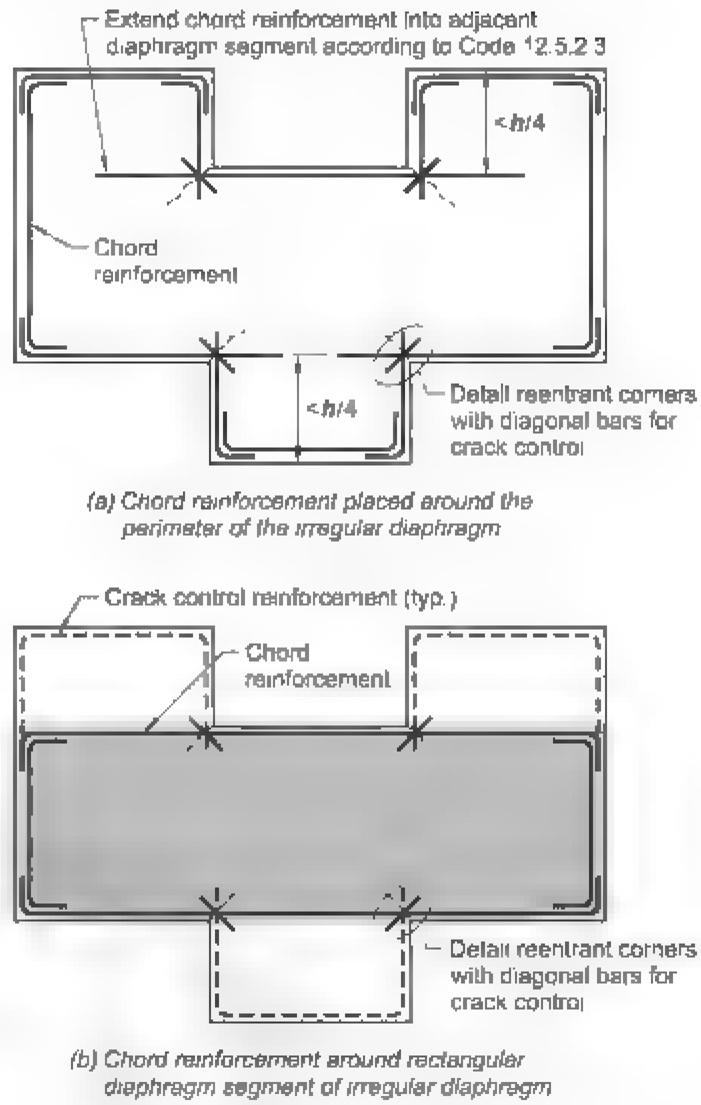


Fig. 8.6d—Chord reinforcement in a diaphragm with balconies.

For connections to lateral moment-resisting frames, collector longitudinal bars need to extend at least ℓ_d into the frame. Additional reinforcement in the frame's beams could be necessary to transfer the force to other columns of the resisting frame (Fig. 8.6c).

The minimum clear spacing between bars is the greatest of 1 in., one bar diameter, d_b, and (4/3) maximum aggregate size d_{agg} (Code Section 25.2). The maximum spacing is the lesser of 18 in. and five times the diaphragm thickness (Code Section 12.7.2.2).



(a) Chord reinforcement placed around the perimeter of the irregular diaphragm

(b) Chord reinforcement around rectangular diaphragm segment of irregular diaphragm

Fig. 8.6e—Chord reinforcement of irregular diaphragms.

Irregular diaphragms and diaphragms with balconies require special detailing requirements. For a diaphragm with a continuous balcony along one side, chord reinforcement can be placed either along the exterior edge of the balcony or along the exterior frame of a building. It is recommended to place chord reinforcement along the exterior frame of a building and additional crack control reinforcement in the exterior edge of the balcony (refer to Fig. 8.6d). For discontinuous balconies, placing chord reinforcement in the individual balconies will result in a discontinuous chord that is not structurally integral. It will create a complex load path (refer to Fig. 8.6d). For both cases, especially in cold climates where freezing and thawing could result in concrete cracking and exposing bars to moisture, which may result in deterioration of bars, chord reinforcement is recommended to be placed in line with the exterior frame.

Irregular diaphragms like those shown in Fig. 8.6e(a) and (b) can have chord reinforcement placed either around the perimeter of the diaphragm or around the designated rectangular diaphragm segment. If chord reinforcement is detailed per Fig. 8.6e(a), then special precautions are required to detail the development of edge reinforcement into the adjacent diaphragm segment beyond the reentrant corners (Code 12.5.2.3). One alternative to this chord arrangement is to place chord reinforcement within the designated diaphragm segment and detail crack control reinforcement in the slab

areas extending from the designated diaphragm, as shown in Fig. 8.6e(b).

8.7—Summary steps

- Determine if diaphragm can be considered as rigid.
- Calculate F_x at each level from structural analysis.
- Evaluate the diaphragm inertial force F_{px} at the floor and roof levels

$$F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px}$$

- Check that the diaphragm inertial force is within the maximum limits.
- Use the larger of F_x and F_{px} to analyze each diaphragm.
- Add shear forces resulting from the transfer of seismic-load-resisting vertical elements or changes in the relative stiffness to F_{px}. The additional forces are multiplied by the redundancy factor ρ, equal to that used in the design of the structure.
- Include torsion but ignore its effect if it reduces shear in the lateral-load-resisting vertical elements.
- Calculate the net shear in the vertical elements due to F_{px}, which is the difference in shear forces resisted by the vertical elements immediately above and below the level of the diaphragm being designed.
- Determine a set of equivalent loads at the diaphragm level that is in equilibrium with the shear forces determined in Step 6.
- Use the equivalent loads to determine the shear and bending moment at critical sections of the diaphragm.
- Compute the shear per unit length to check the shear strength of the diaphragm.
- Provide collectors to transfer the shear that is in excess of force transferred directly into the vertical elements.
- Calculate the shear strength of the diaphragm and compare it to the factored shear force.
- Check collectors (or equivalent widths of slab assumed to act as a collector) and their connections for diaphragm chord forces.
- Extend chords at reentrant corners, if any, to develop the forces calculated at the critical sections.
- Check shear friction at wall-to-slab interface (SEAOC 2005).

REFERENCES

American Society of Civil Engineers

ASCE/SEI 7-10—Minimum Design Loads for Buildings and Other Structures

ASTM International

ASTM A615/A615M-15—Standard Specification for Deformed and Carbon Steel Bars for Concrete Reinforcement

ASTM A706/706M-14—Standard Specification for Deformed and Plain Low-Alloy Steel Bars for Concrete Reinforcement

Authored documents

Moehle, J. P.; Hooper, J. D.; Kelly, D. J., and Meyer, T. R., 2016, "Seismic Design of Cast-in-Place Concrete Diaphragms, Chords, and Collectors: A Guide for Practicing Engineers," *National Earthquake Hazards Reduction Program (NEHRP) Seismic Design Technical Brief No. 3*, second edition, The National Institute of Standards and Technology (NIST) GCR 16-917-42, Gaithersburg, MD, 39 pp.

Structural Engineer's Association of California (SEAOC), 2005, "Design of Concrete Slabs as Seismic Collectors," Seismology and Structural Standards Committee, Structural Engineers of California, Sacramento, CA, May, 15 pp.

Diaphragms

8.8—Examples

Rigid Diaphragm Example 1 *Reinforced concrete diaphragm without an opening*—An eight-story structure with 5 x 6 bays in the North-South (N-S) and East-West (E-W) directions, respectively, is located in a low-intensity earthquake region. Design the fourth level diaphragm for the following data.

<u>Given</u>:

Geometry—
Bays are 14 ft-0 in. x 36 ft-0 in. (Fig. E1.1(b))
Columns: 24 in. x 24 in.
Story height: Refer to Fig. E1.1(a)
Slab thickness: h = 7 in.
Shear wall thickness: t_w = 12 in.
Perimeter beams: W x H = 18 in. x 30 in.

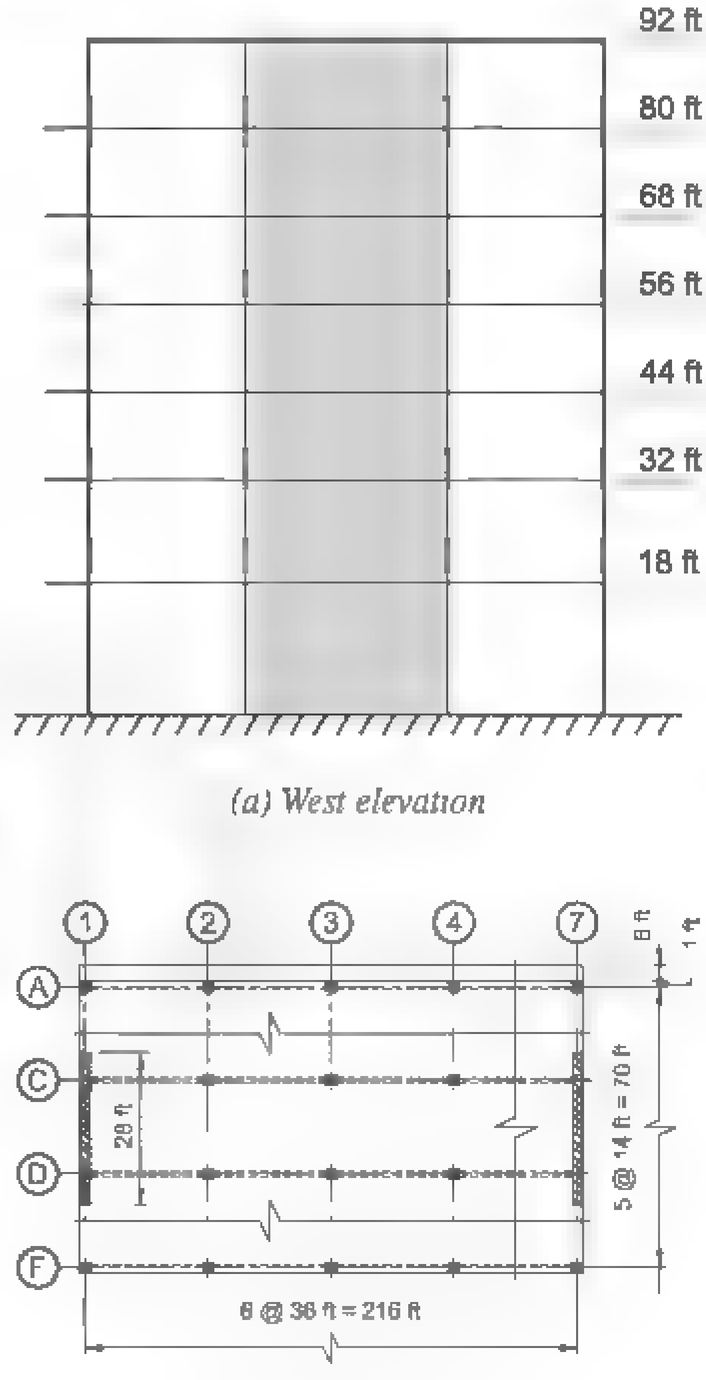
Concrete—
f_c' = 5000 psi
f_y = 60,000 psi

Seismic criteria—
Site class: D
S_s = 0.15 (ASCE/SEI 7, Fig. 22-1)
S_1 = 0.08 (ASCE/SEI 7, Fig. 22-2)
T_L = 12 (ASCE/SEI 7, Fig. 22-14)
R = 4 (ASCE/SEI 7, Table 12.2-1), ordinary reinforced shear wall along Column Lines 1 and 7
R = 5 (ASCE/SEI 7, Table 12.2-1), intermediate moment frame along Column Lines A and F

Building assigned to: Seismic Design Category (SDC) B

Wind criteria—
Building risk category: II ASCE/SEI 7 Table 1.5-1
Importance, I_w = 1 (ASCE/SEI 7, Table 1.5-2)
Wind speed = 115 mph (ASCE/SEI 7, Fig. 26.5-1A)
K_d = 0.85 (ASCE/SEI 7, Table 26.6-1)
Exposure category C (ASCE/SEI 7, Section 26.7)

Topographic effects—
K_{zt} = 1 (ASCE/SEI 7, Section 26.8) Flat
GC_{pi} = 0.18 (ASCE/SEI 7, Table 26.11.1) Enclosed building



(a) West elevation

(b) Fourth level plan

Fig. E1.1—Eight story building.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318). The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi.
Step 2: Slab geometry		
12.3.1.1	Diaphragm thickness must satisfy the requirements for stability, strength, and stiffness under factored load combinations.	
12.3.1.2 7.3.1.1	For simplicity, specify floors and roof slab diaphragm thickness that satisfy the minimum one-way slab thickness, spanning in the short direction ℓ_n = 14 ft with f_y = 60,000 psi and without interior beams. The minimum thickness for exterior panels is $\ell_n/24$, and for interior panels with both ends continuous is $\ell_n/28$. $h_{min} > \ell_n/24$ **Controls**	$h_{min} \geq \dfrac{(14\text{ ft})(12\text{ in./ft}) - 18\text{ in.}}{24} = 6.25\text{ in.}$, say, 7 in.

Step 3. Lateral forces

Seismic and wind force calculations are not shown as they are outside the scope of this Manual.
For seismic design, two sets of design forces are usually specified:

1. F_x is the vertical force distribution to the lateral-force-resisting system (LFRS)
2. F_{px} diaphragm design force at Level x

The eight-story building is analyzed for the seismic and wind effects. Diaphragms are designed for the maximum calculated force. The diaphragm is modeled as rigid in the analysis based on the span-to-depth ratio of 216 ft/70 ft = 3.

Tables E.1 and E.2 compare the lateral wind, W, and seismic, F_{px}, lateral forces. The calculations are not shown as it is outside the scope of this Manual.

Table E.1—North-South direction

Story	Height, ft	Seismic F_{px}, kip	Wind, kip			Controls?*
			WW	LW	Combined	
Roof	92	**118**	31.83	19.89	51	S
7	80	**125**	61.82	39.79	102	S
6	68	**119**	59.74	39.79	100	S
5	56	**108**	57.34	39.79	97	S
4	44	**95**	54.51	39.79	94	S
3	32	87	55.22	43.10	**98**	W
2	18	81	60.21	53.05	**113**	W

Table E.2—East-West direction

Story	Height, ft	Seismic F_{px}, kip	Wind, kip			Controls?
			WW	LW	Combined	
Roof	92	**136**	10.51	6.57	17	S
7	80	**157**	20.42	13.14	34	S
6	68	**157**	19.73	13.14	33	S
5	56	**152**	18.94	13.14	32	S
4	44	**140**	18.00	13.14	31	S
3	32	**126**	18.24	14.24	32	S
2	18	**116**	19.89	17.52	37	S

Note: The controlling force for diaphragm design is represented by S for seismic or W for wind; WW is windward pressure and LW is leeward suction, which are additive.

From the tables above, seismic forces at the fourth floor control the diaphragm design in both directions.

Where $F_x = C_{vx}V$, $C_{vx} = \dfrac{w_x h_x^k}{\sum w_i h_i^k}$, and $F_{px} = \dfrac{\sum F_i}{\sum w_i} w_{px}$ (ASCE/SEI 7, Eq. (12.8-11), (12.8-12), and (12.10-1))

F_{px} is limited to $F_{px,min} = 0.2S_{DS}I_e w_{px}$ and $F_{px,max} = 0.4S_{DS}I_e w_{px}$ (ASCE/SEI 7, Eq. (12.10-2) and (12.10-3))

Step 4. Center of mass (COM) and center of rigidity (COR)

Design fourth-level diaphragm

Take the point of origin at the lower left corner of the building, F1

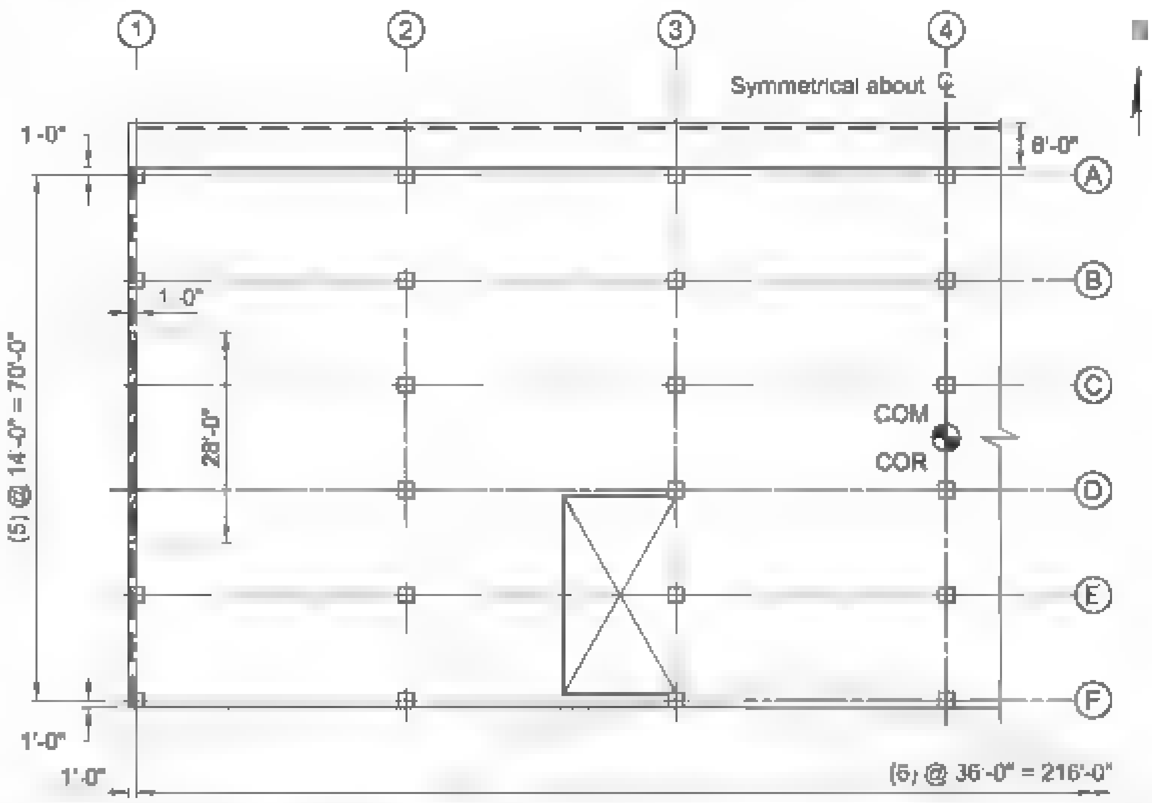


Fig. E1.2—Mass center and rigidity center location

<u>Determine COM</u>

The diaphragm and wall configuration are symmetrical about both x- and y-axes. Therefore, the COM is located at 108 ft-0 in. east of Column Line 1 and 38 ft 0 in. north of Column Line F.

<u>Determine COR</u>

Because of symmetry, the COR and COM coincide.

<u>Accidental torsion</u>

ASCE/SEI 7 commentary Section C12.10.1 considers an additional moment caused by an assumed displacement of COM. A shift of minimum of 5 percent of the building dimension perpendicular to the direction of seismic forces in addition to the actual eccentricity is assumed, referred to as accidental eccentricity.

$e_x = 0 \text{ ft} \pm (0.05)(218 \text{ ft})\ \pm 10.9 \text{ ft}$

$e_y = 0 \text{ ft} \pm (0.05)(78 \text{ ft})\ \pm 3.9 \text{ ft}$

Step 5. Lateral system stiffness

The diaphragm is idealized as a beam whose depth is equal to the full diaphragm depth spanning between idealized rigid supports (shear walls at Column Line 1 and 7 in the N-S direction) or resisted by the building frame in the E-W direction. Therefore, lateral forces are distributed in proportion to the relative stiffnesses of the resisting walls or frames.
Lateral displacement is the sum of flexural and shear displacements.

<u>Wall stiffness in N-S direction</u>

$\Delta = \Delta_{Flexure} + \Delta_{Shear}$

$$\Delta = \frac{Ph^3}{3EI} + \frac{1.2Ph}{AG} \quad \text{where } G \cong 0.4E \text{ and } E = 57{,}000\sqrt{f_c'} = 4{,}030{,}500 \text{ psi}$$

$$\Delta_{Flexure} = \Delta_{Fi} = \frac{P_i h_i^3}{3EI_i} = \frac{P_i h_i^3}{3E\frac{L_i^3 t_i}{12}} = \frac{4P\left(\frac{h_i}{L_i}\right)^3}{Et_i}$$

$$\Delta_{Shear} = \Delta_{Vi} = \frac{1.2Ph}{AG} = \frac{(1.2)Ph_i}{(L_i t_i)0.4E} = \frac{3P\left(\frac{h_i}{L_i}\right)}{Et_i}$$

$\Delta_i = \Delta_{Fi} + \Delta_{Vi}$

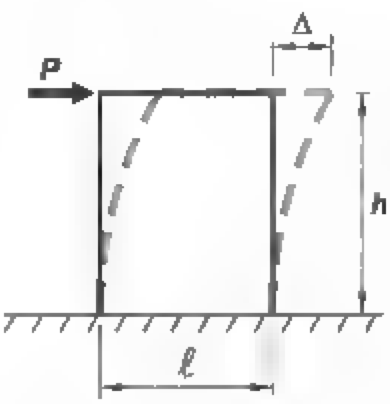


Fig. E1.3—Cantilever wall deflection.

Rigidity $k_i = \sum(1/\Delta_i)$

Wall at CL*	Height h, ft	Length L, ft	h/L	t, in.	$\Delta_{Fi} \times 10^{-3}$, in.	$\Delta_{Vi} \times 10^{-3}$, in.	$\Delta_i \times 10^{-3}$, in.	$k_i = 1/\Delta_i \times 10^{-3}$, in.
1	12	28	0.429	12	$0.0263/E_c$	$0.107/E_c$	$0.1336/E_c$	$7.5E_c$
7	12	28	0.429	12	$0.0263/E_c$	$0.107/E_c$	$0.1336/E_c$	$7.5E_c$

*CL = Column Line.

<u>Equivalent story stiffness of moment frames in the E-W direction</u>

$$k = \frac{12E_c}{h^2\left(\frac{1}{\sum k_c + \sum k_b}\right)}$$, where $k_c = I_c/h$ column stiffness, and $k_b = I_b/\ell$ beam stiffness

$$k_c = \frac{(24\text{ in.})(24\text{ in.})^3}{12(12\text{ ft})(12\text{ in./ft})} = 192\text{ in.}^3$$

$$k_b = \frac{(18\text{ in.})(30\text{ in.})^3}{12(36\text{ ft-}2\text{ ft})(12\text{ in./ft})} = 99\text{ in.}^3$$

In a frame there are seven columns and six beams

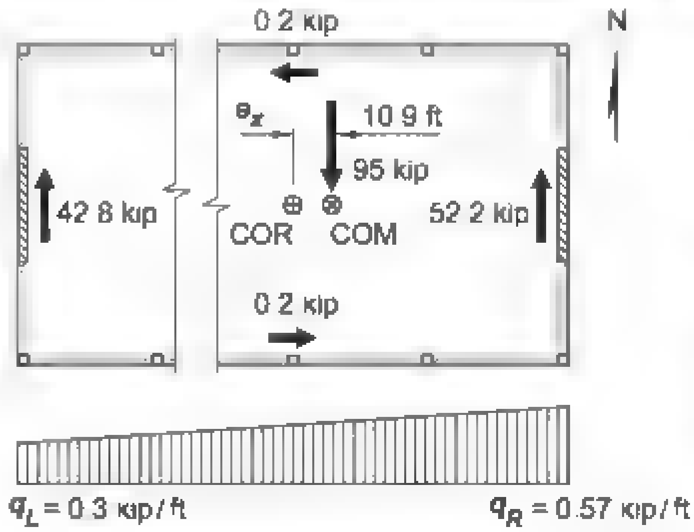
$$k = \frac{12E_c}{((12\text{ ft})(12\text{ in./ft}))^2\left(\frac{1}{(7)(192\text{ in.}^3)+(6)(99\text{ in.}^3)}\right)} = 1.122E_c\frac{1}{\text{in.}}$$

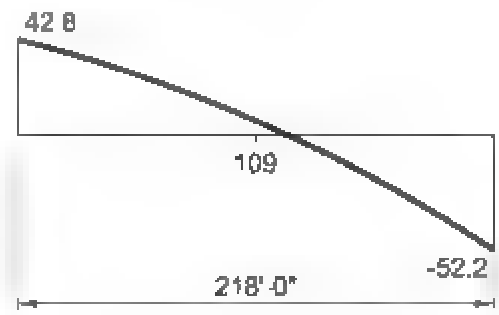
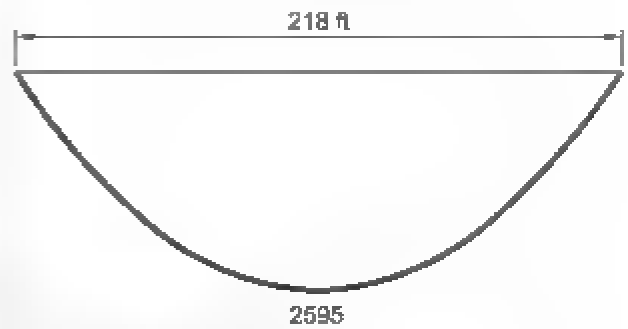
Relative stiffness: Frames = 1 and Walls = 6.7

Step 6: Lateral resisting system forces
The shear wall reactions resist the direct inertial forces F_{px} and forces from accidental torsion. Forces in walls and moment frames are $$F_{uxi} = \frac{k_{ix}}{\sum k_{ix}} F_{px} \pm \frac{k_i d_i}{\sum k_i d_i^2} F_{px} e$$ $$F_{uyi} = \frac{k_{iy}}{\sum k_{iy}} F_{py} \pm \frac{k_i d_i}{\sum k_i d_i^2} F_{py} e_x$$ where d_i is the distance (x_i or y_i) of each wall from the COR $F_{px,y}$ = 95 kip and $F_{px,x}$ = 140 kip are fourth story lateral forces obtained from Step 3 (Tables E.1 and E.2), N-S and E-W directions, respectively. e_x = 10.9 ft and e_y = 3.9 ft are calculated in Step 4. The torsional moment is calculated by multiplying the lateral inertia force by the corresponding eccentricity. NS: $T_y = F_{px,y} e_x$ = (95 kip)(±10.9 ft) = ±1036 ft-kip EW: $T_x = F_{px,x} e_y$ = (140 kip)(±3.9 ft) = ±546 ft-kip $F_u = F_{ui} + F_{ti}$ <u>Lateral force applied in N-S direction</u> F_{py} = 95 kip $$F_{u,wall,max} = \frac{6.7}{6.7+6.7}(95\text{ kip}) + \frac{6.7(218\text{ ft}/2)}{2[(1.0)(39\text{ ft})^2 + (6.7)(109\text{ ft})^2]}(1036\text{ ft-kip}) = 52.2\text{ kip}$$ $$F_{u,wall,min} = \frac{6.7}{6.7+6.7}(95\text{ kip}) - \frac{6.7(218\text{ ft}/2)}{2[(1.0)(39\text{ ft})^2 + (6.7)(109\text{ ft})^2]}(1036\text{ ft-kip}) = 42.8\text{ kip}$$ $$F_{u,MF,max} = \pm\frac{(1.0)(39\text{ ft})}{2[(1.0)(39\text{ ft})^2 + (6.7)(109\text{ ft})^2]}(1036\text{ ft-kip}) = \pm 0.2\text{ kip}$$ <u>Lateral force applied in E-W direction</u> F_{px} = 140 kip $$F_{u,MF,max} = \frac{1.0}{1.0+1.0}(140\text{ kip}) + \frac{(1.0)(78\text{ ft}/2)}{2[(1.0)(39\text{ ft})^2 + (6.7)(109\text{ ft})^2]}(546\text{ ft-kip}) = 70.1\text{ kip}$$ $$F_{u,MF,min} = \frac{1.0}{1.0+1.0}(140\text{ kip}) - \frac{(1.0)(78\text{ ft}/2)}{2[(1.0)(39\text{ ft})^2 + (6.7)(109\text{ ft})^2]}(546\text{ ft-kip}) = 69.9\text{ kip}$$ $$F_{u,wall,max} = \pm\frac{(6.7)(218\text{ ft}/2)}{2[(1.0)(39\text{ ft})^2 + (6.7)(109\text{ ft})^2]}(546\text{ ft-kip}) = \pm 2.5\text{ kip}$$ The difference due to eccentricity in the E-W direction is small. Therefore, use 70 kip.

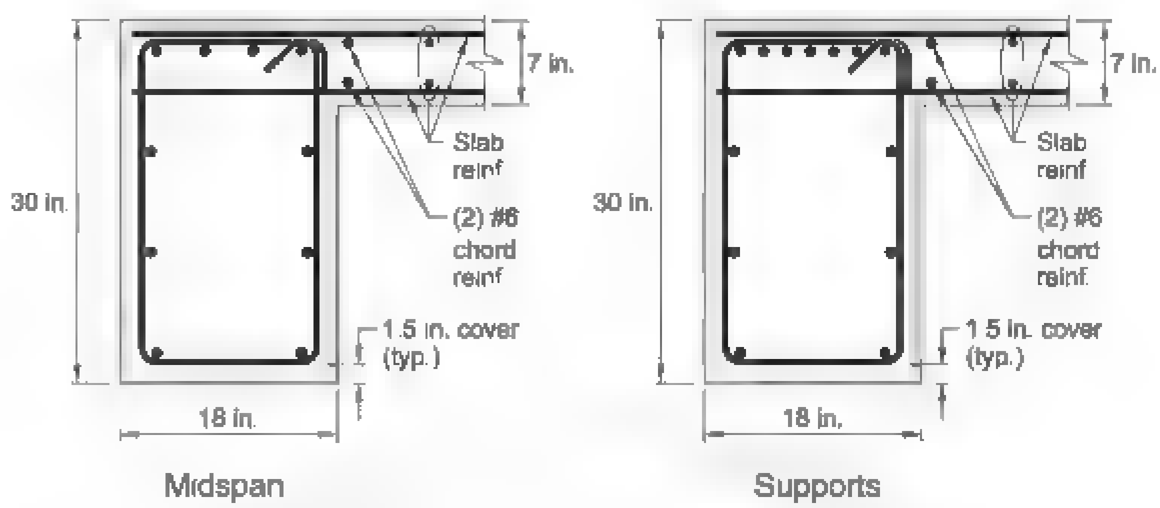
Diaphragms

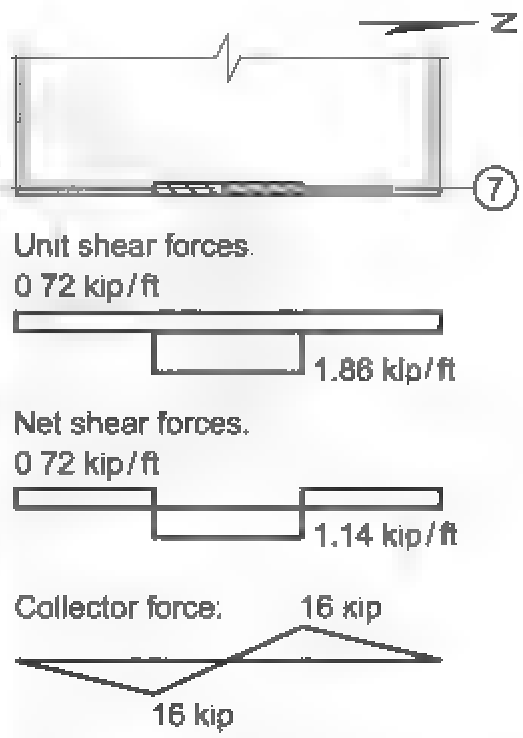
Step 7. Diaphragm shear strength		
	In-plane shear in diaphragm The diaphragm shear strength is calculated from	
12.5.3.3	$V_n = A_{cv}\left(2\lambda\sqrt{f'_c} + \rho_t f_y\right)$ (12.5.3.3)	Nominal shear strength in E-W direction $V_{n,N} = (218\text{ ft})(12\text{ in./ft})(7\text{ in.})\left(2(1.0)\sqrt{5000\text{ psi}} + 0\right)$ $= 2{,}589{,}708\text{ lb} = 2590\text{ kip}$
	Ignoring the strength contribution of reinforcement, $\rho_t = 0$ A_{cv} is the diaphragm gross area less the 6.0 ft overhang (refer to Step 9 for further clarification).	Nominal shear strength in N-S direction $V_{n,E} = (72\text{ ft})(12\text{ in./ft})(7\text{ in.})\left(2(1.0)\sqrt{5000\text{ psi}} + 0\right)$ $= 855{,}316\text{ lb} = 855\text{ kip}$
12.5.3.2 21.2.4.2	Applying the shear strength reduction factor ϕ = 0.75 at the north and south ends along column lines A and F	$\phi V_{n,N} = (0.75)(2590\text{ kip}) = 1940\text{ kip}$
12.5.3.2 21.2.4.1	At the east and west ends along column lines 1 and 7, the shear strength reduction factor, ϕ, must not exceed the least value for shear used for the vertical components of the primary seismic-force resisting system (wall). Therefore, $\phi = 0.6$:	$\phi V_{n,E} = (0.6)(855\text{ kip}) = 513\text{ kip}$
12.5.1.1	Check if factored shear force is less than design shear strength calculated in Step 6.	NS: $\phi V_n = 513\text{ kip} >> F_u = 52\text{ kip}$ **OK** EW: $\phi V_n = 1940\text{ kip} >> F_u = 70\text{ kip}$ **OK**
		Therefore, diaphragm has adequate strength to resist the lateral inertia force and shear reinforcement is not required, $\rho_t = 0$.
12.5.3.4	The nominal shear strength, V_n, must not exceed: $V_n = 8A_{cv}\lambda\sqrt{f'_c}$	$V_n = \dfrac{8(72\text{ ft})(10\text{ in.})(12\text{ in./ft})(1.0)\sqrt{4000\text{ psi}}}{1000\text{ lb/kip}} = 4372\text{ kip}$
	A_{cv} is the diaphragm gross area less the 6.0 ft overhang	By inspection this is satisfied. **OK**

Step 8. Diaphragm lateral force distribution N-S		
	Lateral force is distributed to the walls as follows, refer to Fig. E1.4	
12.4.2.4(a) 12.5.1.3(a)	Diaphragm is idealized as rigid. Design moments, shear, and axial forces are calculated assuming the diaphragm as a beam supported by idealized rigid supports with a depth equal to full diaphragm depth.	
	The wall and frame forces and the assumed direction of torque due to the eccentricity are shown in Fig. E1.4.	
12.4.2.4	The diaphragm force distribution is calculated by using q_L and q_R as the left and right diaphragm reactions per unit length (Fig. E1.4):	*Fig. E1.4— Seismic forces in the lateral-force-resisting systems in the N-S direction*
	Design force: 95 kip	
	Force equilibrium $q_L\left(\frac{L}{2}\right)+q_R\left(\frac{L}{2}\right)=F_{px,der(NS)}$	$q_L\left(\frac{218\text{ ft}}{2}\right)+q_R\left(\frac{218\text{ ft}}{2}\right)=95\text{ kip}$ (I)
	Moment equilibrium $q_L\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)+q_R\left(\frac{L}{2}\right)\left(\frac{2L}{3}\right)=F_{px,der(NS)}\left(\frac{L}{2}+0.05L\right)$	$q_L\frac{(218\text{ ft})^2}{6}+q_R\frac{2(218\text{ ft})^2}{6}=(95\text{ kip})\left(\frac{218\text{ ft}}{2}+10.9\text{ ft}\right)$ (II)
	From statics solve equations (I) and (II) for q_L and q_R.	$q_L+q_R=0.87$ kip/ft (I) $q_L+2q_R=1.44$ kip/ft (II) q_L =0.3 kip/ft and q_L = 0.57 kip/ft

	Find the maximum moment by taking the first derivative of the moment equation expressed as a function of x (unknown distance) $dM/dx = 0$	$x = 114.3$ ft $M_{max} = 2595$ ft-kip
	Draw the shear and moment diagrams (Fig. E1.5). Note: In an Aug. 2010 National Institute and Standards Technology (NIST) report, GCR 10-917-4, "Seismic Design of Cast-in-Place Concrete Diaphragms, Chords, and Collectors," by Moehle et al. states that, "This approach leaves any moment due to the frame forces along column lines A and F unresolved. Sometimes this is ignored or, alternatively, it too can be incorporated in the trapezoidal loading." In this example the small moment due to the frame forces (0.2 kip) is ignored.	 *Diaphragm shear diagram (kip)*  *Diaphragm moment diagram (ft-kip)* *Fig. E1.5—Shear and moment diagrams.*
	In smaller buildings in which seismic demand is low, there are no irregularities, and torsional moments are not significant, the diaphragm shears and moments can be based on a uniformly distributed load, rather than a linearly varying load.	$q = \dfrac{95\text{ kip}}{218\text{ ft}} = 0.44$ kip/ft
	Resulting in a maximum moment of:	$M_{max} = \dfrac{(0.44\text{ kip/ft})(218\text{ ft})^2}{8} = 2614$ ft-kip Note: Both approaches, in this example, result in close maximum moments (less than 1 percent), but at different locations (114.3 ft versus 109 ft). Shear diagram for the second approach is a straight line with equal shear force at both ends. In this example, the detailed approach is presented.
Step 9: Chord reinforcement N-S		
	Maximum moment is calculated above:	$M_u = 2595$ ft-kip
12.5.2.3	Chord reinforcement resisting tension must be located within $h/4$ of the tension edge of the diaphragm.	$h/4 = 72.0$ ft/4 = 18 ft
	Note: Chord reinforcement can be placed either in the exterior edge of the balcony or it can be placed along the exterior frame of CL A. Placing chord reinforcement along the exterior frame is a simpler and cleaner load path for the forces in the diaphragm. Crack control reinforcement should be added in the balcony slab for crack control.	

	Assume tension reinforcement is placed in a 3 ft strip at both north and south sides of the slab edges at CLs A and F	$3\text{ ft} < h/4 = 18\text{ ft}$ **OK**
	<u>Chord force</u> The overhang is placed monolithic with the rest of the slab. Chord forces are usually highest furthest from the geometric centroid, in this case, edge of the overhang. To prevent cracking, place chord reinforcement at the outside edge of the overhang. The maximum chord tension force is calculated at 14.3 ft east of CL 1	
	$T_u = \dfrac{M_u}{B - 3\text{ ft}}$	$T_u = \dfrac{2595\text{ ft-kip}}{72\text{ ft} - 3\text{ ft}} = 37.6\text{ kip}$
12.5.2.2	Tension due to moment is resisted by deformed bars conforming to Section 20.2.1 of ACI 318.	
12.5.1.5	Steel stress is the lesser of the specified yield strength and 60,000 psi	$f_y = 60{,}000\text{ psi}$
12.5.1.1	<u>Required reinforcement</u> $\phi T_n = \phi f_y A_s > T_u$	$A_{s,req'd} = \dfrac{37{,}600\text{ lb}}{(0.9)(60{,}000\text{ psi})} = 0.70\text{ in.}^2$
22.4.3.1	The building is assigned to SDC B. Therefore, Chapter 18 requirements for chord spacing and transverse reinforcement of Section 18.12.7.6 of ACI 318 do not apply	
18.12.7.6	Overstrength factor Ω_o for chord design is not required. Therefore, use the compression stress limit in provision 18.12.7.6 of $0.2f_c'$	
	Required chord width $w_{chord} > \dfrac{C_{chord}}{0.2f_c'h_{diaph}}$	$w_{chord} > \dfrac{37{,}600\text{ lb}}{(0.2)(5000\text{ psi})(7\text{ in.})} = 5.4\text{ in.}$ Less than 3 ft assumed. Therefore, **OK**
	Choose reinforcement	Try two No. 6 chord bars. $A_{s,prov} = 2(0.44\text{ in.}^2) = 0.88\text{ in.}^2$
	Check if provided reinforcement area is greater than required reinforcement area	$A_{s,prov} = 0.88\text{ in.}^2 > A_{s,req'd} = 0.70\text{ in.}^2$

	The engineer has two options for providing chord reinforcement along the exterior frames 1 Excess amount of reinforcement used in the beams to resist gravity loads could be used to resist part of the tensile force of the chord. Additional reinforcement is provided to resist the difference. 2 The chord force is resisted with additional reinforcement. Although the first option is more economical, the second option is detailed in this example Here again the engineer has several options 1 Place chord reinforcement outside the web width 2 Place chord reinforcement within the web width  *Fig. E1 6—Chord reinforcement along CLs A and F*

Step 10: Collector reinforcement N-S		
12.5.4.1	Collector elements transfer shear forces from the diaphragm to the vertical walls at both east and west ends along column lines 1 and 7 (Fig. E1.2). Collector elements extend over the full width of the diaphragm. Unit shear force $v_{u@F} = \frac{F_{u@F}}{B}$	From Step 6, F_u = 52.2 kip
	In diaphragm $v_{u@F} = \frac{F_{px}}{L_{diaph}}$	$v_{u@F} = \frac{52.2\text{ kip}}{72\text{ ft}} = 0.72\text{ kip/ft}$
	In wall $v_{u@F} = \frac{F_{px}}{L_{wall}}$	$v_{u@F} = \frac{52.2\text{ kip}}{28\text{ ft}} = 1.86\text{ kip/ft}$
	<u>Force at diaphragm to wall connection</u> East wall south end $F_{7/D.5} = -(0.72\text{ kip/ft})(22\text{ ft}) = -15.8\text{ kip}$ East wall north end $F_{7/B.5} = -15.8\text{ kip} + (1.14\text{ kip/ft})(28\text{ ft}) = 16.0\text{ kip}$ At diaphragm end $F_{7/A} = +16\text{ kip} - (0.72\text{ kip/ft})(22\text{ ft}) = 0.2\text{ kip}$ ≈ 0 kip due to number rounding. Per collector force diagram, the maximum axial force on the collector is $T_u = C_u = 16$ kip. This force must be transferred from the diaphragm to the collector to the shear wall (Fig. E1.7). The building is assigned to SDC B. Therefore, the collector force and its connections to the shear wall will not be multiplied by the system overstrength factor, $\Omega_o = 2.5$ (ASCE/SEI 7-10, Table 12.2-1).	 *Fig. E1.7—Collector force diagram.*
12.5.4.2	Collectors are designed as tension members, compression members, or both.	

12.5.1.1
22.4.3.1

Tension is resisted by reinforcement as calculated above

Required reinforcement:

$\phi T_n = \phi f_y A_s > T_u$

$$A_{s,req'd} = \frac{T_u}{0.9 f_y} = \frac{16{,}000 \text{ lb}}{(0.9)(60{,}000 \text{ psi})} = 0.3 \text{ in.}^2$$

Although one No. 5 bar suffices, two No. 5 bars are provided to maintain symmetry of load being transferred from the slab into the wall.

18.12.7.6

Check if collector compressive force exceeds $0.2f_c'$

Calculate minimum required collector width using $0.2f_c'$

$$w_{coll} > \frac{C_{Coll}}{0.2 f_c' t_{diaph}}$$

$$w_{coll} > \frac{16{,}000 \text{ lb}}{(0.2)(5000 \text{ psi})(7 \text{ in.})} = 2.3 \text{ in.}$$

Use 12 in. wide collector (same width as shear wall). Provide two No. 5 bars at mid-depth of slab to prevent additional out-of-plane bending stresses in the slab. Space the two No. 5 bars at 8 in. on center starting at 2 in. from the edge of the diaphragm within the 12 in. wide collector/shear wall (Fig. E1.8).

This results in compressive stress on the concrete diaphragm collector being relatively low. The section is adequate to transfer shear stress without additional reinforcement.

The collector compression and tension forces are transferred to the lateral force-resisting system within its width (shear wall). Therefore, no eccentricity is present and no in-plane bending occurs.

12.5.4.1

ACI 318 permits to discontinue the collector along the length of the shear wall where transfer of design collector is not required.

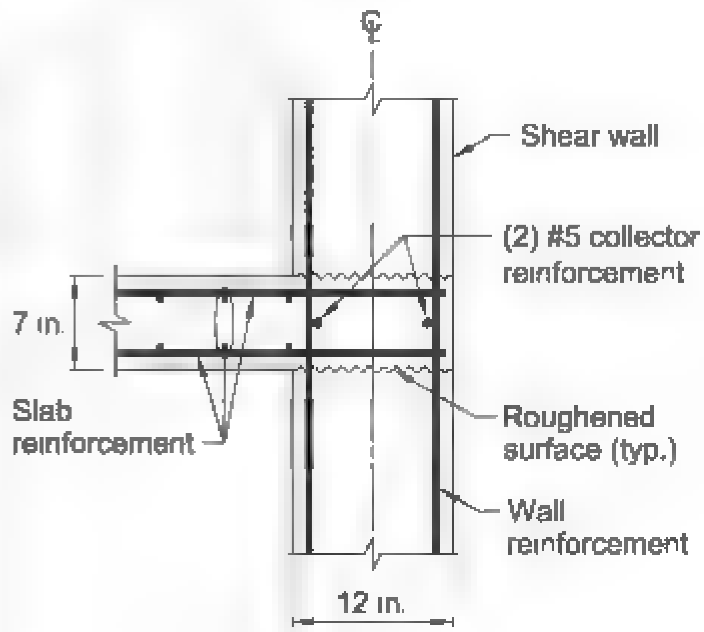


Fig. E1.8—Collector reinforcement.

<u>Check slab shear strength along shear walls</u>

12.4.2.4

Slab shear strength along walls

L = 28 ft and slab thickness t = 7 in.

From

12.5.3.3

$\phi V_c = \phi A_{cv} 2\lambda\sqrt{f_c'}$

$$\phi V_c = (0.75)(2)(1.0)\left(\sqrt{5000 \text{ psi}}\right)(28 \text{ ft})(12)(7 \text{ in.})$$

21.2.4.2

$\phi = 0.75$

ϕV_c = 249,467 lb ≈ 249 kip

12.5.1.1

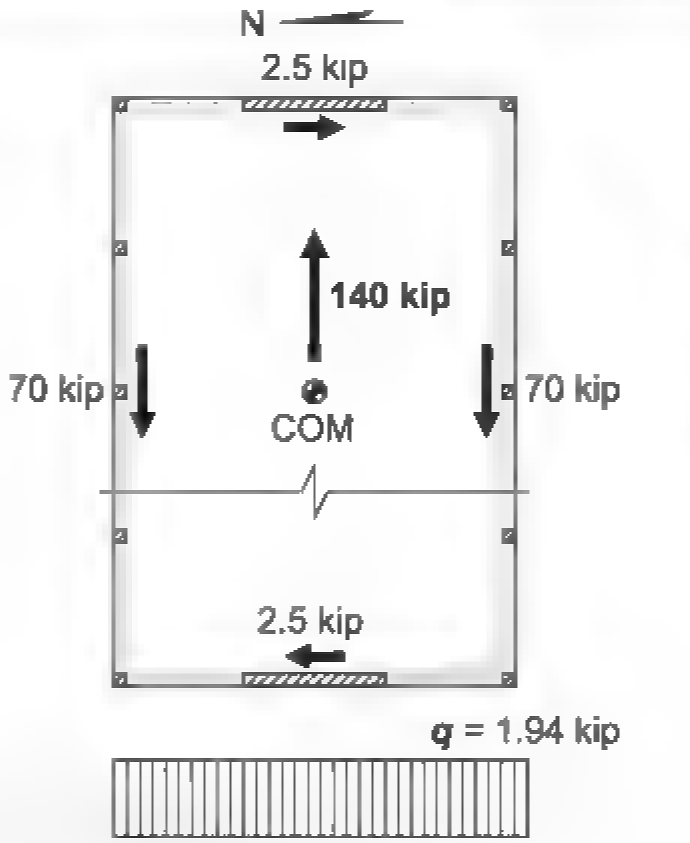
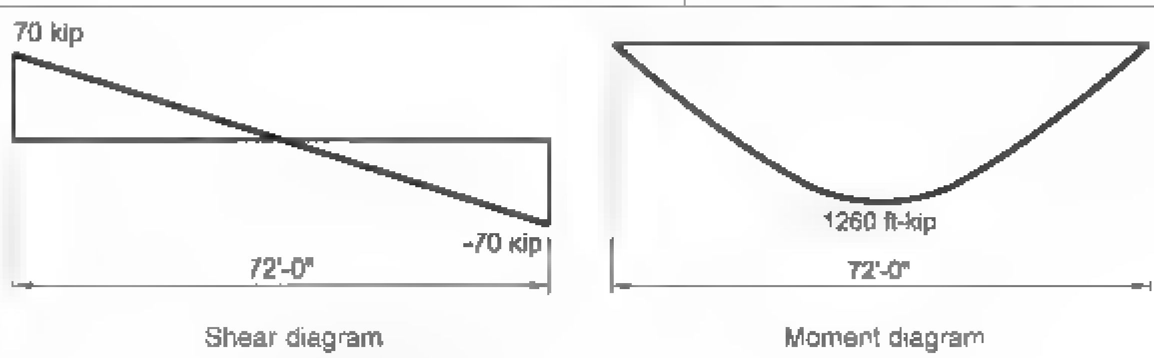
Is the provided shear strength adequate?

ϕV_c = 249 kip > V_u = 52 kip (from Step 7) **OK**

12.5.3.4

By inspection, the diaphragm shear design force satisfies the requirement of Section 12.5.3.4 of ACI 318

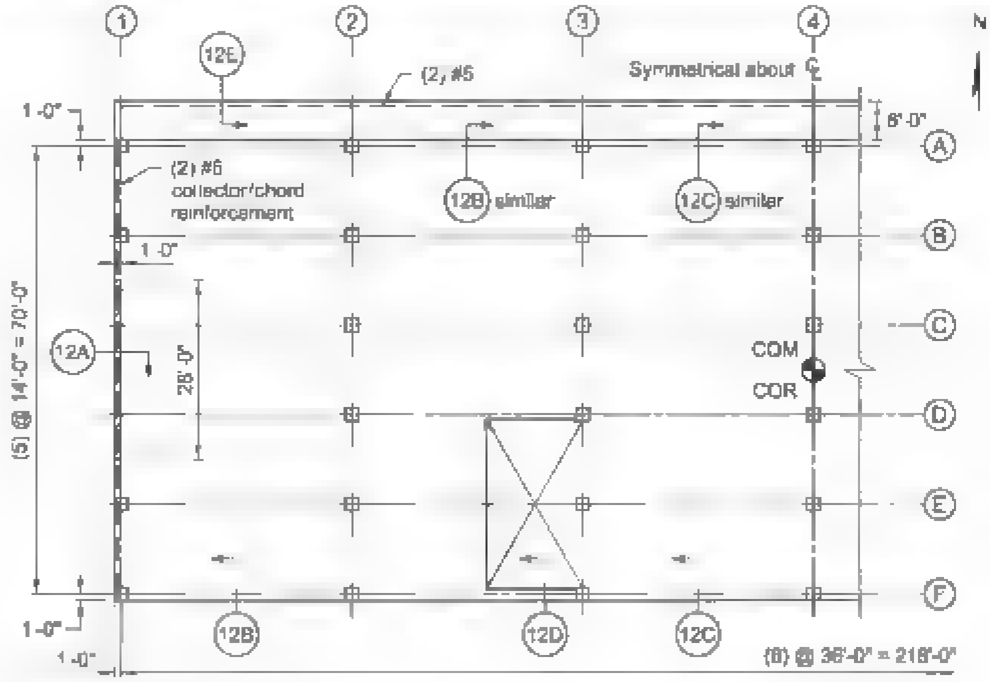
$\phi V_c = \phi A_{cv} 8\lambda\sqrt{f_c'}$

Step 11: Lateral force distribution in diaphragm E-W		
	Design force: 140 kip	 *Fig. E1.9—Seismic forces in the lateral force resisting systems in the E-W direction.*
12.4.2.4(a) 12.5.1.3(a)	Design moments, shear, and axial forces are calculated assuming a simply supported beam with depth equal to full diaphragm length (refer to Fig. E1.9).	
	Because of the negligible effect of accidental torsion, the inertial force is uniformly distributed across the diaphragm width.	$q_u = \left(\frac{140\text{ kip}}{72\text{ ft}}\right) = 1.94\text{ kip/ft}$
		Shear force: $V = (1.94\text{ kip/ft})(36\text{ ft}) = 70\text{ kip}$
	Maximum moment is located at midlength.	$x = 36$ ft
	Draw the shear and moment diagrams (Fig. E1.10).	$M_{max} = \frac{w\ell^2}{8} = \frac{(1.94\text{ kip/ft})(72\text{ ft})^2}{8} = 1260\text{ ft-kip}$
	 *Fig. E1.10—Shear and moment diagrams.*	

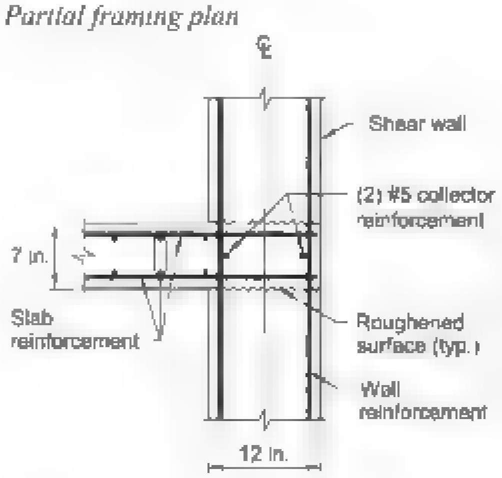
Step 12: Chord reinforcement E-W		
	Calculate chord reinforcement Maximum moment is calculated above (Fig. E1.8)	$M_u = 1260$ ft-kip
12.5.2.3	Chord reinforcement must be located within $h/4$ of the tension edge of the diaphragm.	$h/4 = 218.0\text{ ft}/4 = 54.5$ ft
	Assume tension reinforcement is placed within a 1 ft strip of the slab edge at both east and west sides of the slab	1 ft < $h/4$ = 54.5 ft **OK**
	Chord force The maximum chord tension force is at midspan $T_u = \dfrac{M_u}{L - 1\text{ ft}}$	$T_u = \dfrac{1260\text{ ft-kip}}{218\text{ ft} - 1\text{ ft}} = 5.8$ kip
12.5.2.2	Chord reinforcement Tension due to moment is resisted by deformed bars confirming to Code Section 20.2.1.	
12.5.1.5	Steel stress is the lesser of the specified yield strength and 60,000 psi	$f_y = 60{,}000$ psi
12.5.1.1 22.4.3.1	Required reinforcement $\phi T_n = \phi f_y A_s > T_u$	$A_{s,req'd} = \dfrac{T_u}{0.9 f_y} = \dfrac{5800\text{ lb}}{(0.9)(60{,}000\text{ psi})} = 0.1\text{ in.}^2$
	Along column lines 1 and 7, two No. 5 bars collector reinforcement are provided to resist inertia force in the N-S direction. These bars can be used for chord reinforcement in the E-W direction (refer to Fig. E1.8).	$A_{s,prov} = (2)(0.31\text{ in.}^2) = 0.62\text{ in.}^2 > 0.1\text{ in.}^2$ **OK**
	Maximum shear in the E-W direction occurs at CLs 1 and 7 Unit shear force in frame $v_{u@1,7} = \dfrac{F_{u@1,7}}{L}$	$v_{u@1,7} = \dfrac{70\text{ kip}}{218\text{ ft}} = 0.32$ kip/ft
Step 13. Collector reinforcement		
	Collector along CLs A and F: The continuous reinforced concrete frame over the full length of the building acts as a collector. Note: Provide continuous reinforcement with tension splices (Step 15).	
12.5.3.7	In cast-in-place diaphragms, where shear is transferred from the diaphragm to a collector, or from the diaphragm or collector to a shear wall, temperature and shrinkage reinforcement is usually adequate to transfer that force.	

Step 14: Shrinkage and temperature reinforcement		
12.6.1 24.4.3.2	The minimum area of shrinkage and temperature reinforcement, A_{S+T} $A_{S+T} \geq 0.0018A_g$	 $A_{S+T} = (0.0018)(7\text{ in.})(12\text{ in./ft}) = 0.15\text{ in.}^2/\text{ft}$
24.4.3.3	Spacing of S+T reinforcement is the lesser of $5h$ and 18 in.: (a) $5h = 5(12\text{ in.}) = 60$ in. (b) 18 in. **Controls**	Note: Shrinkage and temperature reinforcement may be part of the reinforcing bars resisting diaphragm in-plane forces and gravity loads. If provided reinforcement is not continuous (placing bottom reinforcing bars to resist positive moments at midspans and top reinforcing bars to resist negative moments at columns), continuity between top and bottom reinforcing bars can sometimes be achieved by providing adequate splice lengths between them
Step 15: Reinforcement detailing		
 12.7.2.1	<u>Reinforcement spacing</u> Minimum and maximum spacing of chord and collector reinforcement must satisfy 12.7.2.1 and 12.7.2.2	
25.2.1	Section 25.2 limits minimum spacing of (a) 1 in. (b) $4/3d_{agg}$ ($d_{agg} = 3/4$ in.) (c) d_b (No. 5)	 Minimum spacing 1.0 in. **Controls** $(4/3)(3/4\text{ in.}) = 1.0$ in. 0.625 in.
18.12.7.7a	The minimum collector reinforcement spacing at a splice must be at least the largest of: (a) Three longitudinal d_b (b) 1.5 in. (c) $c_c \geq \max[2.5d_b, 2\text{ in.}]$	 $3(0.625\text{ in.}) = 1.875$ in. 1.5 in. 2 in. **Controls**
12.7.2.2	Maximum spacing is the smaller of $5h$ or 18 in.	18 in. **Controls**

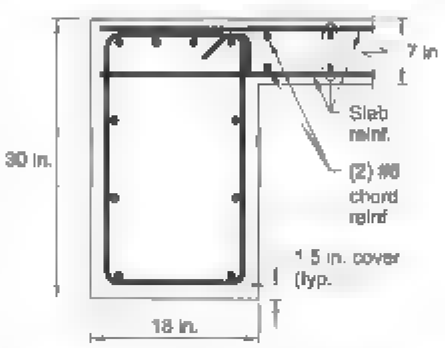
Step 16: Details



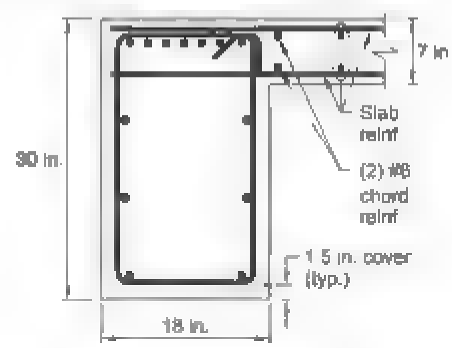
Partial framing plan



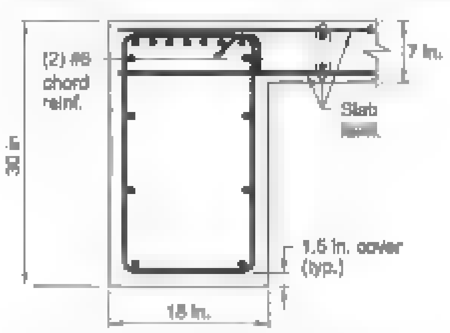
Section 12A—Chord and collector reinforcement at east and west ends of the diaphragm



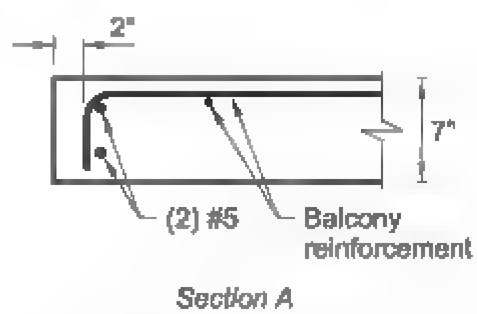
Section 12B—Chord reinforcement at midspan



Section 12C—Chord reinforcement at support



Section 12D—Chord reinforcement within the beam at midspan and support at opening location



Section 12E—Chord reinforcement at overhang

Note: Shrinkage and temperature reinforcement not shown for clarity

Fig. E1.11—Diaphragm chord and collector reinforcement

Rigid Diaphragm Example 2: *Reinforced concrete diaphragm with opening.* Refer to Diaphragm Example 1 for structure and design data. Analyze and design the second level floor diaphragm with a 14 ft 0 in. x 36 ft 0 in. opening as shown in Fig. E2.1. For diaphragm building elevation, material properties, and design criteria, refer to Diaphragm Example 1.

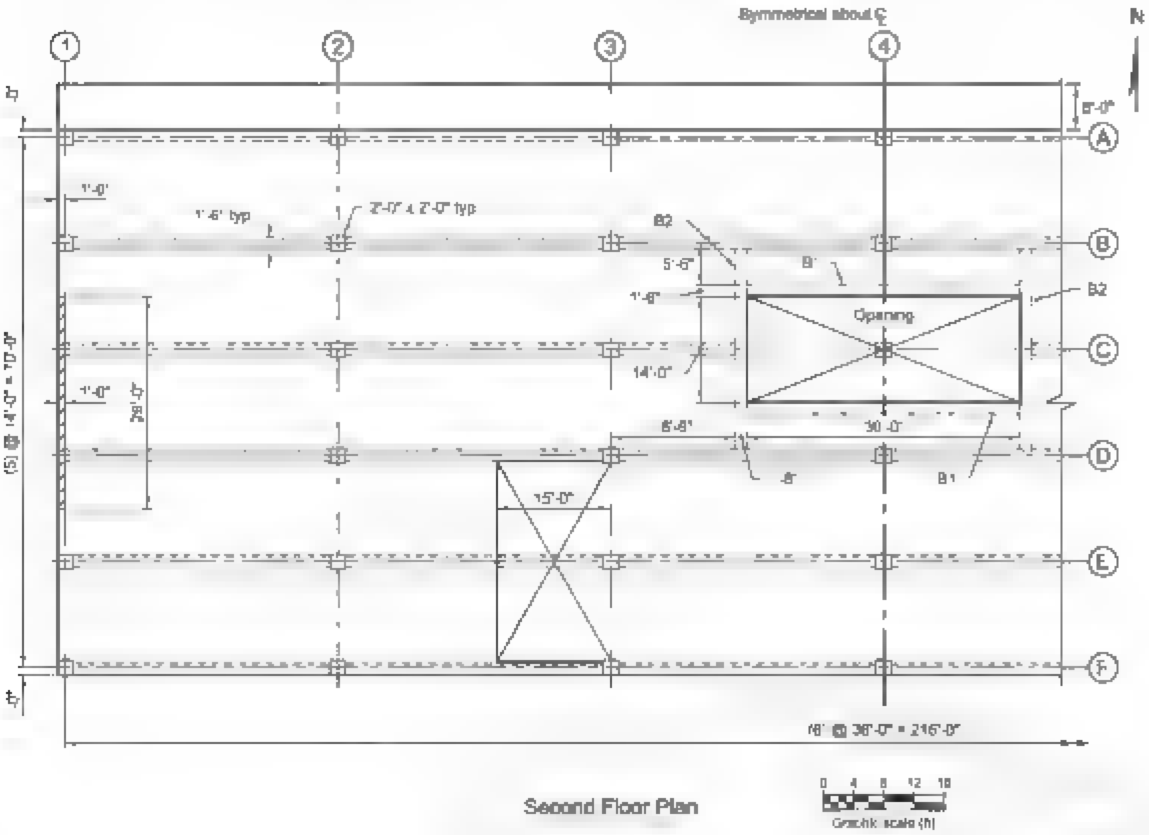


Fig. E2.1—Eight story building.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
	Refer to Rigid Diaphragm Example 1	
Step 2: Slab geometry		
	Satisfied per Rigid Diaphragm Example 1	
Step 3: Lateral forces		
For lateral forces and design forces calculations, refer to Rigid Diaphragm Example 1, Step 3. North South (N-S): 81 kip; although wind load controls (113 kip), the diaphragm will be designed for the seismic load in this example. East West (E-W): 116 kip		

Step 4. Center of mass (COM) and center of rigidity (COR)

Design second-level diaphragm

Take the point of origin at F1 (Fig. E2.2)

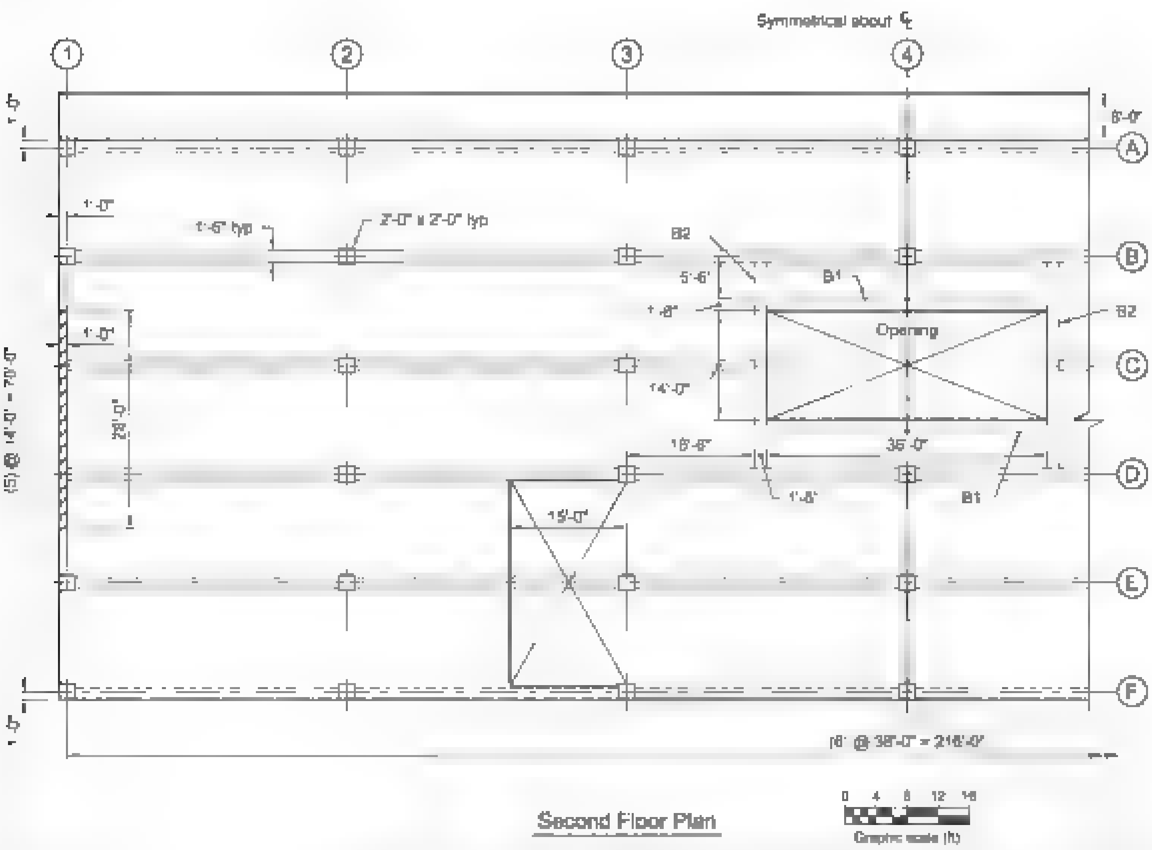


Fig. E2.2—Mass center and rigidity center location excluding accidental torsion.

<u>Determine COM</u>

The COM has shifted to the south because of the opening. Taking the moment area around column line (CL) F

$$y_{COM} = \frac{(78\text{ ft})(218\text{ ft})(37\text{ ft}) - (36\text{ ft})(14\text{ ft})(42\text{ ft})}{(78\text{ ft})(218\text{ ft}) - (36\text{ ft})(14\text{ ft})} = 36.8\text{ ft}$$

Therefore, the COM is located at 108 ft-0 in. east of CL. 1 and 36.8 ft north of CL. F

<u>Determine COR</u>

Because the lateral resisting systems are symmetrical about both axes, the COR is located at
x_{COR} = 218 ft/2 −1 ft = 108 ft and y_{COR} = 78 ft/2 − 1 ft = 38 ft from east of and north of Column Line F1
$\Delta_y = y_{COR} - y_{COM}$ = 38 ft − 36.8 ft = 1.2 ft

<u>Accidental torsion</u>

ASCE/SEI 7, commentary Section C12.10.1 requires an additional moment caused by an assumed displacement of COM. A shift of minimum of five percent of the building dimension perpendicular to the direction of seismic forces in addition to the actual eccentricity is considered, referred to as accidental eccentricity.

e_x = ±(0.05)(218 ft) = ±10.9 ft
e_{y1} = (0.05)(78 ft) = 3.9 ft
e_{y2} = −(0.05)(78 ft) = −3.9 ft

Step 5. Lateral system stiffness calculations

For wall and moment frame stiffness calculations refer to Rigid Diaphragm Example 1, Step 6.

Step 6: Lateral system force distribution

Force in walls and moment resisting frames are given by the following equations

$$F_{ux} = \frac{k_{ix}}{\sum k_{ix}} F_{px} \pm \frac{k_i d_i}{\sum k_i d_i^2} F_{px} e$$

$$F_{uyi} = \frac{k_{iy}}{\sum k_{iy}} F_{py} \pm \frac{k_i d_i}{\sum k_i d_i^2} F_{px} e_x$$

where d_i is the distance (x_i or y_i) of each wall from the COR

F_{py} = 80 kip and F_{px} = 114 kip are second-story lateral forces obtained from Example 1, Step 3 (Table), N-S and E-W directions, respectively

Mass accidental eccentricities are e_x = 10.9 ft, e_{y1} = 3.9 ft + 1.2 ft = 5.1 ft; and e_{y2} = −3.9 ft + 1.2 ft = −2.7 ft are calculated in Step 4 of this example.

The torsional force is calculated by multiplying the lateral inertia force by the corresponding eccentricity

NS: $T_y = F_{py}e_x$ = (81 kip)(±10.9 ft) = ±883 ft-kip

EW: $T_{x1} = F_{px}e_{y1}$ = (116 kip)(+5.1 ft) = +592 ft-kip

$T_{x2} = F_{px}e_{y2}$ = (116 kip)(−2.7 ft) = −313 ft-kip

<u>Lateral force applied in N-S direction</u>

F_{py} = 80 kip

$$F_{u,wall\,max} = \frac{10.5}{(10.5+10.5)}(81\text{ kip}) + \frac{(10.5)(218\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(883\text{ ft-kip}) = 44.5\text{ kip}$$

$$F_{u,wall\,min} = \frac{8.1}{(10.5+10.5)}(81\text{ kip}) - \frac{(10.5)(218\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(883\text{ ft-kip}) = 36.5\text{ kip}$$

$$F_{u,MF} = \pm\frac{(1.0)(78\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(883\text{ ft-kip}) = \pm 0.2\text{ kip}$$

<u>Lateral force applied in E-W direction</u>

(a) F_{px} = 116 kip and e_{y1} = 5.1 ft

$$F_{u,MF,max} = \frac{1.0}{(1.0+1.0)}(116\text{ kip}) + \frac{(1.0)(78\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(592\text{ ft-kip}) = 58.1\text{ kip}$$

$$F_{u,MF,min} = \frac{1.0}{(1.0+1.0)}(116\text{ kip}) - \frac{(1.0)(78\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(592\text{ ft-kip}) = 57.9\text{ kip}$$

$$F_{u,wall} = \pm\frac{(10.5)(109\text{ ft})}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(592\text{ ft-kip}) = \pm 2.7\text{ kip}$$

(b) F_{px} = 116 kip and e_{y2} = 2.7 ft

$$F_{u,MF,max} = \frac{1.0}{(1.0+1.0)}(116\text{ kip}) + \frac{(1.0)(78\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(313\text{ ft-kip}) = 58.1\text{ kip}$$

$$F_{u,MF,min} = \frac{1.0}{(1.0+1.0)}(116\text{ kip}) - \frac{(1.0)(78\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(313\text{ ft-kip}) = 57.9\text{ kip}$$

$$F_{u,wall} = \pm\frac{(10.5)(218\text{ ft}/2)}{(2)[(1.0)(39\text{ ft})^2+(10.5)(109\text{ ft})^2]}(313\text{ ft-kip}) = \pm 1.4\text{ kip}$$

The force distribution in the E-W direction for both calculated eccentricities is small. Therefore, use 58.0 kip

Step 7. Check shear force in diaphragm		
12.5.3.3 12.5.3.2 21.2.4.2 12.5.3.2 21.2.4.1	In-plane shear in diaphragm The diaphragm slab is cast-in-place concrete, therefore, shear strength is calculated from Eq. (12.5.3.3) $V_n = A_{cv}\left(2\lambda\sqrt{f_c'}+\rho_t f_y\right)$ Ignoring the strength contribution of reinforcement, $\rho_t = 0$ A_{cv} is the diaphragm gross area less the 6.0 ft overhang (refer to Step 9 for clarification) Applying the shear strength reduction factor $\phi = 0.75$ at the north and south ends along column lines A and F At the east and west ends along Column Lines 1 and 7, the shear strength reduction factor, ϕ, must not exceed the least value for shear used for the vertical components of the primary seismic-force-resisting system. Therefore, $\phi = 0.75$	Nominal shear strength in E-W direction $V_{n,N} = (218\text{ ft})(12\text{ in./ft})(7\text{ in.})\left(2(1.0)\sqrt{5000\text{ psi}}+0\right)$ $= 2{,}589{,}708\text{ lb} \approx 2590\text{ kip}$ Nominal shear strength in N-S direction $V_{n,E} = (72\text{ ft})(12\text{ in./ft})(7\text{ in.})\left(2(1.0)\sqrt{5000\text{ psi}}+0\right)$ $= 855{,}316\text{ lb} = 855\text{ kip}$ Design shear strength in E-W direction $\phi V_{n,N} = (0.75)(2590\text{ kip}) = 1940\text{ kip}$ Design shear strength in N-S direction $\phi V_{n,E} = (0.75)(855\text{ kip}) = 641\text{ kip}$
12.5.1.1	Check if factored shear force is less than design shear strength calculated in Step 7	NS $\phi V_n = 641\text{ kip} >> F_u = 44.5\text{ kip}$ **OK** EW $\phi V_n = 1940\text{ kip} >> F_u = 58.0$ **OK** Therefore, diaphragm has adequate strength to resist the lateral inertia force and shear reinforcement is not required, $\rho_t = 0$
12.5.3.4	The nominal shear strength, V_n, must not exceed. $V_n = 8A_{cv}\lambda\sqrt{f_c'}$ A_{cv} is the diaphragm gross area less the 6.0 ft overhang.	$V_n = \dfrac{8(72\text{ ft})(10\text{ in.})(12\text{ in./ft})(1.0)\sqrt{4000\text{ psi}}}{1000\text{ lb/kip}} = 4372\text{ kip}$ By inspection this is satisfied. **OK**

Step 8: Second-level diaphragm lateral force distribution N-S

Design force: 81 kip

12.4.2.4(a)
12.5.1.3(a) Diaphragm is idealized as rigid. Design moments and shear axial forces are calculated based on a beam with depth equal to full diaphragm depth satisfying equilibrium requirements.

The wall forces and the assumed direction of torque due to the eccentricity are shown in Fig. E2.3.

12.4.2.4 The distribution of the applied force on the diaphragm is calculated by using q_L and q_R as the left and right diaphragm reactions per unit length (Fig. E2.3):

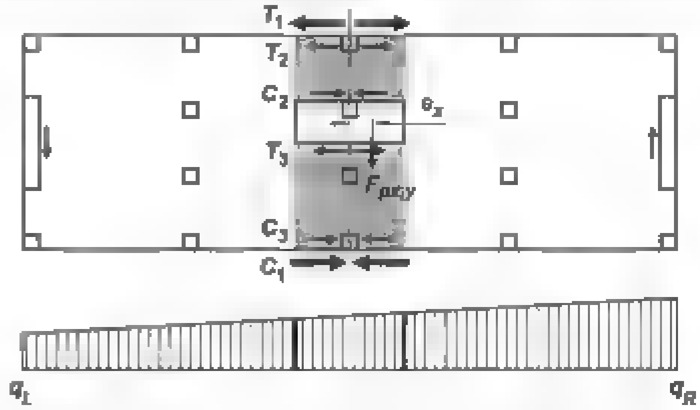


Fig. E2.3—Forces in the structural resisting systems due to a seismic force in the N-S direction.

Force equilibrium

$$q_L\left(\frac{L}{2}\right)+q_R\left(\frac{L}{2}\right) = F_{px,des(NS)}$$

$$q_L\left(\frac{218\text{ ft}}{2}\right)+q_R\left(\frac{218\text{ ft}}{2}\right)=81\text{ kip} \quad \text{(I)}$$

Moment equilibrium

$$q_L\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)+q_R\left(\frac{L}{2}\right)\left(\frac{2L}{3}\right)=F_{px,des(NS)}\left(\frac{L}{2}+0.05L\right)$$

$$q_L\frac{(218\text{ ft})^2}{6}+q_R\frac{2(218\text{ ft})^2}{6} = (81\text{ kip})\left(\frac{218\text{ ft}}{2}+10.9\text{ ft}\right) \quad \text{(II)}$$

From statics solve equations (I) and (II) for q_L and q_R.

$q_L + q_R = 0.74$ kip/ft (I)
$q_L + 2q_R = 1.23$ kip/ft (II)
$q_L = 0.25$ kip/ft and $q_L = 0.49$ kip/ft

Find the maximum moment by taking the first derivative of the moment equation expressed as a function of x (unknown distance) $dM/dx = 0$

$x = 114.45$ ft; $M_{max} = 2213$ ft-kip

Draw the shear and moment diagrams and determine the moment and shear forces at opening (Fig. E2.4).

Note: In an Aug. 2010 National Institute and Standards Technology (NIST) report, GCR 10-917-4, "Seismic Design of Cast-in-Place Concrete Diaphragms, Chords, and Collectors," by Moehle et al. states that, "This approach leaves any moment due to the frame forces along column lines A and F unresolved. Sometimes this is ignored or, alternatively, it too can be incorporated in the trapezoidal loading."

In this example the small moment due to the frame forces (0.2 kip) are ignored.

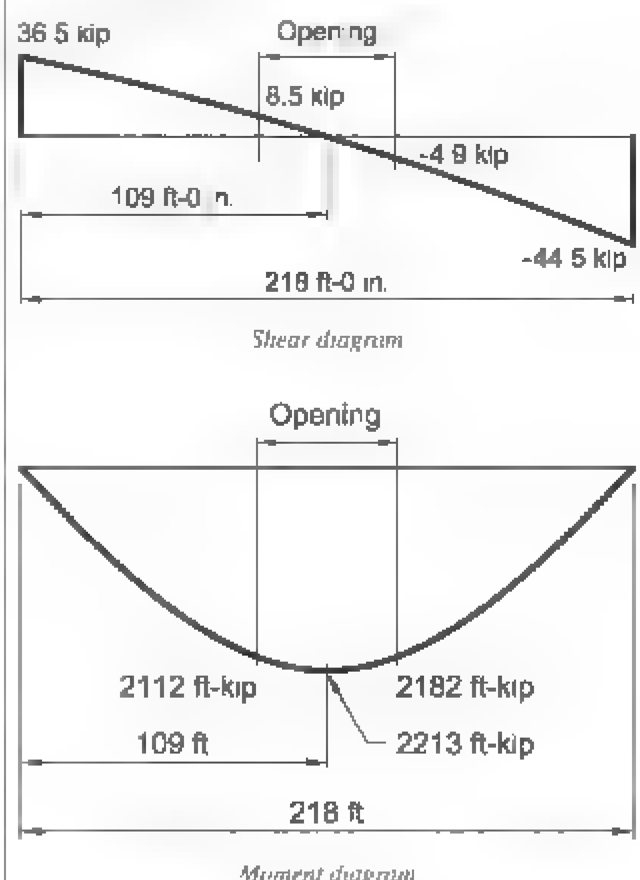


Fig. E2.4—Shear and moment diagrams.

	In smaller buildings in which seismic demand is low, there are no irregularities, and torsional moments are not significant, the diaphragm shears and moments can be based on a uniformly distributed load, rather than a linearly varying load	$q = \dfrac{81\text{ kip}}{218\text{ ft}} = 0.372\text{ kip/ft}$
	Resulting in a maximum moment of:	$M_{max} = \dfrac{(0.372\text{ kip/ft})(218\text{ ft})^2}{8} = 2210\text{ ft-kip}$
	Note: Both approaches, in this example, result in similar maximum moment (2213 ft-kip versus 2210 ft-kip), but at different locations (114.5 ft versus 109 ft). In this example the detailed approach is presented.	
Step 9: Chord reinforcement N-S		
12.5.2.3	Maximum moment obtained from moment diagram.	$M_u = 2213$ ft-kip
	Chord reinforcement resisting tension must be located within $h/4$ of the tension edge of the diaphragm.	$h/4 = 72.0\text{ ft}/4 = 18\text{ ft}$
	Note: Chord reinforcement can be placed either in the exterior edge of the balcony or it can be placed along the exterior frame of CL A. Placing chord reinforcement along the exterior frame is a simpler and cleaner load path for the forces in the diaphragm. Crack control reinforcement should be added in the balcony slab for crack control.	
	Assume tension reinforcement is placed in a 2 ft strip at both north and south sides of the slab edges at CLs 1 and 5	2 ft < $h/4$ = 18 ft **OK**
	<u>Chord force</u> Maximum chord tension force that must be resisted by the chord at midspan is: $T_u = \dfrac{M_u}{B - 2\text{ ft}}$	$T_u = \dfrac{2213\text{ ft-kip}}{72\text{ ft} - 2\text{ ft}} = 31.6\text{ kip}$

Chord forces at opening

The opening in the diaphragm results in local bending of the diaphragm segments on either side of the opening (refer to Fig. E2.5)

1. The diaphragm sections above and below the opening are idealized as fixed end beams
2. The applied loading on the sections above and below the opening are based on the relative mass of each section (1.64.1)
3. The secondary chord forces are calculated based on the internal moment in the diaphragm sections adjacent to the opening
4. The calculated tension and compression secondary chord forces are added to the primary tension and secondary chord forces.

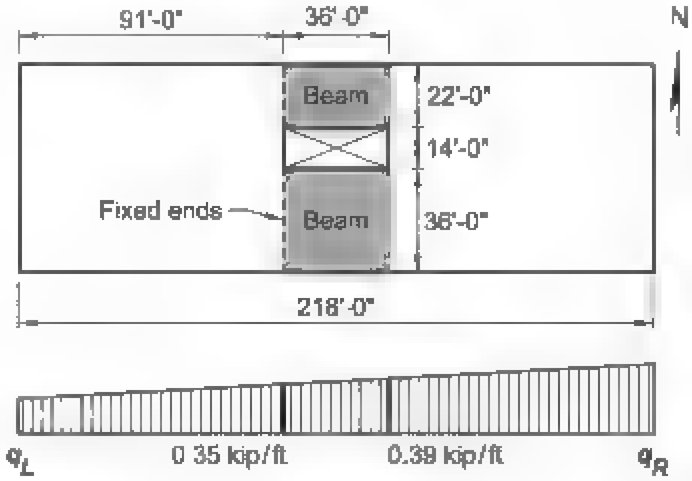


Fig. E2.5—Idealization of sections above and below opening

The opening is located at midlength of the building floor plan in the E-W direction. The total diaphragm forces at left and right edges of the opening are

Force at east and west ends of opening

$$q'_{u@beg} = 0.25\text{ kip/ft} + \frac{(0.49\text{ kip/ft} - 0.25\text{ kip/ft})(91\text{ ft})}{218\text{ ft}} = 0.35\text{ kip/ft}$$

$$q'_{u@end} = 0.25\text{ kip/ft} + \frac{(0.49\text{ kip/ft} - 0.25\text{ kip/ft})(127\text{ ft})}{218\text{ ft}} = 0.39\text{ kip/ft}$$

The load on the north and south section of the diaphragm bound by the opening is distributed according to the ratio of the masses north and south of the opening. Therefore, 38 percent and 62 percent of the overall applied trapezoidal load will be distributed to the north and south section over this portion of the diaphragm, respectively

Force north of opening

$q'_{u@bN} = (0.38)(0.35\text{ kip/ft}) = 0.13\text{ kip/ft}$

$q'_{u@eN} = (0.38)(0.39\text{ kip/ft}) = 0.15\text{ kip/ft}$

The unit forces magnitude at the east and west ends of the opening are close (0.13 kip/ft versus 0.15 kip/ft north of opening) and (0.22 kip/ft versus 0.24 kip/ft south of opening). Therefore, the average unit force of 0.14 kip/ft and 0.23 kip/ft will be used for calculating the diaphragm moment segments north and south of the opening (Fig. E2.6)

Force south of opening

$q'_{u@bS} = (0.62)(0.35\text{ kip/ft}) = 0.22\text{ kip/ft}$

$q'_{u@eS} = (0.62)(0.39\text{ kip/ft}) = 0.24\text{ kip/ft}$

Fixed end moment can be obtained from computer-aided design software programs or from the Reinforced Concrete Design Handbook Design Aid Analysis Tables, which can be downloaded from: https://www.concrete.org/MNL1721Download1

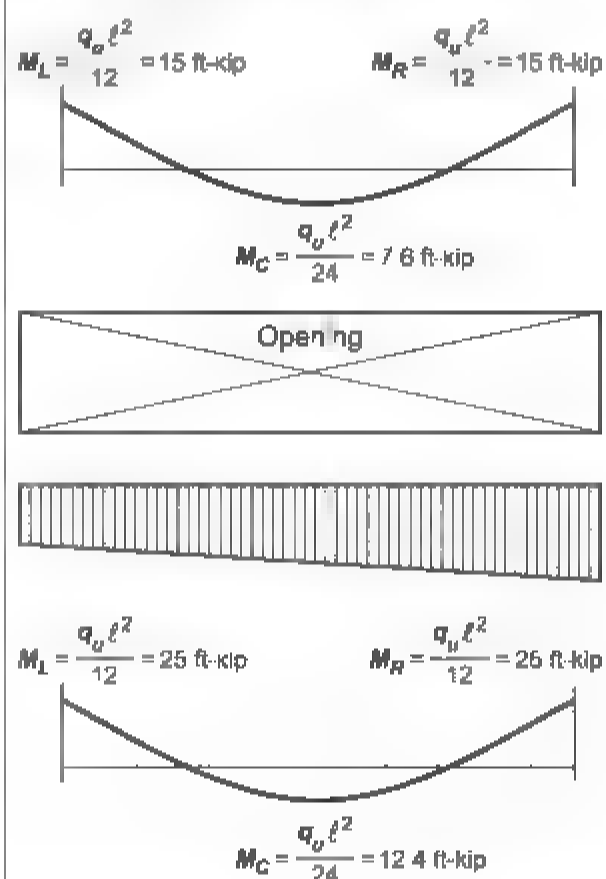


Fig. E2.6—Moment diagram of sections at opening

	The secondary chord forces are obtained from the moment diagram. Assuming a 1 ft strip (< $B/4$) at each end of the span between opening and diaphragm edge $T_{u,opening} = \dfrac{M_u}{D}$	Secondary chord force north of opening $T_{u,op,N} = \dfrac{15\text{ ft-kip}}{21\text{ ft}} = 0.7\text{ kip}$ Secondary chord force south of opening $T_{u,op,S} = \dfrac{25\text{ ft-kip}}{35\text{ ft}} = 0.7\text{ kip}$
12.5.1.1 22.4.3.1	Required reinforcement $\phi T_n = \phi f_y A_s > T_u$	$A_{s,req'd} = \dfrac{700\text{ lb}}{(0.9)(60{,}000\text{ psi})} = 0.01\text{ in.}^2$ Refer to the following discussion.
	The engineer has two options for providing chord reinforcement at the opening 1. The beams are designed for $1.2D + 1.6L$. For seismic, the governing load combination is $(1.2 + 0.2S_{DS})D + 0.5L + E$. The demand from $1.2D + 1.6L$ is usually higher than the gravity portion of the moments under seismic, $(1.2 + 0.2S_{DS})D + 0.5L$. The two loads are proportioned and then the balance reinforcement is used to carry seismic chord/collector forces. 2. The chord force is resisted with additional reinforcement (conservative) In this example the first option is used as the required reinforcement is negligible. Beam top reinforcement is continuous	
	Diaphragm edge Total moment to be resisted is the sum of the main chord force and the secondary chord force: $T_{u,Total} = T_{u1} + T_{u2}$ $T_{u,Total} = T_{u1} + T_{u3}$	$T_{u,total,N} = C_{u,total,N} = 31.1\text{ kip} + 0.7\text{ kip} = 31.8\text{ kip}$, say, 32 kip $T_{u,total,S} = C_{u,total,S} = 31.1\text{ kip} + 0.7\text{ kip} = 31.8\text{ kip}$, say, 32 kip
12.5.2.2	Chord reinforcement. Tension due to moment is resisted by deformed bars conforming to Section 20.2.1 of ACI 318	
12.5.1.5	Steel stress is the lesser of the specified yield strength and 60,000 psi.	$f_y = 60{,}000\text{ psi}$
12.5.1.1 22.4.3.1	Required reinforcement $\phi T_n = \phi f_y A_s \geq T_u$	$A_{s,req'd} = \dfrac{32{,}000\text{ lb}}{(0.9)(60{,}000\text{ psi})} = 0.59\text{ in.}^2$
	The building is assigned to SDC B, therefore, ACI 318 Chapter 18 requirements for chord spacing and transverse reinforcement of 18.12.7.6 do not apply	
18.12.7.6	Overstrength factor, Ω_o, for chord design is not required. Therefore, use the compression stress limit in Provision 18.12.7.6 of $0.2f_c'$ Required chord width: $w_{chord} > \dfrac{C_{Chord}}{0.2f_c'h_{diaph}}$	$w_{chord} > \dfrac{32{,}000\text{ lb}}{(0.2)(5000\text{ psi})(7\text{ in.})} = 4.6\text{ in.}$ Less than the assumed 2 ft. **OK**
	Choose reinforcement	Try two No. 5 chord bars $A_{s,prov} = 2(0.31\text{ in.}^2) = 0.62\text{ in.}^2$
	Check if provided reinforcement area is greater than required reinforcement area	$A_{s,prov} = 0.62\text{ in.}^2 > A_{s,req'd} = 0.59\text{ in.}^2$

Step 10: Collector reinforcement N-S

12.5.4.1 Collectors transfer shear forces from the diaphragm to the vertical walls at both east and west ends along column lines 1 and 7 (Fig. E2.2). Collectors extend over the entire diaphragm width.

Unit shear force

$$v_{u@F} = \frac{F_{u@F}}{B}$$

From Step 6: $F_u = 44.5$ kip

In diaphragm: $v_{u@F} = \dfrac{F_{u@F}}{L_{diaph}}$ $\qquad v_{u@F} = \dfrac{44.5\text{ kip}}{72\text{ ft}} = 0.62\text{ kip/ft}$

In wall: $v_{u@F} = \dfrac{F_{u@F}}{L_{wall}}$ $\qquad v_{u@F} = \dfrac{44.5\text{ kip}}{28\text{ ft}} = 1.59\text{ kip/ft}$

<u>Force at diaphragm to wall connection</u>

West wall south end

$F_{7/D.5} = -(0.62\text{ kip/ft})(22\text{ ft}) = -13.6\text{ kip}$

West wall north end

$F_{7/B.5} = -13.6\text{ kip} + (0.97\text{ kip/ft})(28\text{ ft}) = 13.5\text{ kip}$

At diaphragm end

$F_{7/A} = +13.5\text{ kip} - (0.62\text{ kip/ft})(22\text{ ft}) \approx 0\text{ kip}$

Per collector force diagram, the maximum axial force on the collector is $T_u = C_u = 13.5$ kip. This force must be transferred from the diaphragm to the shear wall (Fig. E2.7).

The collector force and its connections to the shear wall will not be multiplied by the system over strength factor $\Omega_o = 2.5$ (ASCE/SEI 7-10, Table 12.2-1), because this is not a special structural wall.

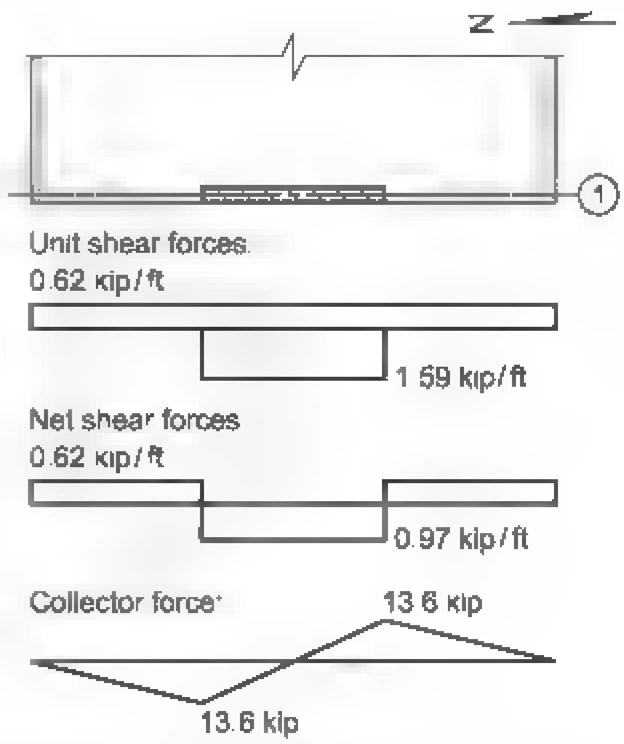


Fig. E2.7—Collector force diagram.

12.5.4.2 Collectors are designed as tension members, compression members, or both.

Tension is resisted by reinforcement as calculated above.

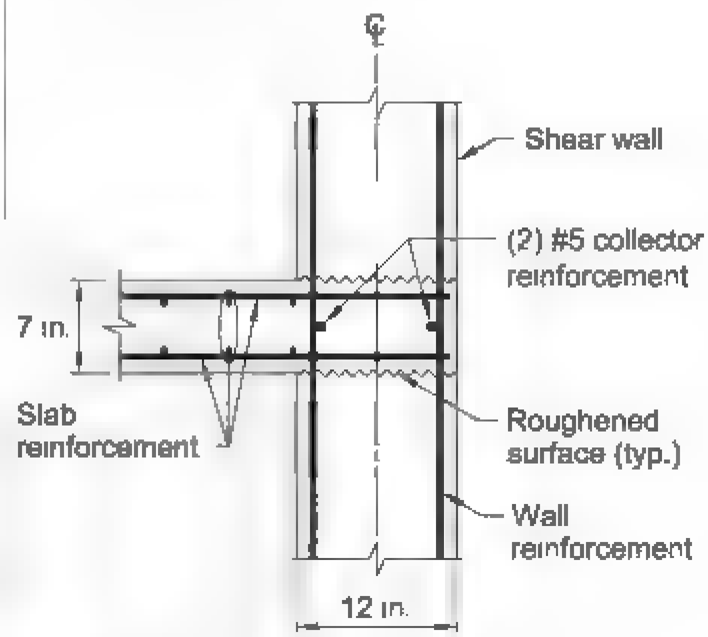
Required reinforcement

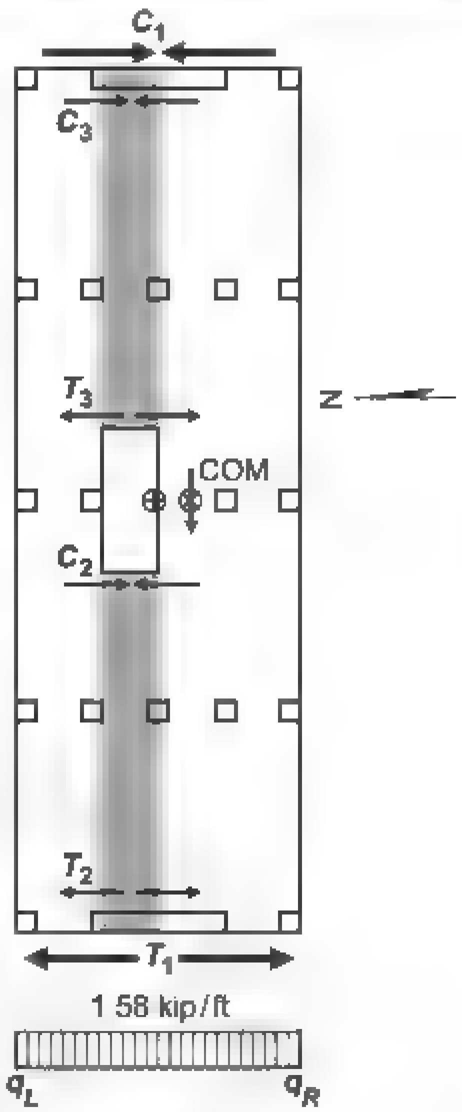
12.5.1.1 $\phi T_n = \phi f_y A_s \geq T_u$

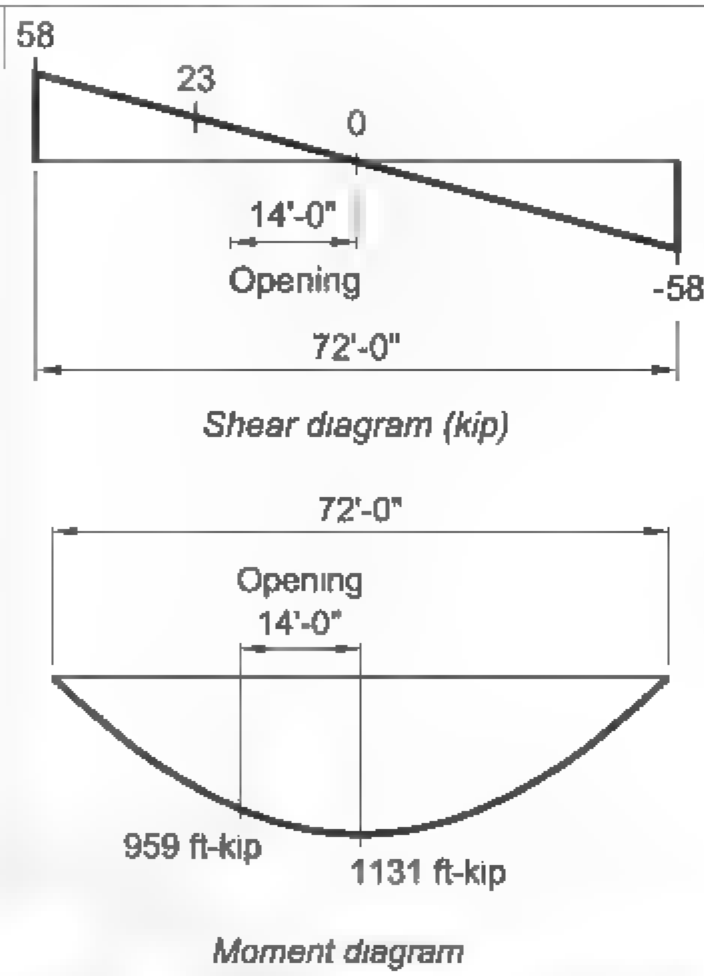
22.4.3.1

$$A_{s,req'd} = \frac{T_u}{0.9 f_y} = \frac{13{,}600\text{ lb}}{(0.9)(60{,}000\text{ psi})} = 0.25\text{ in.}^2$$

Although one No. 5 bar suffices, two No. 5 are provided to maintain symmetry.

18.12.7.6	Check if collector compressive force exceeds $0.2f_c'$. Calculate minimum required collector width using $0.2f_c'$ $w_{coll} > \frac{C_{Coll.}}{0.2f_c' t_{diaph}}$ This results in compressive stress on the concrete diaphragm collector being relatively low. The section is adequate to transfer shear stress without additional reinforcement. The collector compression and tension forces are transferred to the lateral force-resisting system within its width (shear wall). Therefore, no eccentricity is present and no in-plane bending occurs. The Code permits to discontinue the collector along the length of the shear wall where transfer of design collector is not required.	$w_{coll} > \frac{13,600\text{ lb}}{(0.2)(5000\text{ psi})(7\text{ in.})} = 1.9\text{ in.}$ Use 12 in. wide collector (same width as shear wall) Provide two No. 5 bars at mid-depth of slab to prevent additional out-of-plane bending stresses in the slab. Space the two No. 5 bars at 8 in. on center starting at 2 in. from the edge of the diaphragm within the 12 in. wide collector/shear wall (Fig. E2.8)  *Fig. E2.8—Collector reinforcement*
	Check slab shear strength along shear walls	
12.4.2.4	Slab shear strength along walls $L = 28$ ft and slab thickness $t = 7$ in. From	
12.5.3.3	$\phi V_c = \phi A_{cv} 2\lambda\sqrt{f_c'}$	$\phi V_c = (0.75)(2)(1.0)(\sqrt{5000})(28\text{ ft})(12)(7\text{ in.})$
21.2.4.2	$\phi = 0.75$	$\phi V_c = 249,467$ lb ~ 249 kip
12.5.1.1	Is the provided shear strength adequate?	$\phi V_c = 249$ kip $> V_u = 44.5$ kip (from Step 7) **OK**
12.5.3.4	By inspection, the diaphragm shear design force satisfies the requirement of 12.5.3.4. $\phi V_c = \phi A_{cv} 8\lambda\sqrt{f_c'}$	

Step 11 Lateral force distribution in diaphragm E-W		
	Design force 114 kip	
12.4.2.4(a) 12.5.1.3(a)	Design moments, shear, and axial forces are calculated based on a beam with depth equal to full diaphragm length satisfying equilibrium requirements. The wall forces and the assumed direction of torque due to the eccentricity are shown in Fig. E2.9 The distribution of the applied force on the diaphragm is uniform because of the negligible effect of accidental torsion (Fig. E2.9)	*Fig. E2.9—Forces in the structural resisting systems due to a seismic force in the E-W direction.* $q_L = \left(\frac{116\text{ kip}}{72\text{ ft}}\right) = 1.6\text{ kip/ft}$ Shear force: $V = (1.6\text{ kip/ft})(36\text{ ft}) = 58\text{ kip}$ $x = 36\text{ ft}$, $M_{max} = 1131\text{ ft-kip}$

	Maximum moment is taken at midspan. Draw the shear and moment diagrams (Fig. E2.10).	 *Fig. E2.10—Shear and moment diagrams.*
Step 12: Chord reinforcement E-W		
12.5.2.3	Maximum moment is calculated above.	M_u = 1131 ft-kip
	Chord reinforcement resisting tension must be located within $h/4$ of the tension edge of the diaphragm.	$h/4$ = 218.0 ft/4 = 54.5 ft
	Assume tension reinforcement is placed in a 1 ft strip at both north and south sides of the slab edges at CLs 1 and 5.	1 ft < $h/4$ = 54.5 ft **OK**
	<u>Chord force</u> Maximum chord tension force that must be resisted by the chord at midspan. $T_u = \dfrac{M_u}{B - 1\text{ ft}}$	$T_u = \dfrac{1131\text{ ft-kip}}{(218\text{ ft} - 1\text{ ft})} = 5.2\text{ kip}$

Chord forces at opening

The opening in the diaphragm results in local bending of the diaphragm segments on either side of the opening (Fig. E2.11)

1. The diaphragm sections to the east and west of the opening are idealized as fixed end beams.
2. The applied loading on the sections east and west the opening are based on the relative mass of each section (1 1)
3. The secondary chord forces are calculated based on the internal moment in the diaphragm sections adjacent to the opening
4. The calculated tension and compression secondary chord forces are added to the primary tension and secondary chord forces

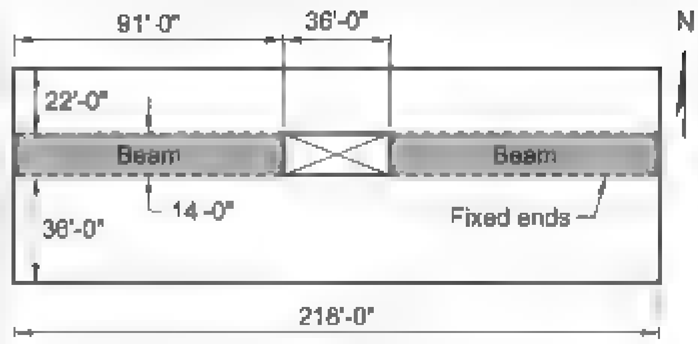


Fig. E2.11—Idealization of sections above and below opening

Force east and west of opening

$$q'_{u@bE} = q'_{u@bW} = \left(\frac{1.6\text{ kip/ft}}{2}\right) = 0.8\text{ kip/ft}$$

The opening is located at mid-length of the building floor plan in the E-W direction. The load on the north and south sections of the diaphragm bound by the opening are equal to one half of the overall applied trapezoidal load over this portion of the diaphragm (Fig. E2.12).

Because forces at both ends of openings are close, a uniform load is assumed

Fixed end moment can be obtained from computer-aided aided design software programs or from Reinforced Concrete Design Handbook Design Aid – Analysis Tables, which can be downloaded at https://www.concrete.org/MNL1721Download1

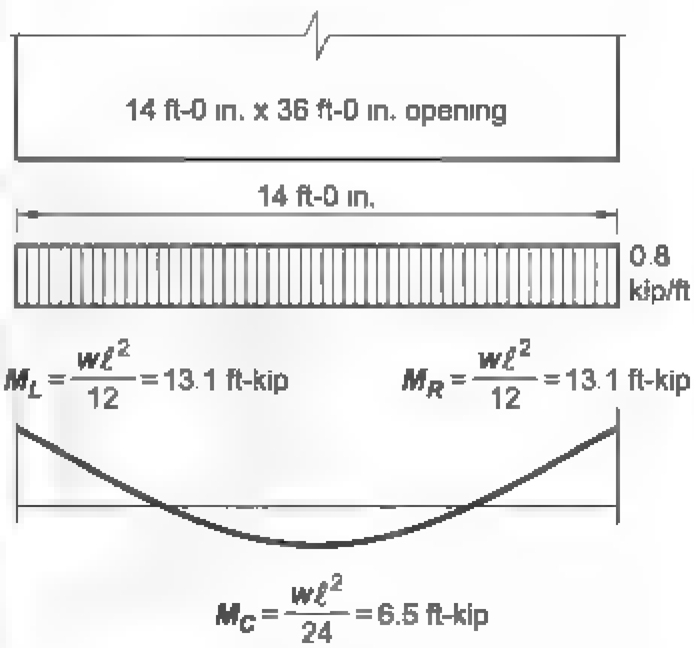


Fig. E2.12—Moment diagram of sections at opening

The secondary chord forces are obtained from the moment diagram. Assuming a 2 ft strip (< B/4) at each end of the span between opening and diaphragm edge

$T_{u,opening} = M_u/D$

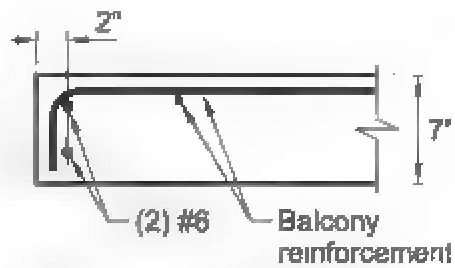
$$T^{+}_{u,E} = \frac{13.1\text{ ft-kip}}{89\text{ ft}} = 0.15\text{ kip}$$

Total moment to be resisted is the sum of the main chord force and the secondary chord force

$T_{u,total} = T_u + T_{u,O2}$

$T_{u,total,N} = C_{u,total,N} = 5.2\text{ kip} + 0.15\text{ kip} = 5.35\text{ kip}$
Use 5.4 kip

12.5.2.2	Chord reinforcement. Tension due to moment will be resisted by deformed bars conforming to Section 20.2.1 of ACI 318	
12.5.1.5	Steel stress is the lesser of the specified yield strength and 60,000 psi	
12.5.1.1 22.4.3.1	Required reinforcement $\phi T_n = \phi f_y A_s \geq T_u$	$A_s = \dfrac{5400\text{ lb}}{0.9(60{,}000\text{ psi})} = 0.1\text{ in.}^2$
	The chord forces north of and south of opening are equal	One No.3 bar satisfies the requirement. The required collector reinforcement in the N-S direction, however, requires two No. 5 bars. Therefore, provided reinforcement is adequate and no additional reinforcement is required.
Step 13. Collector reinforcement E-W		
	Continuous reinforced concrete frame over the full length of the building will act as a collector. Note: Provide continuous reinforcement with tension splices (Step 15)	
12.5.3.7	In cast-in-place diaphragms, where shear is transferred from the diaphragm to a collector, or from the diaphragm or collector to a shear wall, temperature and shrinkage reinforcement is usually adequate to transfer that force	
Step 14: Shrinkage and temperature reinforcement		
12.6.1 24.4.3.2	The minimum shrinkage and temperature Reinforcement, A_{S+T}.	
	$A_{S+T} \geq 0.0018A_g$	$A_{S+T} = (0.0018)(7\text{ in.})(12\text{ in./ft}) = 0.15\text{ in.}^2$
24.4.3.3	Spacing of S+T reinforcement is the lesser of $5h$ and 18 in. $5h = 5(12\text{ in.}) = 60\text{ in.}$ 18 in. **Controls**	Note: Shrinkage and temperature reinforcement may be part of the main reinforcing bars resisting diaphragm in-plane forces and gravity loads. If provided reinforcement is not continuous (placing bottom reinforcing bars to resist positive moments at midspans and top reinforcing bars to resist negative moments at columns), continuity between top and bottom reinforcing bars may be achieved by providing adequate splice lengths between them.

Step 15: Reinforcement detailing		
	Reinforcement spacing	
12.7.2.1	Chord and collector reinforcement minimum and maximum spacing must satisfy 12.7.2.1 and 12.7.2.2	
25.2.1	Section 25.2 requires minimum spacing of (a) 1 in. (b) $4/3d_{agg.}$ (c) d_b No. 5	 Minimum spacing 1.0 in. **Controls** 4/3(3/4 in.) aggregate = 1.0 in. 0.625 in.
18.12.7.7a	Collector reinforcement spacing at a splice must be at least the larger of: (a) At least three longitudinal d_b (b) 1.5 in. (c) c_c > max [$2.5d_b$, 2 in.]	 3(0.625 in.) = 1.875 in. 1.5 in. 2 in. **Controls**
12.7.2.2	Maximum spacing is the smaller of $5h$ or 18 in.	18 in. **Controls**
	Edge reinforcement The opening has four beams around its perimeter. Therefore, the beams reinforcement is adequate to resist the tension forces due to inertial forces and additional reinforcement is not required. Note: If beams are not constructed around the opening perimeter a minimum of two No. 5 is recommended around the opening as shown in the Fig. E2.13 and extended a minimum of its development length. See detailing in Fig. E2.14 and Fig. E2.15	 *Fig. E2.13—Two No. 5 reinforcement around opening.*

Step 16, Details

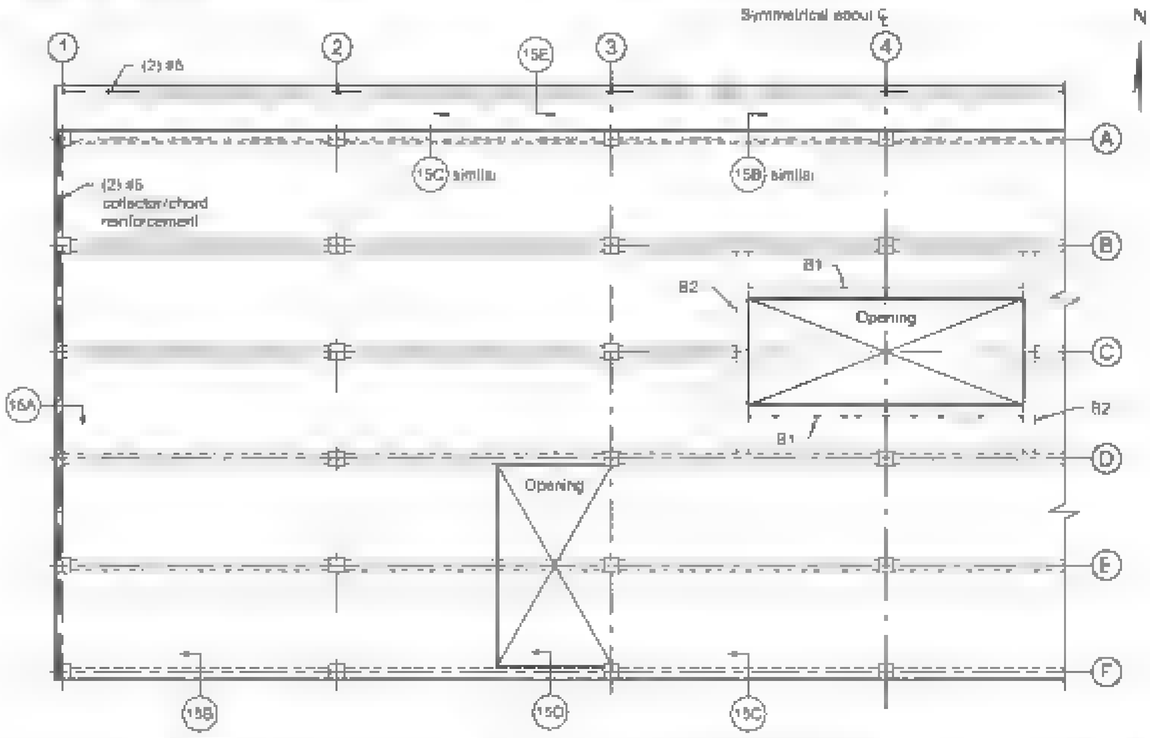


Fig. E2.14—Typical diaphragm to wall section. Note: Slab reinforcement not shown for clarity.

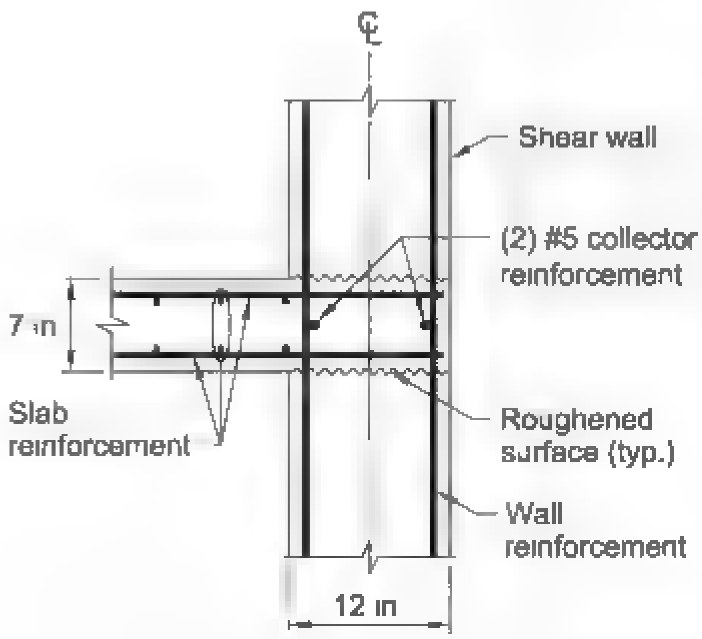


Fig. E2.15a—Collector reinforcement in shear walls along CL 1 and 7.

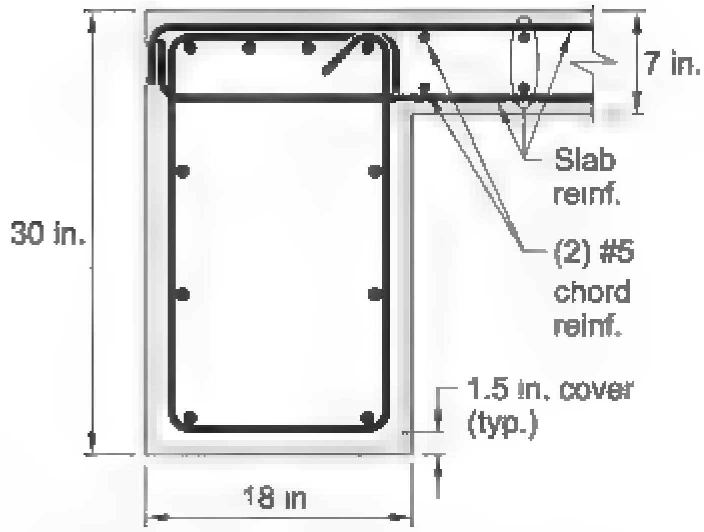


Fig. E2.15b—Chord reinforcement at midspan

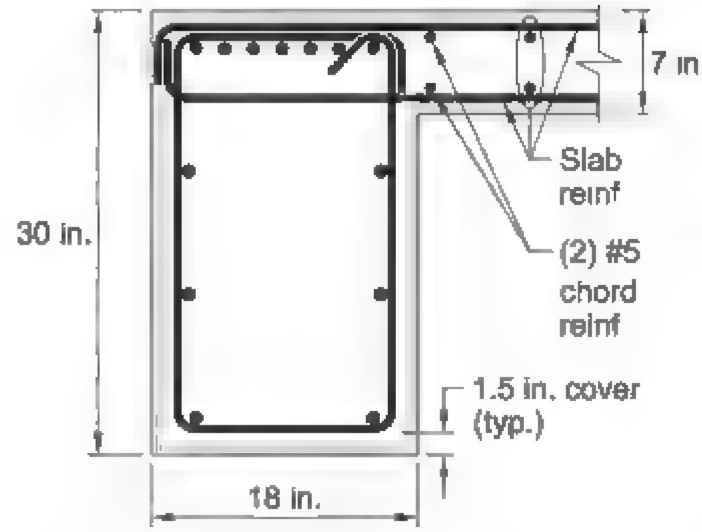


Fig. E2.15c—Chord reinforcement at supports

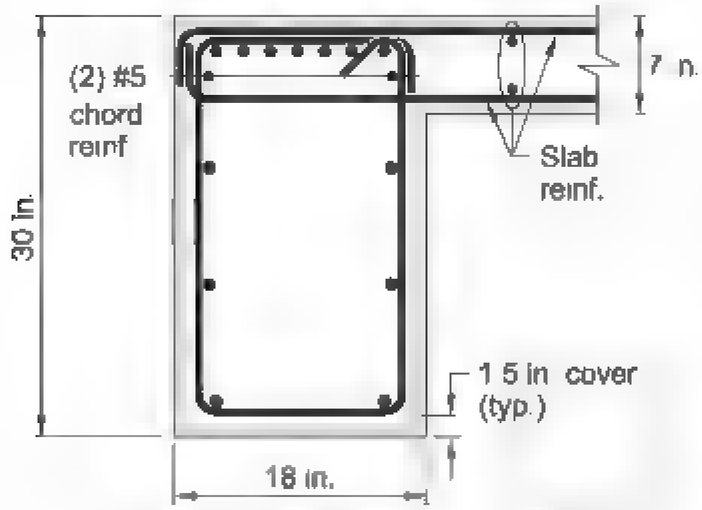


Fig. E2.15d—Chord reinforcement at opening

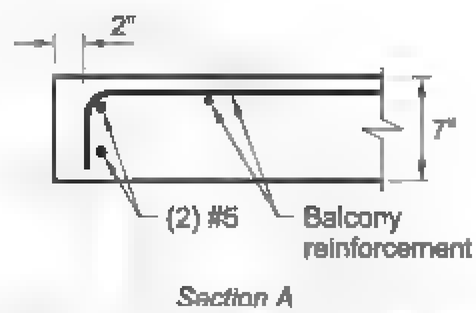


Fig. E2.15e—Crack control reinforcement at balcony edge

Rigid Diaphragm Example 3: *Lateral force distribution of a rigid diaphragm to shear walls*. A three-story wood apartment building is built on a normalweight reinforced concrete one-story slab. The slab is 200 ft x 90 ft with f_c' = 4000 psi and f_y = 60,000 psi. Assume that the structure is located in an active earthquake region Seismic Design Category (SDC) D and that the seismic analysis of the structural analysis based on ASCE/SEI 7, resulting in a base shear coefficient of 0.316. The slab supporting the wood structure is 10 in. thick and the wall lengths, height, and thicknesses are shown as follows. Assume the weight of the wood frame building imparts an equivalent uniform dead load of 135 psf to the slab. In addition, add a 10 psf miscellaneous dead load to the slab. Refer to Fig. E3.1 for geometric information.

This example will determine the seismic forces that are resisted by the shear walls, design the diaphragm, chords, and collectors to resist these forces and transmit them to the walls, and then detail the flatwork accordingly.

Given:

Project data—
Diaphragm size 200 ft 0 in. x 90 ft 0 in.
Wall 1 90 ft 0 in. x 8 in.
Wall 2 30 ft 0 in. x 10 in.
Wall 3 30 ft 0 in. x 10 in.
Wall 4 28 ft 0 in. x 10 in.
Wall 5 40 ft 0 in. x 10 in.
Slab thickness t = 10 in.
Parking structure (top of slab) height is 12 ft above the foundation

Concrete—
f_c' = 4000 psi
f_y = 60,000 psi

Seismic criteria—
SDC D
C_S = 0.316

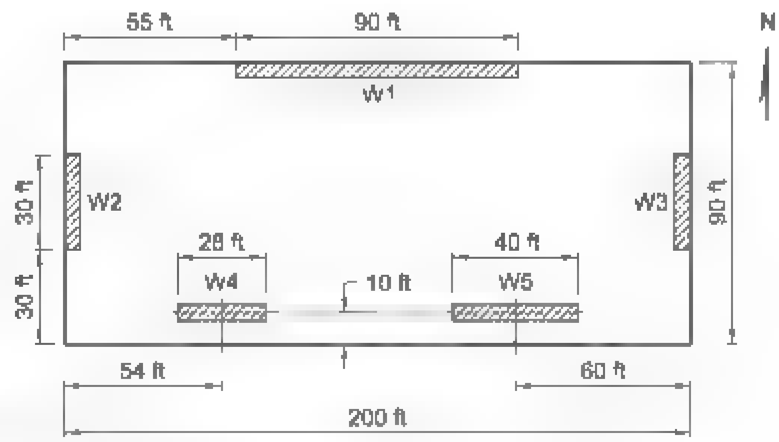


Fig. E3.1—Slab that supports a four-story wood building.

Note: Nonparticipating columns in the lateral-force-resisting system are not shown for clarity.

ACI 318	Discussion	Calculation
Step 1: Material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318).	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied.
	The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes.	Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 4000 psi.
	ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications.	
	There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	
Step 2: Slab geometry		
12.3.1.1 18.13.7.1	Assume that diaphragm thickness satisfies the requirements for stability, strength, and stiffness under factored load combinations. Diaphragm thickness satisfies Chapter 18 minimum thickness requirements.	Given: h = 10 in.

Step 3: Lateral forces		
	The lateral force is obtained by multiplying the self-weight of the reinforced concrete slab, wood frame building dead load, miscellaneous dead load, and the contribution of the shear walls, by the base shear coefficient	
	<u>Gravity loads</u> The reinforced concrete slab self-weight. $W_{slab} = (L)(B)(h)(\gamma_c)$	$W_{slab} = (200\ ft)(90\ ft)(10\ in./12\ in.\ ft)(150\ lb/ft^3)$ $= 2{,}250{,}000\ lb = 2250\ kip$
	Weight of wood frame building dead load and miscellaneous dead load:	$W_D = (135\ psf + 10\ psf)(200\ ft)(90\ ft) = 2610\ kip$
	Total gravity dead load.	$W = 2250\ kip + 2610\ kip = 4860\ kip$
	Shear wall self-weight contribution to diaphragm lateral force calculation is half the wall height.	
	<u>N-S direction</u> $W_i = (L)(H/2)(t_w)(\gamma_c)$	$W_1 = (90\ ft)(12\ ft/2)(8\ in.)/(12\ in./ft)(150\ lb/ft^3)$ $W_1 = 54{,}000\ lb = 54\ kip$ $W_4 = (28\ ft)(12\ ft/2)(10\ in.)/(12\ in./ft)(150\ lb/ft^3)$ $W_4 = 21{,}000\ lb = 21\ kip$ $W_5 = (40\ ft)(12\ ft/2)(10\ in.)/(12\ in./ft)(150\ lb/ft^3)$ $W_5 = 30{,}000\ lb = 30\ kip$
	Total gravity dead load in the N-S direction	$\sum W = 4860\ kip + 54\ kip + 21\ kip + 30\ kip = 4965\ kip$
	<u>E-W direction</u> $W_i = (L)(H/2)(t_w)(\gamma_c)$	$W_2 = (30\ ft)(12\ ft/2)(10\ in.)/(12\ in./ft)(150\ lb/ft^3)$ $W_2 = 22{,}500\ lb = 22.5\ kip$ $W_3 = (30\ ft)(12\ ft/2)(10\ in.)/(12\ in./ft)(150\ lb/ft^3)$ $W_3 = 22{,}500\ lb = 22.5\ kip$
	Total gravity dead load in the E-W direction.	$\sum W = 4860\ kip + 22.5\ kip + 22.5\ kip = 4905\ kip$
	<u>Lateral loads</u> Base shear is obtained from ASCE/SEI 7 Section 12.8.1: $V = C_sW$ C_s is calculated using ASCE/SEI 7 Section 12.8.1.1, not shown here for brevity:	$C_s = 0.316$ given
	The equivalent lateral force distribution over the building height is per ASCE/SEI 7 Eq. (12.8-11)	
	The diaphragm design forces F_{px} are calculated per ASCE/SEI 7 Eq. (12.10-1)	
	F_{px} and F_{py} must be in accordance with ASCE/SEI 7 Eq. (12.10-2) and (12.10-3) Calculations not shown here as it is outside the scope of this Manual Equivalent lateral force at the concrete level is:	$F_x = 363.1\ kip$

	Diaphragm design forces N-S E-W	 F_{py} = 745 kip F_{px} = 726 kip

Note

Conservatively, the weight of all walls–parallel and perpendicular–to the direction of the analysis can be included. In this example, the contribution of wall weights parallel to the applied seismic force is considered in the calculation of diaphragm shears. Walls perpendicular to the applied seismic force are included in determining the lateral force of concrete diaphragms.

Step 4: Center of mass (COM)

Determine center of mass

Assume that the diaphragm is rigid

Assume the (0,0) coordinate is located at the bottom left corner of the diaphragm. Center of mass of walls is shown in Table E.1

Table E.1—Determining shear walls center of gravity

Wall no.	Weight, psf	Length, ft	Area, ft²	Weight, kip	Direction	x_{cg}, ft	Wx_{cg}, ft-kip	y_{cg}, ft	Wy_{cg}, ft-kip
(8 in.)	100	90	540	54	x	100	5400	89.67	4842
2 (10 in.)	125	30	180	22.5	y	0.417	9.38	45	1012.5
3 (10 in.)	125	30	180	22.5	y	199.583	4490.6	45	1012.5
4 ([illegible] in.)	125	28	68	[illegible]	x	54.[illegible]	[illegible]	0	[illegible]
5 (10 in.)	125	40	240	30	x	40.0	4200	0	300
Σ			150				15,234		7377.2

The values of x_{cg} and y_{cg} are the center of mass of each wall. For example:

Wall 1 has the following coordinates: x_{cg} = 55 ft + 90 ft/2 = 100 ft and y = 90 ft – (8 in./12)/2 = 89.67 ft

Wall 2 has the following coordinates: x_{cg} = 0 ft + (10 in./12)/2 = 0.417 ft and y = 30 ft + 30 ft/2 = 45 ft

Center of mass of all walls

$$x_t = \frac{\sum W x_{cg}}{\sum W_t} = \frac{15{,}234\text{ ft-kip}}{150\text{ kip}} = 101.6\text{ ft}$$

$$y_t = \frac{\sum W y_{cg,i}}{\sum W_t} = \frac{7377.2\text{ ft-kip}}{150\text{ kip}} = 49.2\text{ ft}$$

Center of mass of the slab is x_2 = 200 ft/2 = 100 ft and y_2 = 90 ft/2 = 45 ft

Location of center of mass of the slab and walls combined

$$x_m = \frac{\sum W_i x_i}{\sum W_i} = \frac{(4860\text{ kip})(100\text{ ft}) + (150\text{ kip})(101.6\text{ ft})}{4860\text{ kip} + 150\text{ kip}} = 100.05\text{ ft}$$

and

$$y_m = \frac{\sum W_i y_i}{\sum W} = \frac{(4860\text{ kip})(45\text{ ft}) + (150\text{ kip})(49.2\text{ ft})}{4860\text{ kip} + 150\text{ kip}} = 45.13\text{ ft}$$

where 4860 kip and 150 kip are the weight of the slab and walls, respectively

Step 5. Center of rigidity (COR) and lateral system stiffness

Determine center of rigidity

From the lateral analysis, the diaphragm is assumed rigid and therefore, diaphragm flexibility is not considered. Therefore, lateral forces are distributed to shear walls in both directions in proportion to their relative stiffnesses. Lateral displacement is the sum of flexural and shear displacements.

Apply a lateral force of 1 kip is applied at the top of a cantilevered wall as shown in Fig. E3.2. The wall's lateral displacement under a unit load, which is related to its stiffness, is the sum of flexural and shear displacements.

$\Delta = \Delta_{Flexure} + \Delta_{Shear}$

$$\Delta = \frac{Ph^3}{3EI} + \frac{1.2Ph}{AG} \text{ where } G \cong 0.4E \text{ and } E = 3{,}605{,}000 \text{ psi}$$

$$\Delta_{Flexure} = \frac{Ph^3}{3EI} = \frac{Ph^3}{3E\frac{L^3t}{12}} = \frac{4P\left(\frac{h}{L}\right)^3}{Et}$$

$$\Delta_{Shear} = \frac{1.2Ph}{AG} = \frac{(1.2)Ph}{(Lt)0.4E} = \frac{3P\left(\frac{h}{L}\right)}{Et}$$

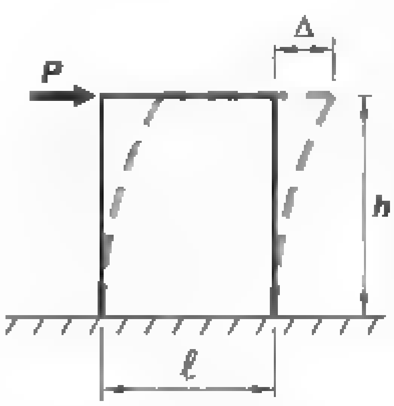


Fig. E3.2—Cantilever wall deflection

Rigidity $k_i = 1/\Delta_i$ (refer to Table E.2)

Table E.2—Determining walls' relative stiffnesses

Wall no.	Height h, ft	Length L, ft	h/L	t, in.	$\Delta_i \times 10^{-4}$, in.	$k_i = 1/\Delta_i \times 10^4$, 1/in.
1	2	90	0.[illegible]333	8	0.[illegible]4	7.043
2	12	30	0.4000	10	0.40	2.476
3	2	30	0.4000	10	0.40	2.476
4	[illegible]	28	0.4286	10	0.44	2.252
5	2	40	0.3000	10	0.28	3.576

Table E.3—Determining walls' rigidity

Wall no.	Direction	x, ft	y, ft	k_{ix}	k_{iy}	$(k_{iy})x$	$(k_{ix})y$
1	x		89.67	7.043			631.55
2	y	0.4[illegible]7	—	—	2.476	1.03	
3	y	99.58			2.476	494.16	
4	x	—	[illegible]0.0	2.252	—	—	22.52
5	x		0.0	3.576			35.76
Σ				12.872	4.952	495.19	689.83

Calculate the system's center of rigidity

$$x_r = \frac{\sum k_{iy} x_i}{\sum k_{iy}} = \frac{495.19\text{ ft/ft}}{4.95/\text{ft}} = 100\text{ ft}$$

$$y_r = \frac{\sum k_{ix} y_i}{\sum k_{ix}} = \frac{689.83\text{ ft/ft}}{12.87/\text{ft}} = 53.6\text{ ft}$$

Torsional eccentricity

The torsional eccentricity is the difference between the system's center of rigidity and its center of mass (Fig. E3.3)

$e_x = x_r - x_m = 100.05\text{ ft} - 100.02\text{ ft} = 0.03\text{ ft}$, which is negligible

$e_y = y_r - y_m = 53.6\text{ ft} - 45.1\text{ ft} = 8.5\text{ ft}$

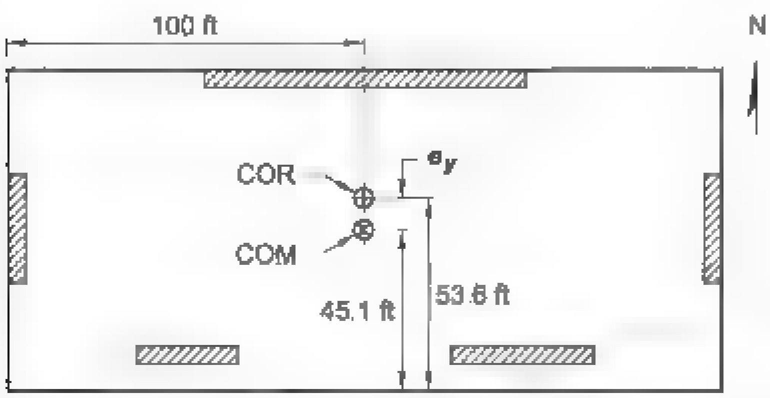


Fig. E3.3—Locations of the system's center of rigidity and center of mass

ASCE/SEI 7 requires shifting the center of mass by a minimum of 5 percent of the building dimension, referred to as accidental eccentricity, in addition to the calculated eccentricity

$e_x = 0\text{ ft} \pm (0.05)(200\text{ ft}) = \pm 10\text{ ft}$

$e_y = 8.5\text{ ft} \pm (0.05)(90\text{ ft})$

$$e_{y1} = 8.5\text{ ft} - 4.5\text{ ft} = 4\text{ ft}$$
$$e_{y2} = 8.5\text{ ft} + 4.5 = 13\text{ ft}$$

Step 6: Lateral resisting system forces

In-plane wall forces due to direct lateral shear force are calculated by:

$$F_{vx} = F_{px}\frac{k_{ix}}{\sum k_{ix}}$$

$$F_{vy} = F_{py}\frac{k_{iy}}{\sum k_{iy}}$$

In-plane wall forces due to torsional moment are calculated by:

$$F_{ti} = \frac{k_i x_i}{\sum k_i x_i^2} T_x$$

$$F_{ti} = \frac{k_i y_i}{\sum k_i y_i^2} T_y$$

The torsional moment is the lateral shear force multiplied by the corresponding eccentricity

NS: $T = F_{px}e_x$ = (745 ft)(±10 ft) = ±7450 ft-kip

EW: $T_x = F_{py}e_{y1}$ = (726 lb)(±4 ft) = ±2904 ft-kip

$T_y = F_{py}e_{y2}$ = (726 lb)(±13 ft) = ±9438 ft-kip

The in-plane diaphragm force is the sum of the direct lateral shear force, F_{vi}, and the torsional moment, F_{ti} (refer to Tables E.4, E.5, and E.6): $F_u = F_{vi} + F_{ti}$

Table E.4—Determining wall shear due to seismic forces in the N-S direction

Wall no.	k_{ix}	k_{iy}	dx_i, ft	dy_i, ft	k_id	$k_i(d)^2$	F_{vi}, kip	F_{ti}, kip	F_{total}, kip	F_{design}, kip	Use, kip
1	7.04	0		36.08	254.09	9166.8	0	27.3	27.3	27.3	27.3
2	0	2.476	–99.6	—	–246.56	24,553.6	372.5	–26.5	26.5	346.0	399.0
3	0	2.476	99.6		246.56	24,553.6	372.5	26.5	26.5	399.0	346.0
4	2.25	0		43.6	98.185	4280.2	0	10.5	10.5	10.5	10.5
5	3.57	0		43.6	155.91	6796.4	0	16.7	16.7	16.7	16.7
Σ	12.87	4.952			0	69,350.5					

Example on calculating dy_i

Wall 1: dy_i = 90 ft – (8 in., 12 in./ft)/2 – 53.6 ft = 36.08 ft

Walls 4 and 5: dy_i = 53.6 ft – 10 ft = 43.6 ft

Table E.5—Determining wall shear due to seismic forces in the E-W direction e_{y1} = 4 ft

Wall no.	k_{ix}	k_{iy}	x_i, ft	y_i, ft	k_id	$k_i(d)^2$	F_v, kip	F_{T2}*, kip	F_{design}, kip
1	7.043	0		36.08	254.09	9166.8	397.2	10.6	386.7
2	0.00	2.476	99.6		246.565	24,553.7	0.0	10.2	0.2
3	0.00	2.476	99.6	—	246.565	24,553.7	0.0	10.2	10.2
4	2.252	0		43.6	98.185	4280.2	127.0	4.1	131.1
5	3.576	0	—	–43.6	–155.91	6796.4	20[illegible].7	6.5	208.[illegible]
Σ	12.87	4.952			0	69,350.9			

Table E.6—Determining wall shear due to seismic forces in the E-W direction e_{y2} = 13 ft

Wall no.	k_{ix}	k_{iy}	x_i, ft	y_i, ft	k_id	$k_i(d)^2$	F_v, kip	F_T*, kip	F_{design}, kip
1	7.043	0		36.08	254.09	9166.8	397.2	34.5	362.8
2	0.00	2.476	99.6		246.565	24,553.7	0.0	33.5	33.5
3	0.00	2.476	99.6		246.565	24,553.7	0.0	33.5	33.5
4	2.252	0		43.6	98.185	4280.2	127.0	13.3	140.4
5	3.576	0		43.6	155.91	6796.4	20[illegible].7	21.2	222.9
Σ	12.87	4.952			0	69,350.9			

where d is the distance (dx_i or dy_i) from the center of each wall to the center of rigidity. F_{x1} is the additional shear force due to eccentricity of 13 ft. F_{x2} is the additional shear force due to eccentricity of 4 ft.

Notes

- dx_i and dy_i are the distances of a wall from the center of rigidity in the x- and y-direction.
- If torsional moment reduces the magnitude of the direct lateral shear on a wall, then it is ignored.

The wall design shear forces are summarized in Table E.7

Table E.7—Summary of wall shear forces due to seismic forces

Wall no.	Wall length, ft	E-W load, kip	N-S, load	Design, shear, kip
1	90.00	387	27	387
2	30.00	33.5	399	399
3	30.00	33.5	399	399
4	28.00	140	11	140
5	40.00	223	17	223

Step 7: Diaphragm shear strength

In-plane shear in diaphragm

The diaphragm nominal shear strength is calculated from Eq. (12.5.3.3)

12.5.3.3 $V_n = A_{cv}\left(2\lambda\sqrt{f_c'} + \rho_t f_y\right)$

North and south

$V_n = (90\text{ ft})(12\text{ in./ft})(10\text{ in.})\left((2)(1.0)\sqrt{4000\text{ psi}}\right)$

$= 1,366,104\text{ lb} = 1366\text{ kip}$

In this example, first check the diaphragm strength without reinforcement; therefore, ignore the strength contribution of reinforcement: $\rho_t = 0$.

East and west

$V_n = (200\text{ ft})(12\text{ in./ft})(10\text{ in.})\left((2)(1.0)\sqrt{4000\text{ psi}}\right)$

$= 3,035,787\text{ lb} = 3036\text{ kip}$

Assume collector length is the full length of diaphragm in both directions.

12.5.3.2
21.2.4.1 Applying the reduction factor $\phi = 0.6$.

NS: $\phi V_n = (0.6)(1366\text{ kip}) = 820\text{ kip}$
EW: $\phi V_n = (0.6)(3036\text{ kip}) = 1821\text{ kip}$

Per ACI 318, ϕ must not exceed the least value for shear used for the vertical components of the primary seismic-force resisting system

12.5.1.1 Check if design shear strength exceeds the factored shear force

$\phi V_n = 820\text{ kip} > V_u = 399\text{ kip}$ **OK**
$\phi V_n = 1821\text{ kip} > V_u = 387\text{ kip}$ **OK**

18.12.9.2 The nominal shear strength, V_n, must not exceed $8A_{cv}\sqrt{f_c'}$

$$\frac{8(10\text{ in.})(90\text{ ft})(12\text{ in./ft})\left(\sqrt{4000\text{ psi}}\right)}{1000\text{ lb/kip}} = 5464\text{ kip}$$

$V_{n,NS} = 1366\text{ kip} < 5464\text{ kip}$ **OK**
$V_{n,EW} = 3036\text{ kip} < 5464\text{ kip}$ **OK**

Step 8. Diaphragm lateral force distribution N-S

12.4.2.4
12.5.1.3

Diaphragm is assumed rigid (ACI 318, Section 12.4.2.4(a)). Therefore, the diaphragm design moments, shears, and axial forces are calculated assuming a simply supported beam with depth equal to full diaphragm depth (ACI 318, Section (12.5.1.3(a)).

The wall forces and the assumed direction of the torsional moment are shown in Fig. E3.4.

Refer to previous Step 5 for calculation of seismic force location.

The seismic force on the diaphragm is distributed within diaphragm as shown in (Fig. E3.4):

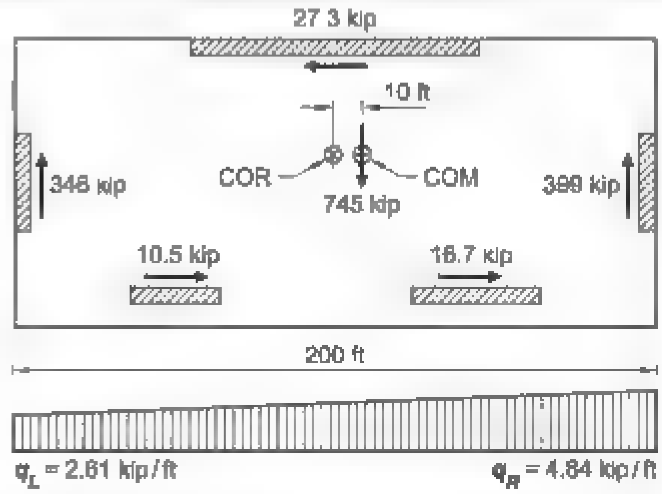


Fig. E3.4—Shear wall forces due to seismic force in the N-S direction.

Force equilibrium

$$q_L\left(\frac{L}{2}\right)+q_R\left(\frac{L}{2}\right)=F_{px,des(NS)} \quad \text{(I)}$$

$$q_L\left(\frac{200\text{ ft}}{2}\right)+q_R\left(\frac{200\text{ ft}}{2}\right)=745\text{ kip}$$

Moment equilibrium (taken around bottom left corner of the diaphragm)

$$q_L\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)+q_R\left(\frac{L}{2}\right)\left(\frac{2L}{3}\right)=F_{px,des(NS)}\left(\frac{L}{2}+0.05L\right) \quad \text{(II)}$$

$$q_L\frac{(200\text{ ft})^2}{6}+q_R\frac{2(200\text{ ft})^2}{6}=(745\text{ kip})\left(\frac{200\text{ ft}}{2}+10\text{ ft}\right)$$

Solve equations (I) and (II) for q_L and q_R

q_L = 2.61 kip/ft, say, 2.6 kip/ft
q_R = 4.84 kip/ft, say, 4.8 kip/ft

Draw the shear and moment diagrams (Fig. E3.5).

Note: In an Aug. 2010 National Institute and Standards Technology (NIST) report, GCR 10-917-4, "Seismic Design of Cast-in-Place Concrete Diaphragms, Chords, and Collectors," by Moehle et al. states that, "This approach leaves any moment due to the frame forces along column lines (CL) A and F unresolved. Sometimes this is ignored or, alternatively, it too can be incorporated in the trapezoidal loading."

F_{max} = 399 kip

M_{max} = 21,106 ft-kip, say, 21,100 ft-kip

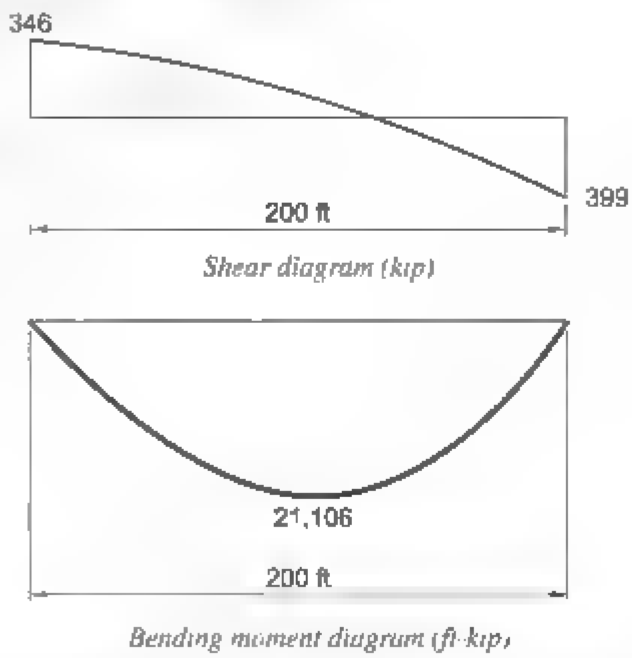


Fig. E3.5—Shear and bending moment diagrams due to a lateral seismic force in the N-S direction.

In smaller buildings in which seismic demand is low, there are no irregularities, and torsional moments are not significant, the diaphragm shears and moments can be based on a uniformly distributed load, rather than a linearly varying load.

q = 745 kip/200 ft = 3.723 kip/ft, say 3.72 kip/ft

Calculate a maximum moment.

$$M_{max}=\frac{(3.73\text{ kip/ft})(200\text{ ft})^2}{8}=18{,}650\text{ ft-kip}$$

	Notes • The difference in maximum moment between the two approaches is 13.3 percent and at different locations (110 ft versus 100 ft). • Shear diagram for the second approach is a straight line with maximum shear force $V = (3.73\ \text{kip/ft})(200\ \text{ft}/2) = 373\ \text{kip}$	
Step 9: Chord reinforcement N-S		
R12.1.1	Assume the slab behaves like a beam with compression and tension forces at the near and far edges, respectively: $C_{chord} = T_{chord} = M/d$	
12.5.2.3	ACI 318 suggests placement of chord reinforcement within an arbitrary width of $h/4$ of the diaphragm tension edge (Fig. E3.6).	$h/4 = 90\ \text{ft}/4 = 22.5\ \text{ft}$ $\Rightarrow d = 90\ \text{ft} - 1/2(22.5\text{ft}) = 78.75\ \text{ft}$
	The maximum chord tension force is:	$T_u = \dfrac{21{,}100\ \text{ft-kip}}{78.75\ \text{ft}} = 268\ \text{kip}$
	Tension due to moment is resisted by deformed bars conforming to Section 20.2.1 of ACI 318. Steel stress is the lesser of the specified yield strength and 60,000 psi.	$f_y = 60{,}000\ \text{psi}$
	Required chord reinforcement area: $\phi T_n = \phi f_y A_s \geq T_u$	$A_s = \dfrac{268\ \text{kip}}{(0.9)(60\ \text{ksi})} = 5\ \text{in.}^2$
	It is, however, recommended to place tension reinforcement close to the tension face. Assume tension reinforcement moment arm is approximately $0.95B$ at both north and south sides of the slab edges: The calculated tension force is	$(0.95)(90\ \text{ft}) = 85.5\ \text{ft}$
	$T_u = \dfrac{M_u}{0.95B}$	$T_u = \dfrac{21{,}100\ \text{ft-kip}}{85.5\ \text{ft}} = 247\ \text{kip}$
	Required tension reinforcement is: $\phi T_n = \phi f_y A_s > T_u$	$A_s = \dfrac{247\ \text{kip}}{(0.9)60\ \text{ksi}} = 4.6\ \text{in.}^2$
18.12.7.6	Per provision 18.12.7.6, the required chord width for the concrete compressive strength limit of $0.2f_c'$ $w_{chord} > \dfrac{C_{chord}}{0.2f_c' h_{diaph}}$	$w_{chord} > \dfrac{247\ \text{kip}}{(0.2)(4000\ \text{psi})(10\ \text{in.})} = 30.9\ \text{in.}$
	Note: The chord force does not need to be increased by the overstrength factor.	and $w_{chord} = 30.9\ \text{in.} < h/4 = 90\ \text{ft}/4 = 22.5\ \text{ft}$ **OK** Say, 32 in.
	Note: Although it is permissible to place bars within 22 ft 6 in. ($h/4$) of diaphragm width, it is recommended to place bars close to the tension end, where it is most effective. Since load is reversible, chord reinforcement is placed at both north and south edges of diaphragm (refer to Fig. E3.6). The final layout of bars will be coordinated with the collector reinforcement due to inertial force in the East West (E-W) direction.	

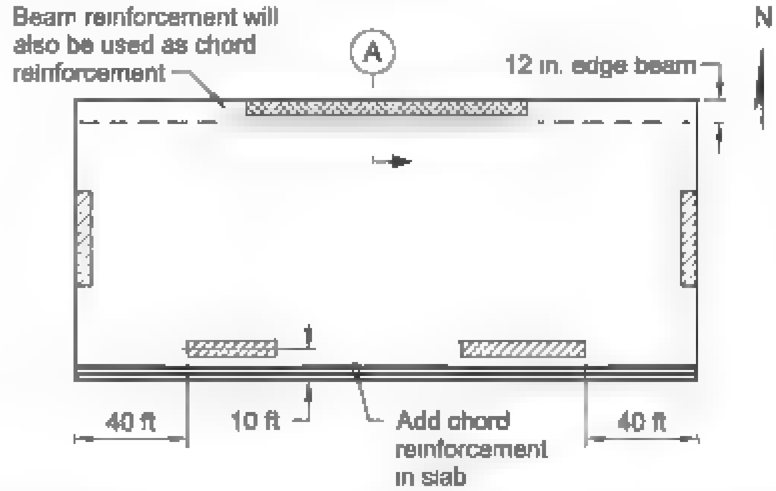


Fig. E3.6—Suggested chord reinforcement at the north and south edges of the diaphragm.

Step 10: Collectors design N-S

	Collectors transfer shear forces from the diaphragm to the vertical walls at both east and west ends (Fig. E3.4). Assume collectors extend over the full width of the diaphragm. Partial depth collectors can be considered, but a complete force path should be designed that is capable of transmitting all forces from the diaphragm to the collector and into the vertical elements.	
12.5.4.1	Unit shear force is the maximum diaphragm shear divided by the diaphragm depth, B = 90 ft.	
	$v_{u@F} = \frac{F_{u@F}}{B}$	From Step 6 (Table E.7): F_u = 399 kip
	In slab:	$v_{u@F} = \frac{399\text{ kip}}{90\text{ ft}} = 4.43\text{ kip/ft}$
	In wall:	$v_{u@F} = \frac{399\text{ kip}}{30\text{ ft}} = 13.3\text{ kip/ft}$
	Check if the concrete shear strength excluding reinforcement exceeds the factored shear.	
	$\phi v_c = \phi 2\sqrt{f_c'}bt_{diaph}$	$\phi v_c = \frac{(0.6)(2\sqrt{4000\text{ psi}})(12\text{ in./ft})(10\text{ in.})}{1000\text{ lb/kip}} = 9.1\text{ kip/ft}$ $\phi v_c = 9.1\text{ kip/ft} < v_u = 13.3\text{ kip/ft}$ **NG**
	Shear reinforcement is, therefore, required. Use the No. 5 at 16 in. on center temperature and shrinkage reinforcement in each direction to increase shear capacity (assuming two-way slab).	$\rho_t = \frac{0.31\text{ in.}^2}{(10\text{ in.})(16\text{ in.})} = 0.00194$ $\phi v_n = 9.1\text{ kip/ft} + (0.6)(0.00194)(10\text{ in.})(12\text{ in.})(60\text{ ksi})$ $= 17.5\text{ kip/ft} > v_u = 13.3\text{ kip/ft}$ **OK**

	<u>Force at diaphragm to wall connection</u> The proportional diaphragm force that the collector transfers to walls connection is (Fig. E3.7) Wall south end $(4.43\text{ kip/ft})(30\text{ ft}) = 132.9\text{ kip}$ Wall north end $-132.9\text{ kip} + (13.3\text{ kip/ft} - 4.43\text{ kip/ft}) \times (30\text{ ft})$ $= 133.2\text{ kip}$ Slab end $+133.2\text{ kip} - (4.43\text{kip/ft})(30\text{ ft}) \sim 0\text{ kip}$	

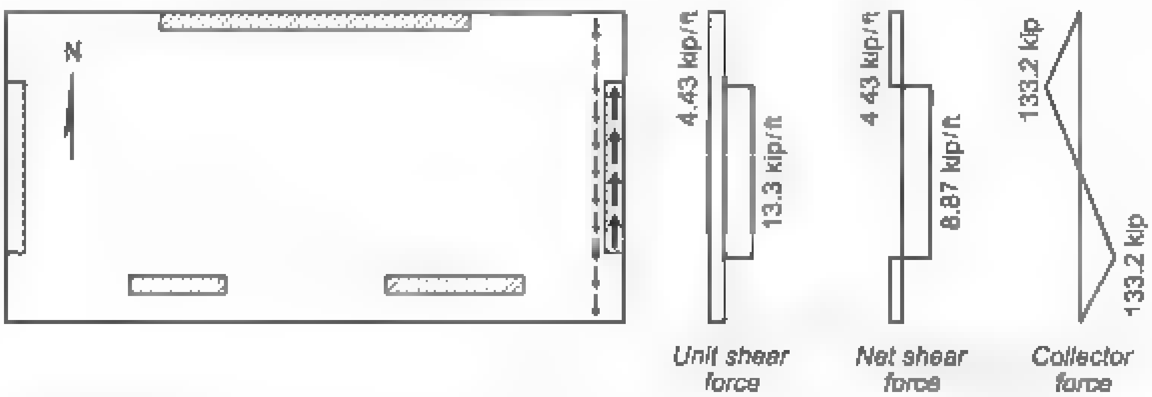


Fig. E3.7—Collector forces due to inertial forces in N-S direction

	The collector factored force that is transferred to the walls is shown in Fig. E3.7.	
18.12.2.1	This collector force is then multiplied by the system overstrength factor $\Omega_o = 2.5$ for building systems with special structural walls in SDC D (ASCE/SEI 7, Table 12.2-1)	$T_u = \Omega_o T_{Coll} = \Omega_o C_{Coll} = (2.5)(133.2\text{ kip}) = 333\text{ kip}$
12.5.4.2 R12.5.4	Collectors are designed as both tension and compression members. There are no beams along the CL 1, so a portion of the slab is used as a collector. The collector width for tension reinforcement is determined by engineering judgement. ACI 318 provides in the commentary that a collector width cannot exceed approximately one-half the contact length between the collector and the vertical element measured from the face of the vertical element.	$b_{eff} = 30\text{ ft}/2 + 10\text{ in.} = 15.8\text{ ft} = 190\text{ in.}$
18.12.7.6	The collector width, however, is chosen such that the limiting stresses are not exceeded. When the tension and compression collector forces are increased by the overstrength factor, then the limiting concrete compression stress is $0.5f_c'$. Calculate the required compressive collector width	$w_{collector} = \dfrac{2.5C_{coll}}{0.5f_c't} = \dfrac{333\text{ kip}}{(2\text{ ksi})(10\text{ in.})} = 16.7\text{ in.}$ $w_{collector} = 16.7\text{ in.} < 190\text{ in.}$ but $w_{collector} = 16.7\text{ in.} > t_w = 10\text{ in.}$ Therefore, part of the slab, b_{eff}, is needed to resist the collector force
22.4.3	Required reinforcement area to resist collector force: $\phi T_n = \phi f_y A_s \geq T_u$	$A_s = \dfrac{\Omega_o T_{coll}}{0.9f_y} = \dfrac{333\text{ kip}}{0.9(60\text{ ksi})} = 6.2\text{ in.}^2$

Note: Collector reinforcement may be varied along the length of the diaphragm based on required strength and terminated where not required. In this example, the reinforcement is extended over the full length of the diaphragm.

A number of bars are placed in line with the wall. The balance is distributed across the width of the collector element. In this case, for the 10 inch thick wall, two No. 8 bars in line with the wall will result in a reasonable bar spacing of 6 in. Therefore, two No. 8 bars are centered on the wall for

$A_{s,line}$ = 1.58 in.^{2}

The balance of the required reinforcement is.

$A_{s,bal}$ = 6.2 in.^{2} – 1.58 in.^{2} = 4.62 in.^{2} distributed over the 15.0 ft wide collector; 4.62 in.^{2}/15.0 ft = 0.31 in.^{2}/ft

Try eight No. 5 top and bottom spaced over 15 ft

$A_{s,prov}$ = (2)(8)(0.31 in.^{2}) = 4.96 in.^{2}
$A_{s,prov}$ = 4.96 in.^{2} > $A_{s,req'd}$ = 4.62 in.^{2} **OK**

Design collector region

The collector geometry results in a moment in the diaphragm section adjacent to the wall because of the eccentricity between the collector and the wall. This moment is solved through shear forces in the diaphragm perpendicular to the collector and bending in the plane of the diaphragm due to eccentric tension and compression forces (Fig. E3.8).

$M_u = T_{dist}e_{ten} + C_{dist}e_{comp} - V\ell_{wall}$

where T_{dist} is the portion of the tension collector force resisted by $A_{s,dist}$; C_{dist} is the portion of the compression collector force resisted by slab outside the wall, and V is the shear strength of the diaphragm.

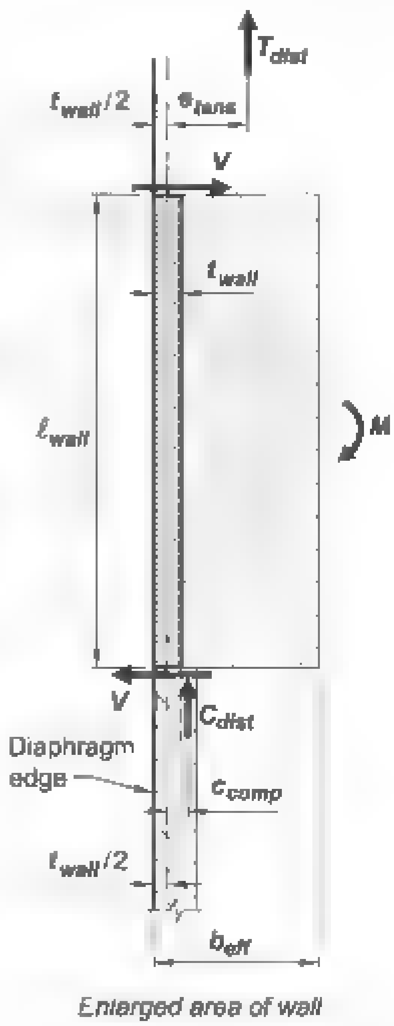


Fig. E3.8—Diaphragm segment plan at East shear wall. West shear wall is symmetric.

Where the collector element is in tension, the concrete contribution to V is neglected, V_c = 0

$\phi V_s = \phi f_y \rho_t t_w (w_{comp} - t_{wall})$

ϕV_s = 0.6(0.00193)(60 ksi)(10 in.)(16.7 in. – 10 in.) = 4.7 kip

Assume No. 5 @ 16 in. on center is provided

$$\rho_t = \frac{0.31 \text{ in.}^2}{(10 \text{ in.})(16 \text{ in.})} = 0.00193 > 0.0018$$

For more in-depth understanding refer to Structural Engineer Association of California (SEAOC) Seismology Committee (2007) "Concrete Slab Collectors," from the Aug. 2008 *SEAOC Blue Book: Seismic Design Recommendations Compilation*, Structural Engineers Association of California, Sacramento.

	Tension force in the slab (outside the wall geometry) is proportional to reinforcement	$T_{dist} = \left(\dfrac{4.62\text{ in.}^2}{6.2\text{ in.}^2}\right)(333\text{ kip}) = 247\text{ kip}$
	The moment arm	$e_{tens} = \dfrac{(15\text{ ft})(12\text{ in./ft})}{2} + \dfrac{10\text{ in.}}{2} = 95\text{ in.}$
	Compression force in the slab (outside the wall geometry) is proportional to collector width	$C_{dist} = \left(\dfrac{16.7\text{ in.} - 10\text{ in.}}{16.7\text{ in.}}\right)(333\text{ kip}) = 137\text{ kip}$
	Moment arm	$e_{comp} = \dfrac{10\text{ in.}}{2} + \dfrac{16.7\text{ in.} - 10\text{ in.}}{2} = 8.35\text{ in.}$
	$M_u = T_{dist}e_{ten} + C_{dist}e_{comp} - V\ell$	$M_u = (247\text{ kip})(95\text{ in.}) + (134\text{ kip})(8.35\text{ in.}) - (4.7\text{ kip})(29.5\text{ ft})(12\text{ in./ft})$ $M_u = 22{,}920\text{ in.-kip}$
	Assume $\ell = 30\text{ ft} - 0.5\text{ ft} = 29.5\text{ ft}$ moment arm	
	Required reinforcement	$A_{s,req'd} = \dfrac{22{,}920\text{ in.-kip}}{(0.9)(60\text{ ksi})(0.9)(29.5\text{ ft})(12\text{ in./ft})}$ $A_{s,req'd} = 1.33\text{ in.}^2$ Use three No. 6 dowels at each end of the wall Refer to Fig. E3.9

	Shear transfer design. A number of bars are placed in line with the wall, which results in transferring portion of the diaphragm force in tension and direct bearing of slab against the wall in compression. The diaphragm and shear transfer interface is designed for the balance of the collector element.	
	Assuming tension forces are distributed in proportion to collector area, V_u for diaphragm and shear transfer design is then calculated as follows	V_u = 247 kip + 134 kip + (4.43 kip/ft)(30 ft) = 514 kip
	6.2 in.^{2} is the required reinforcement area to resist the collector force in prior calculation.	
	1.58 in.^{2} is the area of two No. 8 bars placed in-line with the shear wall	
12.5.3.7	Shear from the diaphragm is transferred by shear friction to the wall with dowels placed perpendicular to the wall-slab interface	
	$V_n = \mu A_{vf} f_y$	
22.9.4.2	Assume that diaphragm slab is placed against hardened wall concrete that is clean, free of laitance, and intentionally roughened to a full amplitude of approximately 1/4 in. From Table 22.9.4.2, μ = 1.0	
21.2.1(b)	Use a reduction factor of ϕ = 0.75, because the shear interface is not a member. Otherwise, ϕ = 0.6	514 kip $\leq \phi V_n$ = (0.75)(1.0)A_{vf}(60 ksi)
	The wall is ℓ_{wall} = 30 ft long.	A_{vf} = 11.42 in.^{2} or A_{vf}/ℓ_{wall} = 11.42 in.^{2}/(30 ft) = 0.38 in.^{2}/ft Try No. 6 at 12 in. on center $A_{s,prov}$ = 0.44 in.^{2}/ft **OK**
	The required diaphragm strength is:	v_u = 514 kip/30 ft = 17.1 kip/ft
	The diaphragm shear strength is the contribution of concrete and reinforcement in prior calculation of this step	
	$\phi v_n = \phi(v_c + v_s)$	ϕv_n = 17.5 kip/ft > v_u = 17.1 kip/ft
22.9.4.4	The value of V_n across the assumed shear plane must not exceed the lesser of the following limits	
	(a) $0.2f_c'A_c$ (b) $(480 + 0.08f_c')A_c$ (c) $1600A_c$	(0.2)(4000 psi) = 800 psi **OK** (480 + 0.08(4000 psi)) = 800 psi **OK** 1600 psi
		Therefore, V_n must not exceed $\frac{(800\text{ psi})(10\text{ in.})(30\text{ ft})(12\text{ in./ft})}{1000\text{ lb/kip}} = 2880\text{ kip} > V_n$

Step 11. Diaphragm lateral force distribution E-W

The wall forces and the assumed direction of torque due to accidental eccentricity are shown in Fig. E3.10.

The distribution of the diaphragm force is calculated by using q_L and q_R as the left and right diaphragm reactions per unit length (Fig. E3.10).

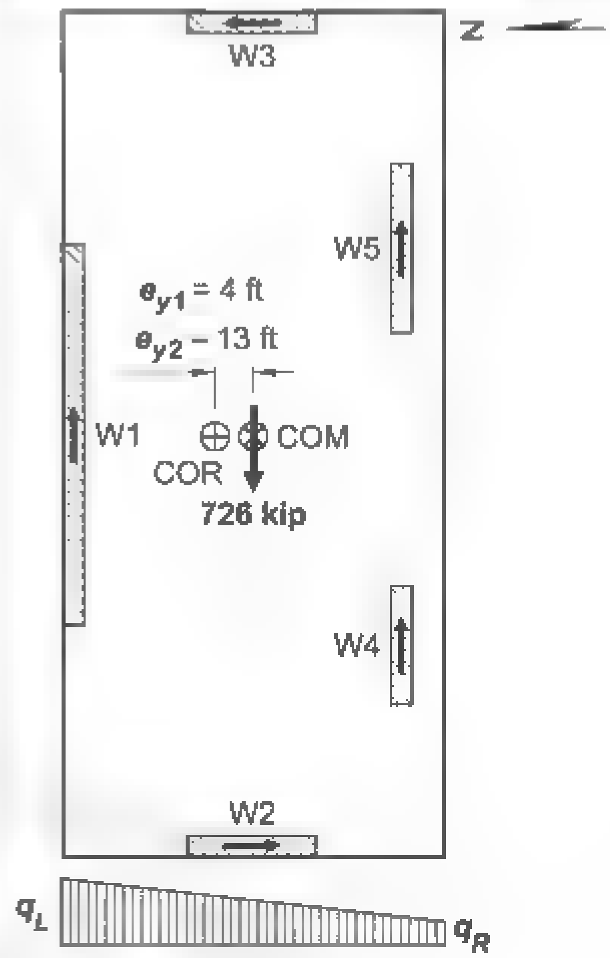


Fig. E3.10—Shear wall forces due to a seismic force in the E-W direction at e_y = 8.5 ft.

Case I
Eccentricity at e_y = 4 ft
Refer to Step 5 of this example for calculation of eccentricity

<u>Force equilibrium</u>

$$q_L\left(\frac{L}{2}\right)+q_R\left(\frac{L}{2}\right)=F_{px,des,EW} \quad \text{(I)}$$

$$q_L\left(\frac{90\text{ ft}}{2}\right)+q_R\left(\frac{90\text{ ft}}{2}\right)=726\text{ kip}$$

<u>Moment equilibrium (taken around CL W1)</u>

$$q_L\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)+q_R\left(\frac{L}{2}\right)\left(\frac{2L}{3}\right)$$
$$=(F_4+F_5)(80\text{ ft})-F_2(200\text{ ft}) \quad \text{(II)}$$

$$q_L\frac{(90\text{ ft})^2}{6}+q_R\frac{2(90\text{ ft})^2}{6}$$
$$=(208\text{ kip}+131\text{ kip})(80\text{ ft})-(10\text{ kip})(200\text{ ft})$$

F_2, F_4, and F_5 are per Table E.5

Solve equations (I) and (II) for q_L and q_R:

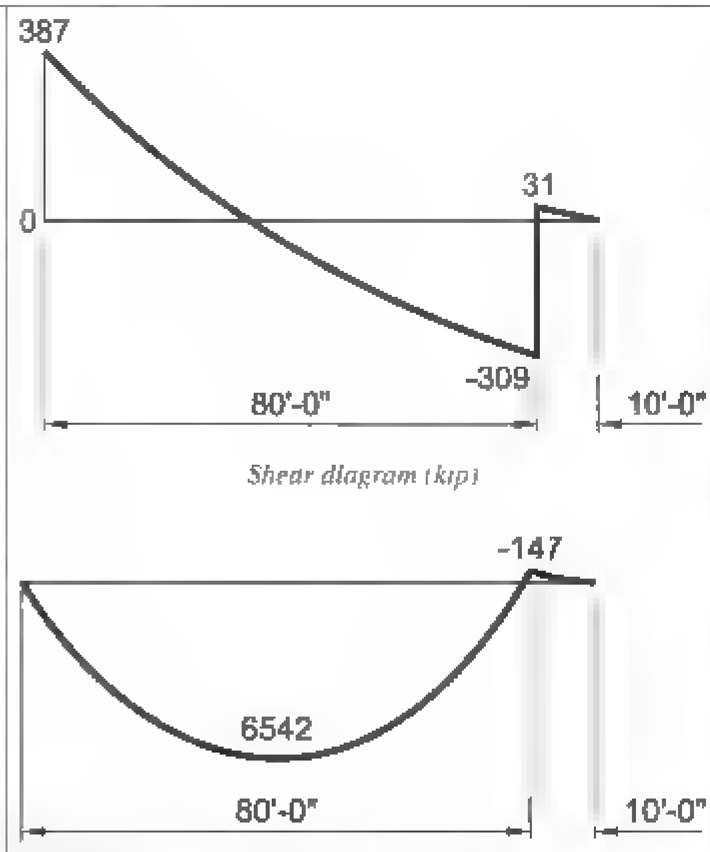
q_R = 2.5 kip/ft
q_L = 13.6 kip/ft

Draw the shear and moment diagrams for the diaphragm assuming simply supported beam behavior (Fig. E3.11).

The maximum moment is located at 40.5 ft from the south end of the diaphragm

V_{max} = 387 kip
M_{max} = 6542 ft-kip

Note: Moehle et al. also state in NIST report number GCR 10-917-4 that, "For a rectangular diaphragm of uniform mass, a trapezoidal distributed force having the same total force and centroid is then applied to the diaphragm. The resulting shears and moments are acceptable for diaphragm design. This approach leaves any moment due to (shear walls perpendicular to the diaphragm inertia lateral force) unresolved, sometimes this is ignored or, alternatively, it too can be incorporated in the trapezoidal loading."



Moment diagram (ft-kip)

Fig. E3.11—Shear and moment diagrams for Case I

Case II
Eccentricity at e_y = 13 ft
Refer to previous Step 5 for calculation of eccentricity

Force equilibrium

$$q_L\left(\frac{L}{2}\right)+q_R\left(\frac{L}{2}\right)=F_{px,des\cdot EW)} \quad \text{(I)}$$

$$q_L\left(\frac{90\text{ ft}}{2}\right)+q_R\left(\frac{90\text{ ft}}{2}\right)=726\text{ kip}$$

Moment equilibrium (taken around CL Wall)

$$q_L\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)+q_R\left(\frac{L}{2}\right)\left(\frac{2L}{3}\right)$$
$$=(F_4+F_5)(80\text{ ft})-F_2(200\text{ ft}) \quad \text{(II)}$$

$$q_L\frac{(90\text{ ft})^2}{6}+q_R\frac{2(90\text{ ft})^2}{6}$$
$$=(223\text{ kip}+140\text{ kip})(80\text{ ft})-(33.5\text{ kip})(200\text{ ft})$$

F_2, F_4, and F_5 are per Table E.6.

Solve equations (I) and (II) for q_L and q_R

q_R = 0.4 kip/ft
q_L = 15.7 kip/ft

Draw the shear and moment diagrams for the diaphragm assuming simply supported beam behavior (Fig. E3.12).

The maximum moment is located at 36 ft from the south end of the diaphragm

V_{max} = 363 kip
M_{max} = 6507 ft-kip, say, 6500 ft-kip

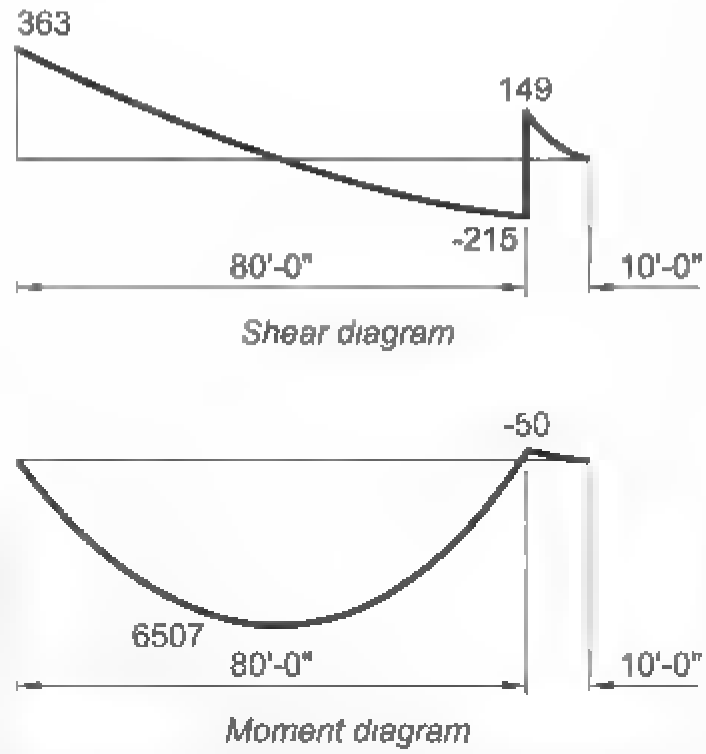


Fig. E3.12—Shear and moment diagrams for Case II

	Case I	Case II
Shear, kip	389	363
Moment, ft-kip	6540	6500

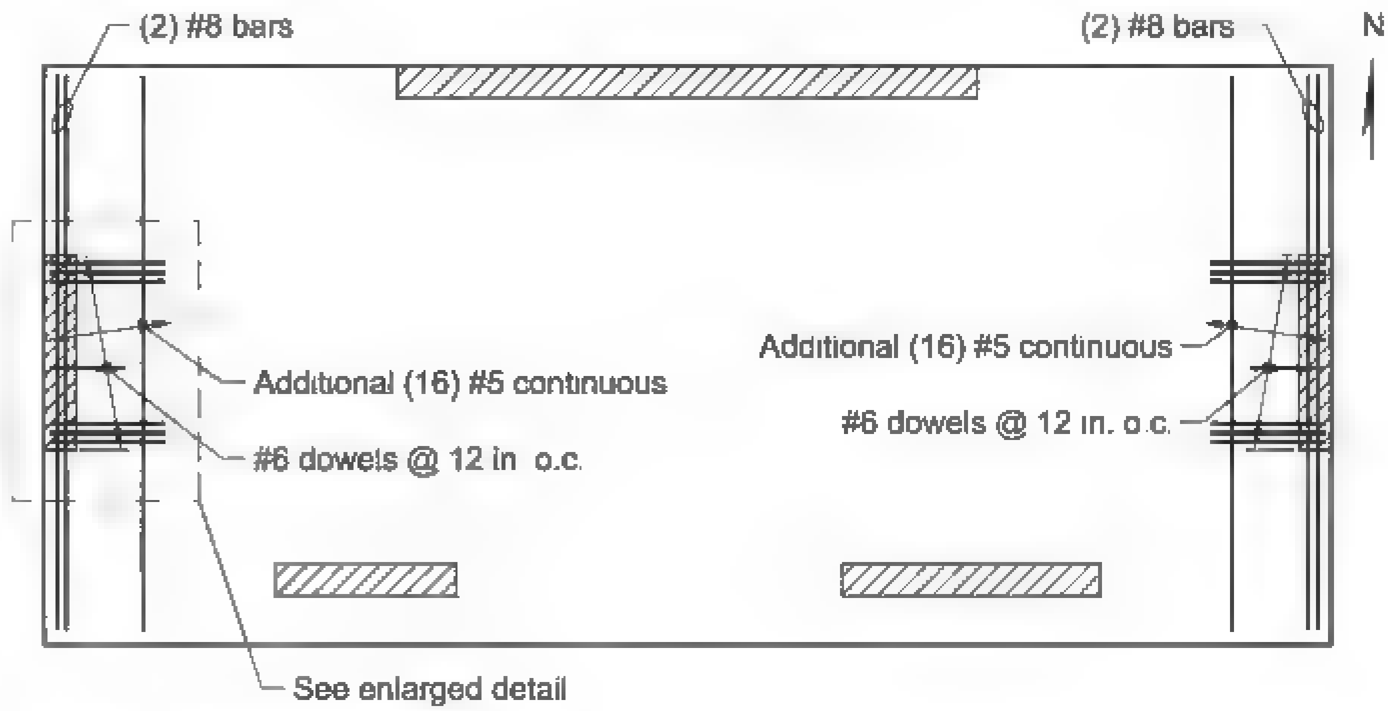
Case I controls. Therefore, design diaphragm in the East-West (E-W) direction for the inertial force obtained from controlling case
Note: Taking the approach of equivalent uniformly distributed inertial force by ignoring the accidental torsion, the corresponding shear and moment forces are

Shear: 317.6 kip at wall W1 and 408.4 kip at walls W4 and W5 combined
Maximum moment: 6252 ft-kip

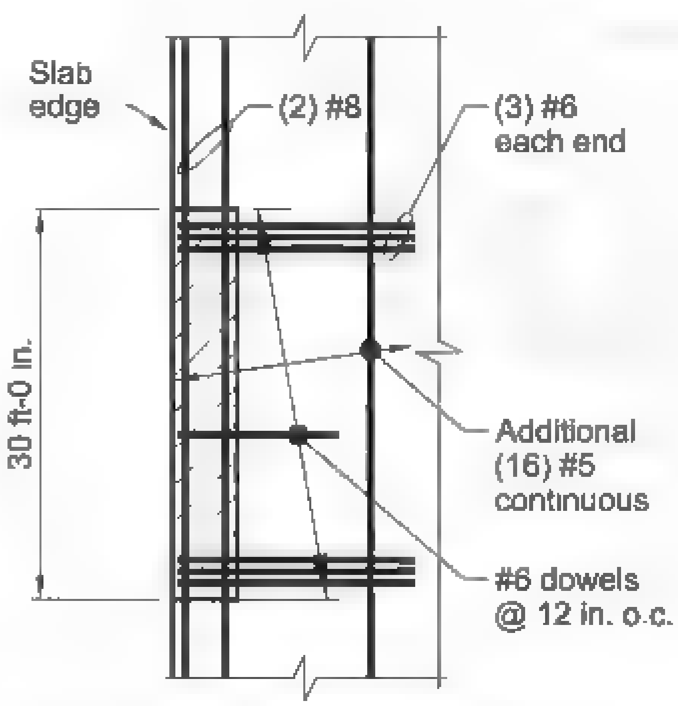
Comparing the moments of the two approaches, we find that the difference is less than 5 percent - negligible.
In smaller buildings in which seismic demand is low, there are no irregularities, and torsional moments are not significant, the diaphragm shears and moments can be based on a uniformly distributed load, rather than a linearly varying load. This example, however, will use the detailed approach applying the five percent accidental torsion.
Force summary in the E-W direction

	Case I	Case II	Controlling case
Maximum moment, ft-kip	6540	6500	[illegible]
W_1 shear force, kip	387	363	I
W_4 shear force, kip	131	140	II
W_5 shear force, kip	208	223	II

Step 12. Chord reinforcement E-W		
R12.1.1	Assume the slab behaves like a beam with compression and tension forces at the near and far edges, respectively: $C_{chord} = T_{chord} = M/d$	
	Chord reinforcement resisting tension must be located within $h/4$ of the tension edge of diaphragm	$h/4 = 200\text{ ft}/4 = 50\text{ ft} < d = 175\text{ ft}$
	Assume that tension reinforcement will be placed within wall thickness. Therefore, moment arm is approximately 200 ft − 1/2 (10 in./12) = 199.58 ft at both east and west sides of the slab edges.	$d = 199.58\text{ ft}$
	<u>Chord force</u> The maximum chord tension force is at midspan	
	$T_u = \dfrac{M_u}{d}$	$T = \dfrac{6540\text{ ft-kip}}{199.58\text{ ft}} = 32.8\text{ kip}$
18.12.7.6	Calculate the required chord width for the calculated concrete compressive strength limit of $0.2f_c'$	
	$w_{chord} > \dfrac{C_{chord}}{0.2f_c't}$	$w_{chord} > \dfrac{32.8\text{ kip}}{(0.2)(4000\text{ psi})(10\text{ in.})} = 4.1\text{ in.}$
	Note: Chord force does not need to be increased by the overstrength factor	
12.5.2.3	Chord reinforcement resisting tension must be located within $h/4$ of the tension edge of diaphragm.	
	Check if required calculated width is less than wall thickness:	$w_{chord} = 4.1\text{ in.} < t_w = 10\text{ in.}$ **OK**
	Tension due to moment is resisted by deformed bars conforming to Section 20.2.1 of ACI 318. Steel stress is the lesser of the specified yield strength and 60,000 psi	$f_y = 60{,}000\text{ psi}$
	<u>Required reinforcement</u> $\phi T_n = \phi f_y A_s > T_u$	$A_{s,req'd} = \dfrac{32.8\text{ kip}}{(0.9)60\text{ ksi}} = 0.61\text{ in.}^2$
	Note: The chord reinforcement at east and west ends is compared to the partial collector reinforcement placed in the east and west walls due to inertia forces in the North-South (N-S) direction. From Fig. E3.9, two No. 8 bars are placed within the wall thickness that exceed the required chord reinforcement. Therefore, **OK**.	



Collector reinforcement at Shear Walls 2 and 3



Enlarged collector reinforcement at Wall 2

Fig. E3.9—Collector reinforcement for lateral force in the N-S direction.

Step 13: Diaphragm shear strength
Refer to Step 7 for diaphragm shear strength

Step 14. Collector design E-W		
12.5.4.1	<u>Wall 1</u> Collectors transfer shear forces from the diaphragm and transfer them axially to wall W1 (Fig. E3.13). In this example, assume collectors extend over the full length of the diaphragm.	
	Unit shear force:	From Step 6 (Table E.5): F_u = 387 kip
	$v_{u@F} = \dfrac{F_{u@F}}{B}$	
	In slab:	$v_{u@F} = \dfrac{387\text{ kip}}{200\text{ ft}} = 1.94\text{ kip/ft}$
	In Wall W1:	$v_{u@F} = \dfrac{387\text{ kip}}{90\text{ ft}} = 4.3\text{ kip/ft}$
12.5.4.1	<u>Walls 4 and 5</u> Collectors transfer shear forces from the diaphragm and transfer them axially to walls W4 and W5 (Fig. E3.13). Collectors extend over the full width of the diaphragm.	
	Unit shear force:	From Step 6 (Table E.6) Slab: F_u = 140 kip + 223 kip = 363 kip
	$v_{u@F} = \dfrac{F_{u@F}}{B}$	$v_{u@F} = \dfrac{363\text{ kip}}{200\text{ ft}} = 1.82\text{ kip/ft}$
		Wall 4: F_u = 140 kip
	In slab:	$v_{u@F} = \dfrac{140\text{ kip}}{28\text{ ft}} = 5\text{ kip/ft}$
		Wall 5: F_u = 223 kip
	In wall:	$v_{u@F} = \dfrac{223\text{ kip}}{40\text{ ft}} = 5.58\text{ kip/ft}$

Step 15: Collector design N-S

Force at diaphragm to wall connection

The proportional diaphragm force that the collector transfers to wall connection is (Fig. E3.13).

Wall 1 west end:
(1.94 kip/ft)(55 ft) = 106.7 kip

Wall 1 east end:
106.7 kip + (2.36 kip/ft)(90 ft) = 106.6 kip

Diaphragm end:
106.6 kip − (1.94 kip/ft)(55 ft) = 0 kip

Wall 4 west end:
(1.82 kip/ft)(40 ft) = 72.8 kip

Wall 4 east end:
72.8 kip + (3.18 kip/ft)(28 ft) = 16.2 kip

Wall 5 west end:
16.2 kip − (1.82 kip/ft)(52 ft) = 78.6 kip

Wall 5 east end:
78.6 kip + (3.76 kip/ft)(40 ft) = 71.8 kip

Diaphragm east end:
+71.8 kip − (1.82 kip/ft)(40 ft) = 0 kip

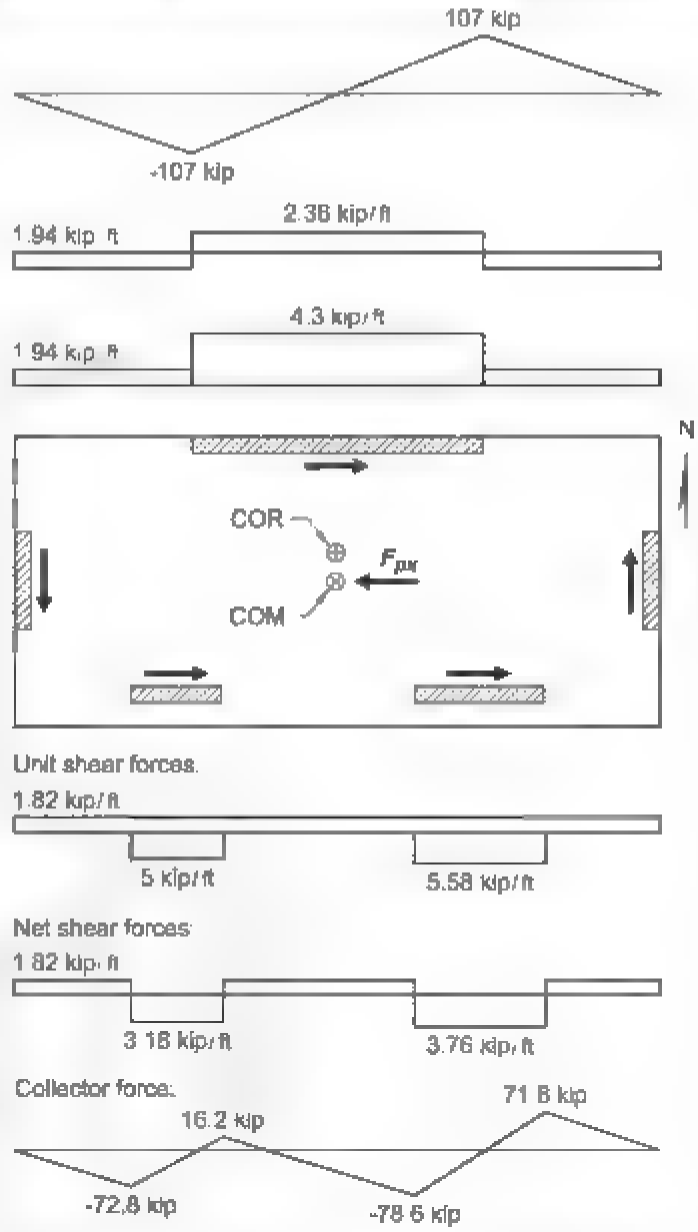


Fig. E3.13—Collector forces in the E-W direction.

18.12.2.1 The collector factored force that is transferred to the walls is shown in Fig. E3.13.

This collector force is then multiplied by the system overstrength factor, $\Omega_o = 2.5$ for building systems with special structural walls in SDC D (ASCE/SEI 7, Table 12.2-1).

	Wall 1		Wall 4		Wall 5	
	West end	East end	West end	East end	West end	East end
$2.5T_u$	268	268	182	41	197	80
$2.5C_u$	268	268	182	41	197	180

Diaphragms

12 5 4 2 Collectors are designed as tension and compression members.

There are no beams along CL 9, so portion of the slab is used as a collector

18 12 7 6 The collector width is determined by engineering judgement and chosen such that the limiting stresses are not exceeded. When the tension and compression collector forces are increased by the overstrength factor, then the limiting concrete compressive stress is $0\ 5f_c'$ Calculate the compressive collector width

$w_{chord} = 2.5C_{coll}/0\ 2f_c't$

	Wall 1		Wall 4		Wall 5	
w_{chord}, in.	7	17	9	7	9 9	9
[illegible] $> t_w$?	Y	Y	N	N	N	N

For Wall 1, the required collector width (17 in) is wider than the wall thickness (8 in) Part of the seismic force is resisted by reinforcement placed in-line with the shear wall to transfer the force directly to the end of the shear wall and direct bearing of slab against wall in compression Therefore, two No. 6 in-line with the wall The balance of seismic force is resisted by reinforcing bars placed along the sides of the wall and uses the slab shear-friction capacity at the wall-to-slab interface to transfer seismic forces to the wall.

For Walls 4 and 5, the maximum required collectors' widths (9 in and 9 9 in , respectively) are narrower than the walls widths (10 in.), therefore, place reinforcement within the walls widths.

Wall 1.

Required area of collector reinforcement:

$\phi T_n = \phi f_y A_s > T_u$

$$A_s = \frac{\Omega_o T_{coll}}{0\ 9 f} = \frac{268 \text{ kip}}{0\ 9(60 \text{ ksi})} = 5 \text{ in}^2$$

Note: Collector reinforcement along the length of the diaphragm may be varied based on required strength and terminated when not required. In this example, the reinforcement is extended over the full length of the diaphragm

R12 5 4 The collector width, as suggested by ACI 318 commentary, is an arbitrary width equal to half the wall width taken from the face of the wall plus the wall width

$b_{eff}\ \ \ \ell_{wall}/2 + t_{wall}$

$b_{eff}\ \ \ 90 \text{ ft}/2 + 8 \text{ in.}/(12 \text{ in./ft}) = 45\ 67 \text{ ft}$

There are several options to detail the collector reinforcement

1 Place bars within the arbitrary width of 45 67 ft. Spreading bars over more than half the diaphragm width is not practical.

2 Use the calculated chord reinforcement in the N S direction to resist the collector inertial forces in the E W direction over h 4 This option is acceptable as the required chord and collector calculated reinforcement is approximately equal, 5 in.^{2}. The collector is wider than the wall, therefore, longitudinal and transverse reinforcement must be provided to transfer forces from the collector into the wall

3 Place bars in a 2 ft 0 in deep by 12 in wide edge beam Collector and chord reinforcement are placed in a beam In this example, this option is used.

	The required reinforcement to resist the gravity dead and live load is calculated to be equal to 3.8 in.^{2}. Required collector reinforcement is 5 in.^{2} calculated above. Therefore, a total of 8.8 in.^{2} must be placed within the beam to resist gravity loads combined with either the calculated chord force or collector force due to inertial forces in the N-S and E-W directions, respectively (refer to Fig. E3.14 section A). Gravity load calculation is not provided in this example. Note: Typically, gravity design is carried out for $1.2D + 1.6L$. For seismic, the gravity loading is $(1.2 + 0.2S_{DS})D + 0.5L$, which is usually less than the previous case. Therefore, it may be possible to count on a portion of provided gravity reinforcement for seismic collectors.	
	The beam (12 in.) is wider than the wall (8 in.). Therefore, the force transferred from the beam to the wall is eccentric (2 in.). Tension and compression forces at both ends of the wall are: Required reinforcement:	$M_u = (268\text{ kip})(2\text{ in.}) = 536\text{ in.-kip}$ $T = C = \dfrac{536\text{ in.-kip}}{(89\text{ ft})(12\text{ ft})} = 0.5\text{ kip}$ $A_s = \dfrac{(0.5\text{ kip})(1000\text{ lb/kip})}{(0.9)(60{,}000\text{ psi})} = 0.01\text{ in.}^2$
	Note: The force is very small and the corresponding reinforcement is negligible. Therefore, it is assumed that the result of the eccentric force between beam and wall centerlines is resisted by the diaphragm. In case the force is large (large eccentricity, large force, or shorter wall length), reinforcement is required and placed and properly developed at both ends of the wall and extending into the diaphragm a minimum length equal to the development length of the bars.	
	Wall 4: Required collector width is less than the wall thickness. Reinforcement may be placed within the wall width.	$A_s = \dfrac{\Omega_o T_{coll}}{0.9 f_y} = \dfrac{182\text{ kip}}{0.9(60\text{ ksi})} = 3.37\text{ in.}^2$ Try four No. 9 bars $A_{s,prov} = 4\text{ in.}^2 > A_{s,req'd} = 2.94\text{ in.}^2$
	Shear friction reinforcement is not required as the collector force is already developed into the wall.	
22.9.4.4	The value of V_n across the assumed shear plane must not exceed the lesser of the following limits: (a) $0.2f_c'A_c$ (b) $(480 + 0.08f_c')A_c$ (c) $1600A_c$	$(0.2)(4000\text{ psi}) = 800\text{ psi}$ **OK** $(480 + 0.08(4000\text{ psi})) = 800\text{ psi}$ **OK** 1600 psi Therefore, V_n must not exceed $\dfrac{(800\text{ psi})(10\text{ in.})(28\text{ ft})(12\text{ in./ft})}{1000\text{ lb/kip}} = 2688\text{ kip} > V_u$
	Wall 5: Required collector width is less than the wall thickness. Reinforcement may be placed within the wall width.	$A_s = \dfrac{\Omega_o T_{coll}}{0.9 f_y} = \dfrac{197\text{ kip}}{0.9(60\text{ ksi})} = 3.65\text{ in.}^2$ Try four No. 9 bars. $A_{s,prov} = 4.0\text{ in.}^2 > A_{s,req'd} = 3.65\text{ in.}^2$
	Shear friction reinforcement is not required as the collector force is already developed into the wall.	

Step 16: Shrinkage and temperature reinforcement		
12.6.1 24.4.3.2	Shrinkage and temperature reinforcement $A_{S+T} \geq 0.0018A_g$	 $A_{S+T} = (0.0018)(10\text{ in.})(16\text{ in./ft}) = 0.288\text{ in.}^2$
24.4.3.3	Spacing of S+T reinforcement is the lesser of $5h$ and 18 in. (a) $5h = 5(15\text{ in.}) = 75\text{ in.}$ (b) 18 in. **Controls**	Note: Shrinkage and temperature reinforcement may be part of the main reinforcing bars resisting diaphragm in-plane forces and gravity loads. If provided reinforcement is not continuous (placing bottom reinforcing bars to resist positive moments at midspans and top reinforcing bars to resist negative moments at columns), the engineer must ensure continuity between top and bottom reinforcing bars by providing adequate splice lengths between them

Step 17: Reinforcement detailing		
18.12.7.7a 25.4.10.2 12.7.3.2	<u>Development</u> Chord and collector reinforcement are extended over full length and width of the edges of the diaphragm. Therefore, development length will be calculated only to determine splice lengths. Development length of shear transfer reinforcement $\ell_d = \left(\dfrac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f_c'}}\right)d_b$	See table below

ℓ_d	ℓ_d, in.	Use ℓ_d, in.
No. 5	23.4	24
No. 6	28.5	30

25.5.2.1	<u>Splices</u> Because the building lengths are longer than a standard shipping length of the No. 8 longitudinal reinforcement, splices will be needed. Use Class B splice: $1.3(\ell_d)$	$\ell_d = (1.3)\dfrac{60{,}000\text{ psi}}{25(1.0)\sqrt{4000\text{ psi}}}(1.0\text{ in.}) = 55.6\text{ in.}$ Say, 56 in. (4 ft 8 in.)
18.12.7.6	The center-to-center spacing of the longitudinal bars for collector and chords at splices and anchorage zones is but not less than 1.5 in. and concrete cover > $2.5d_b$, but not less than 2 in. Therefore, transverse reinforcement is not required.	$3d_b = 3(1.0\text{ in.}) = 3.0\text{ in.}$ minimum spacing $(2.5)(1.0\text{ in.}) = 2.5\text{ in.}$ cover
12.7.2.1	<u>Reinforcement spacing</u> Chord and collector reinforcement minimum and maximum spacing must satisfy 12.7.3.2 and 12.7.3.3	
25.2.1	Section 25.2 requires minimum spacing of (a) 1 in. (b) $4/3d_{agg}$ (c) d_b	For No. 9 bars. Minimum spacing 1.128 in., say, 1.25 in. **Controls**
18.12.7.6	Collector reinforcement spacing at splice must be at least the larger of a. At least three longitudinal d_b b. 1.5 in. c. $c_c > \max[2.5d_b, 2\text{ in.}]$	$3(1.128\text{ in.}) = 3.384\text{ in.}$, say, 3.5 in. **Controls**
12.7.2.2	Maximum spacing is the smaller of $5h$ or 18 in.	18 in. **Controls**

Step 18, Detailing

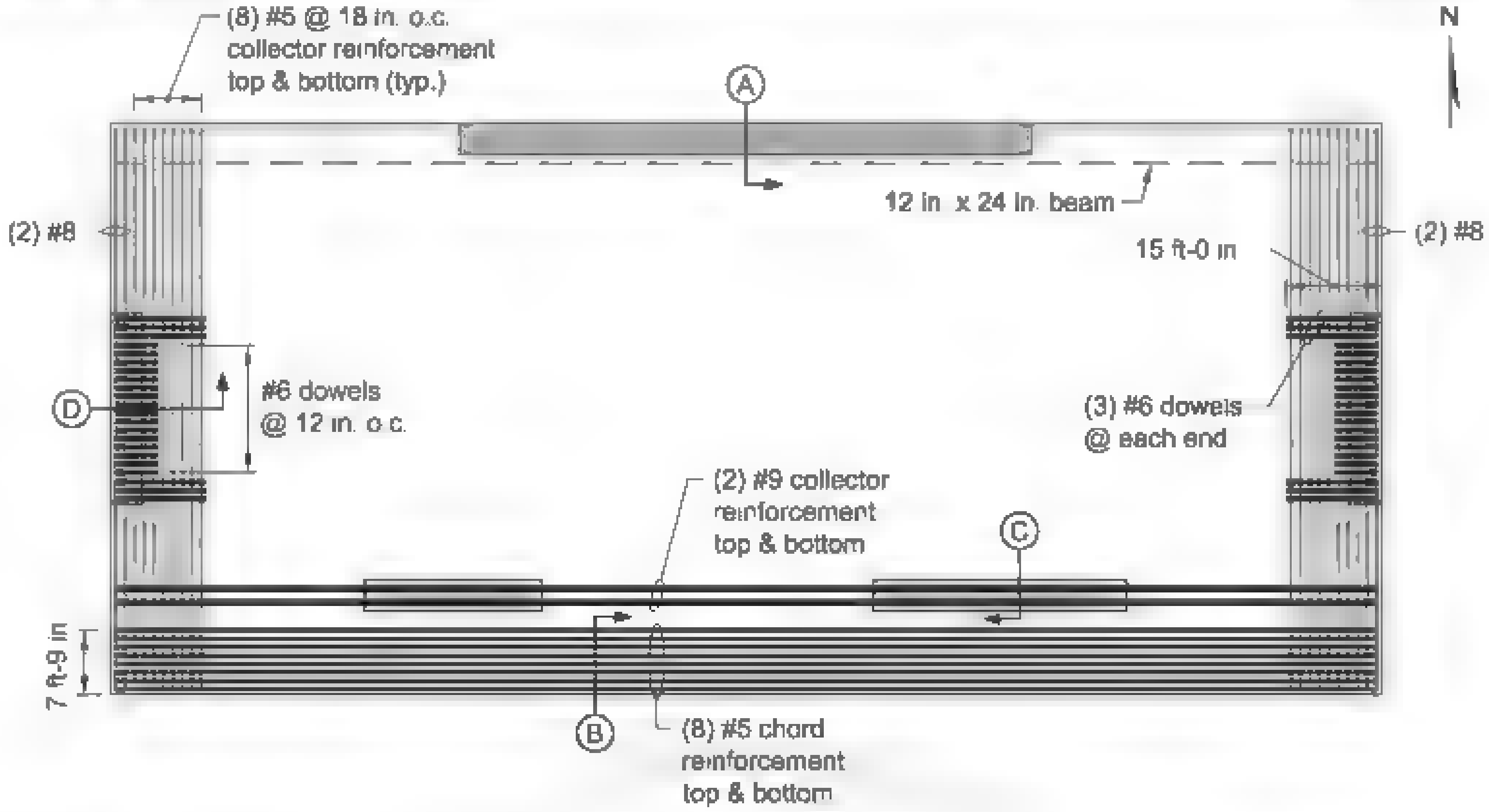
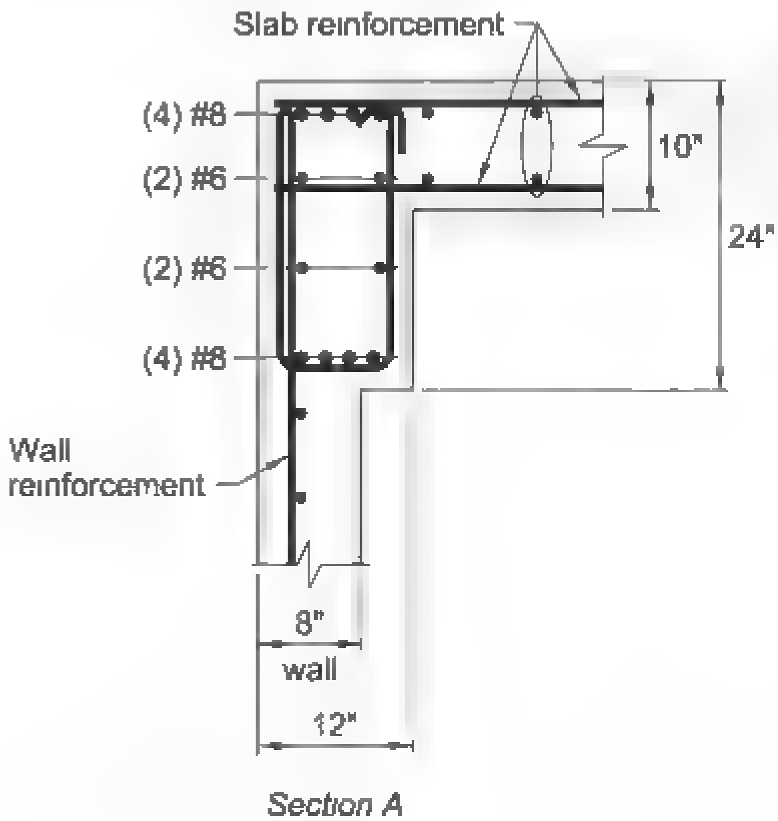
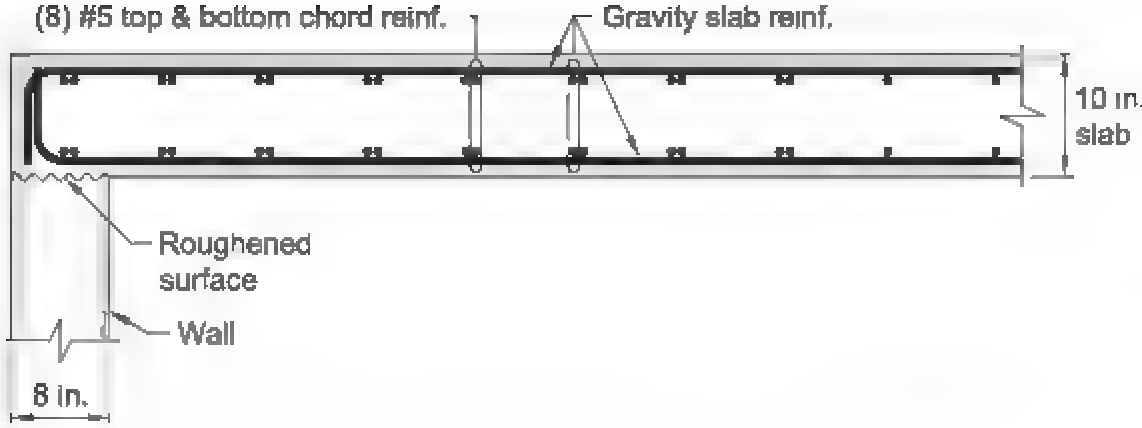


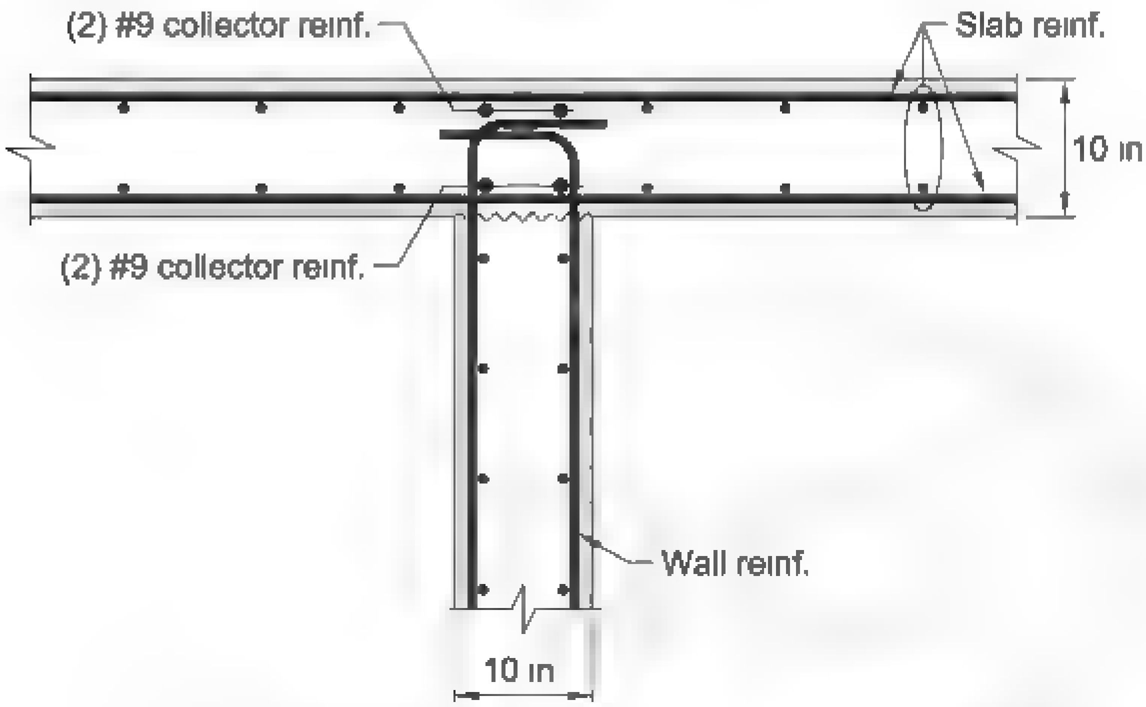
Fig. E3.14—Diaphragm reinforcement detailing



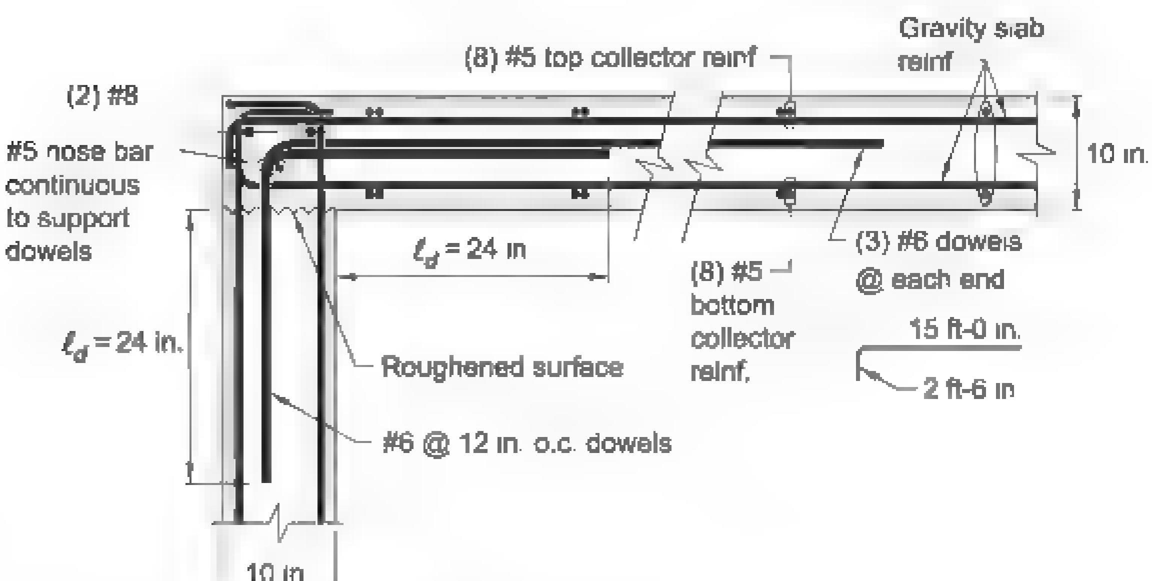
Section A—Edge beam gravity and collector or chord reinforcement



Section B—Chord/collector reinforcement at south end of diaphragm.



Section C—Collector reinforcement along Walls 4 and 5



Section D—Collector reinforcement along Walls 2 and 3

Note: Wall reinforcement not shown for clarity for Walls W4 and W5 the detail is similar, however, alternate dowels to either side of the wall

Step 19: Discussion

There is no consensus among engineers on how to distribute the diaphragm inertia force. Based on discussions with several respected engineers, the main approaches are as follows:

ASCE/SEI 7 Section 12.8.4.2, recommends shifting the center of mass by a minimum of 5 percent of the building dimension in either direction and perpendicular to the seismic loading, referred to as accidental eccentricity. This five percent torsional eccentricity is applied in addition to the calculated geometric eccentricity. However, the ASCE/SEI 7 recommendation is located in the commentary, therefore, it is not mandatory.

This example follows the ASCE/SEI 7 recommendation.

The 5 percent eccentricity is excluded in the analysis. The diaphragm inertia force is uniformly distributed to the diaphragm.

The shear forces in the lateral-force-resisting system due to the equivalent lateral force analysis are compared to the forces due to diaphragm inertia forces. The diaphragm is designed for the larger force.

When center of mass and center of rigidity do not coincide, the lateral-force-is example (W1 (27.3 kip), W4 (10.5 kip), and W5 (16.7 kip)) for a seismic force acting in the N-S direction and (W2 and W3 (22 kip)) for a seismic force acting in the E-W direction.

Drawing the moment diagram shows a discontinuity at the center of rigidity. Moehle et al. also state in *NIST Report No. GCR 10-917-4* that

"For a rectangular diaphragm of uniform mass, a trapezoidal distributed force having the same total force and centroid is then applied to the diaphragm. The resulting shears and moments are acceptable for diaphragm design. Note that this approach leaves any moment due to (shear walls perpendicular to the diaphragm inertia lateral force) unresolved; sometimes this is ignored or, alternatively, it too can be incorporated in the trapezoidal loading."

Other engineers incorporate the moment due to shear walls perpendicular to the diaphragm inertia lateral force. This results in discontinuity (jump) in the moment diagram as shown in Fig. E3.15. Equilibrium in the system is obtained by drawing the moment diagram due to the shear forces in the shear walls perpendicular to the direction of the seismic force (Fig. E3.15).

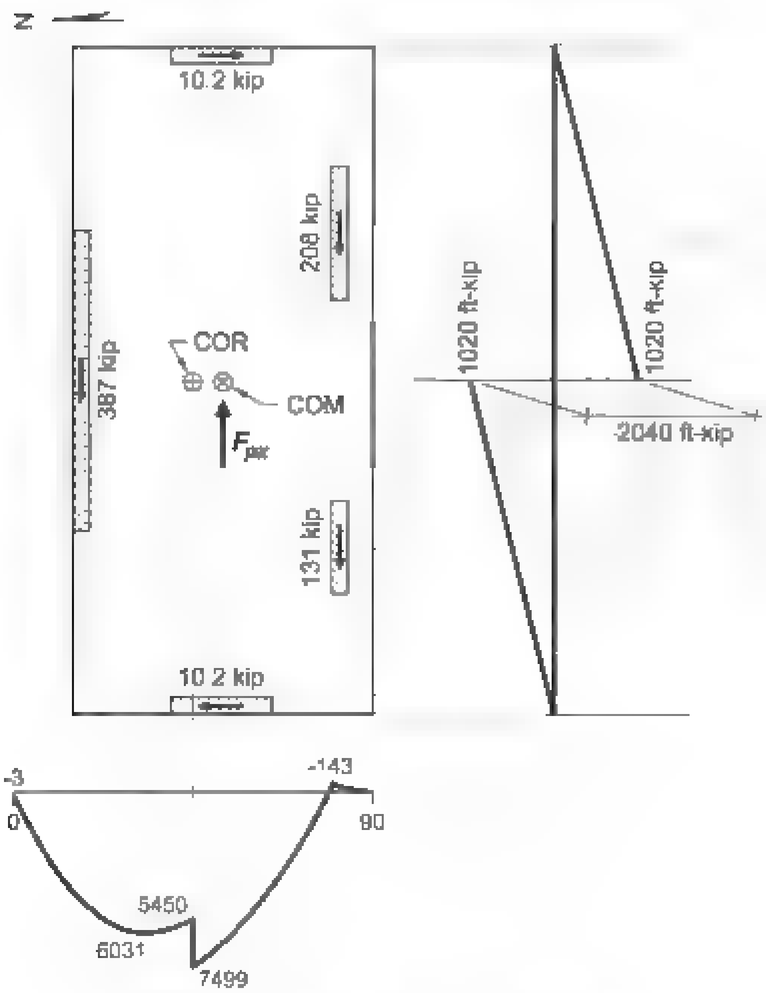


Fig. E3.15—Shear and bending moment diagrams.

In this example, the moment diagram is constructed by incorporating the shear force in the trapezoidal loading for the construction of the moment diagram (Fig. E13.6). Since Case I resulted in a slightly higher moment, the calculations for that case follow.

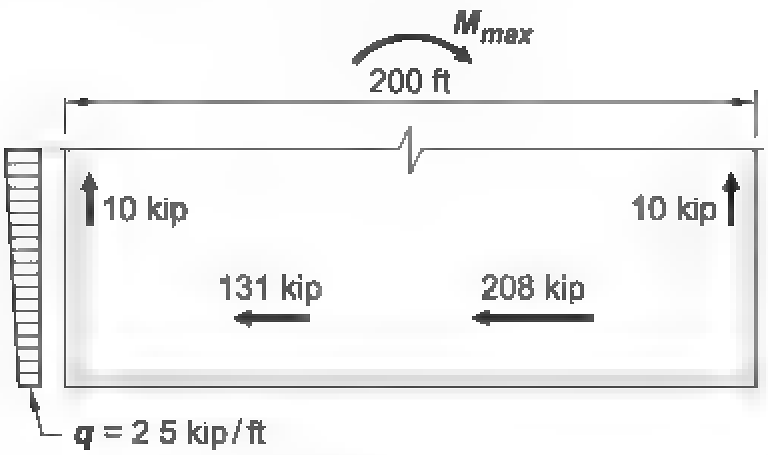


Fig. E3.16—Free body diagram.

$M_x = (131\text{ kip} + 208\text{ kip})(x - 10\text{ ft}) - (2.5\text{ kip/ft})(x^2/3) - [2.5\text{ kip/ft} + (13.6\text{ kip/ft} - 2.5\text{ kip/ft})/60\text{ ft} \times x](x^2/6)$

CHAPTER 9—COLUMNS

9.1—Introduction

The column chapter, Code Chapter 10, follows the organization of the other member chapters: applicability; initial data, analysis to determine the required strength, design area of reinforcement needed to exceed the required strength, check against minimum reinforcement required, and detailing. The analysis must be consistent with Code Chapters 4 through 6. Code Sections 10.1 through 10.4 remind the designer of limits and rules for columns that should be addressed in their analysis model. The requirements for column design start in Code Section 10.5.

A column is always part of the gravity-force-resisting system, and in cast-in-place construction is often part of the lateral force-resisting systems (LFRS). The most common lateral design forces are seismic and wind. For buildings designed to resist seismic forces, the requirements of Code Chapter 10 apply, along with the additional seismic requirements in Code Chapter 18 for columns that are part of an ordinary, intermediate, or special moment frame system. There are also seismic requirements for columns that are not part of an LFRS. The seismic requirements are intended to increase column ductility to accommodate the large displacements that are expected during a maximum design earthquake. For buildings that resist only wind forces, columns are designed by Code Chapter 10 whether they are designated as part of an LFRS or not. There are no additional requirements for wind forces.

This Manual provides some explanation of Code requirements and how they impact a column's design and detailing. A review of basic engineering principles is provided so that a designer can effectively design columns by hand or computer using only this Manual.

9.2—General

The provisions of Code Chapter 10 apply to the design of non prestressed, prestressed, and composite columns. Headings of a section are considered part of the code and care should be taken to notice when a heading limits the following requirements to a particular type of column. The word "non-prestressed" is typically in reference to cast-in-place columns and "prestressed" with precast columns. While ACI 318 covers post-tensioned columns, they are not commonly used. Code Chapter 14 covers the design of plain concrete pedestals.

9.3—Design limits

9.3.1 *General*—Concrete columns offer architects an opportunity to create various cross-sectional shapes for aesthetic purposes. Unusual cross-sectional shapes, however, are more difficult to analyze. Section 10.3 in ACI 318 permits the designer to use an effective cross-sectional area for various situations that allow for a simpler analysis. For example, Section 10.3.1.1 permits the use of an idealized circular section within the outline of the actual section (Fig. 9.3.1). The key point is that using a portion of an oversized column or wall that is easier to analyze is permitted.

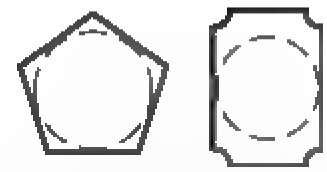


Fig. 9.3.1—Permitted cross section for analysis.

Code Section 10.3.1.2 allows an oversized column to be designed with a smaller effective area, with a lower limit of one-half the total area. The analysis and design assume the smaller effective area, but the column is detailed considering the actual cross section. Note that the minimum area of steel (refer to 9.6 of this Manual) is $0.01A_g$ based on the smaller effective area, but A_g cannot be less than half the area of the actual cross section. Note that the shape of the column can be dictated by the building architecture but it must always meet the requirements of ACI 318.

9.3.2 *Initial sizing*—It is not economical to have a unique design for each column in the building. The following guidelines help in economical column construction.

1. Reuse formwork as much as possible. It is common to use only three or four column sizes for the entire building.

2. Use the same strength of concrete for all columns at a level. Structures under six stories or less commonly use one concrete strength for the full height of the building. Code Section 15.5 has additional design requirements at the floor joints if the concrete strength in the floor system is less than 0.7 times the concrete strength of the column.

3. Proportion the column cross sections and concrete strengths so that reinforcement ratios are in the range of 1 to 2 percent.

4. If higher axial strength is needed, increasing the strength of concrete is usually more efficient than increasing reinforcement area.

The analysis and design of columns is an iterative process. To begin, the designer assumes sectional properties in order to perform an analysis. An initial column area, A_g, can be estimated by dividing the maximum factored axial load by $0.4f_c'$ for ordinary columns or $0.3f_c'$ for columns in high seismic areas. Columns are usually rectangular, square, or round. For the first iteration, a 1 percent reinforcement ratio is evenly distributed around the column perimeter. An effective moment of inertia, I_{eff}, of concrete members is used in the analysis to account for cracking at the nominal condition. The simple I_{eff} values in Table 6.6.3.1.1(a) in ACI 318 are generally used and the cross section properties are assumed to be uniform for the length of the member. With these assumptions, an initial analysis is run and, subsequently, section properties or reinforcement area are adjusted as necessary.

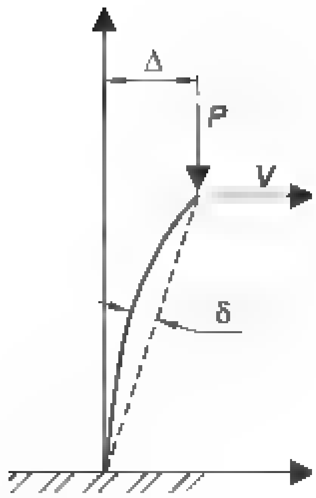


Fig. 9.4.1a—P-Δ effects.

9.4—Required strength

9.4.1 *General*—The required strength is calculated using the factored load combinations in Chapter 5 and analysis procedures in Chapter 6 of the Code. Three methods of analysis: 1) linear elastic first-order analysis, 2) linear elastic second-order analysis, and 3) inelastic analysis are permitted as discussed in Chapter 3 of this Manual. Regardless of the method chosen, all columns must be checked for slenderness.

Slenderness effects are associated with higher slenderness ratios, $k\ell_u/r$. Higher slenderness ratios occur for long columns, columns with a small cross-sectional dimension, or columns with limited end restraint. As a column becomes more slender, larger lateral deformations and deformation along the length occur due to the applied load. The column must support additional moment created by the column axial load acting on the deformed column, also known as second-order moments. There are two types of second-order moments related to the *P*-Δ effects shown in Fig. 9.4.1a: 1) second-order moments due to translation of the column ends (*P*-Δ), and 2) second-order moments due to deflection along the member (*P*-Δ). These second-order moments gradually increase due to this geometric nonlinearity until the column stabilizes. If the slenderness ratio is very high, the column becomes unstable and cannot resist the axial load, refer to Fig. 9.4.1b.

9.4.2 *Slenderness concepts*—A column's degree of slenderness is expressed in terms of its slenderness ratio, $k\ell_u/r$, where ℓ_u is unsupported column length, k is effective length factor reflecting end restraint and lateral bracing conditions, and r is the radius of gyration, reflecting the size and shape of a column cross section.

The column's unsupported length ℓ_u is the clear distance between the underside of the beam, slab, or column capital above, and the top of the floor below. The unsupported length may be different in two orthogonal directions depending on the building geometry. Figure 9.4.2a shows different framing conditions and corresponding unsupported lengths (ℓ_u). Each coordinate and x and y subscript in the figure indicates the plane of the frame in which stability of the column is investigated.

The effective length factor k reflects the column's end restraint and lateral bracing conditions as shown in

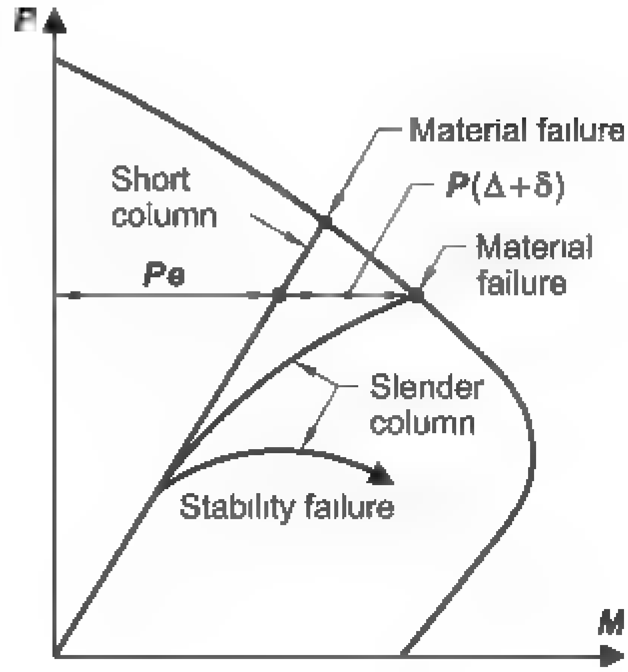


Fig. 9.4.1b—Effects of slenderness on a column.

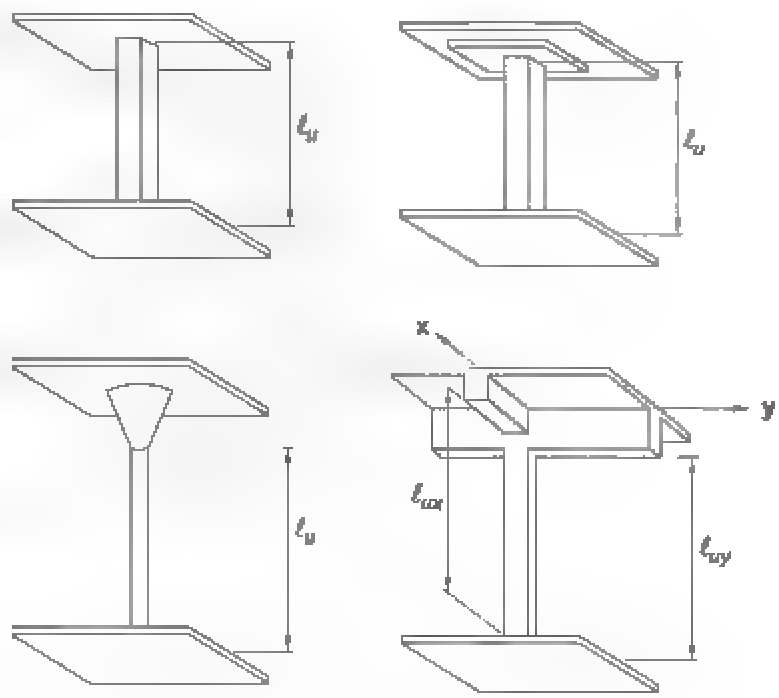


Fig. 9.4.2a—Unsupported column length ℓ_u.

Fig. 9.4.2b. The factor k varies between 0.5 and 1.0 for laterally braced columns, and between 1.0 and ∞ for unbraced columns. Most columns have end restraints that are neither perfectly hinged nor fully fixed. The degree of end restraint depends on the floor stiffness relative to the column stiffness. Jackson and Moreland alignment charts, given in Fig. R6.2.5.1 in the Code, can be used to determine the factor k for different values of relative stiffness at column ends. The stiffness ratios ψ_A and ψ_B used in the charts should reflect concrete cracking, and the effects of sustained loading. Beams and slabs are flexure-dominant members and may crack significantly more than columns, which are compression-dominant members. The reduced moment of inertia values given in Code Section 6.6.3.1.1 should be used to determine k. Tables D1.1 through D1.5 in the supplement to this Manual, ACI Reinforced Concrete Design Handbook Design

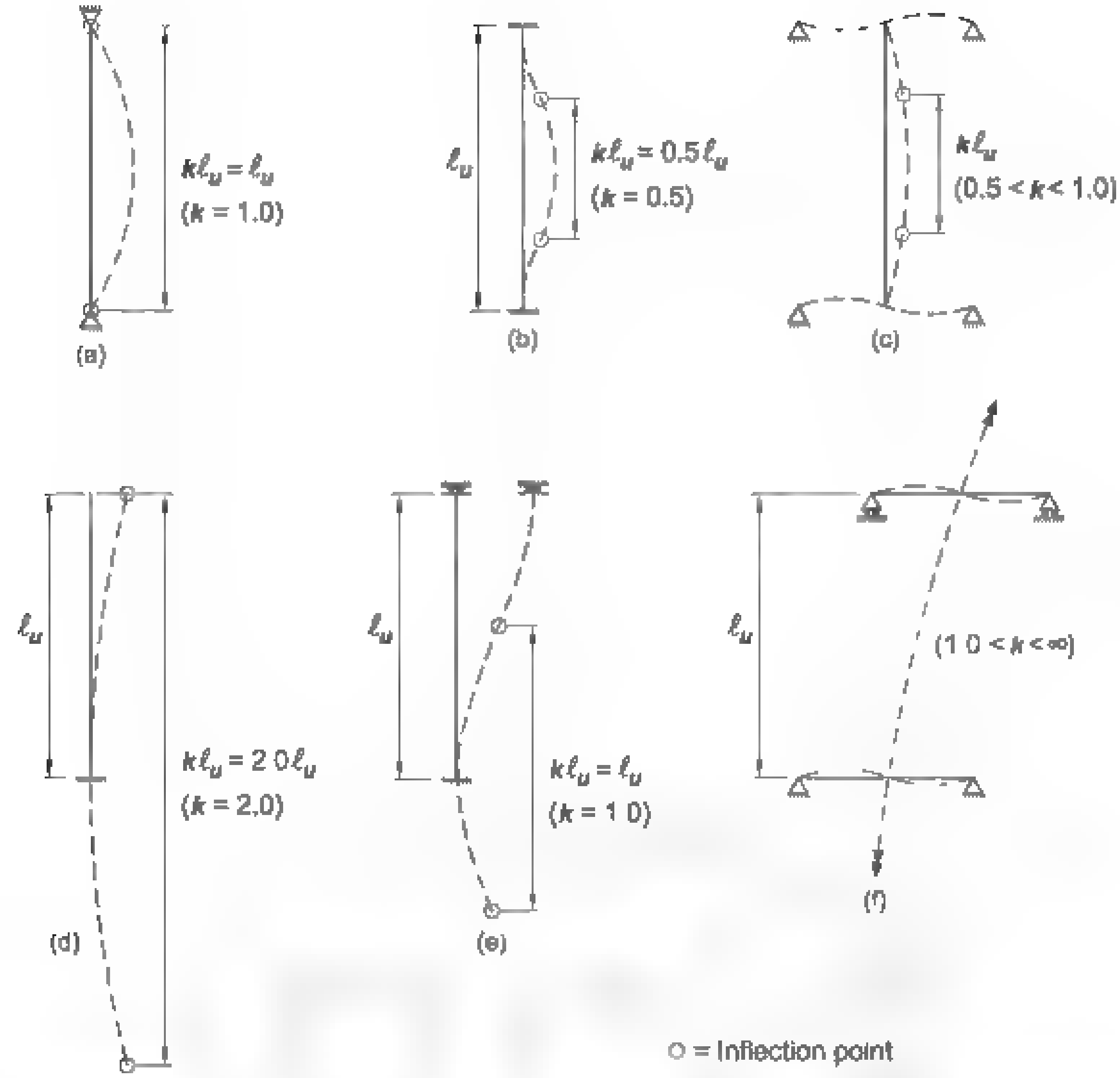


Fig. 9.4.2b—Effective length factor k for columns. (a), (b), and (c) are for nonsway frames, and (d), (e), and (f) are for sway frames.

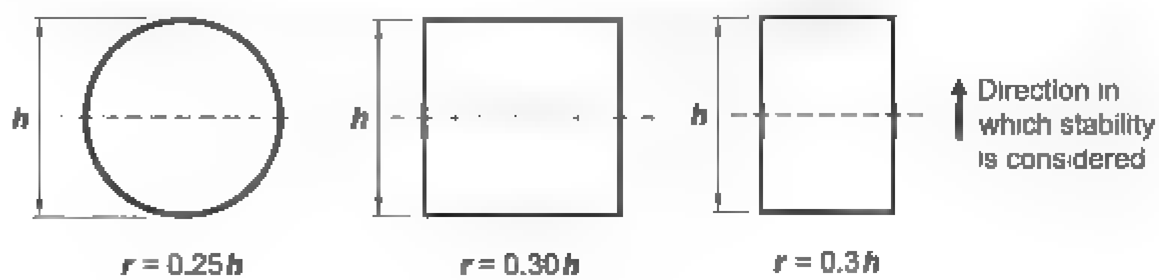


Fig. 9.4.2c—Radius of gyration for circular, square, and rectangular sections.

Aid—Analysis Tables, provide design aids for the calculation of the effective length factor, stiffness, and moment of inertia.

The radius of gyration introduces the effects of cross-section size and shape to slenderness. A section with a higher moment of inertia per unit area produces a lower slenderness ratio and thus a more stable column. The radius of gyration, r, is defined in Section 6.2.5.2 of ACI 318 and shown in the following Eq. (9.4.2).

$$r = \sqrt{\frac{I_g}{A_g}} \qquad (9.4.2)$$

It is permissible to use $r = 0.3h$ for square and rectangular sections, and $r = 0.25h$ for circular sections, where h is the dimension in the direction stability is being considered. This is shown in Fig. 9.4.2c.

9.4.3 *Linear elastic first-order analysis*—For a linear elastic first-order analysis, Code Section 6.6.4 provides a moment magnification method which conservatively accounts for slenderness. This method is shown in Example 9.1 of this Manual. In a linear elastic first-order analysis, the building frame is analyzed once and results are used for input to the moment magnification method.

9.4.3.1 *Sway or nonsway frames*—The designer needs to determine if the column is in a sway or nonsway frame. A frame is nonsway if it is sufficiently supported by lateral bracing, such as structural walls. Structural walls used for

elevator shafts, stairwells, partial building enclosures, or interior stiffening elements provide substantial drift control and lateral bracing. In many cases, even a few structural walls can brace a multi-story, multi-bay building. Sway must be checked for each direction and floor. The Code provides three methods to determine if the lateral stiffness is sufficient to designate the frame as nonsway:

1. Section 6.2.5.1: Columns are nonsway if the gross lateral stiffness of the walls (bracing elements) in a story is at least 12 times the gross lateral stiffness of the columns in that story in the direction considered. This is a simple, conservative hand calculation.

2. Section 6.6.4.3(a): Columns are nonsway if the increase in column end moments due to second-order effects does not exceed 5 percent of the first-order end moments.

3. Section 6.6.4.3(b): Columns are nonsway if the stability index Q does not exceed 0.05 as shown in Eq. (6.6.4.4.1).

$$Q = \frac{\sum P_u \Delta_o}{V_{us} \ell_c} \leq 0.05 \quad (6.6.4.4.1)$$

where $\sum P_u$ is total factored axial load acting on all the columns in a story, V_{us} is total factored story shear, and Δ_o is lateral story drift (deflection of the top of the story relative to the bottom of that story) due to V_{us}. Story drift Δ_o should be computed using section properties taking into account the presence of cracked regions along the member; refer to Code Section 6.6.3.1.

9.4.3.2 *Column slenderness*—The moment magnification method states that for columns in sway or nonsway frames, secondary effects may be neglected if the slenderness ratios are below the limits given in Section 6.5.2.1 in the Code. If these limits are exceeded in a nonsway frame, then second-order effects due to translation (P-Δ) may be ignored, but the second-order effects along the member (P-δ) given in Code Section 6.6.4.5 need to be considered. If the slenderness ratio limits are exceeded in a sway frame, column (P-Δ) and (P-δ) effects are to be considered. Section 6.6.4.6 in the Code is completed first to calculate the amplified end moments due to translation (P-Δ). These modified moments are then used in Code Section 6.6.4.5 to calculate the second-order effects along the member (P-δ).

9.4.4 *Linear elastic second-order analysis*—Many software analysis programs can directly compute the following second-order effects:

(a) Second-order moments (P-Δ) due to the laterally deflected structure

(b) Second-order moments (P-δ) due to deflection along the length of the column

For second-order effects to be included in the analysis, the structure must be reanalyzed after the initial application of load using the deformed geometry rather than the original geometry. The program must be capable of the iterative calculations so that when the final deformations are determined, global equilibrium is also satisfied. The results using this analysis technique will include both first- and second-order load effects, including second-order moments due to sidesway (P-Δ). Member second-order effects (P-δ) can be included by dividing the column into multiple elements to calculate a deflection along its length. Frosch (2011) suggests a method to check a computer program for this capability in ACI Q&A, "Using an Elastic Frame Model for Column Slenderness Calculations." Because of the iterative nature of the analysis technique, the principle of superposition does not apply for calculating the second-order moments. Consequently, applied loads must be factored and combined before use in conducting the analysis.

Code Section 6.7.1.1 states, "A linear elastic second-order analysis shall consider the influence of axial loads, presence of cracked regions along the length of the member, and effects of load duration." The moment magnification method in Section 6.6.4 in the Code accounts for these properties by using stiffness reduction factors, ϕ_K. The method uses two different factors for the two different types of slenderness effects, P-Δ and P-δ; refer to R6.6.3.1.1, R6.6.4.5.2, and R6.6.4.6.2 in the Code. A lower stiffness reduction factor is required for P-δ compared to P-Δ. Computer programs should also account for stiffness reduction in its second-order analysis. These programs should be reviewed for how they account for stiffness reduction.

9.5—Design strength

The majority of reinforced concrete columns are designed to resist flexure, axial force, and shear. Figure 9.5 illustrates strains and stresses in a typical column section subjected to combined moment and axial compression. As can be seen, different combinations of moment and accompanying axial force result in different column nominal strengths and corresponding strain profiles, while also affecting the tension- or compression-controlled behavior. Moment and axial strengths have traditionally been combined into a column interaction diagram for use in design. Interaction diagrams are constructed by computing moment and axial force nominal strengths for different strain profiles using the following equilibrium equations:

$$P_n = C_c + C_{s'} + C_{s2} - T_s \quad (9.5a)$$

$$M_n = C_c x_2 + C_{s'} x_1 + C_{s2(0)} + T_s x_3 \quad (9.5b)$$

As the strains vary using Eq. (9.5a) and (9.5b) from pure compression to pure bending, a nominal strength curve can be created as shown by the outer curve in Fig. R10.4.2.1 in the Code. The nominal strength is adjusted to the design strength by multiplying by the appropriate ϕ factors. The ϕ factor for compression-controlled sections is 0.75 for spirals and 0.65 for other tie configurations. The ϕ factor for all tension-controlled sections is 0.9. This factor varies linearly from 0.65 or 0.75 to 0.9 through the transition zone shown in Fig. 9.5. The design strength curve is shown by the inner curve in Fig. R10.4.2.1 in the Code.

An electronic spreadsheet is provided as a supplement to this Manual to demonstrate how to make an interaction diagram (www.concrete.org/MNL1721Download2). The key points of the diagram are: pure compression (zero

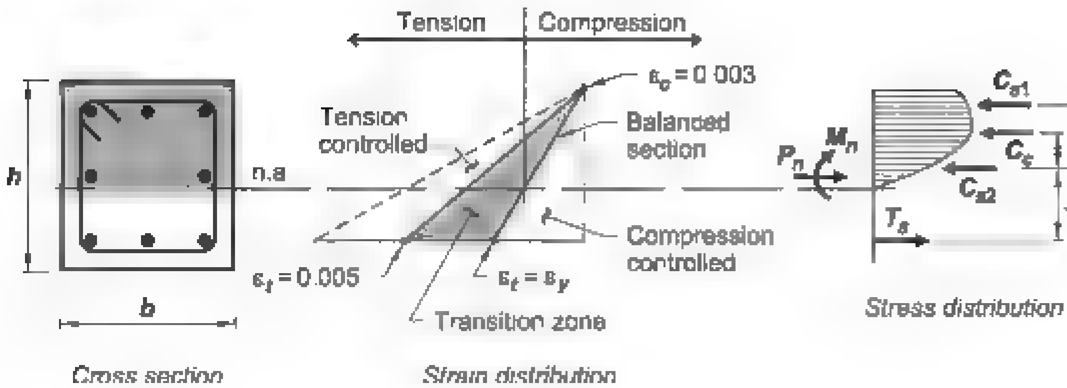


Fig. 9.5—Column section analysis.

moment), pure tension (zero moment), pure bending (zero axial force), extreme tensile reinforcement stress of $0.0f_y$, $0.5f_y$, $1.0f_y$ (balanced point), and maximum usable concrete compressive strain, and the concrete reaches it maximum usable strain.

The previous version of this Manual contained an extensive series of interaction diagrams. Although there are many software programs available for the design of columns, some of the design interaction diagrams have been retained in a supplement to this Manual titled, Reinforced Concrete Design Handbook Design Aid — Analysis Tables. A brief description of how the diagrams are made is given in the supplement along with information on biaxial moments.

Because shear design in columns is similar to beams, review Beam Chapter 8 of this Manual for this information. Significant torsion rarely occurs in columns and, therefore, is not specifically addressed in the columns chapter. For column torsion, the beam chapter should be reviewed.

9.6—Reinforcement limits

The minimum column vertical reinforcement ratio is $0.01A_g$. This amount is enough to keep the reinforcement from yielding due to concrete creep under sustained service loads (Code Section R10.6.1.1). The maximum reinforcement ratio is $0.08A_g$. This amount is approximately the maximum that can be realistically provided at the perimeter of a concrete section that would also meet the minimum cover and spacing requirements. This percentage was set in ACI 318-63 when butt splices were common. For present-day construction, lap splices are more common and for many projects, bars are spliced at the bottom of the column starting at the floor. In this case, the maximum percent of reinforcement is 4 percent because the maximum percentage of reinforcement at a section is 8 percent. If the lap splices are staggered, the maximum percent of reinforcement can increase up to 6 percent.

As stated earlier, usual reinforcing ratios are in the range of 1 to 2 percent. There are cases where more reinforcement is necessary, such as to meet a required strength. If the column design routinely requires reinforcement in the 3 percent range, the designer should consider a different cross section or higher-strength concrete due to the difficulty in fabricating the reinforcement cage and placing concrete.

9.7—Reinforcement detailing

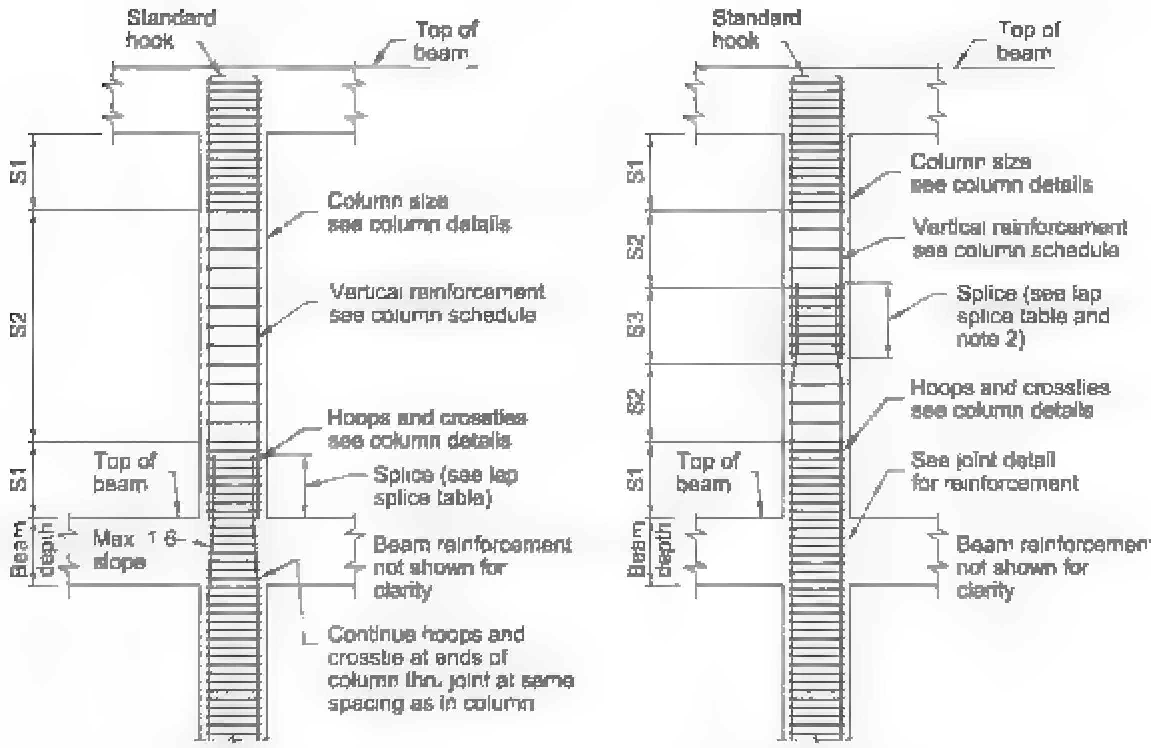
9.7.1 *General*—Section 10.5 in the Code focuses on the calculation of reinforcement area needed to resist design forces and moments. Section 10.6 in the Code provides minimum column reinforcement area. Section 10.7 in the Code provides limitations on the location, spacing, and splicing of longitudinal reinforcement and the location, spacing, geometry, and type of transverse reinforcement. General requirements such as concrete cover, development length, and splice lengths are covered in Code Chapters 20 and 25 for all members. Note that many detailing provisions of this chapter are related to how columns are constructed.

9.7.2 *Longitudinal bars*

9.7.2.1 *Spacing*—The minimum number of bars in a column is given in Code Section 10.7.3.1. Square and rectangular columns must have a minimum of four bars. Circular columns must have six if spiral ties are used. Eight bars are suggested by the Commentary, however, to ensure that the design moment is achieved regardless of the position of reinforcement in the field. The minimum bar spacing is given in Section 25.2 in the Code. A maximum spacing requirement in Code Chapter 10 is not explicitly given. Section 18.7.5.2(e) in the Code, however, states that for columns in a special moment frame, the spacing of longitudinal bars laterally supported by the corner of a crosstie or hoop leg, h_x, shall not exceed 14 in. around the perimeter of the column. Note that this spacing is further reduced to 8 in. for conditions given in Section 18.7.5.2(f).

Typically, bars are evenly spaced around the perimeter, as this helps to create a more stable column cage during construction. Designers will often place bars only at the corners of square or rectangular columns to reduce the field work necessary to place ties. Bundled bars are often needed to meet the required area of steel for this arrangement. Note that Code Section 18.7.5.2 makes this practice impractical for columns in special moment frames due to the 14 in. limitation. If confinement of the concrete core is necessary or desired, evenly spaced longitudinal bars are helpful. If more bars are required to resist flexure at a particular location, some bars can be added to the evenly spaced column bars.

9.7.2.2 *Splicing*—Splice locations, lengths, and types should be included on the structural drawings. Lap splices are the most common splice type due to their ease of fabrication and construction. Mechanical connectors and butt-welded splices are helpful where bar arrangement becomes



Bottom column splice *Mid-height column splice*

Fig. 9.7.2.2a—Column splice locations (Fanella 2007).

congested but they can require additional erection time. End-bearing splices are not common today but are sometimes used in bundled bar arrangements that are only in compression under all load combinations. They are not permitted in high seismic applications. Column splices are usually located at the bottom of columns at each floor if a mid-height splice is not required. Lap splices for columns in buildings assigned to SDC D, E, or F are required to be located in the center half of the column length according to Sections 18.7.4.4 in the Code. Bottom and mid-height column splice locations are illustrated in Fig. 9.7.2.2a.

Lap splices for large bars may extend to one-half the story height, where it may be more economical to lap-splice the bars every other floor (Concrete Reinforcing Steel Institute [CRSI] 2011). The length of lap splices should be noted on the structural drawings. Lap splices vary with the bar diameter, concrete strength, bar spacing, concrete cover, position of the bar, distance from other bars, and if the bar is in tension or compression. Lap splices are not permitted for No. 14 and 18 bars, except for transferring compression (only) to a footing with dowels (Code Section 16.3.5.4).

To maintain bars in the corners of a rectangular column, longitudinal bars that are lap-spliced are usually offset bent into the column above, whether there is a change in column size or not. Circular columns typically need not be offset bent where column size does not change. The slope of the inclined portion of an offset bent bar should not exceed one in six (Code Section 10.7.4.1). Additional ties are required along offset bent bars and are placed not more than 6 in. from the point of the bend (Code Section 10.7.6.4). Typically, three closely spaced ties are sufficient to resist the lateral force created by the bend and one of the ties may be part of the regularly spaced ties. Separate splice bars and more ties may be necessary where the column section changes 3 in. or more. Examples of offset bent splices are illustrated in Fig. 9.7.2.2b. Where there is a reduction of reinforcement, longitudinal bars from the column below are typically terminated within 3 in. of the top of the finished floor (ACI 315R-18), unless design requires otherwise. Column bar area in the column above must be extended from the column below to lap bars above.

Bundled bars are typically groups of larger bars that span two stories. Lap and end bearing splices in bundled bars require staggering the individual bars. For this reason, bundled bar preassembly is more complicated and can create additional erection time to place longitudinal and transverse bars on the freestanding cage.

9.7.3 *Transverse bars*

9.7.3.1 *Column ties*—Standard (nonseismic) arrangements of ties for various numbers of vertical bars are shown in Fig. 9.7.3.1. The one- and two-piece tie arrangements shown provide maximum rigidity for column cages preassembled on the site before erection. The spacing of ties depends on the sizes of longitudinal bars, columns, and of ties. The maximum spacing of ties required for shear is shown in Table 10.7.6.5.2 in the Code.

9.7.3.2 *Spirals*—Spirals are used primarily for circular columns, piers, and caissons. Spiral reinforcement can be plain or deformed bars or wire. The term "spiral" used in the Code is more than a geometric description of a circular tie. It defines the required pitch, reinforcement amount, splicing, and termination, which are listed in Section 25.7.3 in the Code. A continuously wound bar or wire not meeting all of the requirements of 25.7.3 is simply a continuous circular tie. The spiral pitch is between 1 and 3 in., inclusive, and is typically given in 1/4 in. increments. The spiral size and pitch should meet the volumetric reinforcement ratio, ρ_s (Eq. (25.7.3.3) in the Code). The continuation of spirals into floor joints is according to Table 10.7.6.3.2. The minimum diameters to which standard spirals can be formed is given in Table 9.7.3.2 (ACI 315R-18).

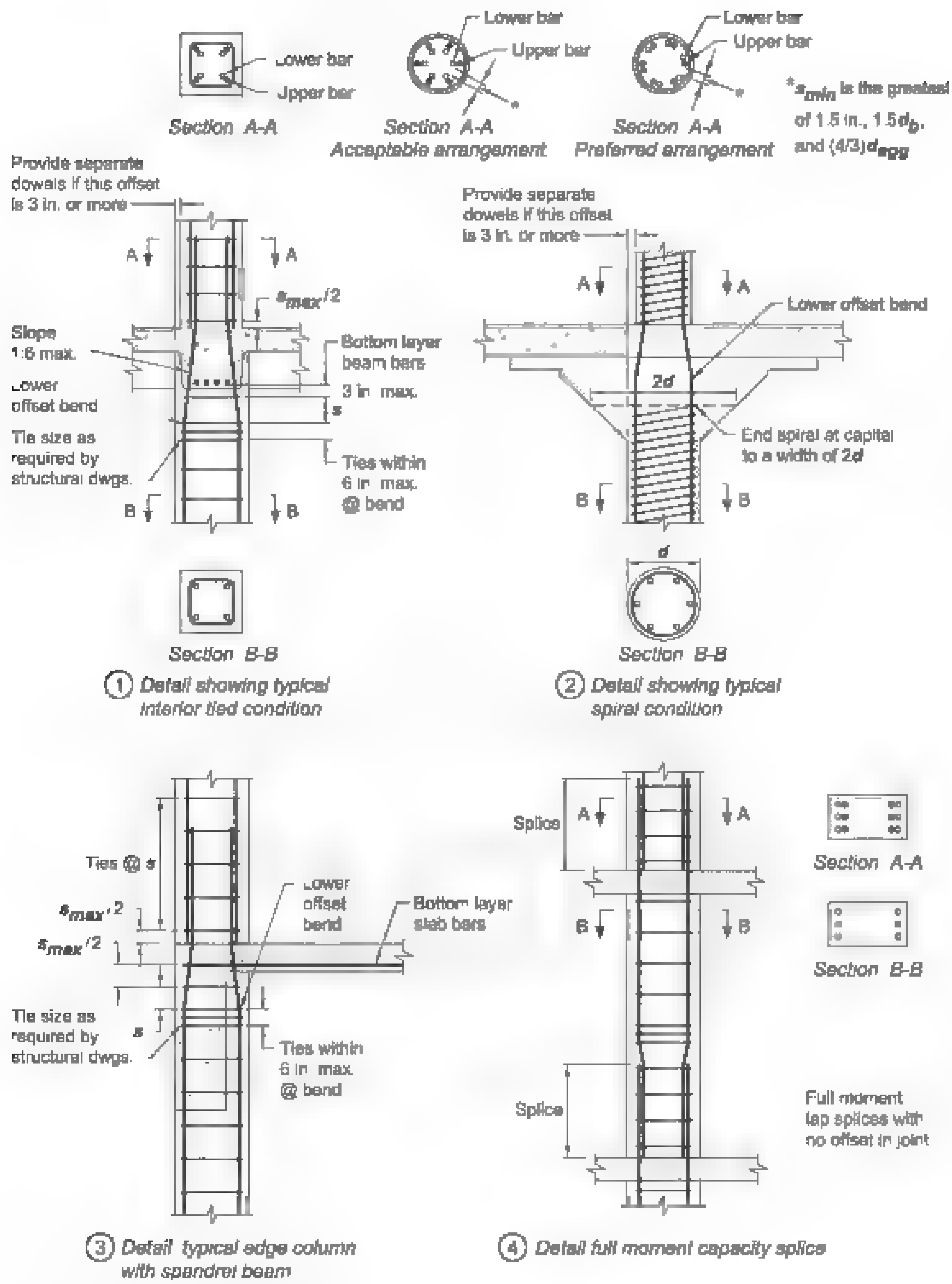


Fig. 9.7.2.2b—Column splice details (ACI 315-99).

Table 9.7.3.2—Minimum diameters of spiral reinforcement

Spiral bar diameter, in.	Minimum outside diameter that can be formed, in.
3/8	9
1/2	12
5/8	15
3/4	30

9.8—Design steps

1. Determine an initial size of the column and amount of reinforcement

(a) Ordinary $A_g = P_{u,max}/0.4f_c'$; High seismic $A_g = P_{u,max}/0.3f_c'$

(b) Shape of column is often dictated by the architect, otherwise, square or round columns are common first estimates

(c) Reinforcement ratios are typically 1 to 2 percent

2. Run initial analysis to detrmine column loads

(a) If second-order effects are accounted by the computer program, go to Step 3

(b) If second-order effects are not accounted by the computer program, use the Moment Magnification method in Section 6.6.4 in ACI 318-14 to calculate these effects

3. Create a moment interaction diagram using an electronic spreadsheet or commercial software. Check the required moment and axial load strengths against the design strength curve.

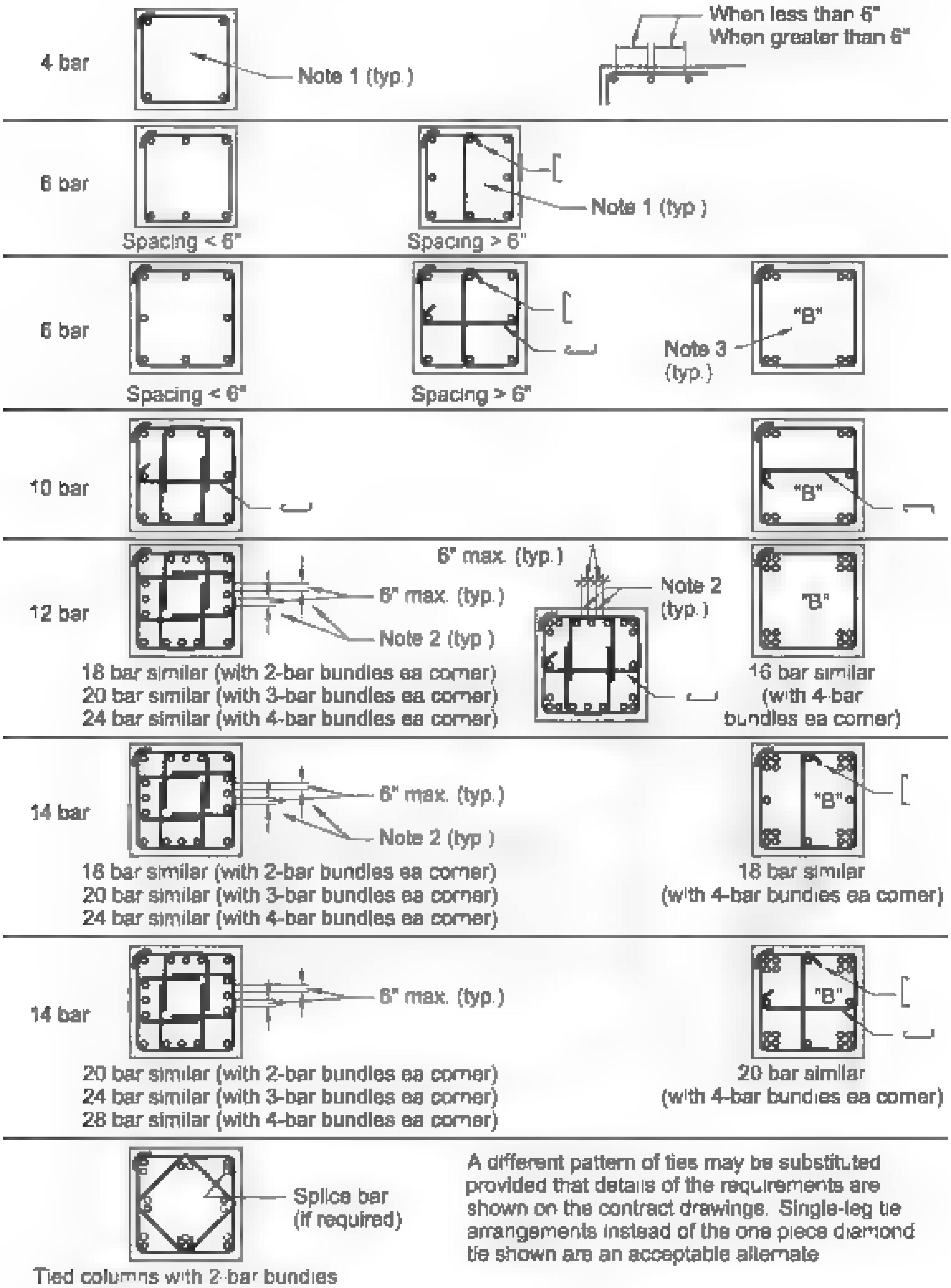


Fig. 9.7.3.1—Standard column ties (ACI 315-99).

4. Make adjustments as necessary to the initial column size and reinforcement. Rerun the analysis, if necessary, until design strength is greater than required strength for all load cases.
5. Check shear strength and minimum shear reinforcement requirements.
6. Detail column longitudinal and transverse requirements showing all bar locations, spacing, splices, and bar terminations. It is common to use typical column details and sections along with a column schedule table.

REFERENCES

American Concrete Institute (ACI)

ACI 315-99—Details and Detailing of Concrete Reinforcement

ACI 318-63—Building Code Requirements for Structural Concrete and Commentary

ACI 318-14—Building Code Requirements for Structural Concrete and Commentary

ACI MNL-17DA-21—Reinforced Concrete Design Handbook Design Aid—Analysis Tables. https://www.concrete.org/MNL1721Download1

ACI MNL-17DAE-21—Interaction Diagram Excel spreadsheet, https://www.concrete.org/MNL1721Download2

9.9—Examples

Columns Example 1: *Column analysis*

Analyze first floor interior column in one direction at location E4, from the example building given in Chapter 1 of this Manual. The moment magnification method is used with a first-order analysis. The column's factored forces and moments are from a first-order frame analysis using hand calculations. The building was also analyzed by first-order and second-order linear elastic methods using commercial software for comparison purposes.

The factored moments are from an analysis of the moment frame along Grid E. Three common controlling load combinations are considered.

Given:

Materials—

Specified concrete compressive strength, $f_c' = 5$ ksi

Specified yield strength, $f_y = 60$ ksi

Modulus of elasticity of concrete, $E_c = 4030$ ksi

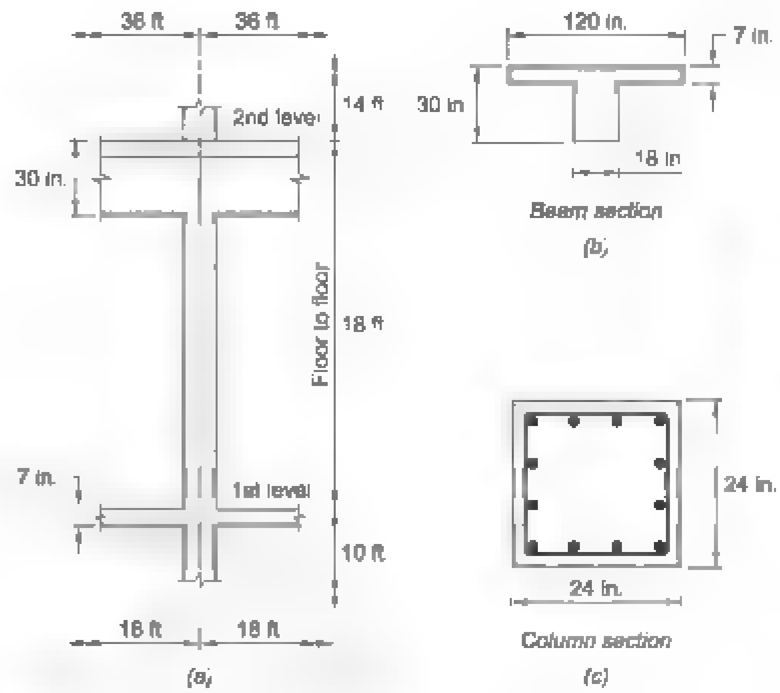


Fig. E1.1—Column geometry and configuration.

Loading—

Loads considered	Dead + live + snow	Dead + wind + live + snow		Dead + EQ + live + snow	
Load Combination	(i) $U = 1.2D + 1.6L + 0.5S$	(ii) $U = 1.2D + 1.0W + 1.0L + 0.5S$		(iii) $U^* = 1.2D + 1.0E + 1.0L + 0.2S$	
Load breakout	$1.2D + 1.6L + 0.5S$	$1.2D + 1.0L + 0.5S$	$1.0W$	$1.2D + 1.0L + 0.2S$	$1.0E$
V_u, kip	0	0	6	0	22
P_u, kip	867	777	0	789	0
$(M_u)_{top}$, kip-in.	24	12	±418	12	±1740
$(M_u)_{bot}$, kip-in.	36	24	±650	24	±2328

*The load factor on D in Load Combination (iii) is increased as required by ASCE/SEI 7 by $0.2S_{DS}$. Note that $\rho = 1.0$ for buildings in SDC B.

Reference: MNL-17 Supplement, Reinforced Concrete Design Handbook Design Aid – Analysis Tables, found at https://www.concrete.org/MNL1721Download1

ACI 318	Discussion	Calculation
Step 1: Determine initial column size		
	Estimate the maximum load at the interior column, E4, using Load combination (i)	
5.3.1	$U = 1.2D + 1.6L + 0.5S$	$P_u = 867$ kip
	Estimate the size of a square column by dividing force by $0.4f_c'$.	$\text{area} = \dfrac{867\text{ kip}}{0.4 \times 5\text{ ksi}} = 434\text{ in.}^2$ $h = \sqrt{434} = 20.8$ in.

6.2.5 6.2.5.1	Complete a rough check on slenderness	$r = 0.3h$ and $\dfrac{k\ell_u}{r} = 45$
	Concrete columns become very slender when $k\ell_u/r$ exceeds 45. Since this column is likely to be in a sway frame, assume a $k = 1.5$ and determine a size that satisfies $k\ell_u/r < 45$	$\ell_u = 18 \times 12 - 30 = 186$ in. Rearranging terms and solve for h $h = \dfrac{1.5 \times 186}{0.3 \times 45} = 20.7$ in.
	It is common for an engineer to choose a large enough column size so the column design is permitted to ignore slenderness. For a frame not braced against sidesway, a $k\ell_u/r$ limit of 22 would allow the engineer to ignore slenderness. This example, however, will consider slenderness and show the full set of calculations needed for a slender column.	Try 24 in. x 24 in. Column formwork is typically arranged to provide columns with even number dimensions. For this design choose a 24 in. square column to ensure a consistent column size for the building, which will improve formwork efficiency
Step 2: Sway or nonsway moment frame		
R6.2.5.3	ACI 318 Fig. R6.2.5.3 helps the engineer determine column analysis options. This example shows the full extent of the provisions when done by hand. These calculations could easily be programmed in a spreadsheet. The first step in the flowchart is to determine if the structure is a sway or nonsway frame.	
	A moment frame that is nonsway greatly reduces the required calculations. The Code provides three options to permit the frame to be considered as nonsway	
6.2.5	1) The stiffness of all the bracing elements in a story are at least 12 times the stiffness of all the columns in the direction of evaluation. Bracing elements generally means walls but braces are sometimes used	There are no bracing elements in the direction of the moment frame.
6.6.4.3(a)	2) The increase of column end moments due to second-order effects does not exceed 5 percent of the first-order end moments.	Second order moments are calculated in Step 4. The results of the calculation show that the second order effects exceed 5 percent
6.6.4.3(b) 6.6.4.4.1	3) Q, in accordance with Section 6.6.4.4.1, does not exceed 0.05. Q is determined for a single story and controlling load combination. Different floors of the same moment frame have different Q values.	$Q = \dfrac{\sum P_u \Delta_o}{V_{us} \ell_c}$ where, $\ell_c = 18 \times 12 - \left(\dfrac{30}{2}\right) + \left(\dfrac{7}{2}\right) = 204.5$ in.
	ℓ_c is the height of the column from center-to-center of the joints.	
	To calculate Q, the load case must consider lateral load and also impose the maximum gravity load. For this example, select Load Combination (iii), which has a larger axial load than Load Combination (ii)	Check Load Combination (iii)
	$\sum P_u$ is the sum all factored column and wall gravity loads at the floor considered for Load Combination (iii). The value was derived from the loads given in Chapter 1 of this Manual	$\sum P_u = 25{,}700$ kip
	V_{us} is the total factored horizontal story shear for Load Combination (iii). The value was calculated from the lateral forces calculated in Chapter 1 of this Manual	$V_{us} = 775$ kip

6.6.3.1.2	Δ_o is the first-order story deflection determined by a linear elastic analysis. For this hand calculation example, a simple approximation of deflection is provided. The first story of a building is often assumed to have a hinge at 0.67 ℓ_c. The following equation provides deflection at a distance ℓ to the hinge. $\Delta = \dfrac{V_{us} \times (\ell)^3}{3\sum EI}$ The value for stiffness should be reduced for cracking. A value of $I = 0.5I_g$ is commonly used for all members in an analysis calculated by hand.	$\Delta_o = \dfrac{V_{us}}{3\sum EI}\left[\left(\dfrac{2\times \ell_c}{3}\right)^3 + \left(\dfrac{\ell_c}{3}\right)^3\right]$ $E_c = 4030$ ksi $I = 24 \times 24^3/12 = 27{,}650$ in.^{4} $\sum EI = 37$ columns $\times 0.5 \times 4030 \times 27{,}650$ $= 2.06 \times 10^9$ kip-in.^{2} $\Delta_o = 0.36$ in.
		$Q = \dfrac{25{,}700 \times 0.36}{775 \times 204.5} = 0.058 > 0.05$ Therefore, this is a sway frame.
	Note that the deflection from a first-order linear elastic analysis from software that accounts for the relative stiffness of all the members is 0.64 in. The advantage of a more accurate calculation of lateral deflection is discussed in further detail in Step 4.	A more accurate first-order analysis shows that $Q = \dfrac{25{,}700 \times 0.64}{775 \times 204.5} = 0.104$
Step 3: Check to see if slenderness can be neglected		
	Compute the slenderness ratio, $k\ell_u/r$. The notation ℓ_u is the unsupported length. The floor-to-floor distance is 18 ft, and the floor beam at the second level is 30 in. deep.	$\ell_u = 18$ ft $\times$ 12 in./ft $-$ 30 in. $= 186$ in.
	To calculate the effective length factor k for the column, the member stiffnesses framing at the top and bottom joints need to be calculated. For rectangular sections, $I = bh^3/12$ For T sections, find the centroid by $y = \dfrac{\sum \text{moment of area}}{\sum \text{area}}$ then use transformation of sections to calculate I.	T Beam (at top of column) $b_f = 120$ in. (calculated in Beam Ex. 1) $b_w = 18$ in. $h_f = 7$ in. $h = 30$ in. $y_{beam} = \dfrac{120\times 7\times 3.5 + 18\times 23\times 18.5}{120\times 7 + 18\times 23} = 8.5$ in. $I_{beam} = \dfrac{120\times 7^3}{12} + (120\times 7\times 5.0^2) + \dfrac{18\times 23^3}{12} + (18\times 23\times 10.0^2) = 84{,}100$ in.^{4}
	See the plan in Chapter 1 for plan dimensions.	Slab (at bottom of column) $h = 7$ in. $b = 14$ ft $\times$ 12 in. $= 168$ in. $I_{slab} = 168 \times 7^3/12 = 4800$ in.^{4} Column (all levels) $h = 24$ in. $b = 24$ in. $I_{col} = 24 \times 24^3/12 = 27{,}650$ in.^{4}

6.6.3.1.1(a)	Calculate adjusted EI values. For this part, more detailed values for EI are used	E_c = 4030 ksi $0.35(E_cI)_{beam} = 0.35 \times 4030 \times 84{,}100$ $= 19 \times 10^6$ kip-in.^{2} $0.25(E_cI)_{slab} = 0.25 \times 4030 \times 4800$ $= 4.8 \times 10^6$ kip-in.^{2} $0.70(E_cI)_{col} = 0.70 \times 4030 \times 27{,}650$ $= 78 \times 10^6$ kip-in.^{2}
R6.2.5	Factor k reflects column end restraint conditions, which depend on the relative stiffness of the columns to the floor members at top and bottom joints. At the top joint, the columns frame into beams, and at the bottom joint, the columns frame into a two-way slab. Find the ratio of column stiffness to beam or slab stiffness $\psi = [(EI/\ell_c)_{col,\ above} + (EI/\ell_c)_{col,\ below}]/[(EI/\ell)_{beam,\ left} + (EI/\ell)_{beam,\ right}]$	Joint at top of column $\psi_A = \dfrac{\dfrac{78\times10^6}{204.5}+\dfrac{78\times10^6}{168}}{\dfrac{119\times10^6}{432}+\dfrac{119\times10^6}{432}}$ $\psi_A = 1.5$ Joint at bottom of column $\psi_B = \dfrac{\dfrac{78\times10^6}{120}+\dfrac{78\times10^6}{204.5}}{\dfrac{4.8\times10^6}{204.5}+\dfrac{4.8\times10^6}{204.5}}$ $\psi_B = 23.0$
R6.2.5	Read k from the nomograph for sway frames.	For a sway frame, $k \approx 2.2$ (Note: For a nonsway frame, $k \approx 0.9$)
6.2.5.2	Determine the radius of gyration, r $r = \sqrt{\dfrac{I_g}{A_g}}$ Note that Section 6.2.5.1 also allows an approximation of 0.3 times the width of the column in the direction of the frame, which would be 0.3 × 24 = 7.2 in. this case.	I_g = 27,650 in.^{4} A_g = 576 in.^{2} $r = \sqrt{\dfrac{27{,}560}{576}} = 6.9$
6.2.5	Check to see if slenderness can be neglected for sway frame. Slenderness for a sway frame can be neglected if $k\ell_u/r \leq 22$	For a sway frame, $\dfrac{2.2\times186}{6.9} = 59 > 22$ Slenderness cannot be neglected
	Note: For a nonsway frame, there are two limits that must be met $\dfrac{k\ell_u}{r} \leq 34+12\left(\dfrac{M_1}{M_2}\right)$ and $\dfrac{k\ell_u}{r} < 40$ Use Load Combination (iii) to find M_1 and M_2	Note: For a nonsway frame $\dfrac{k\ell_u}{r} = \dfrac{0.9\times186}{6.9} = 24$ $24 \leq 34+12\left(\dfrac{1752}{2352}\right) = 40.9$ and $24 < 40$ Therefore, slenderness could be neglected if this was a nonsway frame. For demonstration check this as both sway and nonsway frame

Step 4. Determine second order effects for $P\Delta$, sway

For a sway frame, the secondary moments at the end of the column due to differential movement of the ends of column must be calculated before calculating deformations along the length. Note that secondary moments are often called "$P\Delta$" (uppercase delta) effects by most reference materials, but are labeled "$P\delta_s$" (lowercase delta) in ACI 318.

From Load Combination (iii) above:

	M_{ns}	M_s
	$1.2D + 1.0L + 0.2S$	$1.0E$
M_1, kip-in.	2	–740
M_2, kip-in.	24	+2328

6.6.4.6.1 The Code provides a conservative method to estimate this effect. The equations are

$$M_1 = M_{1ns} + \delta_s M_{1s}$$
$$M_2 = M_{2ns} + \delta_s M_{2s}$$

Where the first-order moments due to gravity loads for a single load combination (M_{1ns}) are added to the first-order moments due to lateral loads (M_{1s}) multiplied by the sway moment magnification factor δ_s.

6.6.4.6.2 The sway moment magnification factor may be determined one of two ways

6.6.4.6.2a

$$\delta_s = \frac{1}{1-Q} \geq 1$$

This expression is commonly used if software determines the first order lateral deflection.

OR

$$\delta_s = \frac{1}{1-0.058} = 1.06$$

6.6.4.6.2b

$$\delta_s = \frac{1}{1-\dfrac{\sum P_u}{0.75\sum P_c}} \geq 1$$

This expression is commonly used in hand calculations.

Note that Section 6.6.4.6.2c indicates that the second order effect may be determined by software that performs a second-order elastic analysis.

$$\delta_s = \frac{1}{1-\dfrac{\sum P_u}{0.75\sum P_c}}$$

where $\sum P_u$ = 25,700 kip (From Step 2)

Columns

6.6.4.4.2 6.6.4.4.4 R6.6.4.6.2	Calculate the critical load P_c from Eq. (6.6.4.4.2). EI_{eff} may be calculated from one of three equations in 6.6.4.4.4. Use Eq. (6.6.4.4.4a) since the column reinforcement is not known at this point of the design. Key points about EI_{eff} are: (a) Commentary Section R6.6.4.4.4, explains the differences in the equations (b) For sway frames, β_{ds} is substituted for β_{dns} and is 0.0 for short term lateral loads (c) I_{se} in Eq. (6.6.4.4.4b) may be calculated using Table D.4.5 in the supplement to this Manual, see reference at the start of this example.	$P_c = \dfrac{\pi^2(EI)_{eff}}{(k\ell_u)^2}$ $(EI)_{eff} = \dfrac{0.4E_cI_g}{1+\beta_{ds}}$ $= \dfrac{0.4\times 4030\times 27,650}{1+0}$ kip-in.^{2} $= 45\times 10^6$ kip-in.^{2} $k = 2.2$ (sway frame) $\ell_u = 186$ in. $P_c = \dfrac{\pi^2\times(45\times 10^6)}{(2.2\times 186)^2} = 2630$ kip $\delta_s = \dfrac{1}{1-\dfrac{25,700}{0.75(37\text{ columns}\times 2630)}} = 1.54$ The large disparity between the two results (1.06 versus 1.54) is unusual. This disparity indicates that the 24 in. column stiffness may be outside normal bounds for stable results, and the engineer should consider increasing the column size.
	Discussion In MacGregor and Hage (1977), it is suggested that Δ_o in Eq. (6.6.4.6.2a) be taken from a first-order analysis using a computer program that accounts for the member stiffnesses. As noted in Step 2, the software lateral deflection is 0.64 in. and the value for Q becomes 0.104. Thus, the revised δ_s for Eq. (6.6.4.6.2a) is $\delta_s = \dfrac{1}{1-0.104} = 1.12$ The MacGregor and Hage (1977) reference is informative and describes the Q method. A few helpful suggestions from the reference are: (a) For Q values between 0.05 and 0.2, the error in second-order moments will be less than 5 percent (b) Δ_o/H (story height) should be less than 1/500 for nonsway frames and less than 1/200 for sway frames at factored loads (c) $\sum P_u/\sum P_{cr}$ in Eq. (6.6.4.6.2b) should be less than 0.2 In this example, $\sum P_u/\sum P_{cr}$ is 0.26 > 0.2, so the results from Eq. (6.6.4.6.2b) are questionable. Q calculated from deflections by a first-order computer analysis is 0.1 which is in the range suggested by MacGregor and Hage, thus, the second order-moments calculated by the Q method should be within 5 percent of the actual. If the Δ_o from the computer analysis was not available, it would be prudent to use Eq. (6.6.4.6.2b) and increase the square column size to satisfy $\sum P_u/\sum P_{cr} < 0.2$	
	Calculate the magnified moments using the software-based Q. Note that only the moments due to lateral loads are magnified.	$\delta_s M_{1s} = 1.12\times 1740 = 1950$ kip-in. $\delta_s M_{2s} = 1.12\times 2328 = 2610$ kip-in.
	Calculate the design moments M_1 and M_2	$M_1 = M_{1ns} + \delta_s M_{1s} = 12 + 1950 = 1962$ kip-in. $M_2 = M_{2ns} + \delta_s M_{2s} = 24 + 2610 = 2634$ kip-in.

6.2.5.3	Check to see if the second-order moments exceed 40 percent of first-order moments.	First order M_1 is 1740 + 12 = 1752 kip-in. 1962 < 1742 × 1.4 = 2718, therefore **OK** First order M_2 is 2328 + 24 = 2352 kip-in. 2610 < 2352 × 1.4 = 3292, therefore **OK**
Step 5. Determine second order effects for $P\delta$, nonsway		
6.6.4.6.4	Second-order effects along the length of the column must be calculated for a sway or nonsway frame where slenderness cannot be neglected. The magnified moments, M_1 and M_2, from Section 6.6.4.6 are used in Section 6.6.4.5.	
6.6.4.5.1	The required moment for design is calculated by multiplying the larger end moment M_2 by the moment magnification factor δ $M_c = \delta M_2$	
6.6.4.5.2	The moment magnification factor δ is calculated by $\delta = \dfrac{C_m}{1-\dfrac{P_u}{0.75P_c}} \geq 1.0$	P_u = 789 kip
6.6.4.5.3	$C_m = 0.6 - 0.4\dfrac{M_1}{M_2}$	M_1 = 1962 kip-in. M_2 = 2634 kip-in. **Controls**
6.6.4.5.4	Check $M_{2,min}$: $M_{2,min} = P_u(0.6 + 0.03h)$	$M_{2,min}$ = 789 (0.6 + 0.03 × 24) = 1041 kip-in., which is less than 2634
R6.6.4.5.3	Notice that M_1/M_2, has been updated to follow the right hand rule, thus, the sign convention has been changed in ACI 318. M_1/M_2 is positive where column bent in double curvature M_1/M_2 is negative where column bent in single curvature	$C_m = 0.6 - 0.4\dfrac{1962}{2634} = 0.30$
6.6.4.4.2 R6.6.4.4.4	The critical buckling load was calculated in Step 4, but β_{ds} was substituted for β_{dns}. It shall now be calculated using β_{dns}. The commentary states that β_{dns} may be assumed to be 0.6. Another common way of calculating β_{dns} is to divide the dead load by transient gravity loads for a given load combination. For Load Combination (iii), the calculation is $\beta_{dns} = \dfrac{1.2D}{1.2D+1.0L+0.2S}$ $= \dfrac{1.2\times517}{1.2\times517+1.0\times151+0.2\times10}$ $= 0.80$	$P_c = \dfrac{\pi^2(EI)_{eff}}{(k\ell_u)^2}$ $(EI)_{eff} = \dfrac{0.4E_cI_g}{1+\beta_{dns}}$ $= \dfrac{0.4\times4030\times27,650}{1+0.80}$ kip-in.^{2} $= 24.8 \times 10^6$ kip-in.^{2} k = 2.2 (sway frame) ℓ_u = 186 in. $P_c = \dfrac{\pi^2\times(24.8\times10^6)}{(2.2\times186)^2} = 1460$ kip

	Magnify the moment for second-order effects along the length of the column.	$\delta = \dfrac{0.3}{1 - \dfrac{789}{0.75 \times 1460}} = 1.07$ $M_c = \delta M_2 = 1.07 \times 2634 = 2820$ kip-in.

Step 6. Summary and discussion

This example calculates the second-order moment for one column, in one direction, for one load case. It is easy to see that this is a time-consuming process. The following table shows a comparison between the moment and axial loads calculated by hand and by computer software for Load Combination (iii).

	Hand (first-order)	Hand (second-order)	Computer (first-order)	Computer (second-order)
P_u, kip	789	789	818	818
M_u, kip-in.	2352	2820	2177	2401

One can see that the hand calculation to find the second-order moment is more conservative than a computer analysis. The second-order moment increase for the computer is 10 percent which is lower than the 18 percent calculated by the moment magnification method. Notice that the Q-method used in Eq. (6.6.4.6.2a) was aided with a Δ_o computed by a computer analysis. If that deflection was not available, Eq. (6.6.4.6.2b) would have been used and total increase of 63 percent would have been required which is more than the 40 percent allowed. Thus, a larger column would have been selected as suggested in the discussion in Step 4.

Columns Example 2 *Column for an ordinary moment frame*—Design and detail the first floor interior column at location E4 from the example building given in Chapter 1 of this Manual (Fig. E2.1). The column is part of an ordinary moment frame. Example 2 is the design of the column analyzed in Example 1. The loads have been modified to match the results of an analysis from commercial software capable of second-order linear elastic analysis.

<u>Given:</u>

Materials

Specified yield strength, $f_y = 60$ ksi

Modulus of elasticity of steel, $E_s = 29{,}000$ ksi

Specified concrete compressive strength, $f_c' = 5$ ksi

Modulus of elasticity of concrete, $E_c = 4030$ ksi

Nominal maximum size of aggregate is 1 in.

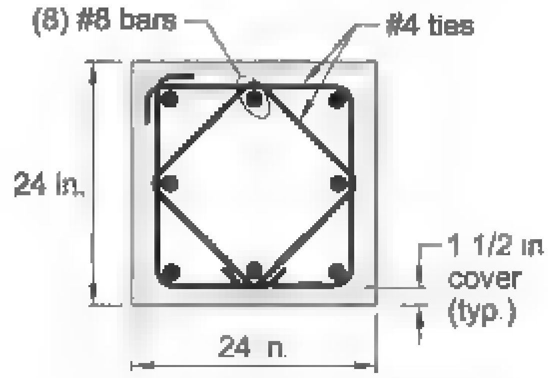


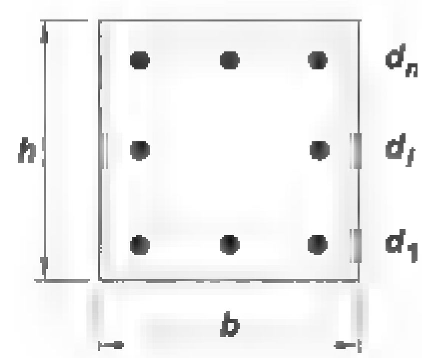
Fig. E2.1—Column section and reinforcement.

Loading—

Load combinations	P_u, kip	M_u, kip-in.	V_u, kip
(i) $U = 1.2D + 1.6L + 0.5S$	890	0	0
(ii) $U = 1.2D + 1.0W + 1.0L + 0.5S$	800	651	5
(iii) $U^* = 1.2D + 1.0E + 1.0L + 0.2S$	818	2401	18
(iv) $U^* = 0.9D + 1.0E + 1.0L + 0.2S$	486	2401	18

*The software adjusts seismic load combinations as required by ASCE/SEI 7.

Reference: MNL-17 Supplement Interaction Diagram Excel spreadsheet, found at https://www.concrete.org/MNL-17/Download?

ACI 318	Discussion	Calculation
Step 1: Find the required area of longitudinal reinforcement		
10.5.2.1 22.2 22.4	The Code references Section 22.4 for the calculation of P_n and M_n. Section 22.4 provides an equation to calculate P_0 and references Section 22.2 for strain limits. The interaction of P_n and M_n is evaluated by making an interaction diagram. A tutorial spreadsheet is provided with this manual that demonstrates how to make a diagram for a given section and reinforcement ratio ρ. This example used the spreadsheet to create an interaction diagram. The generated interaction diagram can be used to determine if the assumed longitudinal reinforcement is satisfactory. The section properties and geometry were determined in the analysis from Column Example 1. The next step is to assume a quantity, size, and location of the longitudinal reinforcement. The design of columns is often an iterative process. The final design may show that a larger section or more reinforcement is needed, in which case, the analysis will need to be revised.	
	The spreadsheet analyzes rectangular or square columns for combined axial and flexural strength. The designer inputs the number (n) of steel layers in the column cross section (Fig. E2.2). The spreadsheet places the first (d_1) and last layer (d_n) as close to the outer face as permitted (concrete cover plus tie bar diameter). The remaining layers (d_i) are evenly spaced between the outer layers.	 *Fig. E2.2—Parameters for spreadsheet analysis.*

10.6.1.1
10.7.3.1

The design moments are not very large, so try the minimum area of reinforcement and assume a uniform distribution of bars around the perimeter. Longitudinal bars are typically larger bars, No. 7 and greater, to make stable column cages for erection and to reduce the number of ties at a section.

$A_{s,min} = 0.01A_g = 0.01 \times 24^2 = 5.76 \text{ in.}^2$

Try 8 bars, one in each corner and one on each side (3 layers: 3 bars, 2 bars, 3 bars)

Area bar needed = 5.76 in.^{2}/8 = 0.72 in.^{2}

At least four bars are required in a rectangular column.

Area of a No. 8 bar is 0.79 in.^{2}, therefore, try eight-No. 8 bars

22.2.2.4.3

The spreadsheet performs a sectional strength analysis using an equivalent rectangular stress distribution according to Section 22.2.2.4. The β is a function of f_c' which is automatically calculated and displayed for the user's information.

Section properties and geometry		
No. layers:	3	
f_c' =	5,000	psi
β =	0.80	
b =	24	in.
h =	24	in.
f_y =	60	ksi
E_s =	29,000	ksi
Tie bar size:	4	
Clear cover to tie =	1.50	in.
Long. bar size:	8	

10.7.6.1.2
25.7.2.2

No. 4 ties are a common starting size since they provide good initial shear strength and are rigid enough to provide column cage stability during erection.

10.7.1.1
20.5.1.3.1

Concrete cover protects the reinforcement from corrosion and provides fire protection.

The spreadsheet calculates the distances to the layers of reinforcement, which is needed for later calculations, by using the tie bar size, cover to tie, and longitudinal bar size.

Number of bars per layer			
Layer	d_i, in.	No. long. bars	A_{si}, in.^{2}
3	2.500	3	2.37
2	12.000	2	1.58
1	21.500	3	2.37
		Σ	6.32

10.7.2.1
25.2.3

The minimum bar spacing is calculated and displayed for the user's information.

Bar spacing checks			
d =	21.50	in.	
c/c bar sp. (h) =	9.50	in.	OK
c/c bar sp. (b) =	9.50	in.	OK
Min. clear sp. (in.) =	1.50	in.	(25.2.3)

10.5.1.2
21.2

10.5.2.1
22.4

Key variables needed for design are displayed for the user's information. The strain limits and ϕ factors change as the column interaction diagram transitions from compression-controlled sections to tension-controlled sections.

Strain definitions	
f_y/E_s =	0.00207
ε_{cu} =	0.003
Ductile Strain =	-0.005
Brittle Strain =	-0.002
$\phi_{tension\text{-}controlled}$ =	0.9
$\phi_{compression\text{-}controlled}$ =	0.65

The interaction diagram is formulated by calculating P_n and M_n for incremental changes in the net tensile strain in the extreme layer of longitudinal tensile reinforcement at nominal strength, ε_t. An example strain of 0.0007 was chosen to illustrate the calculation of P_n and M_n in the following steps.

1. Vary ε_t from pure compression, ε_t equal to f_y/E_s at all layers, to pure tension, $-f_y/E_s$ at all layers.

Find P_n and M_n for ε_t = 0.0007

2. Calculate c, the distance from the extreme compression fiber to the neutral axis, for the given ε_t by using similar triangles

$$c = \frac{-\varepsilon_{cu} \times d_t}{\varepsilon_t - \varepsilon_{cu}}$$

$$c = \frac{-0.003 \times 21.5}{0.0007 - 0.003} = 28.09 \text{ in.}$$

3. Calculate a, the depth of the equivalent stress block.

$$a = \min\begin{bmatrix} \beta_1 \times c \\ h \end{bmatrix}$$

$$a = \min\begin{bmatrix} 0.80 \times 28.09 = 22.47 \text{ in.} \\ h = 24 \text{ in.} \end{bmatrix}$$

Use $a = 22.5$ in.

4. Calculate C_c, the resultant force of the concrete compression block.

$C_c = 0.85 \times a \times b \times f_c'$

$$C_c = 0.85 \times 22.47 \times 24 \times 5 = 2292 \text{ kips}$$

5. Calculate ε_{si}, the strain at each layer of bars.

$$\text{if } c \geq d_i \text{, then } \varepsilon_{si} = \min\begin{bmatrix} \varepsilon_{cu} \times \dfrac{(c - d_i)}{c} \\ \dfrac{f_y}{E_s} \end{bmatrix}$$

$$\text{if } c < d_i \text{, then } \varepsilon_{si} = \max\begin{bmatrix} \varepsilon_{cu} \times \dfrac{(c - d_i)}{c} \\ -\dfrac{f_y}{E_s} \end{bmatrix}$$

For ε_{s1}, ε_{s2}, and ε_{s3}, $c > d$ thus find the minimum of

$$\varepsilon_{s1} = 0.003 \times \frac{(28.1 - 21.5)}{28.1} = 0.000703$$

$$\varepsilon_{s2} = 0.003 \times \frac{(28.1 - 12)}{28.1} = 0.00172$$

$$\varepsilon_{s3} = 0.003 \times \frac{(28.1 - 2.5)}{28.1} = 0.00273 \quad \text{and}$$

$$\frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

Use $\varepsilon_{s1} = 0.000703$, $\varepsilon_{s2} = 0.00172$, and $\varepsilon_{s3} = 0.00207$

6. Calculate F_{si}, the force at each layer of bars.

If $\varepsilon_{si} > 0$, then $F_{si} = \varepsilon_{si} \times A_{si} \times E_s - 0.85 \times A_{si} \times f_c'$

Note that positive strain indicates that the bar is in compression. The force is adjusted to account for the concrete area displaced by the bars.

If $\varepsilon_{si} \leq 0$, then $F_{si} = \varepsilon_{si} \times A_{si} \times E_s$

For F_{s1}, F_{s2}, and F_{s3}, $\varepsilon_s > 0$, thus

$F_{s1} = 0.000703 \times 2.37 \times 29,000 - 0.85 \times 2.37 \times 5$
$= 38.2$ kip

$F_{s2} = 0.00172 \times 1.58 \times 29,000 - 0.85 \times 1.58 \times 5 = 72.1$ kip

$F_{s3} = 0.00207 \times 2.37 \times 29,000 - 0.85 \times 2.37 \times 5 = 132$ kip

7. Calculate P_n

$P_n = C_c + \sum F_{si}$

Note that $P_{n,max}$ is not applied in this spreadsheet until the design strength curve is calculated

$P_n = 2292 + 38.2 + 72.1 + 132 = 2530$ kip

8. Calculate d_i, the moment arms to force C_c and each F_{si}

$$d_{Cc} = \frac{h}{2} - \frac{a}{2}$$

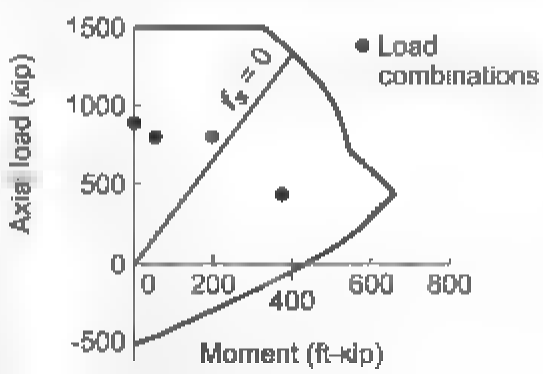
$$d_i = \frac{h}{2} - d_i$$

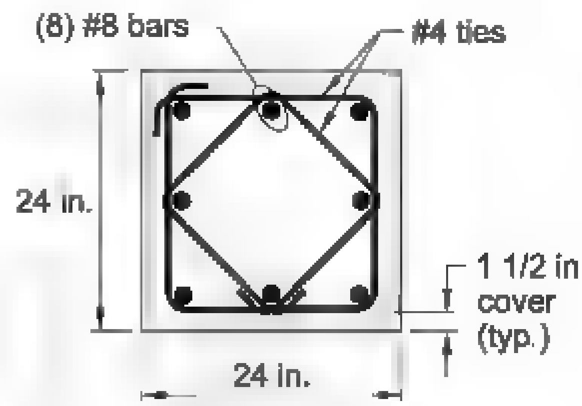
$$d_{Cc} = \frac{24}{2} - \frac{22.47}{2} = 0.77 \text{ in.}$$

$d_1 = 12 - 21.5 = -9.5$ in.

$d_2 = 12 - 12 = 0$ in.

$d_3 = 12 - 2.5 = 9.5$ in.

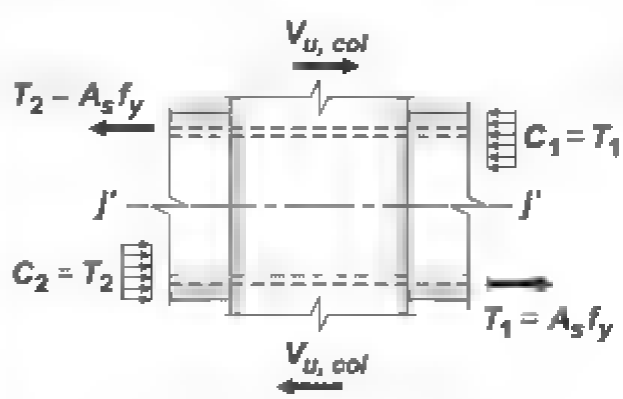
	9. Calculate M, the moment for each force about the center of the section	
	$M_{Cc} = d_{Cc} \times C_c$	$M_{Cc} = 0.77 \times \frac{2292}{12} = 147$ ft-kip
	$M_{si} = d_{si} \times F_{si}$	$M_1 = -9.5 \times \frac{38.2}{12} = -30.2$ ft-kip
		$M_2 = 0 \times \frac{72.1}{12} = 0$ ft·kip
		$M_3 = 9.5 \times \frac{132}{12} = 105$ ft·kip
	10. Calculate M_n	
	$M_n = M_{Cc} + \sum M$	$M_n = 147 - 30.2 + 0 + 105 = 222$ ft-kip
10.5.1.2 21.2 10.5.2.1 22.4.2.1 10.7.5.2.2	The spreadsheet changes the strain in small increments to create a smooth plot of a nominal strength interaction diagram. The values are then multiplied by ϕ and the limits on axial strength are applied to create the design strength interaction diagram. The ϕ factor for compression-controlled sections is 0.65 for ties. The ϕ factor for all tension-controlled sections is 0.9. This factor varies linearly from 0.65 to 0.9 through the transition zone shown which creates the sharp change in the lower part of the curve. For this column example, Fig. E2.3 shows the interaction diagram with the four given load conditions plotted on the graph. Note that points for load combinations (i), (ii), and (iii) are to the left of the $f_s = 0$ line, meaning all the bars remain in compression. Point iv) is for a load combination with lighter gravity loads. It is just to the right of the line, meaning a few bars will be in tension. Thus, a tension splice is required. Another line could be drawn on the diagram showing where the tensile bar stress is $0.5f_y$. That line delineates whether a Class A or B splice is required. For this example, all the bars are going to be spliced at one location so a Class B splice is always required.	Design Strength Interaction Diagram.  *Fig. E2.3—Column design interaction diagram.* Use eight No. 8 bars evenly spaced around the perimeter.
Step 2: Find the required area and geometry of transverse reinforcement		
10.5.3.1 22.5 18.3.3	The Code references Section 22.5 for the calculation of V_n. The P_u given in Load Combination (iii) has the expression $0.2S_{DS}$ applied in the downward direction as required by ASCE/SEI 7. The P_u value shown here is changed to reflect $0.2S_{DS}$ in the upward direction for the lowest axial load associated with this load combination. Note that ℓ_u (186 in.) is greater than $5c_1$ (120 in.), therefore, the additional shear requirement for ordinary moment frame columns does not apply.	Load Combination (iii) $V_u = 18$ kip $P_u = 737$ kip

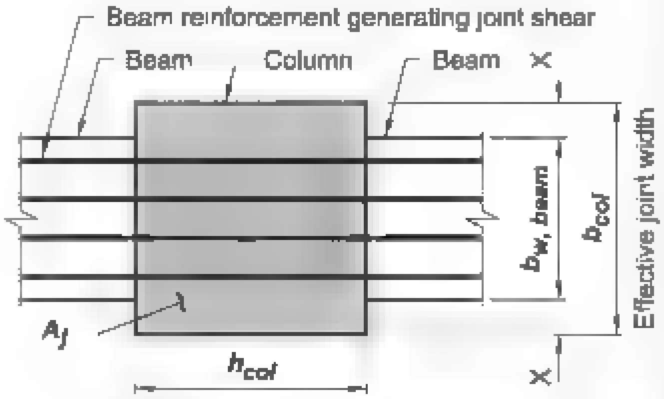
10.7.6.1.2 25.7.2 10.7.6.2	Tie requirements must meet the geometry requirements of 25.7.2 and location requirements of 10.7.6.2	
25.7.2.2	The minimum tie bars size is No. 3 for longitudinal bars No. 10 or smaller, however, a No. 4 tie was chosen as discussed in Step 1	Use No. 4 ties.
25.7.2.1	The minimum spacing is $(4/3)d_{agg}$, however, the maximum tie spacing requirement controls for this example. The maximum spacing shall not exceed the least of: (a) $16d_b$ of the longitudinal bar (b) 48 d_b of the transverse bar (c) h or b	(a) 16×1.00 in. = 16 in. **Controls.** (b) 48×0.50 in. = 24 in. (c) 24 in. Use s = 16 in. on center (o.c.)
25.7.2.3 Fig. R25.7.2.3a	Section 25.7.2.3 requires that lateral support from ties is provided for bars at every column corner and also bars with greater than 6 in. clear on each side. Thus, every vertical bar needs lateral support in this example. Ties with 90-degree standard hooks are acceptable for this case. A diamond-shaped tie is used to support bars along the sides (Fig. E2.4). This is desirable because it provides a stable column cage for erection, however, it becomes a fabrication problem when the column is rectangular and this tie becomes oblong. It is common to use alternative tie geometry, such as cross ties, for columns that are not square.	Use No. 4 ties @ 16 in. o.c.  *Fig. E2.4—Column reinforcement details.*

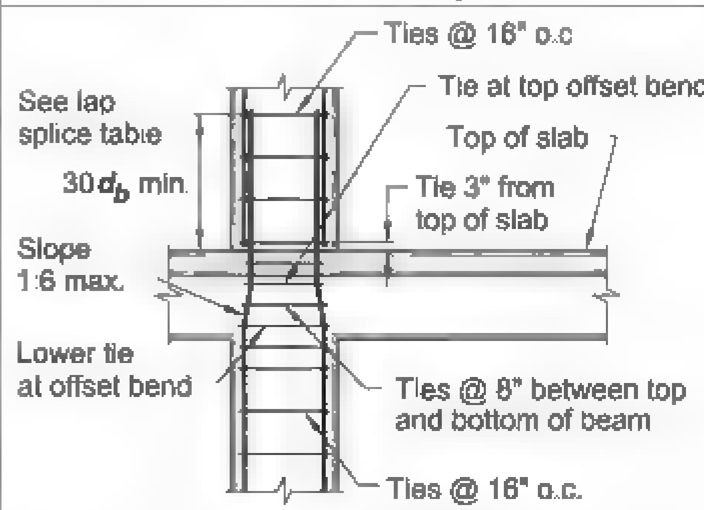
10.5.3.1 22.5.5	Column shear strength is calculated in accordance with Code Section 22.5 and compared to the required strength V_u. If minimum shear reinforcement is not provided, then the concrete contribution to shear strength (V_c) must be determined using Eq. (22.5.5.1c), which depends on the axial load, size effect factor, and quantity of flexural reinforcement. If minimum shear reinforcement is provided, then V_c can be determined using Eq. (22.5.5.1a). Because column ties must be provided, determine if they are sufficient to satisfy minimum shear reinforcement requirements.	V_u = 18 kip
10.6.2	Minimum shear reinforcement is the greater of the following:	
	$0.75\sqrt{f_c'}\dfrac{b_w s}{f_{yt}}$	Check No. 4 ties at 16 in. spacing. $A_{v,prov} = 2(0.2\text{ in.}^2) = 0.4\text{ in.}^2$ $0.75\sqrt{5000\text{ psi}}\dfrac{24\text{ in.}(16\text{ in.})}{60{,}000\text{ psi}} = 0.34\text{ in.}^2$
	$50\dfrac{b_w s}{f_{yt}}$	$50\text{ psi}\dfrac{24\text{ in.}(16\text{ in.})}{60{,}000\text{ psi}} = 0.32\text{ in.}^2$
10.6.2.1		**OK** Column ties No. 4 at 16 in. spacing satisfy the minimum shear reinforcement requirement. If column ties did not satisfy minimum shear reinforcement requirements, then Eq. (22.5.5.1c) for V_c could have been used to determine if $V_u < \phi V_c/2$. If so, then Code Section 10.6.2.1 indicates that minimum shear reinforcement is not required.
22.5.5.1a	Ignoring axial load, and using normalweight concrete, the applicable equation from Table 22.5.5.1a becomes: $\phi V_c = \phi 2\sqrt{f_c'}b_w d$	d = 24 in. – 1.5 in. – 0.5 in. – 0.5(1 in.) = 21.5 in. $\phi V_c = 0.75(2\sqrt{5000\text{ psi}})(24\text{ in.})(21.5\text{ in.}) = 54.7\text{ kip}$ $> V_u = 18\text{ kip}$ **OK**

Step 3: Beam-column joint design

15.2.2 15.5.1	The transfer of column axial force through the joint at the top of the column must be checked. It is common to have higher strength concrete in the columns and walls and a lower strength in the floor system. If the concrete strength in the floor system is less than 70% of the column concrete strength, then the Code provides three options to account for this difference. This building uses the same f_c' for both the columns and floor system so no additional calculations or adjustments are necessary.	
15.2 18.3	Code Section 15.2.1 requires that beam-column joints satisfy the detailing provisions of 15.3 and strength requirements of 15.4. Because this is designated as an ordinary moment frame, the joint must also satisfy applicable requirements of 18.3.	
15.3 15.3.1.1	This column is part of an ordinary moment frame that is part of the seismic-force-resisting system and must satisfy the detailing requirements of Code Sections 15.3.1.2 through 15.3.1.4.	

15.3.1.2 25.7.2 15.3.1.3 15.3.1.4	Provide transverse joint reinforcement in accordance with Code Section 25.7.2 when using ties. Ties have been sized for the column using these same provisions. Continue column ties through joint. Provide at least two sets of ties within the depth of the shallowest beam framing into the joint at a spacing of no more than 8 in. within the depth of the deepest beam.	Thus, at least two ties are required along the depth of the joint at a spacing not greater than 8 in.
18.3.4 15.4.1.1	Calculate column shear force V_u associated with the nominal moments and related shears calculated in accordance with Code Section 18.3.4. The column shear can be approximated with the following equation assuming inflection points occur at the column midheights. $V_{u,col} = \dfrac{\left[\left(M_{nl} + M_{nr}\right) + \left(V_{ul} + V_{ur}\right)\dfrac{h}{2}\right]}{\ell}$ where ℓ is the distance between the mid-height of the column above and below the joint. Note that it is unconservative to ignore the slab for this check; therefore, consider the full effective width, b_f, of the T-Beam in the calculations.	Use the Beam Example 1 in Manual Chapter 7 to calculate the nominal moment strength and associated shears. Use seven No. 7 bars in the top and two No. 7 bars in the bottom. M_{nl} = 6520 kip-in. M_{nr} = 1970 kip-in. V_{ul} = 70 kip V_{ur} = 35 kip Also, $\ell = \dfrac{18\text{ ft} + 14\text{ ft}}{2}12\text{ in./ft} = 192\text{ in.}$ h = 24 in. $V_{u,col} = \dfrac{\left[(6520+1970)+(70+35)\dfrac{24}{2}\right]}{192} = 51\text{ kip}$
	The shear at the center of the joint can be determined using the FBD from Fig. E2.5. $V_j = T_2 + C_1 - V_{u,col}$  *Fig. E2.5—Free-body diagram of beam-column joint.*	T_1 = 2 × 0.60 × 60 = 72 kip T_2 = 7 × 0.60 × 60 = 252 kip C_1 = T_1 = 72 kip C_2 = T_2 = 252 kip V_j = 252 + 72 – 51 = 273 kip

15.4.2.4	The effective area of the joint, A_j, is determined by multiplying the column depth, h, by the effective width, which is the lesser of (refer to Fig. E2.6):	
	(a) $b_{w,beam} + h_{col}$ (b) $b_{w,beam} + 2x$ (c) b_{col} where x is the smaller distance between the edge of the beam and edge of the column.  *Fig. E2.6—Beam-column joint showing effective area of joint, A_j.*	$b_{w,beam} = 18$ in. $h_{col} = b_{col} = 24$ in. $x = \dfrac{24\text{ in.} - 18\text{ in.}}{2} = 3$ in. Effective joint width (a) 18 + 24 = 42 in. (b) 18 + 2 × 3 = 24 in. (c) 24 in. **Controls** $A_j = 24 \times 24 = 576$ in.^{2}
15.4.2.3 15.2.8	Calculate joint shear strength using the applicable equation from Code Table 15.4.2.3. Columns and beams are continuous and joint is not confined by transverse beams according to Code Section 15.2.8. $V_n = 20\lambda\sqrt{f_c'}A_j$	$V_n = \dfrac{20 \times 1.0 \times \sqrt{5000} \times 576}{1000} = 814$ kip $\phi V_n = 0.75 \times 814 = 610$ kip > 273 kip Joint design strength is satisfactory
ACI 352R	For a greater understanding of joint design, reference ACI 352R, "Recommendations for Design of Beam-Column Connections in Monolithic Reinforced Concrete Structures."	
Step 4: Detail the column splice and joint at Level 2		
	It is common to begin lap splices for the vertical reinforcement at the floor level in ordinary moment frames. Since all the splices are at one location, a Class B tension lap length is provided and checked against the compression lap length as a minimum.	

10.7.5.1.3 25.5 25.5.2 10.7.5.2.2 10.7.1.2 25.4.2.4	ℓ_{st} is the greater of: (a) $1.3\ell_d$ (b) 12 in. where, $$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\psi_g}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\right)d_b$$ and $$K_{tr} = \frac{40A_{tr}}{sn}$$	Determine ℓ_{st} for a No. 8 bar: $\psi_t = \psi_e = \psi_s = \psi_g = \lambda = 1.0$ $d_b = 1.0$ in. $c_b = 1.5 + 0.5 + 1.0/2 = 2.5$ in. $n = 3$, number of longitudinal bars along the splitting plane $s = 16$ in., spacing of ties $A_{tr} = 4$ tie legs $\times$ 0.2 in.^{2} = 0.8 in.^{2} $$K_{tr} = \frac{40\times 0.8}{16\times 3} = 0.67$$ $$\left(\frac{c_b+K_{tr}}{d_b}\right) = \left(\frac{2.5+0.67}{1.0}\right) = 3.17 \leq 2.5$$ $$\ell_d = \frac{3}{40}\frac{60{,}000}{1.0\sqrt{5000}}\frac{1.0}{2.5}1.0 = 25.4\text{ in.}$$ $\ell_{st} = 1.3 \times 25.4 = 33.0$ in.
10.7.5.1.3 25.5 25.5.5.1	For f_y equal to 60 ksi, ℓ_{sc} is the greater of: (a) $0.0005f_yd_b = 30d_b$ (b) 12 in. A common way of expressing this splice on the structural drawings is to make a lap splice table and reference it in the detail. Note that it is common to splice at every other story to save time on labor in the field.	Check compression lap splice length $\ell_{sc} = 30 \times 1.0 = 30$ in. For ease of construction, use $\ell_{st} = 33$ in. for all splices.
10.7.4.1 10.7.6.4	Lateral support of offset bends is provided by column ties at bends. Ties need to resist 1.5 times the horizontal tension component of the computed force in the inclined portion of the offset bar. The horizontal component was determined for the bar strength at maximum incline, 1 in 6 (9.5 degrees). The calculations show that one additional No. 4 tie can laterally support one No. 8 bar at the offset.	Nominal vertical bar strength No. 8 $(f_yA_{st}) = 60 \times 0.79 = 47.4$ kip Horizontal tension at the bend $P_u = 1.5 \times 47.4 \times \sin(9.5°) = 11.7$ kip Nominal tie strength No. 4 $(f_yA_{st}) = 60 \times 0.20 = 12$ kips > 11.7 kip Provide one tie leg for each vertical bar at its offset. The current tie detail meets this requirement.
10.7.6.2 10.7.6.2.2 15.3.1.4	The Code requires the first tie starting on any level to be within, $s/2$, of the top of the slab. It is good construction practice to start the tie at 2 or 3 in. from the top of the floor (beginning of the column cage) and then proceed with the typical tie spacing. In any story, the top tie or hoop in the column should be located not more than one-half the tie or hoop spacing below the lowest horizontal reinforcement in the slab. The joint reinforcement spacing of 8 in. determined in Step 3, however, controls. Space ties at 8 in. between top of slab and bottom of beam (Fig. E2.7). Locate the top offset bend at the tie just below the slab. Provide a tie at the bottom offset bend.	 *Fig. E2.7—Beam-column joint and column reinforcement splice details.*

Step 5. Discussion and summary	
	The 24 x 24 in. column is satisfactory for design. The minimum reinforcement, eight No. 8 bars, is sufficient to resist the factored loads and moments. Shear is very low and only the minimum tie area and spacing, No. 4 ties at 16 in. on center, are required for support of the longitudinal reinforcement. On inspection, this column size and reinforcement will work for the remaining floors, since the columns get shorter and the loads decrease. A smaller column will work for the upper floors but the cost of a change in formwork may not overcome the cost of the small amount of concrete that is saved. For this building, the architect may want to save some space on the upper floors and there may be a compromise between functionality and expense.

Columns Example 3: *Column for an intermediate moment frame (IMF)*

Design and detail the second floor interior column (Fig. E3.1) at location E4 from the example building given in Chapter 1 of this Manual. The column is part of an IMF. Example 3 is a continuation of Examples 1 and 2. The loads have been modified to match the results of an analysis from commercial software capable of second-order linear elastic analysis and for Seismic Design Category C.

Given:

Materials—

Specified yield strength, $f_y = 60$ ksi
Modulus of elasticity of steel, $E_s = 29{,}000$ ksi
Specified concrete compressive strength, $f_c' = 5$ ksi
Modulus of elasticity of concrete, $E_c = 4030$ ksi
Normalized maximum size of aggregate is 1 in.

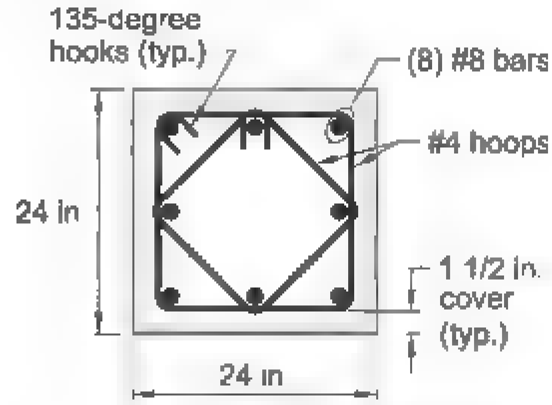


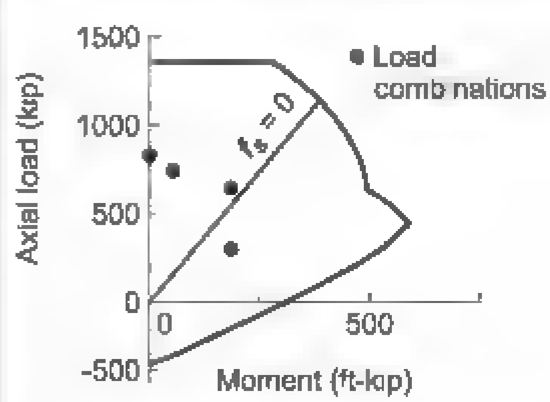
Fig. E3.1—Column section and reinforcement.

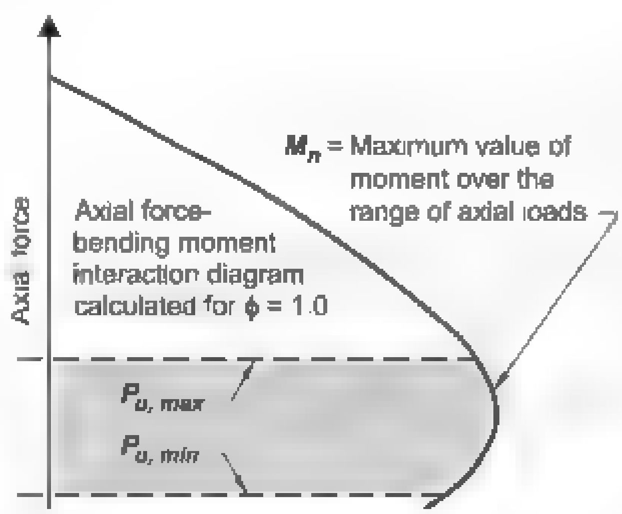
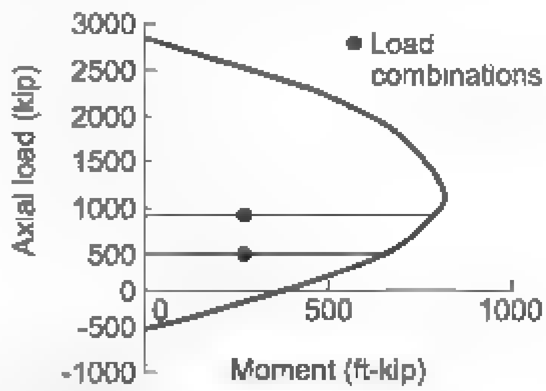
Loading

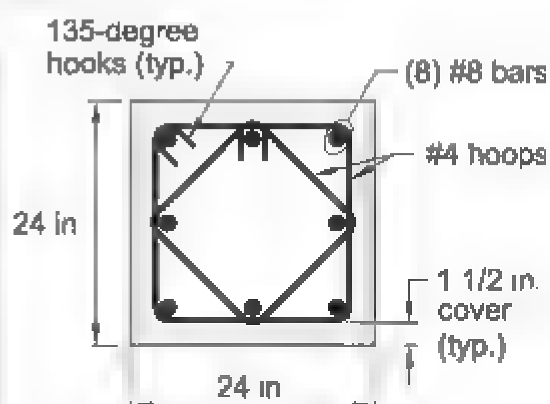
Load Combinations	P_u, kip	M_u, kip-in.	V_u, kips
(i) $U = 1.2D + 1.6L + 0.5S$	890	0	0
(ii) $U = 1.2D + 1.0W + 1.0L + 0.5S$	800	651	5
(iii) $U^* = 1.2D + 1.0E + 1.0L + 0.2S$	848	3228	25
(iv) $U^* = 0.9D + 1.0E + 1.0L + 0.2S$	456	3228	25

*The software adjusts seismic load combinations as required by ASCE/SEI 7.

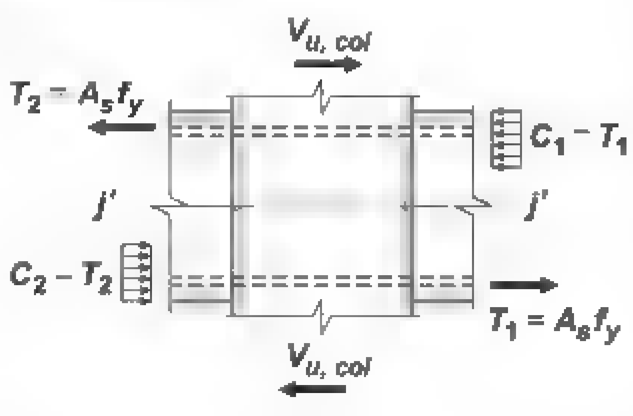
Reference: MNL-17 Supplement. Interaction Diagram Excel spreadsheet found at https://www.concrete.org/MNL17Download2

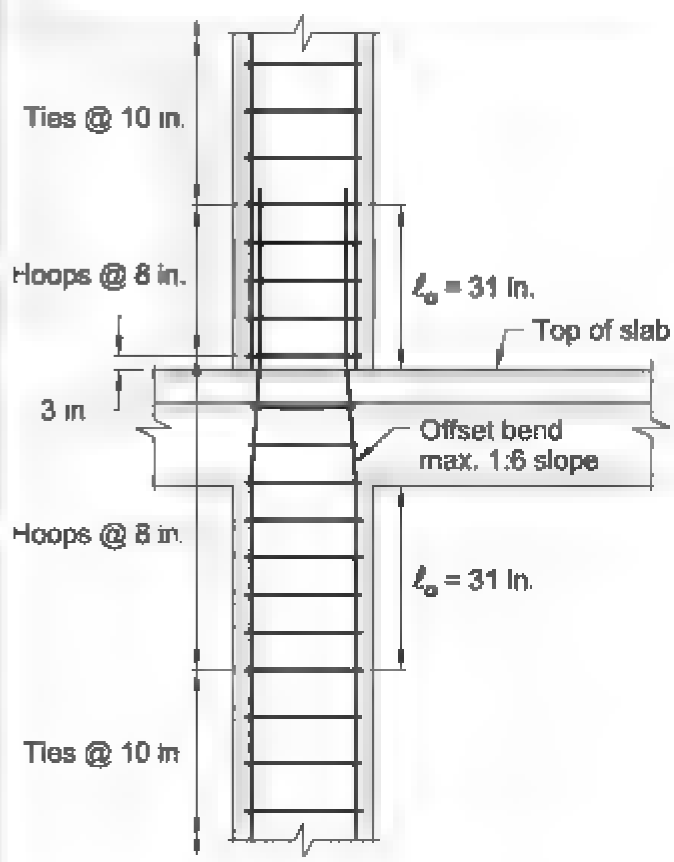
ACI 318	Discussion	Calculation
Step 1. Discussion on modification of Example 2 for an IMF		
10.5.2.1 22.4 22.2	This example demonstrates the column design requirements for an IMF. The design requirements are similar to an ordinary moment frame (OMF) except that hoops are required in the plastic hinge region, ℓ_o, at a reduced spacing. The additional design requirements for an IMF increase the response modification coefficient R from 3 for an OMF to 5 for an IMF. The increase in R results in a decrease in seismic base shear. An IMF is permitted to be used for structures assigned to SDC B and required for SDC C. For the column designed in Examples 1 and 2, the shear is so low that it is not economical to require the extra detailing to gain the benefit of the reduced shear. For this example, the example building is analyzed for a region assigned to SDC C and Load Combination (iii) is revised.	
Step 2: Find the required area of longitudinal reinforcement		
	The same column from Example 2 is checked using the interaction diagram spreadsheet referenced at the start of this example. See Example 2 for a detailed discussion of the calculation of nominal and design interaction diagrams. The diagram on the right is for this column example and it has the required moment and axial force from the four given load combinations plotted (Fig. E3.2).	Design Capacity Interaction Diagram  *Fig. E3.2—Column design interaction diagram.* Eight No. 8 bars evenly spaced around the perimeter is sufficient.

Step 3. Find the required area of transverse reinforcement		
10.5.3.1 22.5 10.6.2	In addition to the detailing requirements of Chapter 18 for IMF, gravity load effects must also be considered. The shear has increased in Load Combination (iii) but it is still very low. The concrete design strength calculated in Example 2 is sufficient to carry the load.	$\frac{\phi V_c}{2} = 27 \text{ kip} \geq V_u = 25 \text{ kip}$
18.4.3.1	For columns in IMFs, there are additional shear requirements. ϕV_n shall be at least the lesser of:	
18.4.3.1(a)	Shear calculated for reverse curvature from the maximum M_n over the range of factored axial loads with lateral forces. The following figure illustrates how to find M_n (National Institute of Standards and Technology (NIST), "Seismic Design of Reinforced Concrete Special Moment Frames: A Guide for Practicing Engineers," NIST GCR 16-917-40) (Fig. E3.4).  *Fig. E3.4—Moment strength over a range of factored axial force.*	The range of nominal moments is shown in the following figure (Fig. E3.3).  *Fig. E3.3—Nominal strength interaction diagram for a range of axial force.*
	Select the point of maximum nominal moment strength between $P_{u,min}$ and $P_{u,max}$. OR	The interaction diagram spreadsheet allows the user to input an axial load value and it will calculate the nominal moment, see the "Select Axial Load" tab. For P_u = 848 kip, M_n = 9373 kip-in. For P_u = 456 kip, M_n = 7994 kip-in. Use, M_n = 9373 kip-in. $V_u = \frac{M_{nt} + M_{nb}}{\ell_u} = \frac{2 \times 9373}{186} = 101 \text{ kips}$
18.4.3.1(b)	Shear with the overstrength factor Ω_o applied.	Ω_o = 3 (Table 12.2-1, ASCE 7) $V_u = 3 \times 25 = 75$ kip **Controls**

10.5.3.1 22.5.8.5.3	Shear reinforcement is required. The equation for shear reinforcement is $V_s = \dfrac{A_v f_{yt} d}{s}$	$\phi V_c/2 = 27\text{ kip} \ngeq V_u = 75\text{ kip}$ **NG** From Example 2 Area of a No. 4 bar = 0.20 in.² A_v = 4 legs × 0.20 = 0.80 in.² d = 24 − 2.5 = 21.5 in. f_{yt} = 60 ksi
10.7.6.5.2 22.5.1.1	When shear reinforcement is required, there is a limit on spacing. The maximum spacing is the lesser of $d/2$ or 24 in. for $V_s \le 4\sqrt{f_c'}b_w d = 4\sqrt{5000} \times 24 \times \dfrac{21.5}{1000} = 145.9$ kip	Assume maximum spacing of $s_{max} = \dfrac{d}{2} = \dfrac{21.5}{2} = 10.75$; use s = 10 in. Calculate V_s assuming that only the hoop placed along column perimeter contributes to shear strength $V_s = \dfrac{0.80 \times 60 \times 21.5}{16} = 64.5$ kips 51.6 < 145.9, thus s_{max} assumption is **OK** $\phi V_n = \phi(V_s + V_c) = 0.75(51.6 + 73) = 93.4$ kip > 75 kip **OK**
10.6.2.2	Check $A_{v,min}$. $A_{v,min}$ shall be at least the greater of: $0.75\sqrt{f_c'}\dfrac{bs}{f_y}$ $50\dfrac{bs}{f_y}$	A_v = 2 legs × 0.2 = 0.4 in.² $\dfrac{0.75 \times \sqrt{5000} \times 24 \times 10}{60{,}000} = 0.21\text{ in.}^2 \le 0.80\text{ in.}^2$ **OK** $\dfrac{50 \times 24 \times 10}{60{,}000} = 0.20\text{ in.}^2 \le 0.80\text{ in.}^2$ **OK**
Step 4: Find the required geometry and spacing of transverse reinforcement		
18.4.3.3 10.7.6.1.2 25.7.4	The tie geometry from Example 2 is acceptable for all locations except in the plastic hinge region, ℓ_o. Section 18.4.3.3 requires that hoops are used in the plastic hinge region, ℓ_o (Fig. E3.5). Hoop geometry is similar to ties but the tie must be closed with seismic hooks at each end. Hoop is further defined in Chapter 2 of the Code. It states that the bend must not be less than a standard 135-degree hook and must have a tail length of at least $6d_b$ or 3 in. Another common tie arrangement is one exterior hoop with one cross tie for the middle bars in each direction. This arrangement could be used if a greater area of shear reinforcement is required because the cross ties could be included in the shear reinforcement area.	Typical section along ℓ_o.  *Fig. E3.5—Column reinforcement details.*
18.4.3.3	The plastic hinge ℓ_o is the greatest of (a) $1/6\ell_u$ (b) h or b (c) 18 in.	Find ℓ_o (a) 1/6 × 186 = 31 in. **Controls.** (b) 24 in. (c) 18 in.

Columns

18.4.3.3	The maximum spacing, s_o, along the plastic hinge region is the smallest of (a) $8d_b$, smallest longitudinal bar (b) $0.5h$ or $0.5b$	Find the maximum spacing for s_o (a) $8 \times 1 = 8$ in. **Controls.** (b) $0.5 \times 24 = 12$ in.
18.4.3.5	Outside ℓ_o the column transverse reinforcement is provided as shown in Example 2 but with a 10 in. spacing for shear	Thus, the transverse reinforcement is No. 4 hoop at 8 in. along ℓ_o and No. 4 ties at 10 in. outside of ℓ_o
Step 5: Beam-column joint design		
15.2 18.4.4	Beam-column joint for this IMF must satisfy design and detailing requirements from both Code Chapters 15 and 18. The column shear for this IMF is larger than that required for the OMF in Example 2. Calculate the required shear and determine if the detailing from Example 2 will work for this design.	
18.4.4.7.2 18.3.4	Calculate column shear force V_u associated with the nominal moments and related shears calculated in accordance with Code Section 18.3.4. The column shear can be approximated with the following equation assuming inflection points occur at the column midheights. $$V_{u,col} = \frac{\left[(M_{nl} + M_{nr}) + (V_{ul} + V_{ur})\dfrac{h}{2}\right]}{\ell}$$ where ℓ is the distance between the mid-height of the column above and below the joint. Note that it is not conservative to ignore the slab for this check, therefore, consider the full effective width, b_f, of the T-Beam in the calculations	Values adapted from Beam Example 1 in Chapter 7 of this Manual. Reinforcement is modified to meet the beam requirements for IMFs. Use seven No. 7 bars in the T-beam flange and three No. 7 bars in the T-beam stem. $M_{nl} = 6520$ kip-in. $M_{nr} = 2960$ kip-in. $V_{ul} = 77$ kip $V_{ur} = 30$ kip Also, $$\ell = \frac{18\text{ ft} + 14\text{ ft}}{2} \times 12\text{ in./ft} = 192\text{ in.}$$ $h = 24$ in. $$V_{u,col} = \frac{\left[(6520 + 2960) + (77 + 30)\dfrac{24}{2}\right]}{192} = 52\text{ kip}$$
	The shear at the center of the joint is $V_j = T_2 + C_1 - V_{u,col}$  *Fig. E3.6—Free-body diagram of beam-column joint*	$T_1 = 3 \times 0.60 \times 60 = 108$ kip $T_2 = 7 \times 0.60 \times 60 = 252$ kip $C_1 = T_1 = 108$ kip $C_2 = T_2 = 252$ kip $V_j = 252 + 108 - 52 = 308$ kip

15.4.2.4	The effective area of the joint, A_j, is determined by multiplying the column depth, h, by the effective width, which is the lesser of (refer to Fig. E2.7). (a) $b_{w,beam} + h_{col}$ (b) $b_{w,beam} + 2x$ (c) b_{col} where x is the smaller distance between the edge of the beam and edge of the column	$b_{w,beam} = 18$ in. $h_{col} = b_{col} = 24$ in. $x = \dfrac{24\text{ in.} - 18\text{ in.}}{2} = 3$ in. Effective joint width (a) 18 + 24 = 42 in. (b) 18 + 2 × 3 = 24 in. (c) 24 in. **Controls** $A_j = 24 \times 24 = 576\text{ in.}^2$
15.4.2.3 15.2.8	Calculate the shear strength of the joint. Table 15.4.2.3 provides V_n for several conditions. Columns and beams are continuous and joint is not confined by transverse beams according to Code Section 15.2.8. $V_n = 20\lambda\sqrt{f_c'}A_j$	$V_n = \dfrac{20 \times 1.0 \times \sqrt{5000} \times 576}{1000} = 814$ kip $\phi V_n = 0.75 \times 814 = 610\text{ kip} > 308$ kip Joint shear strength is sufficient
Step 6: Detail the column splice and joint at Level 2		
18.4.3.4	The required hoop spacing in the plastic hinge region is $s_o = 8$ in. The spacing from the top of slab to the first hoop must be no more than $s_o/2$. Place the first hoop at 3 in. from top of slab similar to example 2	 *Fig. E3.7—Beam-column joint and column reinforcement splice details*
18.4.4.1	Beam-column joints must satisfy the detailing requirements of Codes Sections 15.3.1.2, 15.3.1.3, and 18.4.4.2 through 18.4.4.5, of which only 18.4.4.4 is applicable	
15.3.1.2	Hoops must satisfy the geometric requirements of Code Section 25.7.4	
15.3.1.3	At least two hoops must be provided within the depth of the shallowest beam framing into the joint	
18.4.4.4 18.4.3.3	Hoop spacing in joint must match that of the spacing over the hinge length $s_o = 8$ in. within the height of the deepest beam framing into the joint (Fig. E3.7).	
Step 7: Discussion and summary		
	This example is an extension of Example 2 but the seismic loads were increased for Seismic Design Category C. The difference in design resulted in almost twice as much transverse reinforcement. The requirement to use only hoops resulted in 135-degree hooks instead of 90-degree hooks. The column size and longitudinal reinforcement remained the same.	

Columns Example 4: *Column for a special moment frame (SMF)*

Design and detail the first floor interior column at location E4 from the example building given in Chapter 1 of this Manual. The column is part of an SMF. Example 4 is a continuation of Examples 1, 2, and 3. The loads have been modified to match the results of an analysis from commercial software capable of second-order linear elastic analysis and for Seismic Design Category D.

Given:

Materials—

Specified yield strength, f_y = 60 ksi

Modulus of elasticity of steel, E_s = 29,000 ksi

Specified concrete compressive strength, f_c' = 5 ksi

Modulus of elasticity of concrete, E_c = 4030 ksi

Normalized maximum size of aggregate is 1 in.

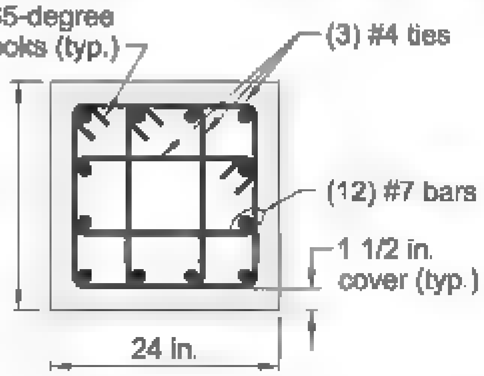
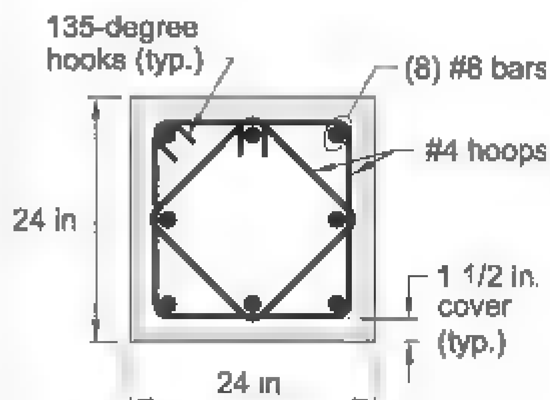


Fig. E4.1—Column section and reinforcement.

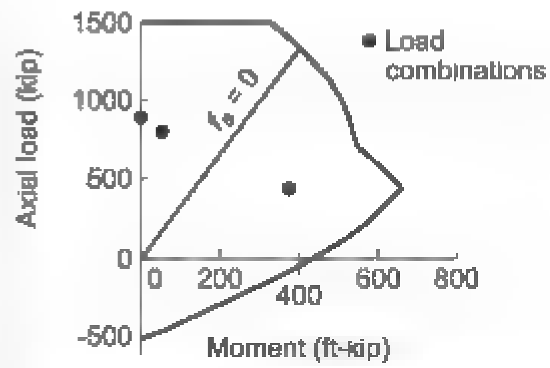
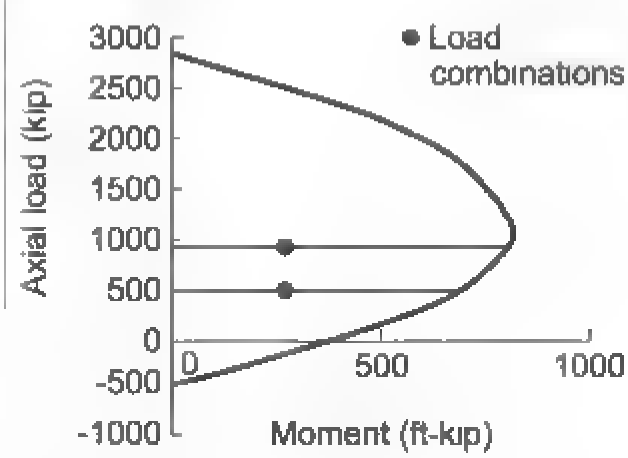
Loading—

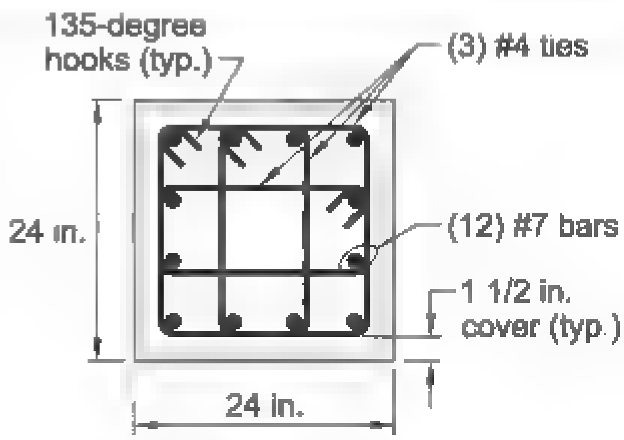
Load combinations	P_u, kip	M_u, kip-in.	V_u, kip
(i) $U = 1.2D + 1.6L + 0.5S$	890	0	0
(ii) $U = 1.2D + 1.0W + 1.0L + 0.5S$	800	651	5
(iii) $U = 1.2D + 1.0E + 1.0L + 0.2S$	872	4491	34
(iv) $U = 0.9D + 1.0E + 1.0L + 0.2S$	432	4491	34

Reference: MNL-17 Supplement: Interaction Diagram Excel spreadsheet found at https://www.concrete.org/MNL172 Download2

ACI 318	Discussion	Calculation
Step 1: Discussion on modification of Example 3 for a special moment frame (SMF)		
	This example demonstrates the column design requirements for an SMF. The additional design and detailing requirements for an SMF increase the response modification coefficient, R, from 5 for an IMF to 8 for an SMF. An SMF is permitted to be used for structures assigned to SDC B and C and required for SDC D, E, and F. For this example, the example building is analyzed for a region assigned to SDC D and Load Combinations (iii) and (iv) are revised. This example starts with the column designed in Example 3.	The following properties are from the intermediate moment frame (IMF) design and are repeated here for information. Column $h = b = 24$ in. Eight No. 8 longitudinal bars No. 4 hoops  *Fig. E4.2—Column section and reinforcement details from Example 3.* Beam $h = 30$ in. $b_f = 120$ in. $b_w = 18$ in. Seven No. 7 long. bars in beam flange Three No. 7 long. bars in beam stem

Step 2: Check dimensional limits and axial load		
18.7.2.1	The column cross section shall satisfy the following: a) The least dimension shall be at least 12 in. b) $b/h \geq 0.4$, where $h \geq b$	$h = 24$ in. ≥ 12 in. **OK** $b/h = 1 \geq 0.4$ **OK**
18.8.2.3(a)	The beam-column joint must be deep enough to prevent excessive slip of the longitudinal beam bars through the column. For normalweight concrete, the joint depth parallel to the beam must be at least 20 times the largest longitudinal bar diameter in the beam.	The largest longitudinal reinforcement in the beam that runs through the joint is a No. 7 $20d_b = 20 \times 0.875 = 17.5$ in. $h = 24 > 17.5$ in. **OK**
18.8.2.3(c)	The depth of the joint shall not be less than one-half the depth of any beam framing into the joint	$\frac{h_{beam}}{2} = \frac{30}{2} = 16$ in. $h_{column} = 24$ in. > 16 in. **OK**
18.7.5.2(f)	The arrangement of the longitudinal reinforcement is affected if $P_u > 0.3A_g f_c'$ or $f_c' > 10{,}000$ psi The value h_x shall not exceed 8 in. if this occurs; otherwise, it shall not exceed 14 in.	$P_u = 872$ kip $A_g = 576$ in.^{2} $872 \text{ kip} > 0.3 \times 576 \times 5 = 864$ kip $5000 \text{ psi} \ngtr 10{,}000 \text{ psi}$ Therefore, h_x shall not exceed 8 in.
	h_x for the cross section in Examples 2 and 3 is $h_x = \frac{[24 - (1.5 \times 2) - (0.5 \times 2) - 1.0]}{2} = 9.5 \text{ in.} \nleq 8 \text{ in.}$ Thus, either the cross-section can be increased or the reinforcement should be rearranged to satisfy this requirement.	The area of steel in the current column is $A_{st} = 8 \times 0.79$ in.^{2} $= 6.31$ in.^{2} Try using 4 bars at each side using No. 7s $A_{st} = 12 \times 0.60$ in.$^2 = 7.20$ in.^{2} $h_x = [24 - (1.5 \times 2) - (0.5 \times 2) - 0.875]/3$ $= 6.4$ in. ≤ 8 in.
	This column is at a lower level and the axial load is slightly greater than $0.3A_g f_c'$. The upper level columns will not exceed this limit. It is therefore recommended to rearrange the reinforcement. The reinforcement for the upper levels can switch back to the previous cross-section if it is found to be more economical	Use twelve No. 7 longitudinal bars evenly spaced around the perimeter

Step 3. Find the required area of longitudinal reinforcement		
10.5.2.1 22.4 22.2 18.7.4.3	Using the Interaction Diagram spreadsheet referenced at the start of this Example, the revised column section is analyzed for the four load combinations given for this example. Code Section 18.7.4.3 limits the development length of longitudinal reinforcement such that $1.25\ell_d \le \ell_u/2$. If this provision is violated, then the development length must be reduced, which is really only practical with a reduction in bar diameter. It is prudent to do a preliminary check of this provision as longitudinal bars are being selected for strength at this stage of the design to avoid major design changes if this provision is checked later in the process.	Design Capacity Interaction Diagram (Fig. E4.3)  *Fig. E4.3—Column design interaction diagram.* Twelve No. 7 bars evenly spaced around the perimeter is sufficient
18.7.4.1	For SMFs, the maximum amount of longitudinal reinforcement, A_{st}, is reduced to $0.06A_g$. The minimum amount of A_{st} stays the same at $0.01A_g$. If lap splices are used, then the limit is reduced to $0.03A_g$.	$0.01A_g = 0.01 \times 24 \times 24 = 5.76\text{ in.}^2$ $0.06A_g = 0.06 \times 24 \times 24 = 34.56\text{ in.}^2$ $A_{st} = 12 \times 0.60\text{ in.}^2 = 7.20\text{ in.}^2$ **OK**
18.7.3.2 18.6.3.2	The flexural strength of column in an SMF must satisfy $\sum M_{nc} \ge \left(\frac{6}{5}\right)\sum M_{nb}$ The beam design is not part of this example. The beam used for the ordinary moment frame (OMF) and IMF does not meet all the requirements for an SMF. The beam is modified here as necessary to fully demonstrate the column and joint design.	From Example 3, the nominal moments in the beam are $M_{nl} = 6520$ kip-in. $M_{nr} = 2960$ kip-in. Add one more No. 7 bar to the bottom of the beam so that the positive moment is at least one-half the negative moment at the joint face. $M_{nl} = 6520$ kip-in. $M_{nr} = 3940$ kip-in.
	M_n is the minimum nominal moment in the range of the interaction curve related to the minimum and maximum axial loads for the seismic load combinations. For more information on how to calculate M_n, refer to Step 2 in Example 3.	Column  *Fig. E4.4—Nominal strength interaction diagram for a range of axial force.*
	The Interaction Diagram Spreadsheet allows the user to input an axial load value and it will calculate the nominal moment, see the "Select Axial Load" tab.	For $P_u = 872$ kip, $M_n = 9720$ kip-in. For $P_u = 432$ kip, $M_n = 8380$ kip-in. Use the lowest value, $M_n = 8380$ kip-in. $(2 \times 8380) \ge \frac{6}{5}(6520 + 3940)$ $16{,}750 > 12{,}550$ **OK**

Step 4. Determine the geometry of the transverse reinforcement		
18.7.4.4 18.7.5.1 18.7.5.5 18.7.5.2	Hoops for rectangular columns or spirals for circular columns are required for the entire column height. The hoops in the plastic hinge zone, ℓ_o, and the splice must meet geometry requirements of Section 18.7.5.2. There are six conditions that geometry of the section must satisfy. The cross section shown in Fig. E4.5 meets these conditions.	 *Fig. E4.5—Column reinforcement details*
	Note that hoops are closed ties with seismic hooks at the end that are at least 135 degrees and a minimum tail length of $6d_b$ or 3 in. Crossties are permitted to support the longitudinal bars between the corners. Where crossties are used they shall be alternated end for end along the longitudinal bar. For this example, every bar needed support. Since there are an even number of bars on each face of the column, overlapping hoops provide the least number of pieces that still provide the necessary support and confinement. Notice that Section 18.7.5.2(f) was checked early in this example. The maximum longitudinal spacing requirements of Section 18.7.5.2(e) and (f) can impact the design of the column as it did in this example.	
Step 5: Determine the maximum spacing of the transverse reinforcement		
18.7.4.4 18.7.5.1 18.7.5.3	The maximum spacing, s_o, along the plastic hinge length, ℓ_o, and splice regions is the smallest of: (a) $0.25h$ or $0.25b$ (b) $6d_b$, smallest longitudinal bar (c) $4+\left(\frac{14-h_x}{3}\right)$ Also, s_o shall not exceed 6 in. and need not be taken less than 4 in.	Maximum spacing for s_o (a) $0.25 \times 24 = 6$ in. (b) $6 \times 0.875 = 5.25$ in. **Controls** (c) $4+\left(\frac{14-6.4}{3}\right) = 6.53$ in.
18.7.5.5	The maximum spacing, s, between ℓ_o and the column splice regions is the smallest of: (a) $6d_b$, smallest longitudinal bar (b) 6 in.	Maximum spacing for s (a) 6 in. (b) $6 \times 0.875 = 5.25$ in. **Controls**
		Use $s = 5.25$ in. along the column height unless noted otherwise from the following checks.

Step 6: Check the minimum amount of transverse reinforcement required along the plastic hinge length, ℓ_o		
18.7.5.4	Since P_u is greater than $0.3A_gf_c'$ as shown in Step 2, A_{sh}/s_ob_c shall be greatest of: (a) $0.3\left(\dfrac{A_g}{A_{ch}}-1\right)\dfrac{f_c'}{f_{yt}}$ (b) $0.09\dfrac{f_c'}{f_{yt}}$ (c) $0.2k_fk_n\dfrac{P_u}{f_{yt}A_{ch}}$ where $k_f=\dfrac{f_c'}{25{,}000}+0.6\geq 1.0$ and $k_n=\dfrac{n_l}{n_l-2}$ n_l is the number of longitudinal bars around the perimeter of the column core that are laterally supported by the corner of hoops or seismic hooks.	$A_{sh}=0.20\text{ in.}^2\times 4=0.80\text{ in.}^2$ $b_c=24-(2\times 1.5)=21\text{ in.}$ $A_g=24\times 24=576\text{ in.}^2$ $A_{ch}=21^2=441\text{ in.}^2$ $k_f=\dfrac{5000}{25{,}000}+0.6=0.8\ngeq 1.0$; use 1.0. $k_n=\dfrac{12}{12-2}=1.2$ (a) $0.3\left(\dfrac{576}{441}-1\right)\dfrac{5}{60}=0.0077$ (b) $0.09\dfrac{5}{60}=0.0075$ (c) $0.2\times 1.0\times 1.2\dfrac{872}{60\times 441}=0.0079$ **Controls** Find the required maximum spacing for this amount of reinforcement $\dfrac{A_{sh}}{s_ob_c}\geq 0.0079$ or $s_o\leq\dfrac{A_{sh}}{0.0079b_c}=\dfrac{0.80}{0.0079\times 21}=4.8\text{ in.}$ Use $s=4.5$ in. along ℓ_o
Step 7: Check the minimum amount of transverse reinforcement required for shear		
18.7.6.1.1	Section 18.7.6.1.1 states "The design shear force V_e shall be calculated from considering the maximum forces that can be generated at the faces of the joints at each end of the column. These joint forces shall be calculated using the maximum probable flexural strengths, M_{pr}, at each end of the column associated with the range of factored axial forces, P_u, acting on the column. The column shears need not exceed those calculated from joint strengths based on M_{pr} of the beams framing into the joint. In no case shall V_e be less than the factored shear calculated by analysis of the structure."	

The first part of Section 18.7.6.1.1 is very similar to how M_n is calculated in Step 3 above, except that $1.25f_y$ is used to generate the interaction diagram (National Institute of Standards and Technology (NIST), "Seismic Design of Reinforced Concrete Special Moment Frames: A Guide for Practicing Engineers," NIST GCR 16-917-40). This modified moment is called the probable moment, M_{pr} (Fig. E4.7).

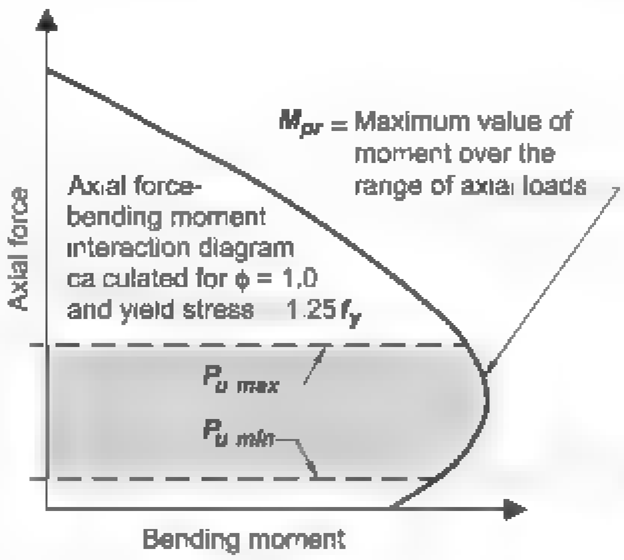


Fig. E4.7—Moment strength over a range of factored axial force.

Using the Interaction Diagram Spreadsheet, generate the curve for f_y = 75 ksi (Fig. E4.6).

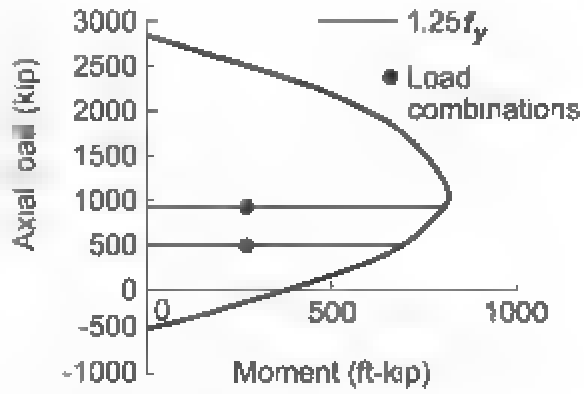


Fig. E4.6—Probable strength interaction diagram for a range of axial force.

Use the "Select Axial Load" sheet in the spreadsheet to find M_{pr}.

For P_u = 872 kip, M_{pr} = 10,284 kip-in.
For P_u = 432 kip, M_{pr} = 9120 kip-in.

Use M_{pr} = 10,284 kip-in.

$$V_e = \frac{M_{prt} + M_{prb}}{\ell_u} = \frac{2 \times 10{,}284}{186} = 111 \text{ kip}$$

The second part of Section 18.7.6.1.1 is similar to the procedure used to determine the column shear in Step 3 of Example 3. Calculate column shear force V_e associated with the probable moments and related shears of the beam. The column shear can be approximated with the following equation assuming inflection points occur at the column mid-heights.

The probable moment strengths and associated beam shear forces are:
M_{pr1} = 8010 kip-in.
M_{pr2} = 4920 kip-in.
V_{e1} = 85 kip
V_{e2} = –22 kip

Also,

$$\ell = \frac{18 \text{ ft} + 14 \text{ ft}}{2} \times 12 \text{ in./ft} = 192 \text{ in.}$$

h = 24 in.

$$V_{e,col} = \frac{\left[\left(M_{pr1} + M_{pr2}\right) + \left(V_{e1} + V_{e2}\right)\frac{h}{2}\right]}{\ell}$$

$$V_{e,col} = \frac{\left[(8010 + 4920) + (85 - 22)\frac{24}{2}\right]}{192} = 71 \text{ kip}$$

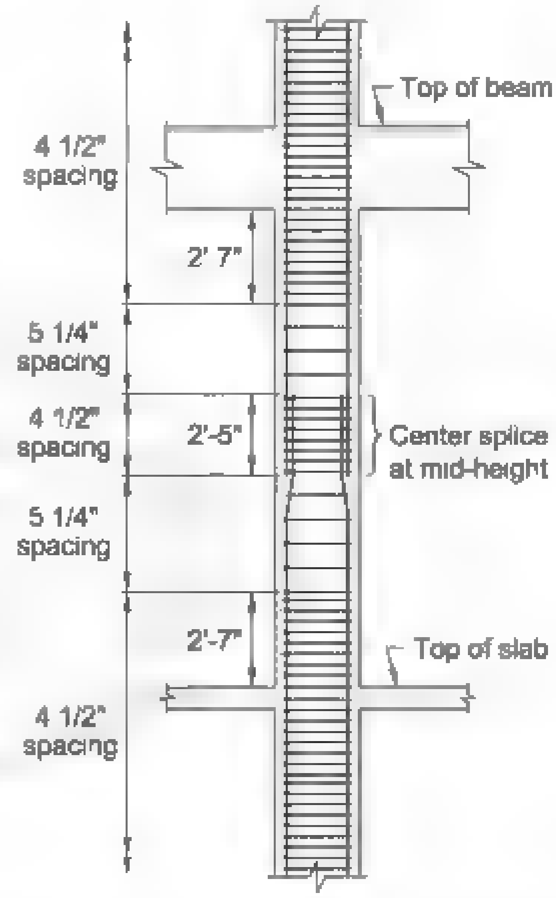
where ℓ is the distance between the mid-height of the column above and below the joint. Note that it is not conservative to ignore the slab for this check; therefore, consider the full effective width, b_f, of the T-beam in the calculations.

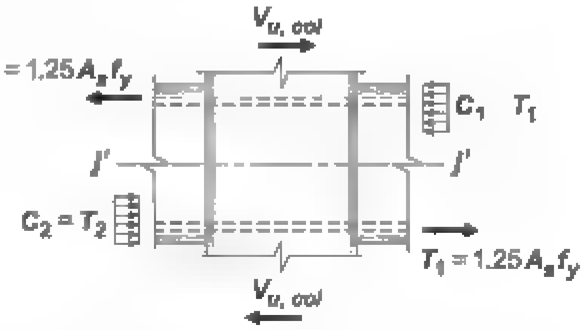
The last part of Section 18.7.6.1.1 states that V_e cannot be less than V_u from the analysis.

The maximum V_u from the load combinations
V_u = 34 kip

Columns

	Although the requirement permits the use of the shear calculated for the second part of Section 18.7.6.1.1, for the purpose of demonstration the shear related to M_{pr} of the column will be used to check the design shear strength of the column.	Use $V_e = 111$ kip
10.5.3.1 22.5.8.5.3	Determine the maximum spacing of hoops required for shear	From Columns Example 2. $V_c = 73$ kip $\phi = 0.75$ Area of No. 4 bar $= 0.20$ in.^{2} $A_v = 4$ legs $\times 0.20 = 0.80$ in.^{2} $d = 24 - 2.44 = 21.56$ in.
22.5.1.1	The equation for shear reinforcement is $V_s = \dfrac{A_v f_{yt} d}{s}$ $\phi V_n = \phi(V_s + V_c) > V_e$	$s = 5.25$ in. from Step 5 $V_s = \dfrac{0.80 \times 60 \times 21.56}{5.25} = 197$ kips $\phi V_n = 0.75(197 + 73) = 203$ kip $> V_e = 111$ kip
Step 8: Summarize the amount and spacing of transverse reinforcements along the height of the column		
18.7.5.1	The plastic hinge length, ℓ_o, is the greatest of: (a) h (b) $1/6\ell_u$ (c) 18 in. ℓ_o is starts at the top of the slab and bottom of the beam and extends toward the middle of the column	Find ℓ_o (a) 24 in. (b) 1/6 x 186 = 31 in. **Controls.** (c) 18 in. From Step 6, $s = 4.5$ in. Along ℓ_o, use No. 4 hoops at 4.5 in.
18.7.4.3	Lap splices shall be placed within the center half of the column. The hoop spacing along the splice length must comply with Sections 18.7.5.2 and 18.7.5.3 but not 18.7.5.4. In the calculation of hoop spacing for ℓ_o regions, 18.7.5.4 controlled but the difference is so small in this case that the same spacing is used	Along the splice, use No. 4 hoops at 4.5 in.
10.7.5.1.3 25.5 25.5.2 10.7.5.2.2 25.4.2.4	ℓ_{st} is the greater of: (a) $1.3\ell_d$ (b) 12 in. where, $\ell_d = \dfrac{3}{40} \dfrac{f_y}{\lambda\sqrt{f'_c}} \dfrac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)} d_b$ and $K_{tr} = \dfrac{40A_{tr}}{sn}$	Determine ℓ_{st} for a No. 7 bar: $\psi_t = \psi_e = \psi_s = \psi_g = \lambda = 1.0$ $d_b = 0.875$ in. $c_b = 1.5 + 0.5 + 0.875/2 = 2.44$ in. $n = 4$, number of bars along the splitting plane $s = 4.5$ in., spacing of ties $A_{tr} = 4$ tie legs $\times 0.2$ in.^{2} $= 0.8$ in.^{2} $K_{tr} = \dfrac{40 \times 0.8}{4.5 \times 4} = 1.78$ $\left(\dfrac{c_b + K_{tr}}{d_b}\right) = \left(\dfrac{1.94 + 1.78}{0.875}\right) = 4.25 < 2.5$ $\ell_d = \dfrac{3}{40} \dfrac{60{,}000}{1.0\sqrt{5000}} \dfrac{1.0}{2.5} 0.875 = 22.3$ in. $\ell_{st} = 1.3 \times 22.3 = 29.0$ in.

18.7.4.3	Limit bar size (and development length) to control potential bond splitting failure of longitudinal bars using the following limit	
	$1.25\ell_d < \ell_u/2$	$1.25(22.3\text{ in.}) = 27.9\text{ in.}$ $< 0.5 \times 186\text{ in.} = 93\text{ in.}$ OK
10.7.5.1.3 25.5 25.5.5.1	For f_y equal to 60 ksi, ℓ_{sc} is the greater of (a) $0.0005 f_y d_b = 30d_b$ (b) 12 in.	Check compression lap splice length $\ell_{sc} = 30 \times 0.875 = 26.25\text{ in.}$
10.7.5.2.1		For ease of construction, use $\ell_{sc} = \ell_{st} = 29\text{ in.}$ for all splices and splice at mid-height at every level
	The end result is hoop spacing of 4.5 in. in ℓ_o regions and along the splice length. The remainder of the column has a spacing of 5.25 in. (Fig. E4.8)	
18.7.4.4 18.2.7 18.2.8	Note that mechanical and welded splices are permitted. Type 1 mechanical splices and welded splices are not permitted within a distance equal to twice the column depth from the bottom of beam or top of slab. Type 2 mechanical splices do not have a location restriction for cast-in-place SMFs.	*Fig. E4.8—Column longitudinal and transverse reinforcement details*
Step 9: Beam-column joint design		
18.8 18.8.2.1	The column shear force at the joint is calculated in Step 7. The column shear is calculated from the probable moment in the beams where the flexural tensile reinforcement is stressed to $1.25f_y$ (Fig. E4.9).	$V_{e,col} = 71\text{ kip}$

	The shear at the center of the joint is $V_u = T_2 + C_1 - V_{u,col}$  *Fig. E4.9—Free-body diagram of beam-column joint*	$T_1 = 4 \times 0.6 \times 60 \times 1.25 = 180$ kip $T_2 = 7 \times 0.875 \times 60 \times 1.25 = 315$ kip $C_1 = T_1 = 180$ kip $C_2 = T_2 = 315$ kip $V_u = 315 + 180 - 71 = 424$ kip
15.4.2.4	The effective area of the joint, A_j, is determined by multiplying the column depth h by the effective width, which is the lesser of (refer to Fig. E2.7): (a) $b_{w,beam} + h_{col}$ (b) $b_{w,beam} + 2x$ (c) b_{col} where x is the smaller distance between the edge of the beam and edge of the column	$b_{w,beam} = 18$ in. $h_{col} = b_{col} = 24$ in. $x = \dfrac{24 \text{ in.} - 18 \text{ in.}}{2} = 3$ in. Effective joint width (a) 18 + 24 = 42 in. (b) 18 + (2 × 3) = 24 in. (c) 24 in. **Controls** $A_j = 24 \times 24 = 576$ in.^{2}
18.8.4.3 15.7.8 21.2.4.4	Calculate the shear strength of the joint. Table 18.8.4.3 provides V_n for several conditions. Columns and beams are continuous and joint is not confined by transverse beams according to Code Section 15.2.8.	$V_n = \dfrac{20 \times 1.0 \times \sqrt{5000} \times 576}{1000} = 814$ kip $\phi V_n = 0.85 \times 814 = 610$ kip > 424 kip Joint is **OK**

Step 10: Discussion and summary

This example is an extension of Examples 2 and 3 but the seismic loads where increased for a Seismic Design Category D. The difference in design resulted in a rearrangement of the longitudinal bars and a different hoop arrangement. The amount of transverse reinforcement is almost 4 times that of an OMF and about twice that of an IMF. The column size was sufficient for all three types of moment-resisting frames.

Columns Example 5:

Redesign column from Example 4 (E4) assuming that the column is not part of the seismic-force-resisting system (carries gravity load only). The loads have been modified to match the results of an analysis from commercial software capable of second-order linear elastic analysis and for Seismic Design Category D.

Given:

Materials—

Specified yield strength, f_y = 60 ksi

Modulus of elasticity of steel, E_s = 29,000 ksi

Specified concrete compressive strength, f'_c = 5 ksi

Modulus of elasticity of concrete, E_c = 4030 ksi

Normalized maximum size of aggregate is 1 in.

Loading—

Load combinations	P_u, kip	M_u, kip-in.	V_u, kip
(i) $U = 1.2D + 1.6L + 0.5S$	890	0	0
(ii) $U = 1.2D + 1.0W + 1.0L + 0.5S$	800	651	5
(iii) $U^* = 1.2D + \delta_u$	872	2204	15
(iv) $U^* = 0.9D + \delta_u$	432	2204	15

*The software adjusts seismic load combinations as required by ASCE/SEI 7.

Reference: MNL-17 Supplement: Interaction Diagram Excel spreadsheet found at https://www.concrete.org/MNL172.Download?

ACI 318	Discussion	Calculation
Step 1: Members not part of seismic-force-resisting system		
4.4.6.5	Current practice is such that a few moment frames are designed and detailed to contribute to the seismic-force-resisting system with the remaining moment frames in the system providing gravity load support only. In reality, however, in monolithically cast-in-place concrete systems, all structural members will participate in the structural response to lateral loads.	
4.4.6.5.1	When conducting the structural analysis of the system, the effect of the structural members assumed not to be part of the seismic-force-resisting system (also known as "nonparticipating members") on the structural response must be considered. Ignoring the contribution of these members may not necessarily be conservative and should be incorporated into the structural analysis.	
4.4.6.5.2	In addition, it is imperative that the nonparticipating columns be detailed to accommodate the story drifts and forces generated as the building sways under the design event while maintaining gravity load strength and stability. Failure to provide this capability has resulted in building collapses in past earthquakes. Code Section 4.4.6.5.2 indicates that the consequences of damage to nonparticipating columns must be considered for SDC B through F.	
4.4.6.5.3	For nonparticipating columns in SDC D through F, the member must meet the applicable requirements of Code Chapter 18.	

Step 2. Analysis of nonparticipating columns		
18.14.2.1	Column must be evaluated for the effect of vertical ground motion acting simultaneously with the design story drift (δ_u). This will be incorporated into the dead load effect in the analysis	
18.14.3.1 18.14.3.2	Analyzing the column for the effects of δ_u is optional. If the column is analyzed for the effects of δ_u and the induced moments and shears do not exceed the design moment and shear strength of the column, then Code Section 18.14.3.2 must be satisfied	
18.14.3.3	If the effects of δ_u are not calculated or if the induced moments or shears exceed their respective design strengths, then Code Section 18.14.3.3 must be satisfied. Detail this column for both cases and compare	
Step 3: If effect of story drift on nonparticipating column is not calculated		
18.14.3.3	Detail nonparticipating column assuming that the the induced moments and shears are not calculated. This is also the procedure in the case where the moment or shear effect, or both, exceed the column strength	
18.14.3.3a	Materials, mechanical splices, and welded splices must satisfy the requirements of special moment frames in Code Sections 18.2.5 through 18.2.8	
18.14.3.3c	Columns must satisfy Code Sections 18.7.4, 18.7.5, and 18.7.6	
18.14.3.3d	Joints must satisfy Code Section 18.4.4	
18.7.4.1	Provide a minimum area of longitudinal reinforcement	Check original column design from Example 2, 24 in. square with eight No. 8 bars A_{st} = 8(0.79 in.^{2}) = 6.32 in.^{2} 0.01(24 in.)^2 = 5.76 in.^{2} < 6.32 in.^{2} 0.06(24 in.)^2 = 34.56 in.^{2} > 6.32 in.^{2} (check 12.64 in.^{2} if lap splices are used) **OK**

18.7.4.3	To control the potential for bond splitting failure, select bar size such that the bar development length satisfies the following equation. $1.25\ell_d \leq \ell_u/2$	
25.4.2.1	Determine required development length using simplified formulas from Table 25.4.2.3 for No. 7 bars and larger -and- Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b.	$d_b = 1$ in. < clear cover $2d_b = 2$ in. < clear bar spacing ~8 in. $\lambda = 1.0$
25.4.2.3	$\ell_d \geq \left(\dfrac{3f_y\psi_t\psi_e\psi_g}{40\lambda\sqrt{f_c'}}\right)d_b$	Bars are oriented vertically $\psi_t = 1.0$
25.4.2.1(b)	$\ell_d \geq 12$ in.	Bars are uncoated $\psi_e = 1.0$
25.4.2.5	ψ_t Casting position factor ψ_e Epoxy coating factor ψ_g Reinforcement grade factor	Bars are Grade 60 $\psi_g = 1.0$ Required development length $\dfrac{3(60{,}000\text{ psi})(1.0)(1.0)(1.0)}{40(1.0)\sqrt{5000}\text{ psi}}(1\text{ in.}) = 63.6$ in. $1.25(63.6\text{ in.}) = 79.5\text{ in.} < 0.5(186\text{ in.}) = 93$ in. **OK**
18.7.4.4	Place lap splice within center half of the column length and enclose with transverse reinforcement	
18.7.5.1	Place transverse reinforcement over length of plastic hinge ℓ_o from each joint face.	Plastic hinge length was calculated for the same column configuration in Example 4. $\ell_o = 31$ in. Hoop size and spacing was calculated for the same column configuration in Example 4. $\ell_o = 31$ in.
Step 4: If effect of story drift on nonparticipating column is calculated		
18.14.3.2	Detail nonparticipating column assuming that the induced moments and shears were calculated and do not exceed the column strength	
18.14.3.2b	Columns must satisfy 18.7.4.1 and 18.7.6	
18.7.4.1	Provide a minimum area of longitudinal reinforcement.	Use final column design from Example 4 24 in. square with twelve No. 7 bars. $A_{st} = 12(0.6\text{ in.}^2) = 7.20\text{ in.}^2$ $0.01(24\text{ in.})^2 = 5.76\text{ in.}^2 < 7.20\text{ in.}^2$ $0.06(24\text{ in.})^2 = 34.56\text{ in.}^2 > 7.20\text{ in.}^2$ (splices are used) (check 14.40 in.^{2} if lap splices are used) **OK**

18.7.6.1.1	Nonparticipating column must be designed for the shear force generated by the frame side sway. To ensure that the design shear force V_e is large enough, the maximum probable moment strength of the columns is calculated using the interaction diagram developed using $1.25f_y$.	Use the results from the shear check in Step 7 of Example 4, which resulted in a maximum hoop spacing of 5.25 in.
18.14.3.2b	Hoops satisfying Code Section 25.7.4 must be provided over the full length of the column with a spacing not to exceed the lesser of $6d_b$ of the smallest enclosed bar and 6 in.	6(0.875 in.) = 5.25 in. < 6 in. **OK**. Hoop spacing limitations also satisfy shear reinforcement requirement. Use maximum spacing of 5.25 in. over full length of column
18.7.5	Determine hoop requirements over plastic hinge length ℓ_o.	
18.7.5.1	Place transverse reinforcement over length of plastic hinge ℓ_o from each joint face.	Use the plastic hinge length that was calculated for the same column configuration in Example 4 ℓ_o = 31 in.
18.7.5.2	Transverse reinforcement configuration requirements for hinge length in nonparticipating columns need only satisfy (a) through (e). Because (f) is not required to be checked, cross ties with alternating 135- and 90-degree hooks can be used in lieu of double hoops.	Use No. 4 overlapping double hoops with seismic hooks to match those of Example 4. All longitudinal bars are supported by a hoop corner.
18.14.3.2c	Where nonparticipating columns resist large axial loads (factored column force greater than $0.35P_o$), transverse reinforcement must satisfy the confinement and bar restraint requirements of Code Table 18.7.5.4.	$P_o = 0.85(5000\text{ psi})[(24\text{ in.})^2 - 7.20\text{ in.}^2] + 60\text{ ksi}(7.20\text{ in.}^2) = 2849\text{ kip}$ $P_u = 872\text{ kip} < 0.35(2849\text{ kip}) = 997.1\text{ kip}$ **OK**. No need to provide detailing noted in Code Table 18.7.5.4
18.14.3.2d	Beam-column joints shall satisfy Code Chapter 15	Use joint detailing developed in Example 2

Step 5: Detailing and discussion

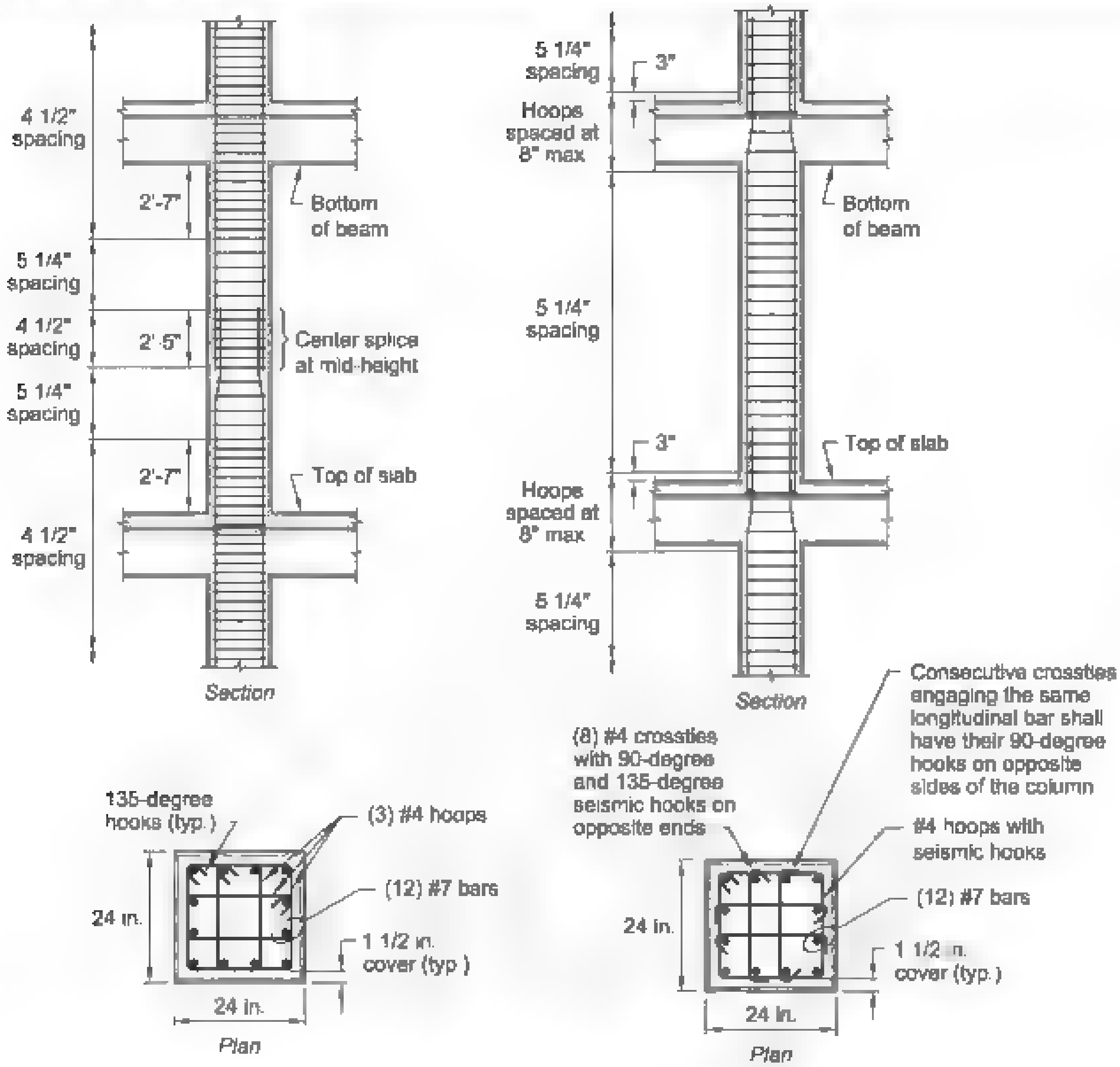


Fig. E5.1—Details of nonparticipating columns with: (a) load effect not considered or when induced moments and shears exceed the column design moment or shear strength; and (b) load effect considered and moments and shears do not exceed the column design moment and shear strength.

Details for a nonparticipating column designed using the two indicated approaches are shown in Fig. 5.1. For buildings in SDC D and higher, the Code allows some reduction in detailing requirements for nonparticipating columns, which are not considered to contribute to the lateral resistance of the structure. The Code allows the designer to choose whether or not to consider the load effect of the design drift on the nonparticipating column.

When the designer chooses not to consider the load effect, the column detailing approaches that of a special moment frame (left figure).

Where the column is designed to resist the effects of the design drift, however, some reduction in detailing requirements is provided. One significant difference is that the splices may be placed at the top or bottom of the column rather than in the middle third of the length. This is true only if shears and moments in columns under displacement δ_u are checked, and the shear and moment do not exceed the design shear and moment. So, in those circumstances it would be allowable to place the splices near the top or bottom of columns.

As a practical matter, however, engineers may find that the design strength of some columns will satisfy the design drift requirements while others will not; this will result in some splices placed at the top of the slab while others must remain in the middle third. Having some column splices at midheight and other splices at the slab within the same story presents practical problems for the contractor and inspectors. Mixing splice locations within a single floor does not really improve efficiency or lower labor costs and may well lead to errors in splice placement, which are very difficult to correct. Furthermore, the locations of columns with midheight splices may change from story to story. Consequently, designers should carefully consider whether taking advantage of Code Section 18.14.3.2 to position column splices just above the floor slab will be the most effective design choice.

CHAPTER 10—STRUCTURAL REINFORCED CONCRETE WALLS

10.1—Introduction

The scope of Code Chapter 11 includes nonprestressed and prestressed cast-in-place and precast reinforced concrete walls. In addition to Chapter 11, Code Section 18.10 provides design and detailing requirements for special cast-in-place walls forming part of the seismic-force-resisting system.

Reinforced concrete structural walls are common in buildings and are typically part of a building's lateral-force-resisting system (LFRS) due to their high in-plane stiffness. Although structural walls are also part of the gravity-force-resisting system, they are often lightly axially loaded. They are designed and detailed to resist the combined effects of gravity and lateral forces.

Walls that are part of the LFRS are commonly referred to as shear walls (ASCE/SEI 7) because they resist a large portion of the total lateral forces acting on the structure through in-plane shear. In the Code, however, all walls, with the exception of cantilever retaining walls, are referred to as structural walls.

The Code and ASCE/SEI 7 have coordinated requirements and identify two categories of LFRS for non-prestressed, cast-in-place walls.

1. An ordinary cast-in-place structural wall, permitted in seismic design categories (SDC) A, B, and C, which is designed and detailed in compliance with Code Chapter 11.

2. A special structural wall is designed and detailed in compliance with Chapters 11 and 18 of the Code. Special walls are required in SDC D, E, and F, but can be constructed in all seismic categories.

Seismic requirements are intended to increase wall strength and ductility to accommodate the large displacement demands expected under the design earthquake load effects. Walls are laterally connected to diaphragms and vertically to foundations or support elements. In seismic and nonseismic design, the connections to diaphragms are designed to remain elastic, and the energy from the lateral forces is dissipated by the structural wall. In seismic design, the connection to the foundation is often the point of maximum wall moment where yielding of vertical reinforcement is expected.

10.2—General

10.2.1 *Distinguishing a column from a wall*—The section geometry of a wall can be so similar to a column that the question of when a rectangular column becomes a wall is often deliberated. For special moment frames, columns are defined as having a minimum aspect ratio of 0.4 in Code Section 18.7.2.1(b). Although this limit is necessary to achieve the expected behavior, it might not be the best limit for the consideration of column or wall design. Expected behavior and construction constraints of the member are often the keys to answering the question of wall or column. Columns usually have high axial loads and their shear behavior is similar to beams. Walls usually have low axial loads and their shear behavior is similar to one-way slabs for out-of-plane loads, with predominantly shear behavior for in-plane loads. For further discussion and information regarding unique shear behavior for in-plane loads in walls, refer to Moehle (2015).

10.2.1.1 *Longitudinal reinforcement*—In general, longitudinal bars require lateral support to prevent buckling of the bars due to axial compression. In a wall, if longitudinal reinforcement is required for axial strength, or if A_{st} exceeds $0.01A_g$, then Code Section 11.7.4.1 states that the longitudinal reinforcement must be supported by transverse ties. This requirement could be used as a practical limit to determine whether the member should be designed as a column or a wall. If the wall requires heavy reinforcement (exceeding $0.01A_g$), a tie bar would be required at every intersection of longitudinal and transverse reinforcement, which would significantly affect the required amount of construction labor. Designing this same member as a column could be more practical.

10.2.1.2 *Shear aspect ratios*—The next limit to consider is shear. Most walls have a length-to-thickness ratio of at least 6. For these aspect ratios, it is easy to see how the shear behavior will differ from a column for either in-plane or out-of-plane loads. For smaller aspect ratios of approximately 2.5 to 6, the member is designed either as a wall or column, depending on the shear force applied and the direction of shear, except as limited by Code Section 18.7.2.1(b). For aspect ratios under 2.5, the member is likely to be designed as a column. Further discussion on this topic is given in Garcia (2003).

10.2.2 *Wall layout*—Shear walls should be located within a building plan to efficiently resist lateral loading. Locating shear walls in the center of each half of the building is generally a good location for resisting lateral forces (Fig. 10.2.2(a)). This arrangement, however, can restrict architectural use of space.

Although shear walls are commonly located at the ends of a building, such wall locations will increase slab restraint and shrinkage stresses, especially in long buildings and buildings such as parking structures that are exposed to large temperature changes (Fig. 10.2.2(b)). Symmetrical wall arrangements provide optimal flexural and torsional stiffness. Walls at the perimeter resist torsional forces most effectively. Walls away from the perimeter, however, could be associated with a higher tributary inertial mass and, consequently, larger axial force from gravity loads may be required to resist uplift or overturning. Walls away from the perimeter are less efficient in resisting horizontal torsion effects.

An unsymmetrical arrangement results in exaggerated torsional response due to eccentricity between the center of mass of the diaphragm and center of stiffness of the shear walls (Fig. 10.2.2(c)). Such a shear wall layout must be designed explicitly for torsion. A symmetrical arrangement is preferable because of the reduced torsional response and to avoid the additional design complications of a torsional analysis.

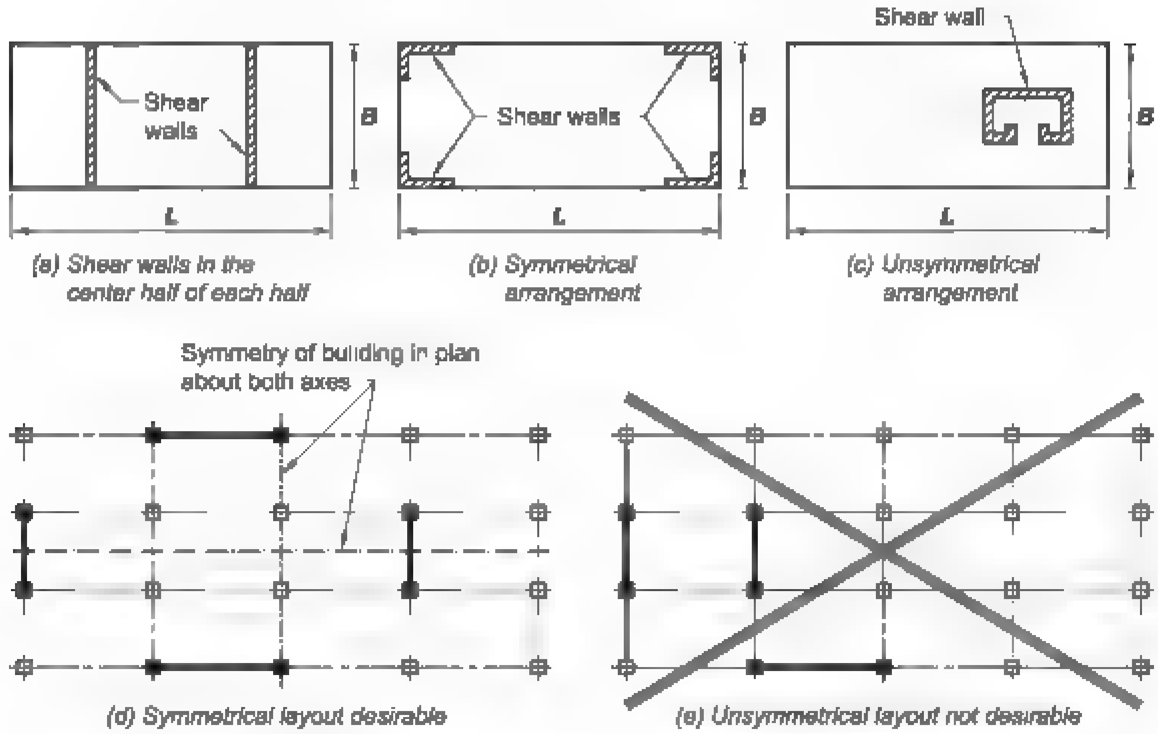


Fig. 10.2.2—Shear wall layouts

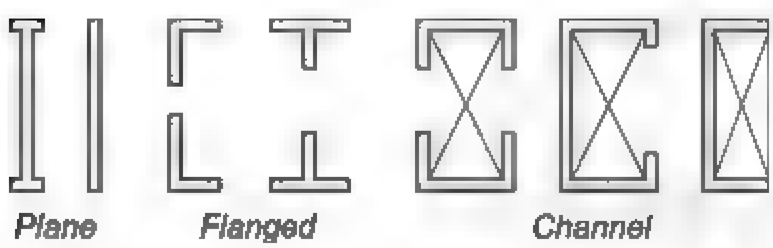


Fig. 10.2.3a—Shear wall cross sections.

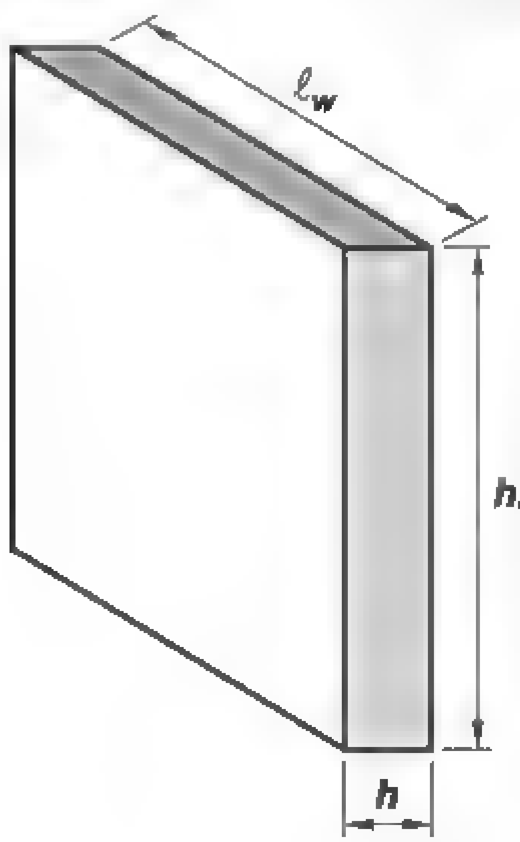


Fig. 10.2.3b—Defining shear wall height, length, and thickness

10.2.3 *Wall configurations*—Shear walls could have several geometric configurations, including plane, flanged, or channel sections. A plane wall section is rectangular with or without enlarged ends (boundary elements). Flanged shear walls are often T- or L-shaped sections. Reinforced concrete walls around building elevator core shafts and stairwells are typically in a C- or U-shape (channel shear wall) (Fig. 10.2.3a).

Notation used to describe the wall dimensions are shown in Fig. 10.2.3b, where h is the wall thickness, h_w is the wall height, and ℓ_w is the wall length.

10.2.4 *Wall type*—The selection of a shear wall type is based on several factors, including functionality, constructability, economy, and seismic performance (Moehle et al. 2011). For low-rise buildings (Fig. 10.2.4(a)), squat, solid walls are predominantly used ($h_w/\ell_w < 2$). As the building increases in height, the wall height to length increases ($h_w/\ell_w > 2$), which increases the slenderness of the walls (Fig. 10.2.4(c)). Perforated walls (Fig. 10.2.4(b)) are acceptable, but depending on a wall's opening percentage, the wall strength could be reduced. A row of vertically aligned openings in a slender wall results in dividing the wall in two sections, termed "coupled walls" because they behave as two individual continuous wall sections connected by coupling beams (Fig. 10.2.4(d)).

10.2.5 *Design limits*—Minimum wall thicknesses are as shown in Table 10.2.5.

For walls designed by the alternative method for out-of-plane loads using Code Section 11.8, ACI 551.2R provides the following suggested slenderness limits for the initial estimate of wall thickness:

(a) One layer of reinforcement at the center of the wall, $h_w/h \leq 50$

(b) Two layers of reinforcement, one at each face of the wall, $h_w/h \leq 65$

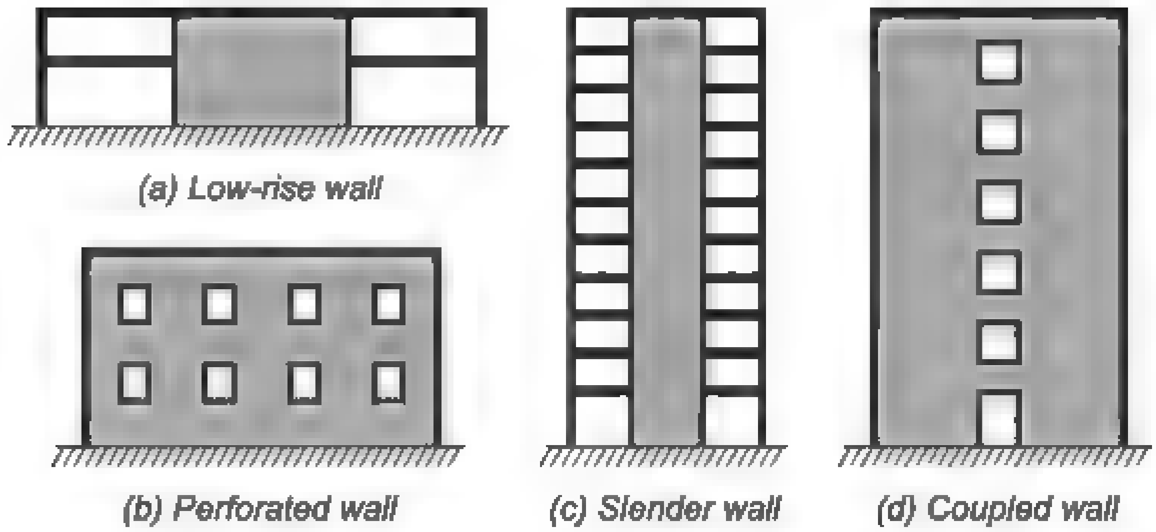


Fig. 10.2.4—Shear wall types

Table 10.2.5—Minimum wall thickness h (ACI 318-14, Table 11.3.1.1)

Wall type		Minimum thickness h	
Bearing*	Greater of:	4 in.	(a)
		1/25 the lesser of unsupported length and unsupported height	(b)
Nonbearing	Greater of:	4 in.	(c)
		1/30 the lesser of unsupported length and unsupported height	(d)
Exterior basement and foundation*	7.5 in.		(e)

*Only applies to walls designed in accordance with the simplified design method of 11.5.3

The Code does not provide separate thickness limits for structural walls resisting in-plane lateral forces. The NEHRP Technical Brief No. 6 (Moehle et al. 2011) suggests the following minimum wall thickness.

(a) Special structural walls: 8 in.
(b) Special structural walls with boundary elements—
  i) Boundary element: 12 in.
  ii) Wall: 10 in.
(c) Coupled shear walls—
  i) Coupling beam designed as a special moment beam: 14 in.
  ii) Coupling beam designed with diagonal reinforcement: 16 in.

While ASCE/SEI 7 provides drift limits for seismic loads, it does not specifically address limits on drift under wind loads, but rather indicate that wind effects should not impair the serviceability of the structure. In general, under wind loads, the relative lateral deflection in any one story should not exceed 1/500 of the story height. In cases where measures are implemented to prevent damage to rigid nonstructural elements, such as cladding, the limit may be increased to 1/400 of the story height.

10.3—Required strength

Chapter 5 in the Code provides the load combinations necessary to design a shear wall for moment, shear, and axial force. Section 12.4 in ASCE/SEI 7 has additional seismic load combinations to consider and load effects, such as the overstrength factor Ω_o.

Table 10.3—Overstrength factor Ω_v at critical section (Code Table 18.10.3.1.2)

Condition	Ω_v	
$h_{wcs}/\ell_w > 1.5$	Greater of	M_{pr}/M_u[1]
		1.5[2]
$h_{wcs}/\ell_w \le 1.5$	1.0	

[1]For the load combination producing the largest value of Ω_v.

[2]Unless a more detailed analysis demonstrated a smaller value, but not less than 1.0.

The in-plane earthquake load effects for shear walls are typically obtained from a lateral load analysis using one of the analysis methods listed in the following, along with the appropriate load factors. In addition, the design shears for special structural walls that are slender ($h_{wcs}/\ell_w > 1.5$) must be amplified by Ω_v to account for flexural overstrength at critical sections where longitudinal reinforcement is expected to yield (Code Section 18.10.3.1.2) (Table 10.3). h_{wcs} is the height of the entire wall above the critical section in inches. The effect that moment overstrength has on the shear forces is illustrated in Fig. 10.3. Because nominal and probable flexural strength will depend on axial force, M_{pr}/M_u will vary for different load combinations. Consequently, all possible combinations should be investigated to ensure that the largest value of Ω_v is obtained. For walls with $h_{wcs}/\ell_w > 2.0$, the design shears must also be amplified by ω_v to account for the dynamic amplification caused by higher vibration modes (Code Section 18.10.3.1.3)

$$\omega_v = 0.9 + \frac{n_s}{10} \qquad n_s \le 6$$
$$\omega_v = 1.3 + \frac{n_s}{30} \le 1.8 \qquad n_s > 6 \qquad (18.10.3.1.3)$$

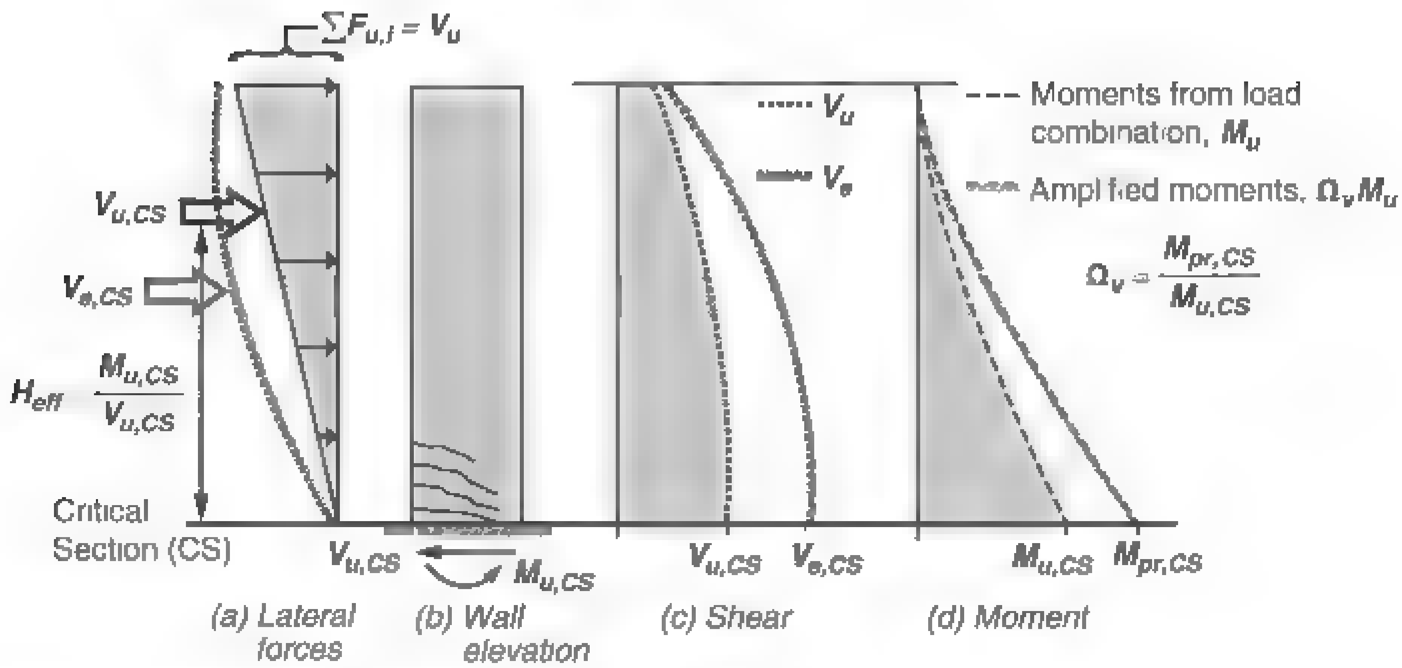


Fig. 10.3—Determination of shear demand for slender walls (Moehle et al. 2011).

Table 10.3.1a—Design coefficients for shear walls in bearing wall systems

Seismic-force-resisting system	Response modification coefficient R	Deflection amplification factor C_d
Special reinforced concrete shear walls	5	5
Ordinary reinforced concrete shear walls	4	4
Intermediate precast shear walls	4	4
Ordinary precast walls	3	3

Table 10.3.1b—Design coefficients for shear walls in building frame systems

Seismic-force-resisting system	Response modification coefficient R	Deflection amplification factor C_d
Special reinforced concrete shear walls	6	5
Ordinary reinforced concrete shear walls	5	4.5
Intermediate precast shear walls	5	4.5
Ordinary precast walls	4	4

where n_s is the number of stories of the wall and shall not be taken less than the quantity $0.007h_{wcs}$. The product of Ω_v and ω_v need not exceed 3.0 (Code Section 18.10.3.1).

10.3.1 *Methods of analysis*—ASCE/SEI 7 allows for three different types of analysis for determining the lateral seismic forces:

1. Equivalent Lateral Force Analysis (ELF): The equivalent lateral force analysis is the simplest method of analysis and is sufficient for many structures using an optional approximate fundamental period T_a, which can be conservative. ASCE/SEI 7 places restrictions on the use of the ELF method in seismic design categories D, E, and F.

Table 10.3.1c—Design coefficients for shear walls in dual systems with special moment frames capable of resisting at least 25 percent of prescribed seismic forces

Seismic-force-resisting system	Response modification coefficient R	Deflection amplification factor C_d
Special reinforced concrete shear walls	7	5.5
Ordinary reinforced concrete shear walls	6	5

2. Modal Response Spectrum Analysis (MRS): The modal response spectrum analysis accounts for the elastic dynamic behavior of the structure and determines the building period. The calculated base shear can be less than the base shear calculated using the ELF method. The base shear, however, should be scaled to a minimum of 85 percent of the ELF base shear.

3. Nonlinear Response History Analysis (NRHA): ASCE/SEI Chapter 16 and Code Appendix A provide requirements for conducting nonlinear response history analysis, which is a form of dynamic analysis in which response of the structure to ground motion data is generated using time-step numerical integration. Material and geometric nonlinear behavior can be incorporated into the response history analysis by modifying the structure's stiffness matrix during the progression of the analysis to account for the changes in element stiffness associated with hysteretic behavior and P-Δ effects. R, C_d, and Ω_o coefficients, which are used in linear procedures, are not applied because nonlinear analysis directly accounts for the effects represented by these coefficients. Due to inconsistencies in the details of the analysis procedures between those of the Code and Chapter 16 of ASCE/SEI 7, the Code

requirements control. ASCE/SEI 7 requires that a linear analysis be conducted in accordance with Chapter 12 as a supplement to the nonlinear response history analysis.

ASCE/SEI 7 Table 12.2-1 provides the required response modification coefficient R and deflection amplification factors C_d required in the analysis. The relevant coefficients are listed for shear walls in Tables 10.3.1a through 10.3.1c.

Sections 6.6, 6.7, and 6.8 in the Code address first-order elastic analysis, second-order elastic analysis, and second-order inelastic analysis, respectively. Section 18.2.2.1 in the Code requires that the interaction of all structural and nonstructural members that affect the linear and nonlinear response of the structure to earthquake motions be considered in the analysis.

For flanged walls, the effective flange width of a wall varies depending on the anticipated deformation level and whether it is in tension or compression. Tests (Wallace 1996) have shown that the effective flange width increases with increasing drift level, and the effectiveness of a flange in compression differs from that for a flange in tension. The value used for the effective compression flange width has little effect on the strength and deformation capacity of the wall; therefore, to simplify design, a single value of effective flange width based on an estimate of the effective tension flange width is used in both tension and compression.

Section 18.10.5.2 in the Code defines the effective flange width that extends from the face of the web of L-, T-, C-, or other flanged sections as the lesser of one-half the distance to an adjacent wall web and 25 percent of the total wall height. The full flange width, and not the effective flange width, may be used in determining the tributary gravity loads that resist uplift (ASCE/SEI 7).

10.4—Design strength

Walls are a versatile building element used in a variety of ways that determine the design approach for the wall. A wall is typically very long in one plan dimension, compared to the orthogonal dimension making the wall slender. This slenderness can control the design if there are large loads applied laterally along the smaller wall dimension h. Loads applied in this direction are commonly called out-of-plane loads. Loads applied laterally along the larger wall dimension ℓ_w are commonly called in-plane loads. Rarely do out-of-plane and in-plane loading have to be considered at the same time, though axial loads are always present. The designer typically designs a wall for the two conditions discussed for axial load with out-of-plane and those with in-plane loads.

10.4.1 *Design for axial load*—Wall design for axial load is similar to column design. The wall slenderness is considered by using the moment magnification method in Code Section 6.6.4 for a first-order analysis, or by using a computer program that accounts for P-Δ effects using a second-order analysis. The design is completed according to Code Section 22.4, which can be quickly evaluated using an interaction diagram generated by software or an electronic spreadsheet. If the resultant of all axial loads is located in the middle third of the wall thickness h, then a simplified equation is permitted in Code Section 11.5.3, where moment

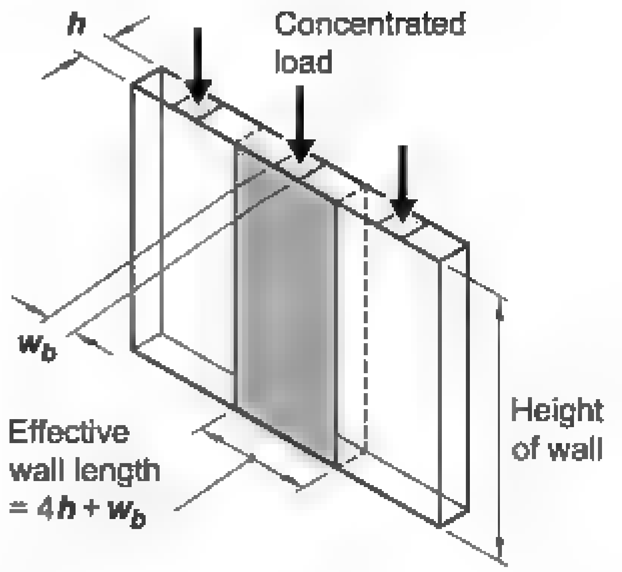


Fig. 10.4.1—Portion of wall considered effective in resisting axial load.

can be ignored and P_n is directly calculated. The wall section considered effective to resist a concentrated gravity load is the width of the bearing plus four times the wall thickness (Fig. 10.4.1). The effective width cannot cross a vertical wall joint unless the joint is designed to transfer the load (Code Section 11.2.3.1).

10.4.2 *Axial and out-of-plane loads*—Walls should be analyzed for combined axial and out-of-plane loads. For walls that are part of a multi-story building lateral load system, combined axial and out-of-plane loads rarely control the design of the wall. The design of walls with primarily out-of-plane loads can be completed according to Code Section 22.4 and slenderness checked using Code Section 6.6.4 for a first-order analysis, or by using a computer program that accounts for P-Δ effects using a second-order analysis.

For a tall one-story building with long shear walls, such as a warehouse, combined axial and out-of-plane loads typically control the wall design. There are many one-story commercial buildings that use exterior concrete walls to support roof loads from the adjacent interior bay and resist lateral out-of-plane and in-plane loads. These buildings are typically 40 to 60 ft in height to allow for rack storage or second-story mezzanines. The wall thickness is usually made as slender as possible for economy. The design of these walls is typically completed in accordance with Code Section 11.8, which is an alternative method of slender wall analysis. Another option is to conduct a finite element analysis (FEA) that accounts for P-Δ effects.

The alternative method has several limitations as stated in Code Section 11.8.1.1. Limits that generally control use of this method:

(a) P_u at midheight cannot exceed $0.06f_c'A_g$

(b) Out-of-plane deflection at service loads cannot exceed $\ell_c/150$

A comprehensive discussion of this method, including its derivation, limitations, use, and worked examples, is given in ACI 551.2R.

10.4.3 *Axial and in-plane loads, squat walls*—Walls are typically part of the LFRS due to their large in-plane stiffness. In squat walls ($h_w/\ell_w < 2$), the predominant wall

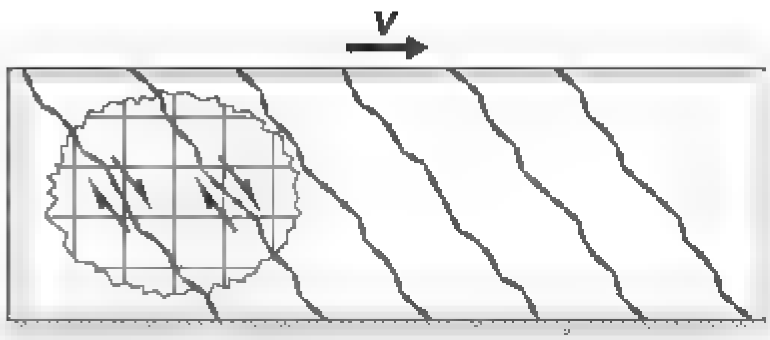


Fig. 10.4.3—Shear in squat walls.

failure mode is diagonal shear. Shear applied to the top of the wall is delivered to the base through compressive struts. Diagonal cracks form along the struts at inclined angles of approximately 38 degrees (Barda et al. 1977), as shown in Fig. 10.4.3. The vertical (longitudinal) reinforcement is mostly effective in resisting this type of shear through shear friction. Separation at the crack engages the vertical reinforcement in tension, creating a clamping force and increased resistance to shear. The vertical reinforcement is fully effective for wall height-to-length ratios (h_w/ℓ_w) of 0.5 or less. As this ratio increases above 0.5, the horizontal (transverse) reinforcement begins to provide a portion of the resistance. Where the height-to-length ratio exceeds 2.5, the horizontal reinforcement provides most of the shear strength and the shear behavior is more like a beam.

This change in shear behavior is accounted for in the minimum reinforcement requirement according to Eq. (11.6.2) of the Code.

$$\rho_\ell \geq 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_t - 0.0025) \quad (11.6.2)$$

where ρ_ℓ shall be at least 0.0025. ρ_t is calculated according to Code Section 11.5.4.8. If this value is less than a reinforcement ratio of 0.0025, then the minimum reinforcement ratio for both horizontal and vertical reinforcement is 0.0025. If the required shear reinforcement exceeds 0.0025, Eq. (11.6.2) of the Code assures that enough longitudinal (vertical) reinforcement is provided for shear-friction resistance in squatter walls. At $h_w/\ell_w < 0.5$, ρ_ℓ computed from Eq. (11.6.2) could exceed ρ_t required by 11.5.4.8. However, Eq. (11.6.2) does not require that ρ_ℓ exceed ρ_t calculated by Code Section 11.5.4.8. At $h_w/\ell_w > 0.5$ and < 2.5, Eq. (11.6.2) provides a minimum amount of longitudinal reinforcement that changes linearly from ρ_t required at an h_w/ℓ_w of 0.5 to h_w/ℓ_w of 2.5. At $h_w/\ell_w > 2.5$, the transverse (horizontal) reinforcement is fully engaged and a minimum a ρ_ℓ of 0.0025 is provided.

For ordinary structural walls, the axial and flexural strength is calculated according to Code Section 22.4, as discussed in 10.4.2 of this Manual. In-plane shear strength V_n is calculated according to Code Section 11.5.4.3. This in-plane shear strength has the same form as the shear strength equation used in Code Section 18.10.4.1 for structural walls resisting seismic loads and is discussed in more detail in the following. According to Code Section 11.5.4.2, the nominal shear strength V_n must be less than

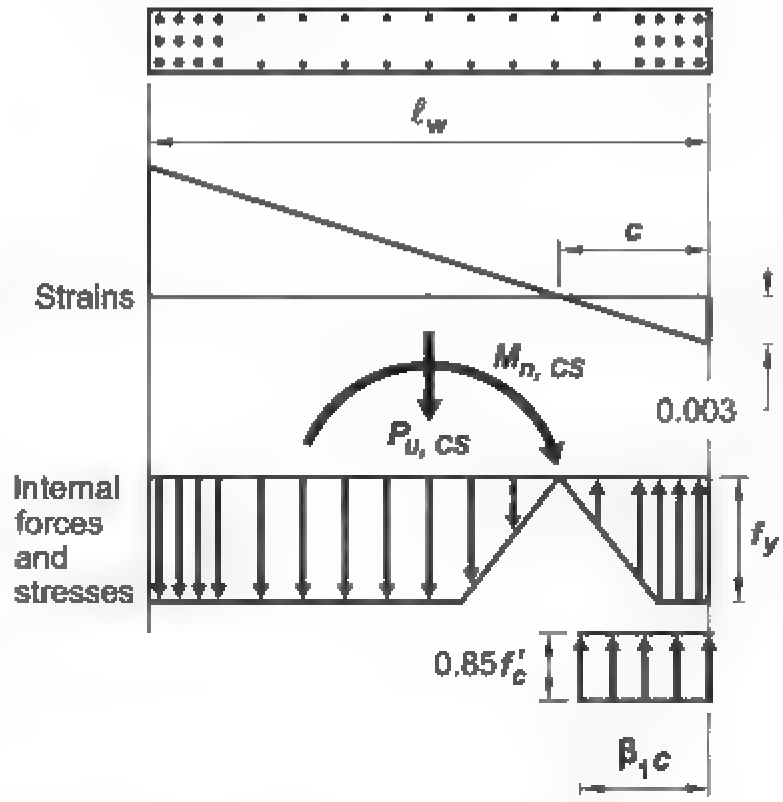


Fig. 10.4.4—Calculation of neutral axis depth c (Moehle et al. 2011).

$$V_n \leq 8\sqrt{f_c'}hd$$

For special structural walls, the axial and flexural strength may also be calculated according to Code Section 22.4, which is discussed in 10.4.2 of this Manual. For walls dominated by flexural action ($h_w/\ell_w > 2.0$), the displacement-based approach of Code Section 18.10.6.2 may be used to determine the need for boundary elements. For all other conditions, the stress-based method of Code Section 18.10.6.3 may be used to determine if boundary elements are necessary. If boundary elements are not necessary, the design of a special structural wall is similar to an ordinary structural wall. A more detailed discussion about boundary elements is given in 10.4.4 of this Manual. Key aspects of squat wall shear design for both special structural walls and ordinary structural walls are:

(a) Shear stress is calculated over A_{cv}. A_{cv} is typically the length of the wall, ℓ_w, multiplied by the width of the wall web, h (Code Section 18.10.4.1).

(b) Minimum reinforcement is approximately the same:

i) For ordinary structural walls, the minimum reinforcement is according to Table 11.6.1 of the Code for V_u less than $0.5\phi V_c$ or $\phi h d\lambda\sqrt{f_c'}$

ii) For special structural walls, minimum ρ_t is according to Table 11.6.1, where V_u does not exceed $\lambda\sqrt{f_c'}A_{cv}$ (Code Section 18.10.2.1). Otherwise, the minimum reinforcement ratio for ρ_t and ρ_ℓ is 0.0025 (Code Section 18.10.2.1).

(c) For special structural walls with $h_w/\ell_w < 2.0$, ρ_ℓ shall be at least ρ_t (Code Section 18.10.4.3).

(d) For V_u greater than $2\lambda\sqrt{f_c'}A_{cv}$ or $h_w/\ell_w > 2.0$, two curtains of reinforcement are required (Code Section 18.10.2.2).

(e) Except at the top of a wall, longitudinal reinforcement must extend at least 12 ft above the point where it is no longer required but need not extend more than the bar development length above the next floor (Code Section 18.10.2.3(a))

(f) At locations where yielding of longitudinal reinforcement is expected, development lengths shall be 1.25 times the values calculated for f_y in tension (Code Section 18.10.2.3(b))

10.4.4 *Axial and in plane loads, slender walls*—The term "slender walls" is used if the predominant failure mode is flexure. Slender walls often require boundary elements, which offer increased flexural strength, enhanced curvature capacity, and better distribution of flexural cracks that promote increased displacement capacity (Moehle 2015). For walls designed only by Code Chapter 11, the design for axial and flexural strength is according to Code Section 22.4, as discussed in 10.4.2 of this Manual. For special structural walls, Chapter 18 of the Code requires special boundary elements (SBE) under specific conditions. There are two methods for determining the need for SBE: the displacement method of Code Section 18.10.6.2 and the stress-based method of Code Section 18.10.6.3. Either method can be used, but the displacement method is the preferred method of design according to the NEHRP Technical Brief No. 6 (Moehle et al. 2011). This method assumes that the wall is effectively continuous from the base of the structure to the top of the wall and that the wall is dominated by flexural action at a critical yielding section.

An SBE is required where

$$\frac{1.5\delta_u}{h_{wcs}} \geq \frac{\ell_w}{600c} \qquad (18.10.6.2a)$$

where δ_u is the design displacement; h_{wcs} is the height of entire wall above the critical section; ℓ_w is length of wall, and c is the largest neutral axis depth calculated for the factored axial force and nominal moment strength consistent with the direction of the design displacement δ_u. The ratio δ_u/h_{wcs} should not be taken less than 0.005. This equation was derived based on the ultimate conditions shown in Fig. 10.4.4 (Moehle et al. 2011). The 1.5 multiplier to δ_u was added in the 2014 Code to more accurately evaluate the deflection of the wall at the maximum considered earthquake. The limit of $\delta_u/h_{wcs} > 0.005$ was modified in the 2014 Code. This lower limit provides increased deformation capacity for a range of stiffer walls that did not previously require boundary elements. This method also has an additional requirement for the termination of the transverse reinforcement at special boundary elements, if required. SBE transverse reinforcement must extend vertically above and below the critical section at least the greater of ℓ_w and $M_u/4V_u$, except at the wall base as noted in Code Section 18.10.6.4(j). At the foundation, the boundary element ties or hoops must extend into the foundation 12 in., or if the edge of the boundary element is within one-half the foundation depth from an edge of the footing, ties or hoops must extend into the foundation or support a distance equal to the development length of the largest vertical bar in the boundary element (Code Section 18.13.2.4). The longitudinal reinforcement of the boundary element must be adequately developed in the foundation.

The stress-based method is used for irregular walls or walls with disturbed regions, for example, around openings, according the NEHRP Technical Brief No. 6 (Moehle et al. 2011). The method requires special boundary elements if the effective compressive stress at the wall ends or around openings exceeds $0.2f_c'$. The boundary elements may be discontinued if the effective compressive stress is less than $0.15f_c'$. The stresses are computed using a model that is based on a linear elastic analysis.

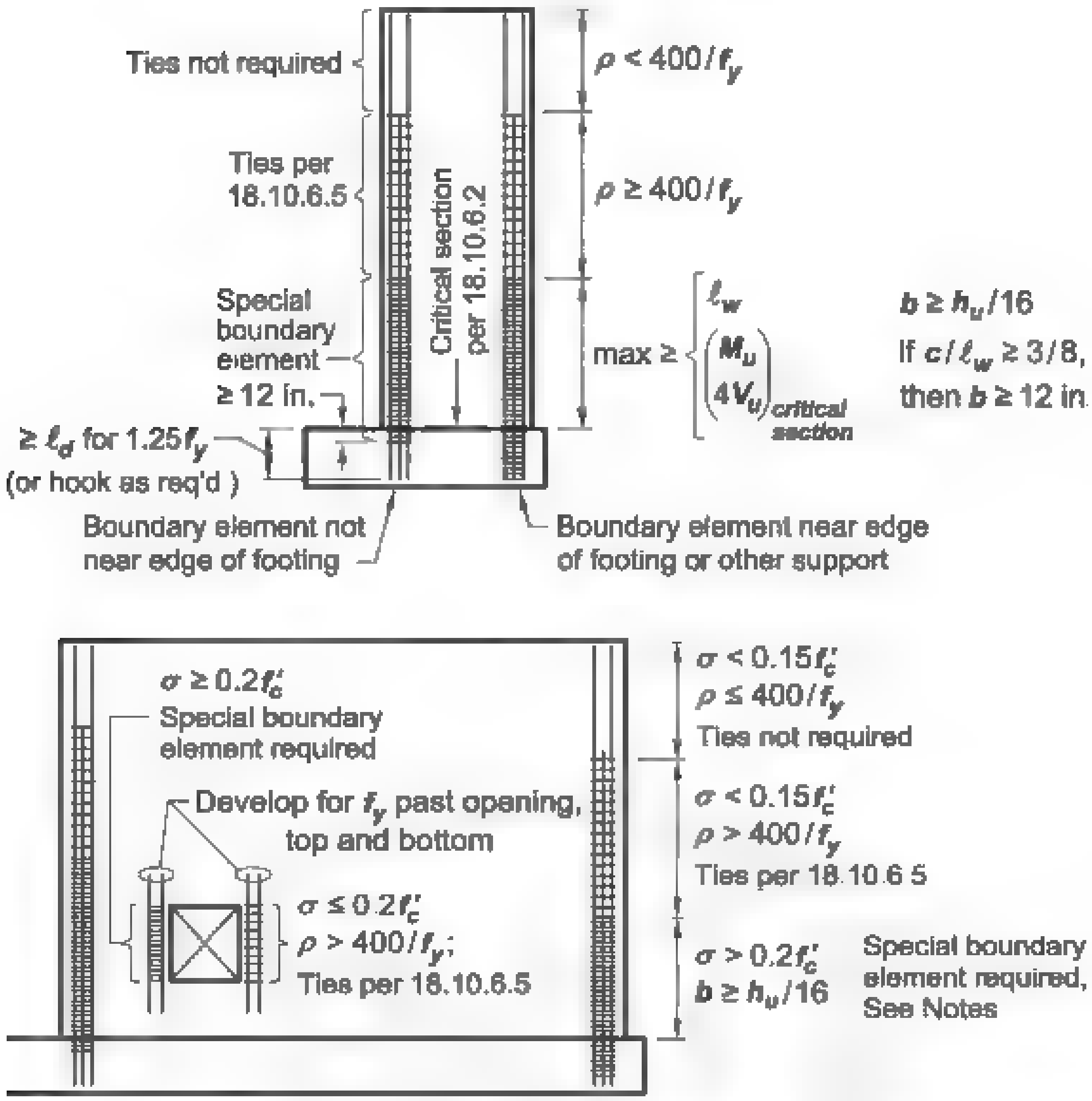
10.4.5 *Special boundary elements*—Special boundary elements (SBE) are required if the limits in Code Section 18.10.6.2 or 18.10.6.3 are not met. SBE is defined in Code Section 18.10.6.4. Where the limits in Code Section 18.10.6.2 or 18.10.6.3 are met, SBE reinforcement is still required as defined in Code Section 18.10.6.5. If an SBE is not required, boundary transverse reinforcement is required if the longitudinal reinforcement ratio at the wall boundary, ρ, exceeds $400/f_y$. Boundary reinforcement is also required for the stress-based method where the effective compressive stress is between $0.15f_c'$ and $0.2f_c'$ according to Code Section 18.10.6.3. The requirements for size and detailing of these requirements are described in Fig. 10.4.5. Horizontal reinforcement in structural walls with boundary elements must be anchored into the core of the boundary element with hooks, headed bars, or straight embedment and also must extend to within 6 in. of the end of wall (Code Section 18.10.6.4k)

10.4.6 *Vertical wall segments and wall piers*—A vertical wall segment is any portion of a wall that is bounded by the outer edge of a wall and an edge of an opening, or the portion of a wall bounded horizontally by the vertical edges of two openings. Wall piers are vertical wall segments. According to Chapter 11 of the Code, the design of nonseismic vertical wall segments is the same as that of walls. For special structural walls designed according to Chapter 18 of the Code, there are additional requirements. The nominal shear strength is reduced for the total cross section of a wall at the vertical wall segments. The calculated V_n may not exceed $8\sqrt{f_c'}A_{cv}$ for the total A_{cv}, as shown in Fig. 10.4.6. For an individual vertical wall segment, V_n may not exceed $10\sqrt{f_c'}A_{cv}$.

Vertical wall segments are designed as walls, columns, or wall piers according to the segment geometry as summarized in Table 10.4.6. In many cases, the special structural wall requirements apply. If the wall segment is designed as a column, Code Section 18.10.8.1 requires that the special detailing of Code Sections 18.7.4, 18.7.5, and 18.7.6 for columns be applied. Wall piers are a subset of vertical wall segments as defined in Table 10.4.6. They may be designed as special columns or by alternative requirements given in Code Section 18.10.8. Wall piers designed to these alternative requirements require

(a) V_e that can develop M_{pr} at the ends of the column or Ω_o times the factored shear determined by analysis (Code Sections 18.7.6.1 and 18.10.8.1)

(b) Hoops at a spacing not greater than 6 in.



Notes: Requirement for special boundary element is triggered if maximum extreme fiber compressive stress $\sigma \geq 0.2f_c'$. Once triggered, the special boundary element extends until $\sigma < 0.15f_c'$. Since $h_w/\ell_w \leq 2.0$, 18.10.6.4(c) does not apply.

Fig. 10.4.5—Summary of boundary element requirements for special walls (Fig. R18.10.6.4.2 of ACI 318-14).

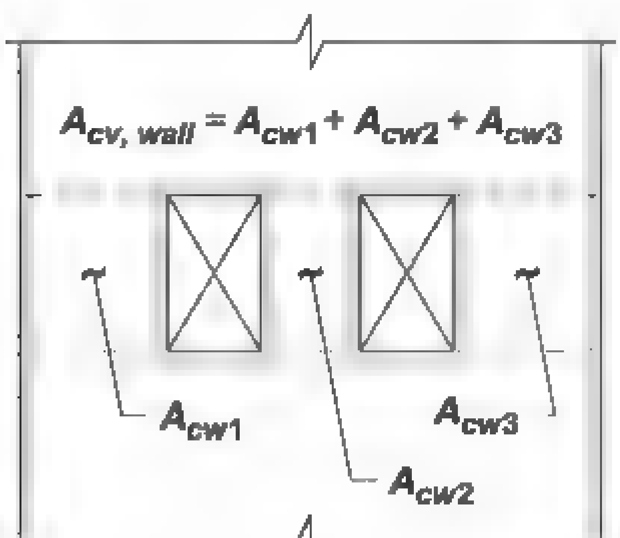


Fig. 10.4.6—Shear strength for vertical wall segments.

(c) Checking to see if the pier should include a special boundary element

(d) Horizontal reinforcement above and below the wall pier to transfer the design shear into the adjacent wall segments

Table 10.4.6—Governing design provisions for vertical wall segments* (Table R18.10.1 in the Code)

Clear height of vertical wall segment/length of vertical wall segment (h_w/ℓ_w)	Length of vertical wall segment/wall thickness (ℓ_w/b_w): $(\ell_w/b_w) < 2.5$	$2.5 < (\ell_w/b_w) < 6.0$	$(\ell_w/b_w) > 6.0$
$h_w/\ell_w < 2.0$	Wall	Wall	Wall
$h_w/\ell_w \geq 2.0$	Wall pier required to satisfy specified column design requirements; refer to Code Section 18.10.8.1	Wall pier required to satisfy specified column design requirements or alternative requirements; refer to Code Section 18.10.8.1	Wall

*h_w is the clear height, ℓ_w is the horizontal length, and b_w is the width of the web of the wall segment.

10.4.7 *Horizontal wall segments and coupling beams*—A horizontal wall segment is any portion of a wall that is bound by the outer edge of a wall and an edge of an opening, or the portion of a wall bound by the horizontal edges of two

openings. According to Chapter 11 of the Code, the design of nonseismic horizontal wall segments is the same as that of walls. Horizontal wall segments in special structural walls are designed as special structural walls according to Chapter 18 of the Code. If horizontal wall segments are part of a coupled special structural wall system, the segment is called a coupling beam. Three categories of coupling beams are defined in the Code:

(a) If $\ell_n/h \geq 4$, the coupling beam is designed as a beam in a special moment frame

(b) If $\ell_n/h < 2$ and $V_u \geq 4\lambda A_{cv}$, the beam is designed with diagonally placed bars for a more effective transfer of shear through the member

(c) For other cases, the beam may be designed either as a special moment frame beam or with diagonally placed bars

The design of a coupling beam is beyond the scope of this Manual. For more information, reference Moehle et al. (2011) and Moehle (2015).

10.5—Detailing

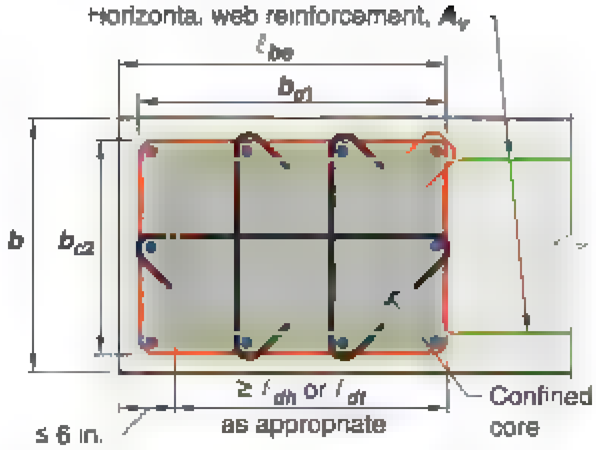
Structural walls are, in general, thin, tall, wide members with reinforcement in both the horizontal (transverse) and vertical (longitudinal) directions. Properly designed and detailed shear walls in buildings have resisted seismic forces and sidesway effectively in past earthquakes.

If shear walls are the only members in the LFRS, they usually behave as cantilever beams, fixed at the base. They transfer moments, shear, and axial forces to the foundation. If the LFRS includes a stiff frame, the wall could behave more like a column, depending on relative stiffnesses and shear wall locations. In such cases, the shear wall will usually collect the large majority of shear, but the shear wall moments may be much less due to frame action.

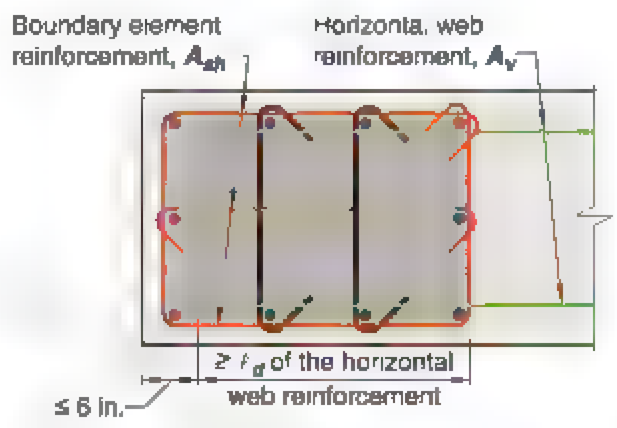
Reinforcement placed in the horizontal and vertical directions resists in-plane shear forces and limits cracking. For taller walls, the vertical reinforcement also serves as flexural reinforcement. If significant moment strength is required, additional reinforcement is placed at the ends of a wall or within boundary elements (Fig. 10.5a and 10.4.5).

Although one curtain of reinforcement is permitted for ordinary shear walls with thickness of 10 in. or less (Code Section 11.7.2.3), two curtains of reinforcement are recommended where possible. It is advantageous to place the transverse reinforcement as the exterior layer to prevent longitudinal reinforcement from buckling and to provide better confinement to the concrete. The casting position of transverse reinforcement assumes more than 12 in. of concrete below the bar for the calculation of development and splice lengths (ψ_t = 1.3).

In ordinary structural walls, the first splice of vertical reinforcement typically occurs immediately above the foundation, where wall longitudinal reinforcement laps with foundation dowel bars. These dowels, lapped with the wall bars, provide the critical mechanism of transferring tension and shear forces from the wall to the foundation. The dowels should extend into the foundation with enough depth to be fully developed for tension. For constructability purposes, dowels with 90-degree hooks should extend to the bottom of



(a)
Option with standard hooks or headed reinforcement



(b)
Option with straight developed reinforcement

Fig. 10.5a—Development of wall horizontal reinforcement in confined boundary element.

the foundation where they can be tied firmly to the foundation bottom reinforcement.

In special structural walls, lap splices are prohibited in boundary regions near critical sections where longitudinal reinforcement is expected to yield as a result of lateral displacements (Fig. 10.5b). They must be avoided for a distance ℓ_d below the critical section and the lesser of h_{sx} or 20 ft above the critical section.

Structural walls in SDC D, E, or F have additional detailing requirements outlined in Section 18.10.2.3 of the Code. Wall longitudinal (vertical) reinforcement must be developed by ensuring that the bars continue past the point at which they are no longer required such that they are fully developed. In previous codes, the reinforcement was extended past the cutoff point a distance of $0.8\ell_w$, which is an approximation of d for in-plane bending of the wall. This approach was analogous to extending bars d past the theoretical cutoff point in beams and is based on the "tension-shift" that can occur as a result of the influence of shear cracking on moment. Structural walls, however, behave as deep beams and extending reinforcement for $0.8\ell_w$ could reach over several floors, which was deemed overly conservative. The Code now indicates that bar termination should be specified so that bars extend above elevations where they are no longer required to resist design flexure and axial force (Fig. 10.5c). Bars are required to extend ℓ_d above the next floor level or no more than 12 ft for cases with large story heights. Bar terminations should be placed well away from critical sections where

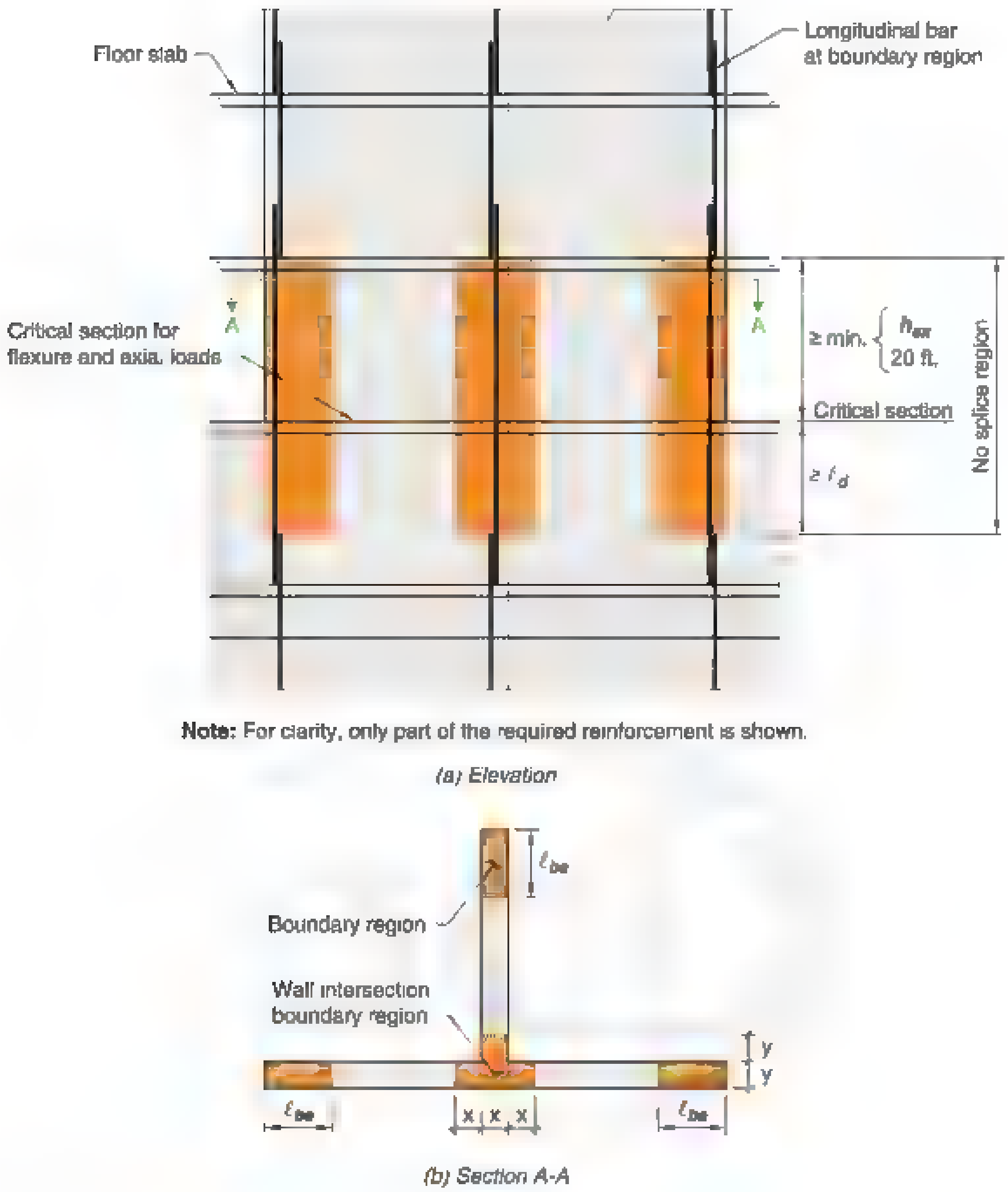


Fig. 10.5b—Wall boundary regions where lap splices are not permitted (Code Fig. R18.10.2.3).

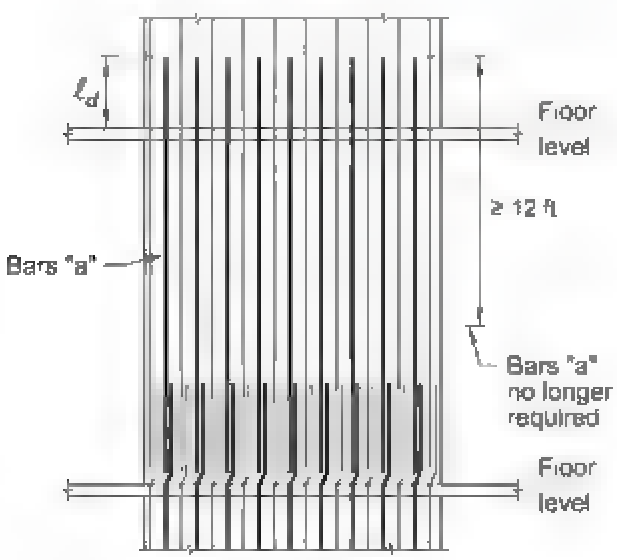


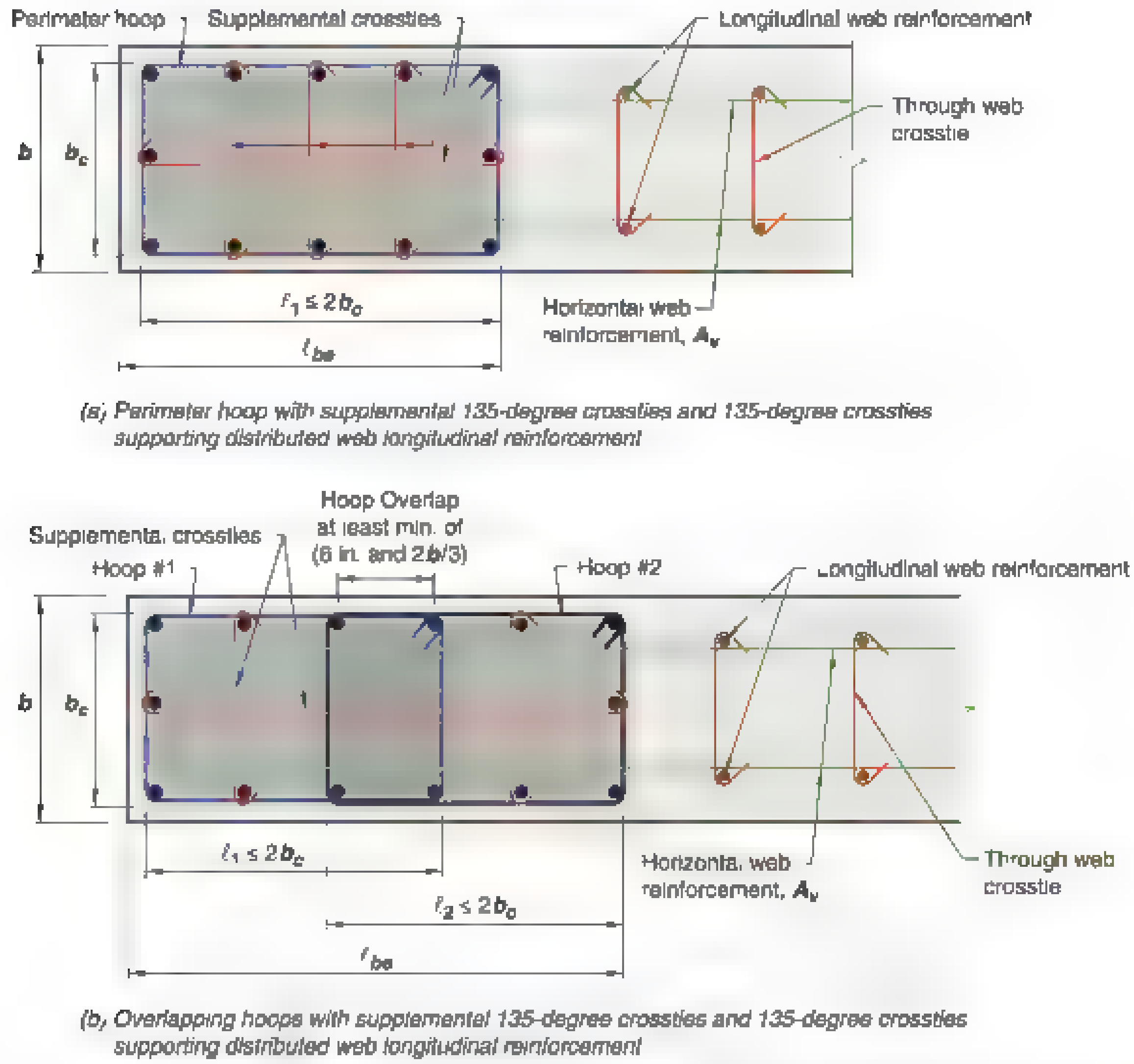
Fig. 10.5c—Termination of longitudinal wall reinforcement.

yielding of longitudinal reinforcement is expected (usually at the base of the wall) and should be accomplished in a gradual manner over the wall height.

At locations where longitudinal reinforcement is likely to yield, the bars must be developed or spliced for an assumed yield stress of $1.25f_y$.

Where special boundary elements are not required and where the longitudinal reinforcement ratio exceeds $400/f_y$, the boundary transverse reinforcement must satisfy most of the requirements for special moment frame columns (Code Sections 18.7.5.2(a) through (e)). In addition, the vertical spacing of the transverse reinforcement must be in accordance with Table 18.10.6.5(b). The spacing of the transverse reinforcement is smaller at the base of the wall for a distance ℓ_w or $M_u/4V_u$. The required spacing decreases as the yield strength of the longitudinal reinforcement increases from 60 to 100 ksi. Above this region, the spacing widens until the reinforcement ratio drops below $400/f_y$, where transverse reinforcement is not required (Fig. 10.4.5).

Where special boundary elements are required, the boundary transverse reinforcement must satisfy most of the requirements for special moment frame columns—Code



(a) Perimeter hoop with supplemental 135-degree crossties and 135-degree crossties supporting distributed web longitudinal reinforcement

(b) Overlapping hoops with supplemental 135-degree crossties and 135-degree crossties supporting distributed web longitudinal reinforcement

Fig. 10.5d—Boundary transverse reinforcement requirements for special boundary elements (Code Fig. R18.10.6.4a).

Sections 18.7.5.2(a) through (d) and 18.7.5.3—except that Section 18.7.5.3(a) is to be one-third of the least dimension of the boundary element. In addition, Code Section 18.10.6.4(f) must be used to detail transverse reinforcement as illustrated in Fig. 10.5d.

10.6—Summary

Structural walls have two main advantages:

1. They are relatively easy to construct because reinforcement detailing of walls is straightforward.

2. Because of their inherent stiffness, they usually minimize sway and damage in structural and nonstructural elements, such as glass windows and building contents, for buildings exposed to high lateral loads.

Structural walls have disadvantages, including these two:

1. Shear walls can create interior barriers that interfere with architectural and mechanical requirements.

2. Shear walls carry large lateral forces resulting in the possibility of large overturning moments. Attention is required at the wall-foundation interface and foundation design.

REFERENCES

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Authored documents

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Garcia, L. E., 2003, "Concrete Q&A: 318-02 Questions," *Concrete International*, V. 25, No. 10, Oct., p. 120.

Moehle, J. P., 2015, *Seismic Design of Reinforced Concrete Buildings*, McGraw-Hill Education, New York, 760 pp.

Moehle, J. P.; Ghodsi, T.; Hooper, J. D.; Fields, D. C.; and Gedhada, R., 2011, "Seismic Design of Cast-in-Place Concrete Special Structural Walls and Coupling Beams: A Guide for Practicing Engineers," *NEHRP Seismic Design Technical Brief No. 6*, National Institute of Standards and Technology, Gaithersburg, MD, 37 pp.

Wallace, J. W., 1996, "Evaluation of UBC-94 Provisions for Seismic Design of RC Structural Walls," *Earthquake Spectra*, V. 12, No. 2, May, pp. 327-348. doi: 10.1193/1.1585883

10.7—Examples

Shear Wall Example 1: *Seismic Design Category B wind*: The reinforced concrete shear wall in this example is nonprestressed. This shear wall is part of the lateral force-resisting-system (a shear wall is at each end of the structure) in the North-South (N-S) direction of the hotel (Fig. E1.1). Material properties are selected based on the code limits and requirements of Chapters 19 and 20 (ACI 318), engineering judgment, and locally available materials. The structure is analyzed for all required load combinations by an elastic 3D finite element analysis software model that includes shear wall - frame interaction. The resultant maximum factored moments and shears over the height of the wall are given for the load combination selected. This example provides the shear wall design only at the base.

<u>Given:</u>
P_u = 1015 kip

In-plane—
V_u = 235 kip
M_u = 18,600 ft-kip

Out-of-plane—
V_u = 16 kip
M_u = 60 ft-kip

Material properties—
f_c' = 5000 psi
f_y = 60,000 psi

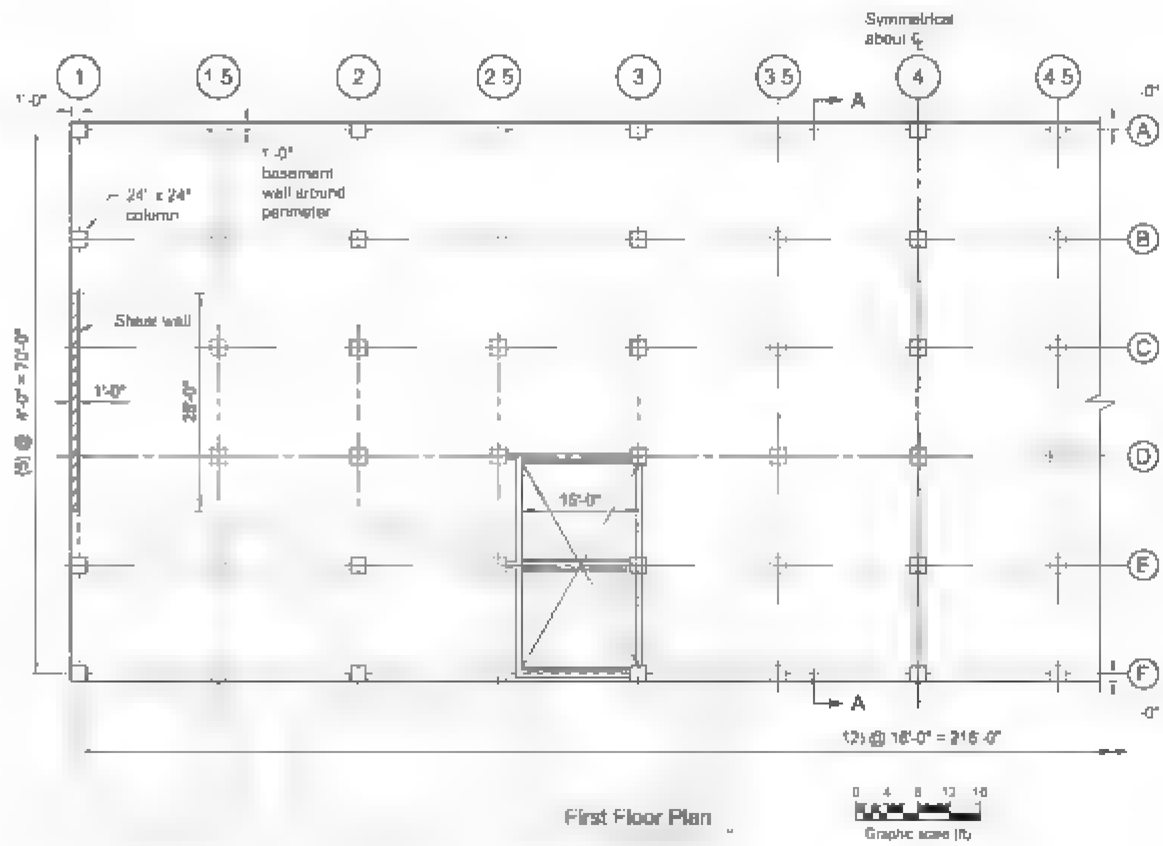


Fig. E1.1—Building floor framing plan: first floor.

This example uses the Interaction Diagram spreadsheet aid found at https://www.concrete.org/MNL1721Download2

ACI 318	Discussion	Calculation
Step 1: Geometry		
11.3.1	This wall design example follows the requirements of Code Section 11.5.2, and, therefore, the wall thickness does not need to meet the requirements of Table 11.3.1.1 (ACI 318). The thickness equations (a) and (b) of Table 11.3.1.1 can, however, provide an indication that the thickness chosen is appropriate.	
	From Table 11.3.1.1, the wall thickness must be at least the greater of 4 in. and the lesser of 1/25 the lesser of the unsupported height of the wall (18 ft for the first elevated floor) and the unsupported length of the wall (28 ft from end to end of the wall).	The unsupported height controls, 18 ft < 28 ft $h_{req'd}$ = (18 ft)(12 in./ft)/25 = 8.64 in. Example shear wall h = 12 in. > $h_{req'd}$ = 8.64 in. **OK**
20.5.1.3.1	A 12 in. wall is used in this design and the wall is assumed to be exposed to weather on the exterior of the structure. Concrete cover is 1-1/2 in., which is in accordance with Table 20.5.1.3.1 (ACI 318).	

Step 2. Loads, load patterns, and analysis of the wall		
11.4 6.9	The structure is analyzed using the assumptions and requirements of 11.4	
	The structure was analyzed using 3D elastic Finite Element Analysis (FEA) software that follows the analysis requirements of Section 11.4 and Chapters 5 and 6 of ACI 318 for loading and analysis, respectively refer to Fig. E1.2 and E1.3 for in-plane flexure and in-plane shear along the height of the wall, respectively	The maximum factored axial force, flexural moment, and shear force at the base of the wall are listed in the given section at the start of this example

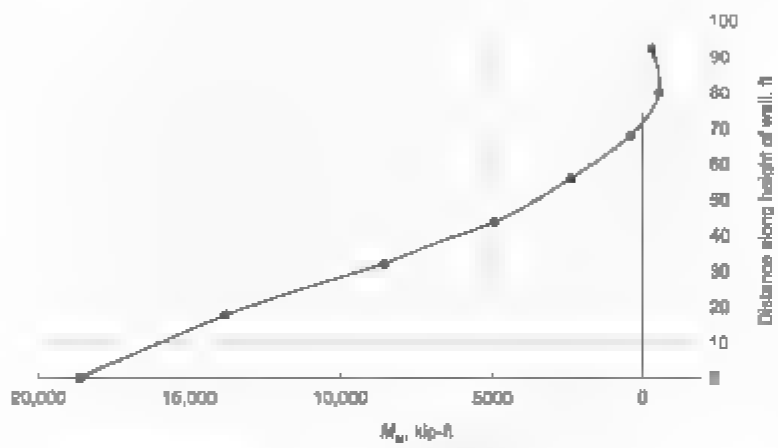


Fig. E1.2—In-plane flexure along the height of the wall

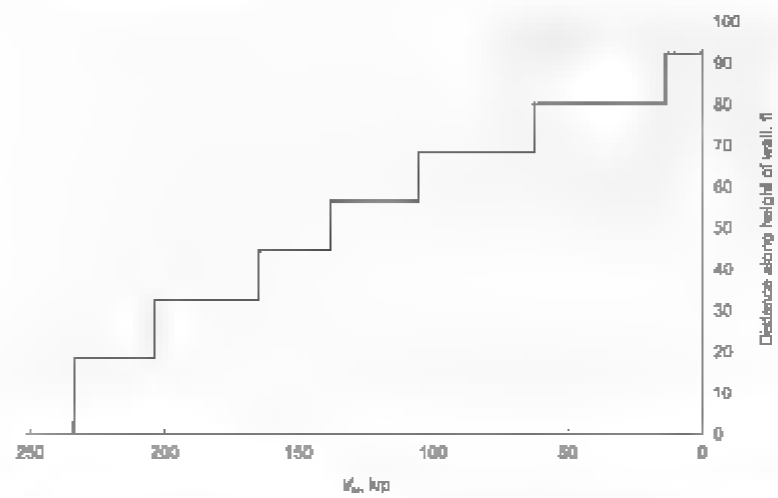


Fig. E1.3—In-plane shear along the height of the wall

Step 3: Concrete and steel material requirements		
11.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318)	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied.
	The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications.	Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi
	There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor	

11.2.1.2	The reinforcement must satisfy Chapter 20 of ACI 318. The designer determines the grade of bar and if the bar should be coated by epoxy, galvanized, or both.	By specifying the reinforcement grade and any coatings, and that the reinforcement shall be in accordance with ACI 301, Chapter 20 of ACI 318 requirements are satisfied. In this case, assume Grade 60 bar and no coatings.
Step 4. Axial and flexural design strength		
11.5 11.5.2	The combined axial and flexural design strength of a shear wall can be determined using an interaction diagram similar to a column interaction diagram.	An initial interaction diagram is made using No. 5 bars at 12 in. spacing throughout the wall (refer to *Note below). It is assumed that all of the longitudinal reinforcement is effective in resisting in-plane flexure.
	The wall interaction diagram is generated using the Interaction Diagram spreadsheet (link in the given section of this example). Refer to Column Example 9.2 in this Manual for additional information about the Interaction Diagram spreadsheet.	The first pair of No. 5 bars is assumed to be at 3 in. from the end of the wall, the second pair is placed at 12 in. from the end of the wall, and the remaining pairs at 12 in. spacing. The different spacing at the end of the wall is to allow for cover on the end pairs of bars in the wall and to force the reinforcement to be symmetrical. The reinforcement is symmetrical about the center of the wall and this bar layout is applied to both ends of the wall.
	To estimate an initial reinforcement area, the wall is assumed to be a cantilever and the amount of flexural reinforcement necessary to resist the moment is calculated.	Fig. E1.4 shows the resulting design strength interaction diagram. The design strength interaction diagram includes the ϕ-factor. The Interaction Diagram spreadsheet contains a sheet named "Select Axial Load." When the user enters a P_n, the sheet calculates the associated maximum M_n on the interaction diagram curve and plots a point on the interaction diagram to show that point. This point is named the "Input Point" on the interaction diagram. The input point of P_n corresponding to a P_u of 1015 kips calculates a point on the design strength interaction diagram M_u of 24,600 ft-kip. The input point is plotted as a solid triangle. The open triangle indicates where the example load resultants are and shows that this iteration does satisfy required strength. Further iterations are unnecessary.
	*Note: For constructability, No. 5 bars are selected for the vertical reinforcement. Smaller bars will work for the strength of the wall, but are often too flexible to efficiently work with in a vertical position.	

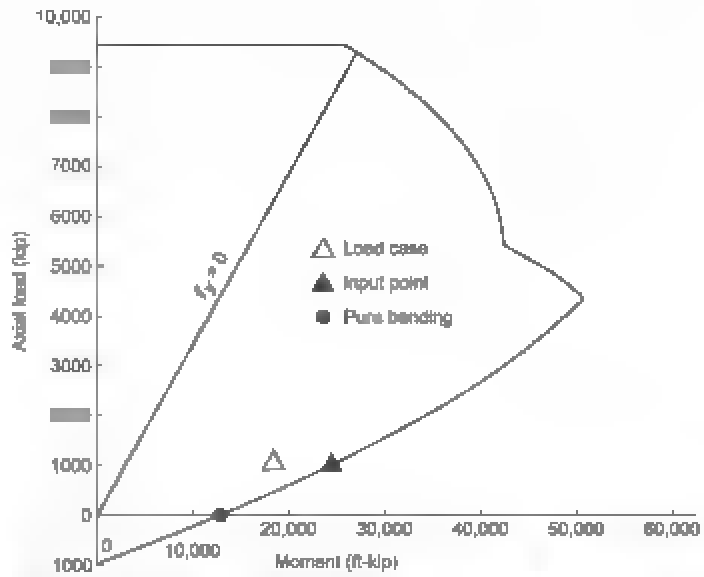


Fig. E1.4—Design strength interaction diagram.

Step 5: Reinforcement limits		
11.5.4	Shear walls resist both in-plane and out-of-plane shear. The out-of-plane shear is small and, by inspection, the wall shear strength is assumed adequate. In-plane shear strength will be calculated using Code Sections 11.5.4.2 through 11.5.4.4	
11.5.4.2	Check limiting nominal shear strength using the following equation. This provision limits shear strength to prevent crushing of diagonal compression struts. $8\sqrt{f_c'}A_{cv}$	A_{cv} = 12 in.(336 in.) = 4032 in.^{2} $\phi V_{n,max} = 0.75(8)\sqrt{5000}$ psi(4032 in.^{2}) = 1711 kip > V_u = 235 kip **OK**
11.5.4.3	Nominal shear strength is dependent on the aspect ratio of the shear wall and the quantity of transverse reinforcement. $V_n = (\alpha_c\lambda\sqrt{f_c'} + \rho_t f_{yt})A_{cv}$ where: $\alpha_c = 3$ for $h_w/\ell_w \leq 1.5$ $\alpha_c = 2$ for $h_w/\ell_w \geq 2.0$ α_c varies linearly between 3 and 2 for $1.5 < h_w/\ell_w < 2.0$	Try ignoring contribution of reinforcement to shear strength to determine if the capacity is sufficient. h_w/ℓ_w = 92 ft/28 ft = 3.3 > 2 ∴ use $\alpha_c = 2$ $V_n = 2(\sqrt{5000}$ psi)(4032 in.^{2}) = 570 kip ϕV_n = 0.75(570 kip) = 428 kip > V_u = 235 kip **OK**. Wall strength is sufficient for in-plane shear without considering transverse reinforcement.

Step 6. Flexure design strength (out-of-plane)		
11.5.1 11.5.3.1 21.2.2	As shown in Step 4, the layers of No. 5 vertical wall reinforcement satisfies the interaction equation for in-plane bending.	
	The resultant of the out-of-plane moment, $M_u = 60$ ft-kip is within the middle third of the wall. This allows Section 11.5.3.1 to be used to check the out-of-plane strength of the wall	Eccentricity of the resultant load $(1015\text{ kip})(e) = 60$ ft-kip $e = 0.7$ in. $e < 2$ in.
	$P_n = 0.55(f_c')(A_g)\left[1-\left(\frac{k\ell_c}{32h}\right)^2\right]$	$P_n = 0.55(5\text{ ksi})(12\text{ in.})(336\text{ in.})\left[1-\left(\frac{(0.8)(202\text{ in.})}{32(12\text{ in.})}\right)^2\right]$
	From Table 21.2.2(b), use axial strength reduction factor $\phi = 0.65$	$P_n = 9120$ kip $\phi P_n = (0.65)(9120\text{ kip}) = 5920$ kip 5920 kip > 1015 kip **OK**
Step 7. Reinforcement limits		
11.6	Minimum reinforcement is based on the magnitude of the applied in-plane shear. Determine if V_u exceeds the following expression	
	$0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv}$	$\alpha_c = 2$ as determined in step 5
21.2.1	Strength reduction factor for shear	$\phi = 0.75$ $0.5(0.75)2\sqrt{5000}\text{ psi}(4032\text{ in.}^2) = 214\text{ kip}$ $V_u = 235$ kip > 214 kip Use Code Section 11.6.2 for minimum reinforcement.
11.6.2	Because the factored shear is sufficiently large, the minimum longitudinal and transverse reinforcement must satisfy the following expressions	Consider the reinforcement (No. 5 at 12 in. spacing, each face) used to satisfy the axial load and in-plane moment. Use the same size and spacing for horizontal reinforcement
	$\rho_\ell \geq 0.0025 + 0.5(2.5 - h_w/\ell_w)(\rho_t - 0.0025)$ $\rho_\ell \geq 0.0025$ $\rho_t \geq 0.0025$	$\rho_{t,prov} = \frac{2(0.31\text{ in.}^2)}{12\text{ in.}(12\text{ in.})} = 0.0043 > 0.0025$ **OK**
	but ρ_ℓ need not exceed ρ_ℓ required for strength by 11.5.4.3	$\rho_{\ell,prov} = \frac{2(0.31\text{ in.}^2)}{12\text{ in.}(12\text{ in.})} = 0.0043$ $\rho_{\ell,requ}$ is the greater of the following: $0.0025 + 0.5(2.5 - 2)(0.0043 - 0.0025) = 0.0030$ or 0.0025 But need not be greater than ρ_ℓ required for strength, which is zero. therefore $\rho_{\ell,requ} = 0$
		No. 5 at 12 in. spacing in each direction and each face satisfy minimum reinforcement requirements for shear

Step 8. Reinforcement detailing		
11.7 11.7.2.1 11.7.3.1	Placing continuous No. 5 bars in each face and direction at 12 in. meets the detailing requirements of 11.7.2.1 and 11.7.3.1. No. 5 bars were selected in the horizontal direction for ease of construction	Reinforcement is not required for shear strength, therefore, the maximum spacing for vertical bars cannot exceed $3h$ (36 in.) or 18 in. The 12 in. spacing of vertical bars are less than these limits Similarly, the maximum spacing for horizontal bars cannot exceed $3h$ (36 in.) or 18 in. The 12 in. spacing of horizontal bars are less than these limits.
11.7.2.3	$h > 10$ in., therefore two layers are required	Two layers are provided having equal reinforcement area
11.7.4.1	If the area of vertical reinforcement exceeds $0.01A_g$, or if the reinforcement is needed to resist axial loads, ties are required to confine the vertical reinforcement. The reinforcement ratio for the flexural vertical reinforcement at the wall ends needs to be calculated to determine if ties are required.	The vertical flexural reinforcement used in the design strength interaction diagram is two No. 5 bars at 12 in. on center spacing. The A_g within this length is 144 in.^{2} A_{st} is 0.62 in.^{2} The ratio of A_{st} to A_g is 0.0043. This is less than the 0.01 and therefore ties are not required by 11.7.4.1 The maximum factored axial load is 1015 kips or 1,015,000 lb and the maximum factored moment is 18,600 ft-kip or 223,200,000 in.-lb. The factored axial stress on the concrete due to the combined loads is σ = 1,015,000 lb/[(12 in.)(28 ft)(12 in./ft)] + 223,200,000 in.-lb × 336 in./37,933,056 in.^{4} = 2229 psi This is below the design strength of concrete and thus steel is not needed to resist the axial load. Therefore, ties are not required by Section 11.7.4.1 Refer to Fig. E1.5 and E1.6 for wall elevation and section cut at the ends of the wall

Walls

Step 9 Detailing

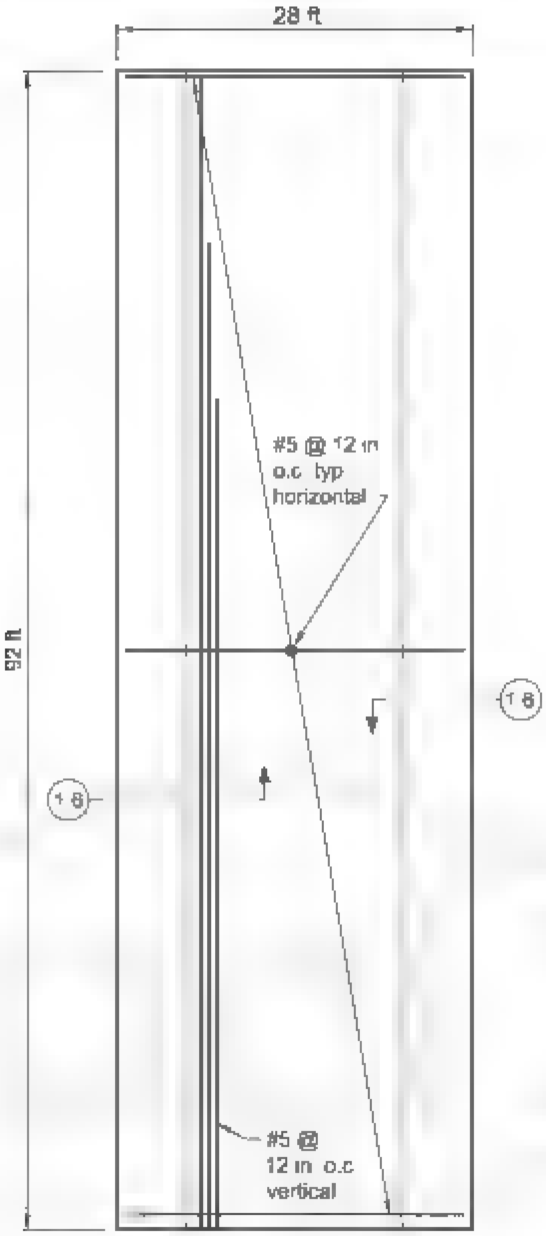


Fig. E1.5—Vertical bar distribution.

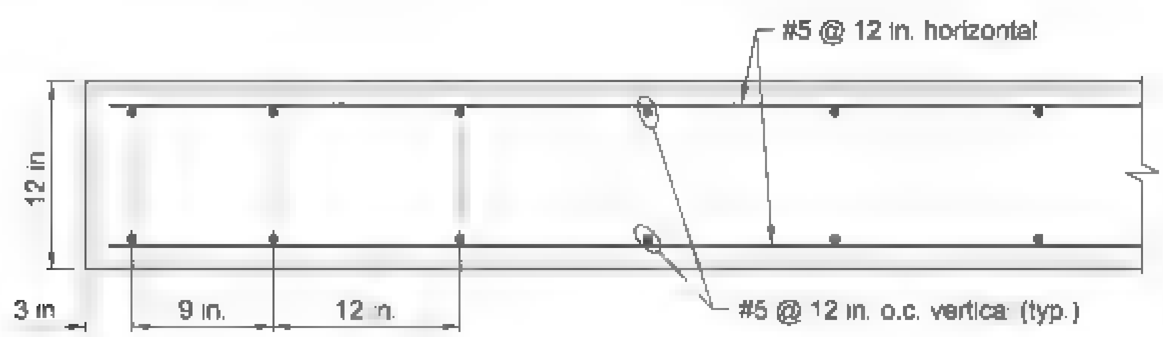


Fig. E1.6—Plan reinforcement layout

Shear Wall Example 2: *Seismic Design Category D*

The reinforced concrete shearwall in this example is nonprestressed. This shearwall is part of the lateral force-resisting system (a shearwall is at each end of the structure) in the North-South (N-S) direction of the hotel (Fig. E2.1). Material properties are selected based on the limits and requirements of Chapters 19 and 20 (ACI 318), engineering judgment, and locally available materials. The structure is analyzed for all required load combinations by an elastic 3D finite element analysis software model that includes shearwall-frame interaction. The resultant maximum factored moments and shears over the height of the wall are given for the load combination selected. This example provides the shearwall design and detailing at the base of the wall.

__Given__

Forces and moments at the wall base—

P_u = 1015 kip

In-plane—

V_u = 470 kip

M_u = 37,200 ft-kip

Material properties—

f_c' = 5000 psi

f_y = 60,000 psi

Out-of-plane—

V_u = 32 kip

M_u = 120 ft-kip

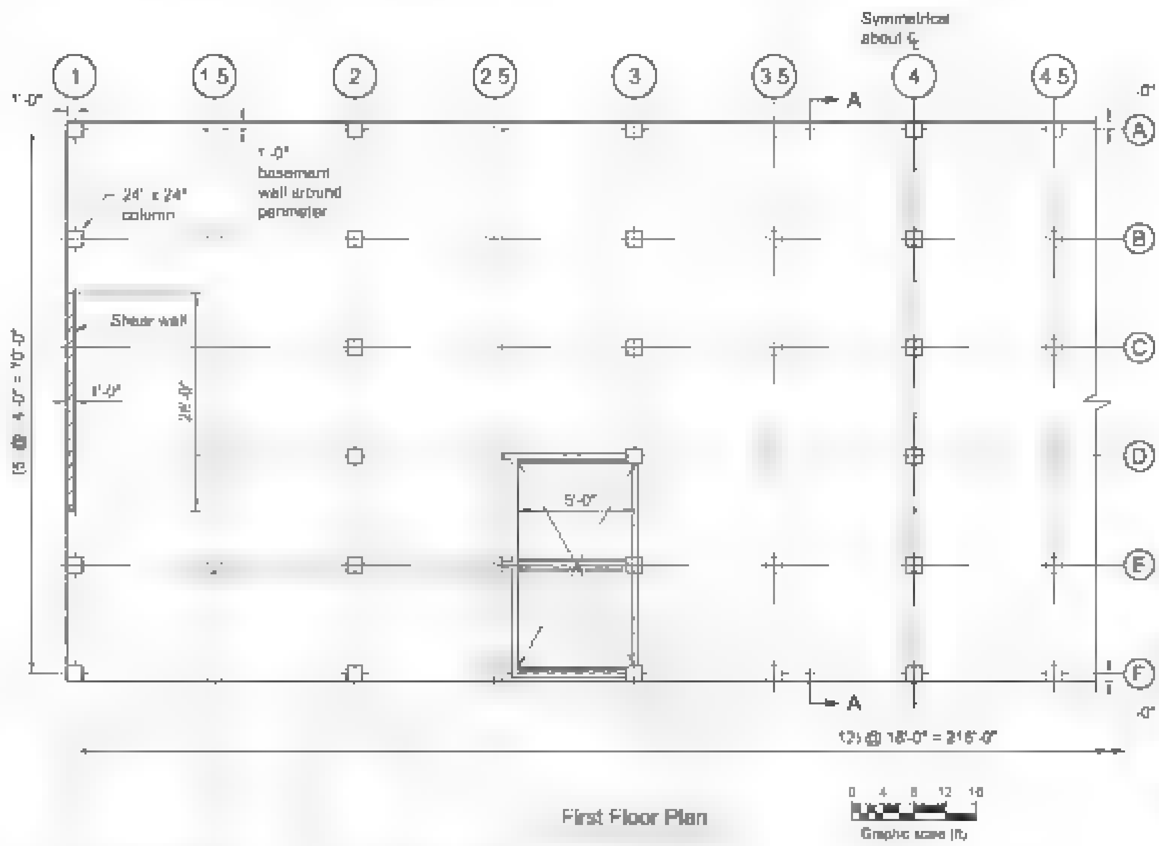


Fig. E2.1—Building floor framing plan, first floor.

This example shows the design and detailing of a special structural shear wall due to in-plane forces, including a seismic boundary element at the wall's edge. In addition, the design strength for the out-of-plane forces is verified. In this example, only one loading condition is checked. In a typical design, several load combinations require checking.

This example uses the Interaction Diagram spreadsheet aid found at https://www.concrete.org/MNL1721Download2

ACI 318	Discussion	Calculation
Step 1 Geometry		
11.3.1	This wall design example follows the requirements of Code Chapter 18 and, therefore, does not need to meet the requirements of Table 11.3.1.1 (ACI 318). However, the thickness equations (a) and (b) of Table 11.3.1.1 can provide an indication that the thickness chosen is an appropriate design starting point. Note that where special boundary elements are required, the special boundary element will be thicker. From Table 11.3.1.1, the wall thickness must be at least the greater of 4 in. and the lesser of 1/25 the lesser of the unsupported height of the wall (18 ft for the first elevated floor) and the unsupported length of the wall (28 ft from end-to-end of wall).	$h_{req'd}$ = (18 ft)(12 in./ft)/25 = 8.64 in. Example shear wall h = 12 in. > $h_{req'd}$ = 8.64 in. **OK**
20.5.1.3.1	A 12 in. wall is used in this design and the wall is assumed to be exposed to weather on the exterior of the structure. Concrete cover is 1-1/2 in., which is in accordance with Table 20.5.1.3.1 (ACI 318).	
Step 2. Loads, load patterns, and analysis of the wall		
11.4	The structure is analyzed using the assumptions and requirements of Section 11.4. The structure was analyzed using 3D elastic Finite Element Analysis (FEA) software that follows the analysis requirements of Section 11.4 of ACI 318 and Chapter 5 and 6 for loading and analysis, respectively (Fig. E2.2 and E2.3 for in-plane flexure and in-plane shear along the height of the wall, respectively).	The maximum factored axial force, flexural moment, and shear force at the base of the wall are listed in the given section at the start of this example.

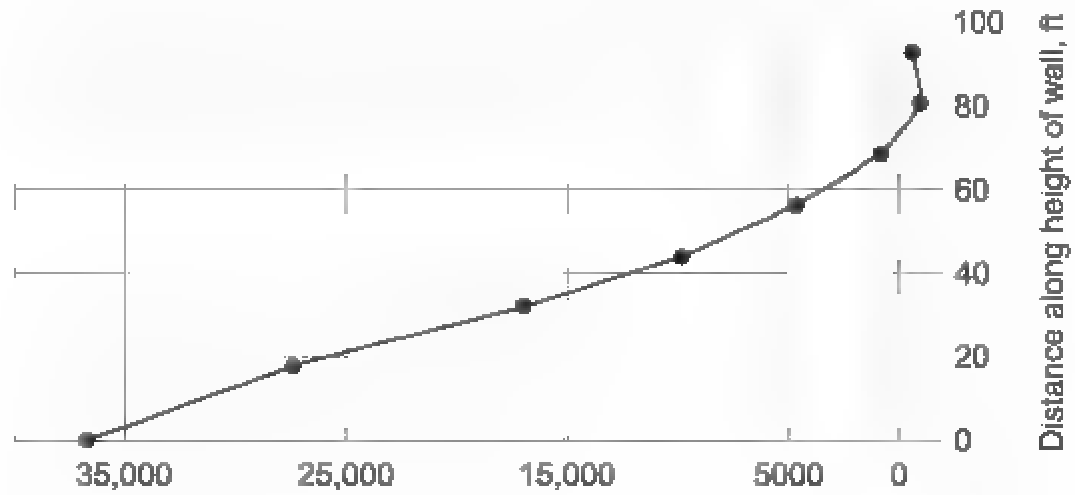


Fig. E2.2—In-plane flexure along the height of the wall

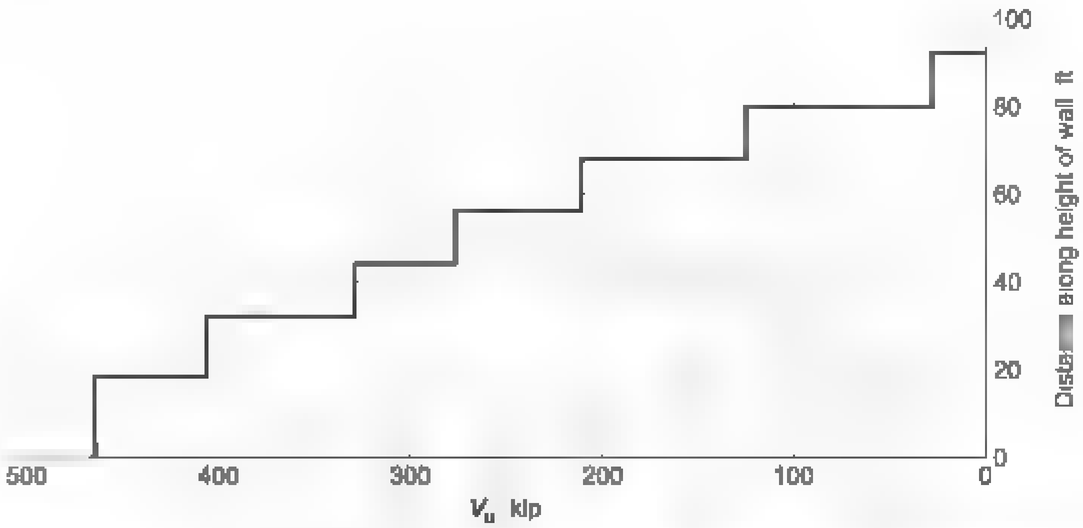


Fig. E2.3—In-plane shear along the height of the wall

Step 3: Concrete and steel material requirements

11.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318).	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied.
	The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes.	Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi.
	ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications.	
	There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	
11.2.1.2	The reinforcement must satisfy Chapter 20 of ACI 318. The designer determines the grade of bar and if the reinforcement should be coated by epoxy or galvanized, or both.	By specifying the reinforcement grade and any coatings, and that the reinforcement shall be in accordance with ACI 301-10, Chapter 20 (ACI 318) requirements are satisfied. In this case, assume Grade 60 bar and no coatings.

Step 4a. Axial and flexural interaction diagram (general)

The combined axial and flexural design strength of a shearwall is determined using an interaction diagram similar to a column interaction diagram.

The wall interaction diagram is generated using the Interaction Diagram spreadsheet (link in the given section of this example).

Refer to Column Example 9.2 in this Manual for additional information about the Interaction Diagram spreadsheet.

Step 4b. Axial and flexural interaction diagram (in-plane)

11.1.2 Section 11.1.2 requires that special structural walls be designed in accordance with Chapter 18 of ACI 318. Chapter 18 covers all requirements necessary to design the wall.

18.10.5 Flexure and axial loads are to be designed in accordance with Code Section 22.4. Longitudinal reinforcement within effective flange widths, boundary elements, and wall web are to be considered effective if they are developed.

Requirements of Code Section 18.10.5 are met through the flexural and axial interaction diagram design process in Step 4b.

The code places geometric limits on wall flanges.

This wall is rectangular in plan and does not have end flanges.

To estimate an initial reinforcement area, the wall is assumed to behave as a cantilever and the amount of flexural reinforcement necessary to resist the moment is calculated.

An initial interaction diagram is generated using No. 8 bars at 12 in. spacing throughout the wall. It is assumed that all of the longitudinal reinforcement is effective to resist in-plane flexure.

The first pair of No. 8 bars is assumed to be at 3 in. from the end of the wall, the second pair is placed at 12 in. from the end of the wall, and the remaining pairs at 12 in. spacing. The wall is symmetrical about the center of the wall and this bar layout is applied at both ends of the wall.

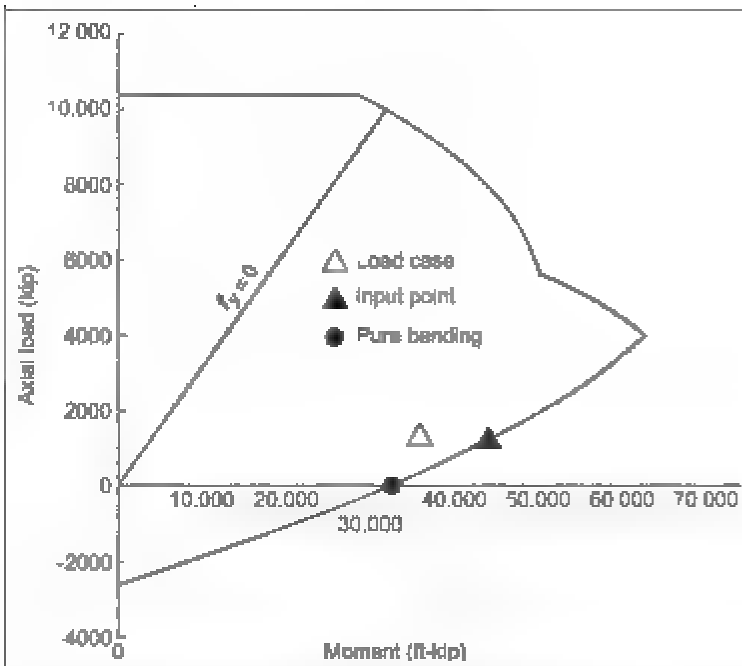


Fig. E2.4—Design strength interaction diagram.

Figure E2.4 shows the resulting design strength interaction diagram. The design strength interaction diagram includes the ϕ-factor. This spreadsheet contains a sheet named "Select Axial Load." When the user enters a P_u, the sheet calculates the associated maximum ϕM_n on the design strength interaction diagram curve and plots a point on the design strength interaction diagram. It also generates a corresponding maximum M_n on the nominal strength interaction diagram (not shown). This point is called the "Input Point" on the interaction diagram. The input point of P_u of 1015 kips calculates a maximum ϕM_n point on the interaction diagram of 40,200 ft-kips. The input point is plotted as a solid triangle along the interaction curve. The example has a P_u of 1015 kip and an M_u of 37,200 ft-kip. The open triangle indicates where the example P_u and M_u are and shows that this iteration does satisfy required strength, therefore, further iterations are unnecessary.

Step 4c. Axial, flexural, and shear (out-of-plane)		
11.5.1	As shown in Step 4b, the layers of No. 8 vertical wall reinforcement satisfies the interaction equation for in-plane bending	
11.5.3.1	The resultant of the out-of-plane moment, $M_u = 120$ ft-kip is within the middle third of the wall. This allows Code Section 11.5.3.1 to be used to check the out-of-plane strength of the wall.	Eccentricity of the resultant load (1015 kip)(e) = 120 ft-kip $e = 1.4$ in. $e < 2$ in.
	$P_n = 0.55(f_c')(A_g)\left(1-\left(\frac{k\ell_c}{32h}\right)^2\right)$	$P_n = 0.55(5\text{ ksi})(12\text{ in.})(336\text{ in.})\left(1-\left(\frac{(0.8)(202\text{ in.})}{32(12\text{ in.})}\right)^2\right)$
21.2.2	From Code Table 21.2.2(b) use axial strength reduction factor: $\phi = 0.65$	$P_n = 9090$ kip $\phi P_n = (0.65)(9090\text{ kip}) = 5900$ kip 5900 kip ≥ 1015 kip
11.5.5.1 22.5.5.1	Out-of-plane shear strength of walls is treated similar to that of one-way slabs. Shear in shallow one-way slab systems rarely controls the design. The concrete contribution to shear strength for members without minimum shear reinforcement includes the size effect, longitudinal reinforcement, and applied axial force. For members with effective depth $d < 10$ in., the size effect factor is 1.0. To conservatively simplify this equation use 1/2 of the lowest longitudinal reinforcement ratio from Code Section 11.6 (0.0012 × 0.5) and ignore the axial load. The 1/2 factor accounts for evenly dividing the reinforcement between each face. This reduces Code Equation 22.5.5.1c to:	Effective depth is based on 1.5 in. clear cover and placement of No. 4 horizontal bars on outside face of cage $d = 12\text{ in.} - 1.5\text{ in.} - 0.5\text{ in.} - 8/16\text{ in.} = 9.5$ in.
	$8(1.0)(1.0)(0.0012 \times 0.5)^{1/3}\sqrt{f_c'}b_w d \approx \frac{2}{3}\sqrt{f_c'}b_w d$	$\phi V_c = 0.75\frac{2}{3}\sqrt{5000}\text{ psi}(336\text{ in.})(9.5\text{ in.}) = 113$ kip $>> V_u = 32$ kip **OK**

Step 5: Reinforcement requirements		
18.10.1	This structure is using a special structural wall as part of the seismic-force-resisting system to resist the lateral loads due to earthquake	Two curtains of steel are used, the distributed reinforcement ratios are met, and the forces are determined within code allowed analysis methods. Therefore, Code Sections 18.10.1 through 18.10.3 (ACI 318) are met
18.10.2	The distributed web reinforcement ratios, ρ_ℓ and ρ_t, for structural walls must be at least 0.0025, except that if V_u does not exceed $A_{cv}\lambda\sqrt{f_c'}$, then ρ_ℓ and ρ_t are permitted to be reduced to the values in Section 11.6.	This example provides No. 6 bars in the horizontal direction in each face at 12 in. This provides 0.88 in.^{2} per foot in the horizontal direction. The transverse reinforcement ratio is ρ_t = 0.88 in.^{2}/(12 × 12) in.^{2} = 0.0061 > 0.0025 **OK** This example provides No. 8 bars in the vertical direction in each face at 12 in. This provides 1.58 in.^{2} per foot in the vertical direction ρ_ℓ = 1.58 in.^{2}/(12 × 12) in.^{2} = 0.0110 > 0.0025 **OK**
18.10.2.2	The Code requires two curtains of distributed reinforcement if: $V_u > 2A_{cv}\lambda\sqrt{f_c'}$ or $h_w/\ell_w > 2.0$	h_w/ℓ_w > (92 ft)/(28 ft) = 3.3 > 2 Two curtains are required and are provided. **OK**
Step 6: Design shear force		
18.10.3.1	The factored shear force may be determined from lateral load analysis with appropriate factored load combinations. For shear walls that are dominated by flexural behavior, the factored shear must then be amplified to account for flexural overstrength at the critical section where yielding of longitudinal reinforcement is expected. In addition, amplification may be appropriate for higher mode effects in taller structures. The final design shear force is $V_e = \Omega_v\omega_v V_u \leq 3V_u$	Factored shear from analysis presented in step 2 V_u = 470 kip
18.10.3.1.2	The overstrength factor (Ω_v) is calculated based on the ratio of probable moment to factored moment for walls with flexure dominated behavior. Code Table 18.10.3.1.2 indicates that flexure dominated behavior occurs in walls with height to length ratios greater than 1.5. In such cases the overstrength factor is greater than one	Overstrength factor (Ω_v) from Code Table 18.10.3.1.2 is based on the height above the critical section, which is located at the base of the wall. Therefore, h_{wcs}/ℓ_w = 92 ft/28 ft = 3.3 >1.5 overstrength factor is greater than 1.0 Determine the flexural overstrength factor using a modified interaction diagram in which the yield strength of the reinforcement is taken as $1.25f_y$ and the strength reduction factors are set to 1.0. It is important to include all longitudinal reinforcement that is contributing to flexural wall strength from both the boundary elements and wall. The modified diagram is shown in Fig. E2.5. Because the probable flexural strength will vary with axial load, select the load combination that maximizes M_{pr}. This point is shown on the interaction diagram M_{pr}/M_u = 51,900 kip-ft/37,200 kip-ft = 1.4 <1.5 Use Ω_v = 1.5

18.10.3.1.3 Higher modes of dynamic response can contribute significantly beyond the fundamental mode in walls with height to length ratios greater than 2.0. The dynamic amplification factor is intended to account for such behavior using the following equations based on the number of stories (n_s).

$$\omega_v = 0.9 + \frac{n_s}{10} \qquad n_s \le 6$$

$$\omega_v = 1.3 + \frac{n_s}{30} \le 1.8 \qquad n_s > 6$$

where n_s should be no less than $0.007h_{wcs}$

Dynamic amplification for 8 stories is

$n_s = 8$

0.007(92 ft)(12 in./ft) = 7.7 in. < 8 Use $n_s = 8$

$$\omega_v = 1.3 + \frac{8}{30} = 1.57$$

$V_e = 1.5(1.57)(470\text{ kip}) = 1107\text{ kip} < 3V_u = 1410\text{ kip}$ **OK**

Use a design shear force of $V_e = 1107$ kip

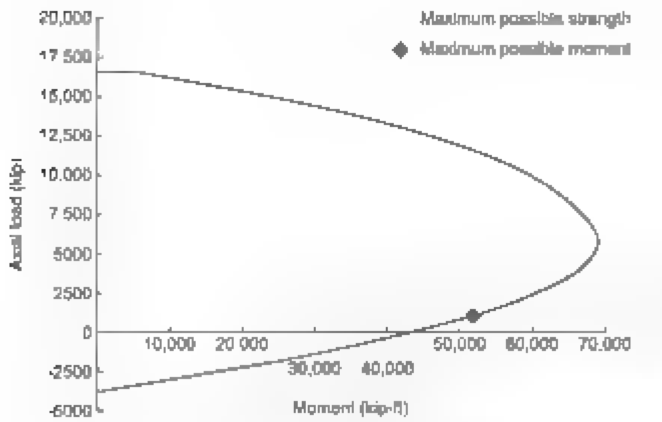


Fig. E2.5—Interaction diagram to determine maximum probable moment at base of shear wall.

18.10.4 The shear strength of special structural walls is affected by the height to length ratio of the wall. The code limits V_n to:

$$V_n = A_{cv}\left(\alpha_c\lambda\sqrt{f'_c} + \rho_t f_{yt}\right)$$

where α_c is 2.0 for $h_w/\ell_w > 2.0$ and varies between 2.0 and 3 for $h_w/\ell_w < 2.0$

$V_n = 4032\text{ in.}^2(2\sqrt{5000\text{ psi}} + (0.0061)(60,000\text{ psi}))$
$= 2045$ kip

where
$A_{cv} = (12\text{ in.})(28\text{ ft})(12\text{ in./ft}) = 4032\text{ in.}^2$

21.2.4, 21.2.4.1 The ϕ factor for special structural walls is determined by Code Sections 21.2.4 and 21.2.4.1. From the analysis of the structure, the maximum axial load under seismic loading combinations for this wall is approximately 1200 kip.

$P_u = 1200$ kip
From the interaction diagram, M_n corresponding to P_u of 1200 kip is
$M_n = 41,860$ ft-kip
V_u from a moment of 41,860 ft-kip is
$V_u = 2 \times 41,860\text{ ft-kip}/18\text{ ft} = 4650$ kip

18.10.4.4 In this example, the code limits on shear strength based on concrete strength of
$V_n < 10A_{cw}\sqrt{f'_c}$
will also limit the ϕ-factor to 0.6.

Max shear:
$V_n = 4032\text{ in.}^2(10\sqrt{5000\text{ psi}}) = 2851$ kip

The V_u calculated from the nominal moment strength of the shear wall is greater than the maximum code allowed shear strength. Therefore, use a ϕ factor of 0.6.

$\phi V_n = (0.6)(2045\text{ kip}) = 1227\text{ kip} > 1107\text{ kip}$ **OK**

Use No. 8 vertical at 12 in. on center, each face for flexure and axial strength and No. 6 horizontal at 12 in. on center, each face for shear strength.

Step 7: Special boundary elements		
18.10.6, 18.10.6.1	Special boundary elements (SBE) are often required in special structural walls to resist the large compression forces at the ends of the walls during an earthquake event. The need for SBE is evaluated in accordance with Code Sections 18.10.6.2 or 18.10.6.3. The requirements of 18.10.6.4 and 18.10.6.5 must also be satisfied.	
18.10.6.2	In tall, slender walls where the inelastic response of the wall is dominated by flexure ($h_{wcs}/\ell_w \geq 2.0$), the wall should be proportioned and reinforced to behave as such. This may require that the ends of the wall be designated as a special boundary element, which requires specific geometric proportions and reinforcing details to ensure stable inelastic behavior under the design lateral displacement (δ_u).	
	Special structural walls that are effectively continuous from the base of the structure to the top of the wall and are designed to have a single critical section for flexure and axial loads must meet the requirements of 18.10.6.2(a) and (b).	Wall is continuous from base of structure to top of wall. **OK** Wall is designed to have a single critical section for flexure and axial loads. **OK**
	According to Section 18.10.6.2(a) an SBE is required where: $\frac{1.5\delta_u}{h_{wcs}} \geq \frac{\ell_w}{600c}$ where δ_u/h_{wcs} need not be taken less than 0.005	The depth of the neutral axis, c, can be determined from the interaction diagram software and is found to be $c = 67.9$ in. In addition to shear and moment, the structural analysis software output includes deflection data. The elastic deflection at the top of the wall from the software is 2.4 in. This must be amplified by $C_d = 5$ from ASCE/SEI 7. $\frac{\ell_w}{600c} = \frac{336\text{ in.}}{600(67.9\text{ in.})} = 0.00825$ $\frac{\delta_u}{h_{wcs}} = \frac{5(2.4\text{ in.})}{92\text{ ft}} = 0.0109 > 0.005$ **OK** $\frac{1.5\delta_u}{h_{wcs}} = \frac{1.5(5)(2.4\text{ in.})}{92\text{ ft}} = 0.0163 > 0.00825$ SBE is required
	SBE is required and must meet the requirements of 18.10.6.2(b), which indicate that the transverse reinforcement must extend vertically above and below the critical section at least the greater of ℓ_w and $M_u/4V_u$ except as permitted in Code section 18.10.6.4(i), which are requirements for transverse reinforcement for web vertical reinforcement. This will be checked with the other boundary element requirements of that Section.	$\ell_w = 336$ in. Use factored moment and shear at base of wall from analysis results. $\frac{M_u}{4V_u} = \frac{41{,}860\text{ kip-ft}}{4(470\text{ kip})} = 267.2$ in. Extend SBE transverse reinforcement above and below the critical section at least 336 in. (28 ft)

	If SBE are required by Code Section 18.10.6.2(a), then one of the following must be satisfied $b \geq \sqrt{0.025c\ell_u}$ $\delta_c/h_{wcs} > 1.5\delta_u/h_{wcs}$, where $\frac{\delta_c}{h_{wcs}} = \frac{1}{100}\left(4 - \frac{1}{50}\left(\frac{\ell_w}{b}\right)\left(\frac{c}{b}\right) - \frac{V_u}{8\sqrt{f'_c}A_{cv}}\right)$ where δ_c/h_{wcs} need not be taken less than 0.015	$\sqrt{0.025(67.9\text{ in.})(336\text{ in.})} = 23.9\text{ in.}$ Calculate the drift capacity of the wall $\frac{\delta_c}{h_{wcs}} = \frac{1}{100}\left[4 - \frac{1}{50}\left(\frac{336\text{ in.}}{16\text{ in.}}\right)\left(\frac{67.9\text{ in.}}{16\text{ in.}}\right) - \frac{1107\text{ kip}(1000)}{8\sqrt{5000}\text{ psi}(12\text{ in.})(336\text{ in.})}\right] = 0.0035$ $\frac{\delta_c}{h_{wcs}} = 0.0035 < \frac{1.5\delta_u}{h_{wcs}} = \frac{1.5(5)(2.4\text{ in.})}{92\text{ ft}(12)} = 0.0163$ **NG** Drift capacity of the wall is not sufficient. Increase SBE width to 16 in. and recalculate drift capacity $\frac{\delta_c}{h_{wcs}} = \frac{1}{100}\left[4 - \frac{1}{50}\left(\frac{336\text{ in.}}{16\text{ in.}}\right)\left(\frac{67.9\text{ in.}}{16\text{ in.}}\right) - \frac{1107\text{ kip}(1000)}{8\sqrt{5000}\text{ psi}(12\text{ in.})(336\text{ in.})}\right] = 0.0173 > 0.0163$ **OK** Although the neutral axis depth will decrease slightly with the increase in width, this will serve to improve the drift capacity. Conservatively use the same neutral axis depth for this check.
18.10.6.4(a), (b), (c), (d)	Code Section 18.10.6.4 (a) through (d) impose additional geometric requirements upon the special boundary elements. Code Section 18.10.6.4 states the following.	
	Where an SBE is required by Code Section 18.10.6.2 or 18.10.6.3, (a) through (k) must be satisfied. (a) SBE must extend horizontally from the extreme compression fiber a distance at least the greater of $c - 0.1\ell_w$ and $c/2$, where c is the largest neutral axis depth calculated for the factored axial force and nominal moment strength consistent with δ_u.	Code Section 18.10.6.4(a) requires that the SBE extend a minimum from the extreme compression fiber $c - 0.1\ell_w = 67.85\text{ in.} - (0.1)(336\text{ in.}) = 34.25\text{ in.}$ or $c/2 = 67.85\text{ in.}/2 = 33.925\text{ in.}$ Round the length of the SBE to 34 in.
	(b) Width of the flexural compression zone, b, over the horizontal distance calculated by 18.10.6.4(a), including flange if present, shall be at least $h_u/16$.	Code Section 18.10.6.4(b) is satisfied by making the wall thicker over the SBE to meet the requirement of $h_u/16 = 216\text{ in.}/16 = 13.5\text{ in.}$ Therefore, the SBE thickness of 16 in. determined previously is adequate.
	(c) For walls or wall piers with $h_w/\ell_w \geq 2.0$ that are effectively continuous from the base of structure to top of wall, designed to have a single critical section for flexure and axial loads, and with $c/\ell_w \geq 3/8$, width of the flexural compression zone b over the length calculated in Code Section 18.10.6.4(a) shall be greater than or equal to 12 in.	Boundary element thickness = 16 in. > 12 in. **OK**
	(d) In flanged sections, the SBE shall include the effective flange width in compression and shall extend at least 12 in. into the web.	Code Section 18.10.6.4(d) does not apply because this is not a flanged section.

18.10.6.4(e)	Code Section 18.10.6.4(e) imposes spacing requirements on the transverse reinforcement. The SBE transverse reinforcement shall satisfy Section 18.7.5.2(a) through (d) and Section 18.7.5.3, except that the transverse reinforcement spacing limit of Section 18.7.5.3(a) must be one-third of the least dimension of the SBE. The maximum vertical spacing of transverse reinforcement in the SBE should not exceed that in Table 18.10.6.5(b).	Code Section 18.10.6.4(e) requires that the geometry and spacing of the vertical and crosstie reinforcement meet most of the requirements of a special moment frame column.
18.10.6.4(e) 18.7.5.2	Transverse reinforcement shall be in accordance with (a) through (d) (a) Transverse reinforcement shall comprise either single or overlapping spirals, circular hoops, or rectilinear hoops with or without crossties. (b) Bends of rectilinear hoops and crossties shall engage peripheral longitudinal reinforcing bars. (c) Crossties of the same or smaller bar size as the hoops shall be permitted, subject to the limitation of Code Section 25.7.2.2. Consecutive crossties shall be alternated end for end along the longitudinal reinforcement and around the perimeter of the cross section.	Code Section 18.7.5.2 requires that the transverse SBE reinforcement satisfy essentially the same requirements as those of a special moment frame column.
18.10.6.4(e) 18.7.5.2 25.7.2.2 25.7.2.3	(d) Where rectilinear hoops or crossties are used, they should provide lateral support to longitudinal reinforcement in accordance with Code Sections 25.7.2.2 and 25.7.2.3. 25.7.2.2 limits hoop size based on the longitudinal bar size and 25.7.2.3 limits the spacing of laterally supported longitudinal bars.	No. 4 transverse bar is satisfactory for longitudinal bar sizes up to No. 11. Hoops are required to be arranged so that all corner bars and alternate longitudinal bar are laterally supported by the corner of a tie with an included angle of not more than 135 degrees. No. 4 hoops will be provided such that all longitudinal bars are enclosed by hoop corners. No unsupported bar is to be farther than 6 in. clear on each side along the tie from a laterally supported bar. There are no laterally unsupported bars.
18.10.6.4(e) 18.7.5.3	18.7.5.3 Spacing of transverse reinforcement shall not exceed the smallest of (a), (b), and (d). (a) One-fourth of the minimum column dimension (b) For Grade 60, six times the diameter of the smallest longitudinal bar (d) s_o, as calculated by: $s_o = 4 + \left(\frac{14 - h_x}{3}\right)$ (18.7.5.3) The value of s_o from Eq. (18.7.5.3) shall not exceed 6 in. and need not be taken less than 4 in.	Code Section 18.10.6.4(e) modifies 18.7.5.3(a) to one-third the least dimension of the boundary element. 16 in./3 = 5.33 in. 18.7.5.3(b) (6)(1 in.) = 6 in. 18.7.5.3 (c) Use vertical bars at the corners of the SBE, but assume that no bars are placed between the corner bars on the short face of the boundary element. Use 1.5 in. clear cover over No. 4 transverse reinforcement. Calculate h_x based on this spacing. h_x = 16 in. – 2(1.5 in.) – 2(0.5 in.) – 1.0 in. = 11 in. $s_o = 4 + \left(\frac{14 - 11}{3}\right) = 5.0$ in. Choose a spacing of 4 in. for the transverse reinforcement in the SBE. All longitudinal bars in the SBE are engaged by a crosstie or rectilinear tie.

18.10.6.4(e) 18.10.6.5(b)	The maximum vertical spacing of transverse reinforcement in the SBE should not exceed that in Code Table 18.10.6.5(b). For Grade 60 reinforcement, the maximum spacing within the inelastic region near the critical section is the lesser of $6d_b$ and 6 in. At other locations, the maximum spacing is the lesser of $8d_b$ and 8 in.	Within the greater of ℓ_w = 336 in. and $M_u/4V_u$ = 268 in. The spacing is limited to 6(1 in.) = 6 in. 6 in. maximum At other locations 8(1 in.) = 8 in. 8 in. maximum s_o = 4 in. < 6 in. and 8 in. Use 4 in. spacing for transverse reinforcement up to 336 in. (28 ft) above the critical section and 8 in. spacing above that
18.10.6.4(f)	Transverse reinforcement and spacing between laterally supported longitudinal bars around the perimeter of the SBE, h_x, must be arranged so that h_x does not exceed the lesser of 14 in. and two-thirds of the SBE thickness.	Check spacing of longitudinal reinforcement 14 in. > h_x = 11 in. **OK** 2/3(16 in.) = 10.6 in. < h_x = 11 in. **NG** Use vertical No. 8 in the middle of each short SBE face. Use two intermediate vertical No. 8 bars in each long face of the SBE face. This will give a spacing of (34 in. – 2(1.5 in.) – 1 in.)/3 sp. = 10 in. < 10.6 in. **OK** For this example, the interaction diagram calculations were modified to account for the added longitudinal reinforcement needed to satisfy transverse and longitudinal reinforcement detailing in the SBE. Reduction in the amount of longitudinal reinforcement is possible above the plastic hinge region where SBE is not required. Rather than extending the SBE longitudinal reinforcement to the top of the wall, it may be more economical to drop longitudinal bars and recalculate the flexural and axial strength. See Code Commentary Fig. R18.10.6.4(c)(a).
	Crossties or corner of a hoop are required to provide lateral support to the longitudinal bars. Each end of the crossties must have a seismic hook.	To be considered laterally supported, vertical bars must be enclosed by crossties or hoops with seismic hooks
	Hoop leg lengths are limited to two times the SBE thickness. In addition, adjacent hoops must overlap at least the lesser of 6 in. and two-thirds the boundary element thickness	Try a single hoop for the SBE. b_c = 16 in. – 2(1.5 in.) = 13 in. ℓ = 34 in. – 2(1.5 in.) = 21 in. ℓ = 21 in. < 2bc = 2(16 in.) = 32 in. **OK** Single hoop is permissible. Use crossties for bars in the short face. Use smaller stacked hoop for bars in long face of SBE

18.10.6.4(g) Quantity of transverse reinforcement is to be based on Code Table 18.10.6.4(g). For rectilinear hoops, the quantity must be the greater of the two following expressions:

$$0.3\left(\frac{A_g}{A_{ch}}-1\right)\frac{f_c'}{f_{yt}}$$

$$0.09\frac{f_c'}{f_{yt}}$$

Code commentary R18.10.6.4(g) defines the following terms:

$A_g = \ell_{be}b$

$A_{ch} = b_{c1}b_{c2}$

where ℓ_{be} and b are the SBE outside dimensions and b_{c1} and b_{c2} are the dimensions of the boundary element core assuming the cover has spalled (Fig. 10.5e).

No. 4 hoops and crossties at 4 in. spacing for compliance with 18.10.6.4(g).

$A_g = (16\text{ in.})(34\text{ in.}) = 544\text{ in.}^2$

$b_{c1} = 34\text{ in.} - 2(1.5\text{ in.}) = 31\text{ in.}$

$b_{c2} = 16\text{ in.} - 2(1.5\text{ in.}) = 13\text{ in.}$

$A_{ch} = (13\text{ in.})(31\text{ in.}) = 403\text{ in.}^2$

Required transverse reinforcing ratio is the greater of the following:

$$0.3\left(\frac{544\text{ in.}^2}{403\text{ in.}^2}-1\right)\left(\frac{5000\text{ psi}}{60{,}000\text{ psi}}\right) = 0.00875$$

$$0.09\left(\frac{5000\text{ psi}}{60{,}000\text{ psi}}\right) = 0.00750$$

Use 0.00875. Determine the number of No. 4 hoop and tie legs perpendicular to dimension b_c.

$A_{sh}/sb_c \geq 0.00875$

$$\frac{n_{leg}(0.2\text{ in.}^2)}{(4\text{ in.})(13\text{ in.})} \geq 0.00875$$

$$n_{leg} \geq \frac{0.00875(4\text{ in.})(13\text{ in.})}{(0.2\text{ in.}^2)} = 2.3$$

At least three legs are required. Two legs are provided by the boundary element hoop and one leg is provided by the crosstie for the No. 8 bars at mid-thickness of SBE.

18.10.6.4(h) Specify the concrete within the thickness of the floor system at the SBE location to have compressive strength at least 0.7 times f_c' of the wall.

18.10.6.4(i), 25.7.2.2 For a distance above and below the critical section as required by Code Section 18.10.6.2(b), vertical reinforcement in the web must have lateral support provided by the corner of a hoop or by a crosstie with seismic hooks at each end. Transverse reinforcement should have a vertical spacing not to exceed 12 in. and diameter satisfying Code Section 25.7.2.2.

Specify No. 4 crossties or stacked hoops on the No. 8 vertical reinforcement at a spacing of 12 in. No. 4 bars satisfy Code Section 25.7.2.2 for bar size. No. 4 bars are typically the smallest diameter bar specified for this application due to the possible lack of availability of No. 3 bars.

18.10.6.4(j)	Code Section 18.10.6.4(j) requires that the transverse reinforcement extend into the wall support base when the critical section occurs at the wall base	This section is satisfied by extending the transverse reinforcement a minimum of 12 in. into the foundation element below the base of the wall.
18.10.6.4(k)	Horizontal reinforcement must be developed within the confined core of the special boundary element using standard hooks or heads. For this example use standard hooks with 90 degree bend. The tails of the hooks can be turned up or down in the boundary element to avoid conflict and maintain cover	
25.4.3	Check the hook length of the No. 6 bar using the following equations:	
25.4.3.1	$\ell_{dh} \geq \left(\dfrac{f_y\psi_e\psi_r\psi_o\psi_c}{55\lambda\sqrt{f_c'}}\right)d_b^{1.5}$ $\ell_{dh} > 8d_b$ $\ell_{dh} > 6$ in.	$\lambda = 1.0$ Bars are uncoated $\psi_e = 1.0$ Bar spacing = 12 in. > 6(0.625 in.) = 3.75 in. $\psi_r = 1.0$ Side cover normal to plane of hook is greater than $6d_b$ 6 (0.625 in.) = 3.75 in. $\psi_o = 1.0$ Concrete strength less than 6000 psi $\psi_c = \dfrac{5000}{15,000} + 0.6 = 0.933$ Required hook development length $\dfrac{60,000\text{ psi}(1.0)(1.0)(0.93)}{55(1.0)\sqrt{5000}\text{ psi}}(0.625)^{1.5} = 7.1$ in. Available hook length is 34 in. – 1.5 in. – 6 in. = 26.5 in. > 7.1 **OK**
25.4.3.2	ψ_e – Coating factor ψ_r – Confining reinforcement factor ψ_o – Location factor ψ_c – Concrete compressive strength factor	

Walls

18.10.6.5	At the elevation where the SBE is no longer required according to Code Section 18.10.6.2 or 18.10.6.3, the wall boundary (edge) must meet the requirements of Code Section 18.10.6.5	For formwork consistency and repetition, maintain the geometry of the SBE from bottom to top of wall. Assuming that architectural constraints are not a concern, however, one option to improve formwork economy is to design the special shear wall with a constant thickness of 16 in. for the entire wall height. This would avoid reentrant formwork corners, which can increase formwork and reinforcement placement complexity and costs.
18.10.6.5(a)	Where the factored shear is greater than $\lambda\sqrt{f_c'}A_{cv}$, horizontal reinforcement terminating at the edges of structural walls is to have a standard hook engaging the edge reinforcement or the edge reinforcement must be enclosed by U-stirrups having the same size and spacing as, and spliced with, the horizontal reinforcement	Not applicable since horizontal reinforcement will be embedded in the boundary element core over the entire height of the wall
18.10.6.5(b)	If the maximum longitudinal reinforcement ratio at the wall boundary exceeds $400/f_y$, boundary transverse reinforcement shall satisfy Code Section 18.7.5.2(a) through (e) over the distance calculated in accordance with Code Section 18.10.6.4(a). The vertical spacing of transverse reinforcement at the wall boundary shall be in accordance with Table 18.10.6.5(b).	Boundary element longitudinal reinforcement can be reduced in higher elevations of the wall by recalculating the moment and axial strength interaction diagram with fewer bars in the boundary element Ties have already been checked to satisfy Code Section 18.7.5.2(a) through (d). Check Code Section 18.7.5.2(e) Spacing h_x is not to exceed 14 in. around the perimeter of the column. Hoops and crossties from SBE will be continued to the top of the wall Vertical spacing of the transverse reinforcement according to Code Table 18.10.6.5(b) was determined previously and should not exceed 8 in. outside the plastic hinge region
18.10.7, 8, and 10	Do not apply	
18.10.9	Code Section 18.10.9.1 Construction joints in structural walls shall be specified according to Code Section 26.5.6, and contact surfaces shall be roughened consistent with condition (b) of Code Table 22.9.4.2 of ACI 318 Final sketch of structural wall using the special boundary elements	Code Section 18.10.9 is satisfied by specifying in the construction documents that all construction joints in the wall be roughened to approximately a 1/4 in. amplitude. Figures E2.6 and E2.7 show the final configuration of the wall if special boundary elements were required.

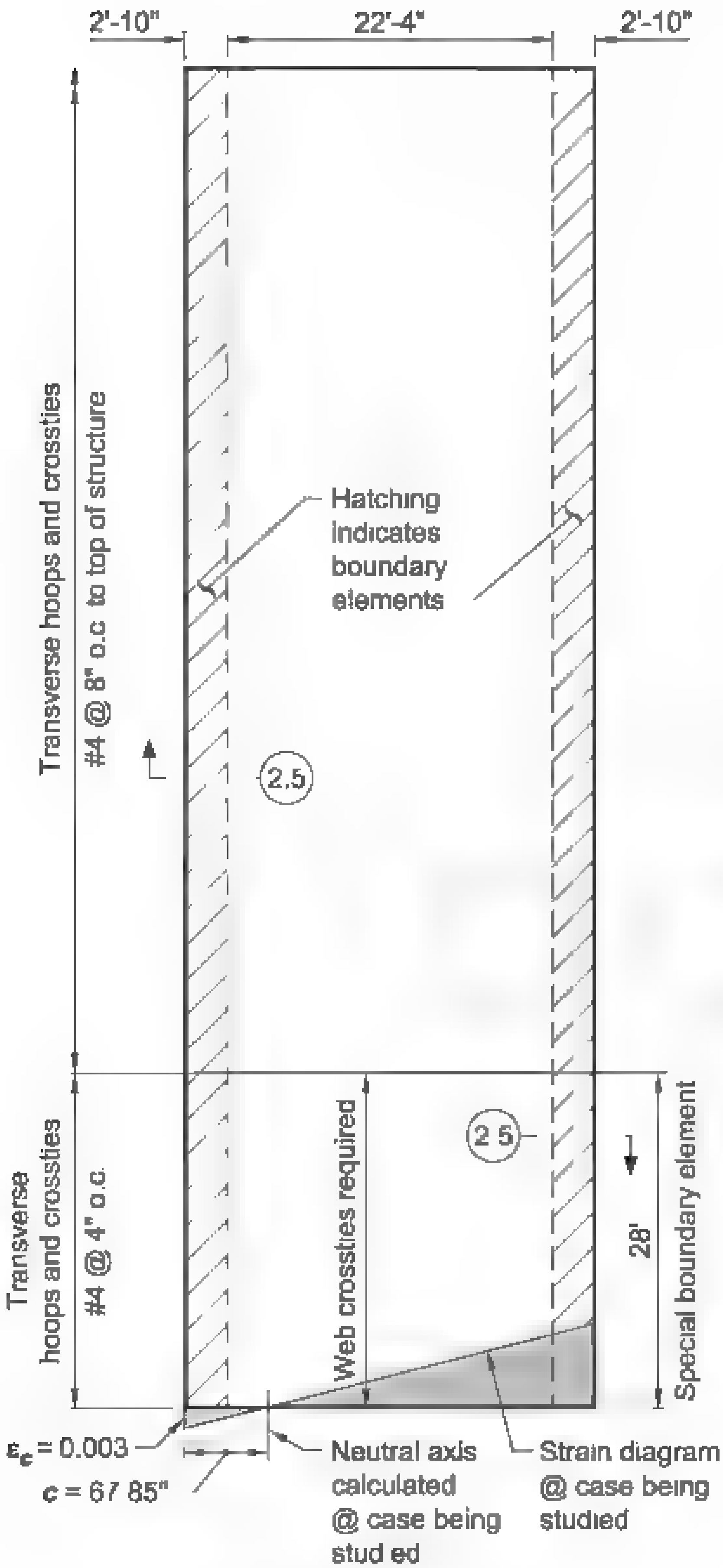


Fig. E2.6—Elevation of wall with special boundary elements.

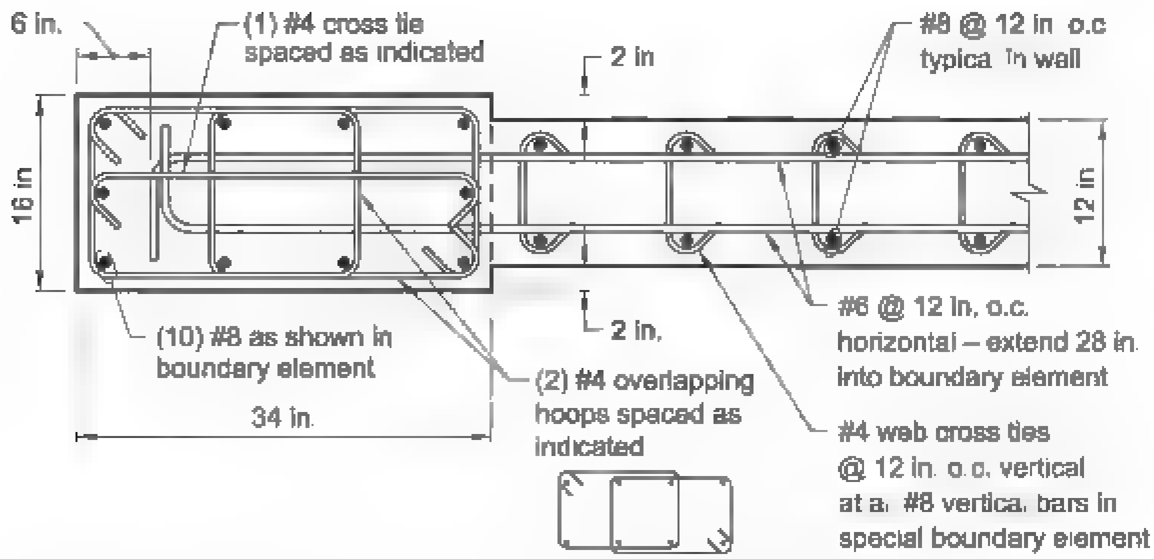


Fig. E2.7—Final layout of special boundary element reinforcement.

CHAPTER 11—FOUNDATIONS

11.1—Introduction

The foundation is an essential building system that transfers column and wall forces to the supporting soil. Foundation design requires detailed knowledge of the relevant geotechnical and structural design requirements. As such, foundation design is typically a collaborative effort between the geotechnical engineer and structural engineer.

Subsurface investigation is typically conducted by a local geotechnical engineer in accordance with the governing general building code. The structural engineer may communicate the arrangement and characteristics of the load-bearing elements along with the preliminary loads to help guide the subsurface investigation. Unless available, the geotechnical engineer will then conduct a site survey and subsurface investigation. In some cases, it may not be clear if the subsurface conditions will permit the use of shallow foundations or require deep foundations. If a shallow foundation is assumed, then soil borings may not penetrate deep enough to provide the knowledge necessary to design deep foundation element, should it be deemed necessary. Either during or following the site investigation, selection of the foundation system is made based on local soil conditions, structural design requirements, and experience of local contractors, among other reasons. Depending on the soil properties and building loads, the engineer may choose to support the structure on a shallow or deep foundation system.

Shallow foundation systems include isolated footings that support individual columns (Fig. 11.1(a) and (b)), combined footings that support two or more columns (Fig. 11.1(c)), strip footings that support walls (Fig. 11.1(d)), ring footings (Fig. 11.1(e)) that support a tank wall, mat footings (Fig. 11.1(f)) that support several or all columns or walls, and deep foundations (Fig. 11.1(g)).

Deep foundations are typically composed of a single pile or group of piles that are embedded in the ground to provide stable transfer of structural loads to the soil. The pile cap or grade beam transfers structural load (usually from columns or walls) into the deep foundation members. The cap is usually relatively thick in comparison to its plan dimensions; this results in a stiff element that evenly distributes axial load to the individual deep foundation members in the group. The pile cap also stabilizes the members in the plane of the cap and may also transfer horizontal forces such as those resulting from wind or earthquakes.

Deep foundation members are slender structural elements installed in the ground using several materials or combinations of materials. They are installed by impact driving, jacking, vibrating, jetting, drilling, grouting, or combinations of these techniques. Deep foundation members are difficult to summarize and classify because there are many types, and new types are still being developed. The information in this chapter was derived primarily from ACI 543R-12. The reader is referred to this document for more in-depth coverage of concrete deep foundation members. Deep foundation design provisions were added to the Code in 2019 for members in seismic design category C through F. These provisions are based in part on similar provisions that were in ASCE/SEI 7 and IBC.

In this chapter, isolated, combined, and continuous footing examples are presented. In addition, both drilled or augered pile (uncased) and precast pile designs are presented. A pile cap design is presented in the Strut-and-Tie chapter of this Manual.

11.2—Footing design

Footing design typically consists of four steps.

1. Determine the necessary soils parameters based on the requirements of the applicable general building code. This step is often completed by consulting with a geotechnical engineer who furnishes information in a geotechnical report. Important information that a geotechnical report should include are the:

- Subsurface profile, which provides physical characteristics of soil, groundwater, rock, and other soil elements
- Shear strength parameters to determine the stability of sloped soil
- Frost depth to determine the bearing level of footing below frost penetration level
- Unit weights, which is the weight of soil and water per unit volume, used to determine the additional load on a footing structure when backfilled
- Recommended foundation types
- Bearing capacity, which is the maximum allowable pressure that a footing is permitted to exert on the supporting soil; the size of the footing or pier is based on allowable loads
- Predicted settlement, which is the anticipated vertical movement of a footing over time
- Pile capacity curves
- Liquefaction, which is an important soil characteristic if the building is located in an active seismic area

2. Analyze the building's structure under service loads (Code R13.2.6.1) and factored loads (Code 5.3.1) to calculate moments and forces on the columns and walls at the footing level; the service load analysis is used to calculate footing bearing areas and the factored load analysis to design the footing.

3. Select the footing geometry so that the soil parameters are not exceeded. The following are typical parameters:

(a) Calculated bearing pressures are assumed to be uniform or to vary linearly; bearing pressure is measured in units of force per unit area, such as pounds per square foot

(b) The effect of anticipated differential vertical settlement between adjacent footings on the superstructure are considered

(c) Footings need to be able to resist sliding caused by any horizontal loads

Foundations

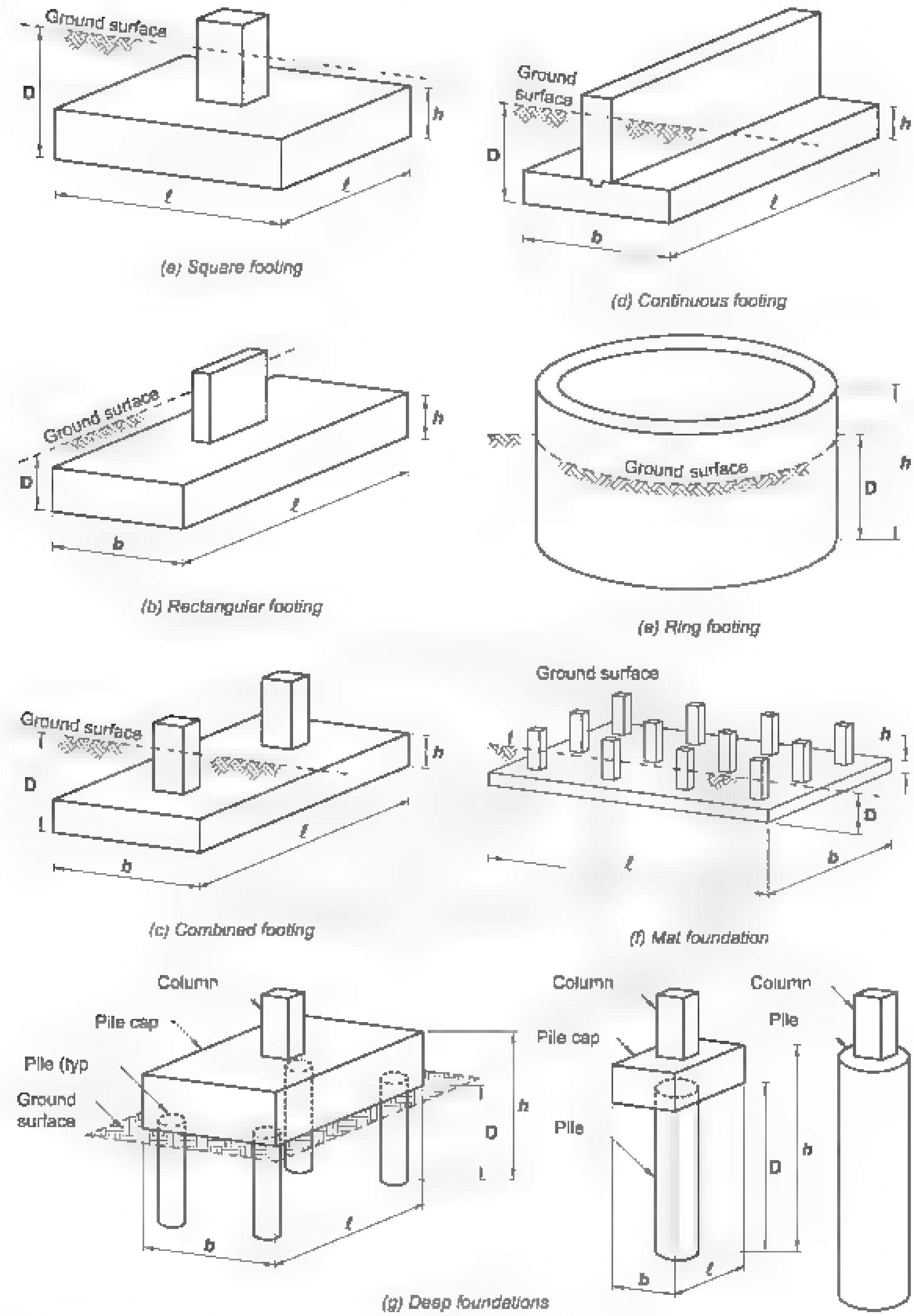


Fig. 11.1—Foundation types.

(d) Shallow footings, assumed not to be able to resist tension, should be able to resist overturning moments from compression reactions only; overturning moments are commonly caused by horizontal loads

(e) Local conditions or site constraints, such as proximity to property lines or utilities, are adequate

4. Code Chapter 13 indicates that one-way shallow foundations including strip footings, combined footings, and grade beams should be designed and detailed according to applicable sections of Code Chapters 7 and 9. Two-way isolated footings should be designed and detailed according to applicable sections of Code Chapter 7 and 8. The Code does not specify which of the provisions of these chapters are applicable to the design of footings. As part of this step, the previously selected geometry is checked against strength requirements of the reinforced concrete sections.

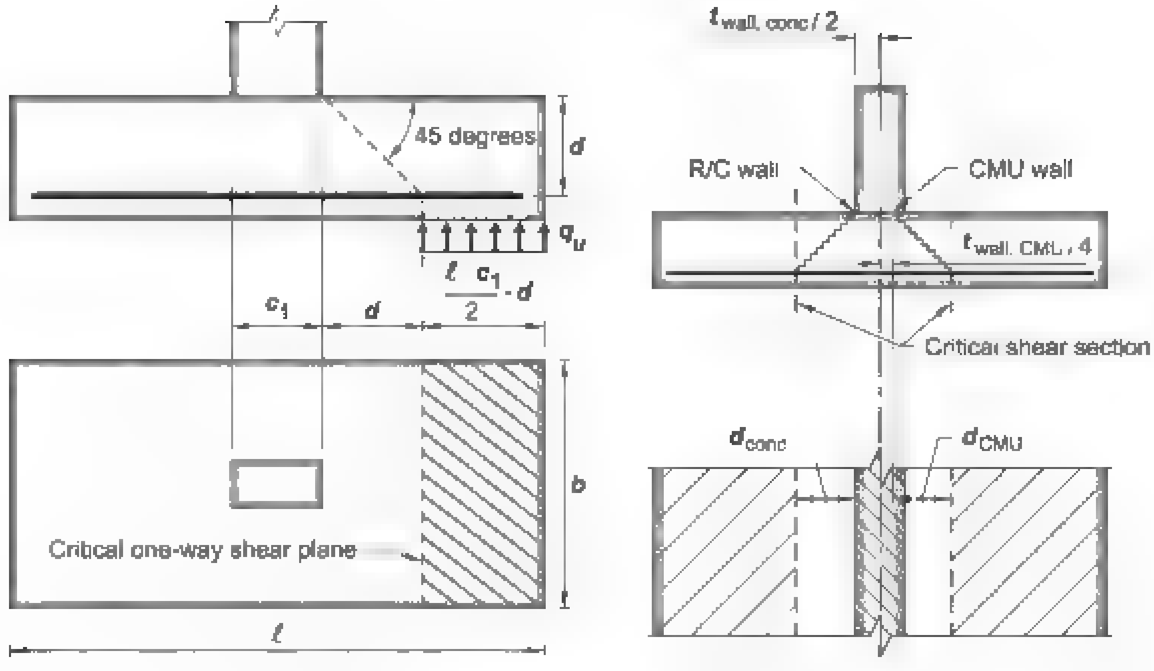
The step-by-step structural design process for concentrically loaded isolated footings follows.

11.3—Design steps

1. Find service dead and live column loads: Code R13.2.6. The footing geometry is selected using service loads.

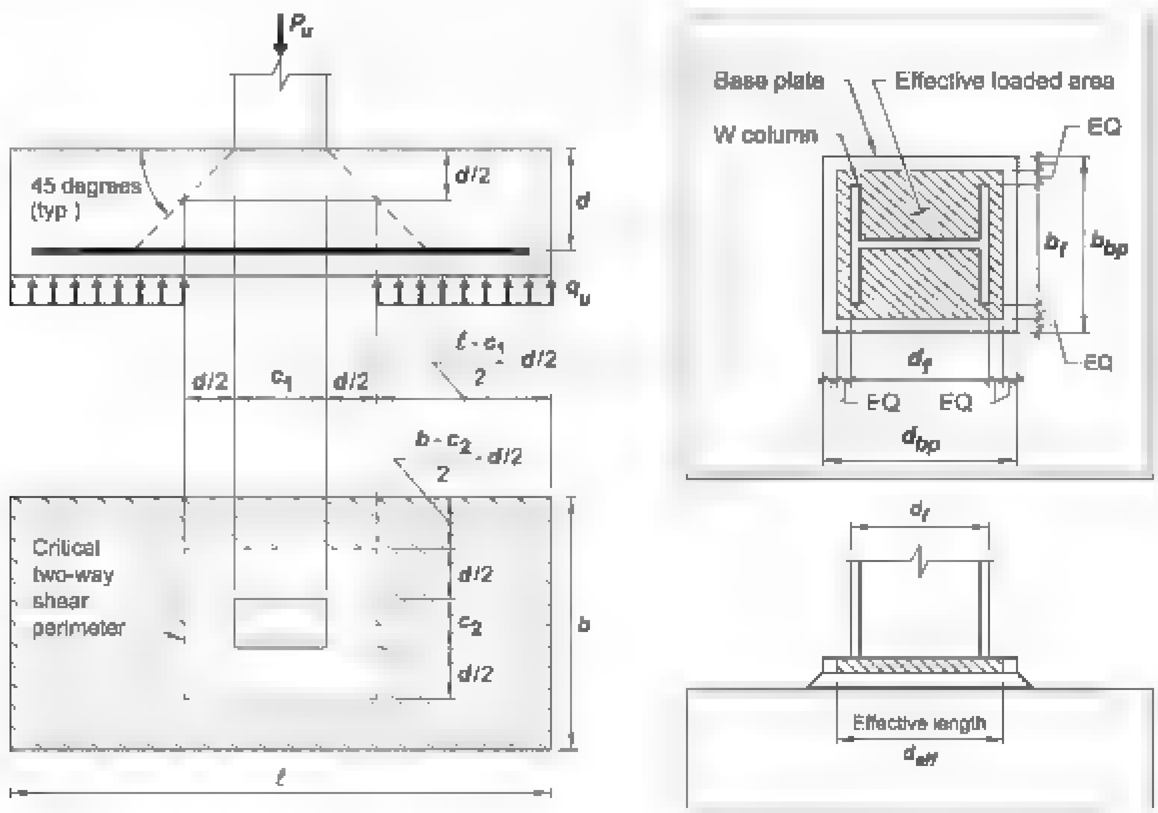
D = service dead load from column

L = service live load from column



(a) Column (b) Reinforced concrete wall or masonry wall

Fig. 11.3a—One-way shear critical section in footings.



(a) Reinforced concrete column (b) Steel column with base plate

Fig. 11.3b—Two-way shear critical section in footings.

$P = D + L$

$A_{req} = P/q_{all}$

For square footings, $\ell \geq \sqrt{A_{req}}$

For rectangular footings, choose one of the sides from site constraints and calculate the other such that: $b \times \ell \geq A_{req}$

2. Calculate the design (factored) column load U: Code 5.3.1

3. Obtain the allowable soil pressure q_{net}. Because soil and concrete unit weights are close (120 and 150 lb/ft³, respectively), the footing self weight may initially be ignored.

4. Calculate the soil pressure based on initial footing base dimensions

Square footing: $q_u = U/\ell^2$

Rectangular footing: $q_u = U/\ell b$

5. Check one-way (beam) shear:

The critical section for one-way shear extends across the width of the footing and is located at a distance d from the face of a column or wall (Fig. 11.3a(a) and Fig. 11.3a(b) left side), Code 8.4.3.2. The shear is calculated assuming the footing is cantilevered away from the column or wall, Code 8.5.3.1.1.

For masonry walls, the critical section for moment is located halfway between the wall center and the face of the

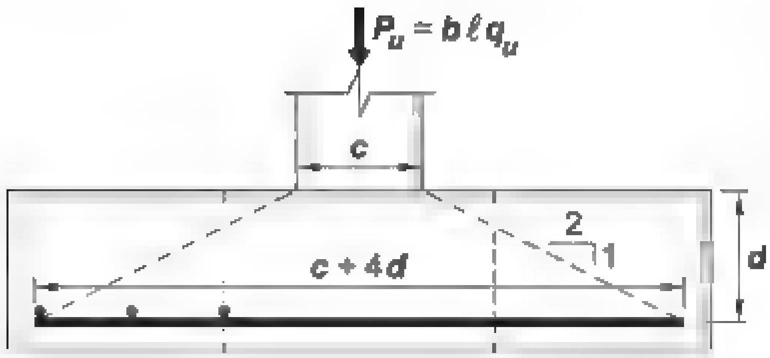


Fig. 11.3c—Column load distribution in footing.

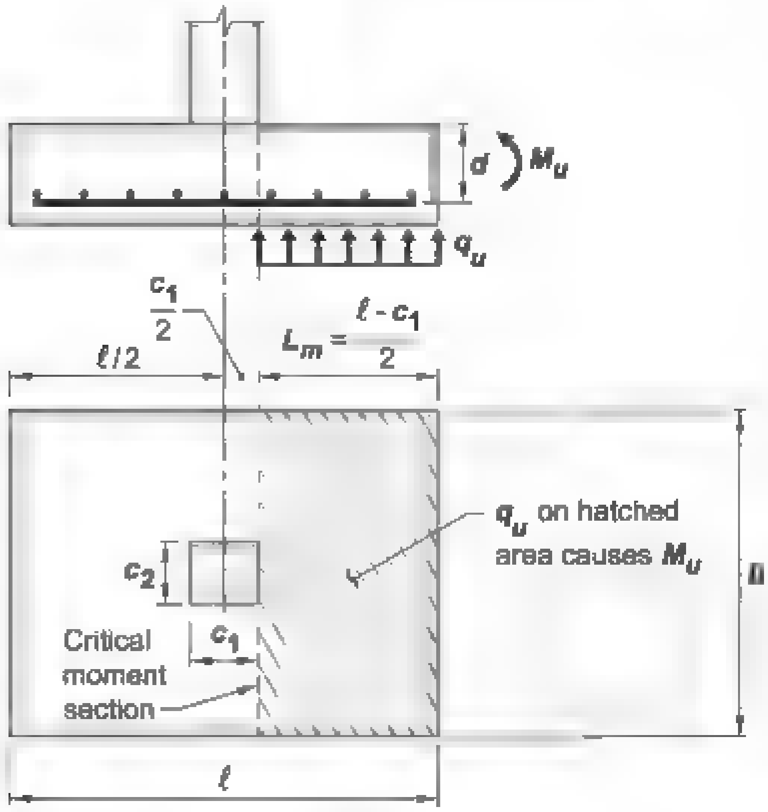


Fig. 11.3d—Moment critical section in footing at reinforced concrete column face.

masonry wall (Fig. 11.3a(b) right side), according to Code Section 13.2.7.2.

For one-way shear strength of isolated footings, the designer is led from Code Chapters 7 and 8 to Code 22.5. The size effect factor for both one- and two-way shear strength may be neglected as indicated in Code 13.2.6.2. Furthermore, minimum shear reinforcement is required where $V_u > \phi V_c$. Isolated footings, however, are typically not designed with shear reinforcement. Consequently, one-way shear strength will oftentimes control the footing thickness. This is particularly true because the change in the shear strength provisions that occurred in ACI 318-19. If shear reinforcement is not provided, size effect is ignored, and axial force is small, then Code equation 22.5.5.1(c) becomes

$$V_c = 8(\rho_w)^{1/3}\sqrt{f'_c}b_w d$$

where ρ_w is the flexural reinforcement ratio ($A_s/b_w d$). An approximation of the concrete contribution based on the minimum flexural reinforcement (Code 7.6.1.1) can be derived by substituting the minimum area of flexural reinforcement into the previous equation. Because the minimum reinforcement is specified as 0.18% of the gross concrete sectional area and ρ_w is based on the effective depth of the section, the ratio of effective depth to member thickness must be known. Assuming a 3 in. clear cover, the following equation provides a conservative estimate of the concrete contribution for a footing up to about 4 ft thick

$$V_c = \sqrt{f'_c}b_w d$$

which is one-half of the concrete contribution contained in previous versions of the Code. This equation is useful for initial estimates of footing thickness and should be confirmed with final calculations using the appropriate equations. If the required footing thickness is excessive, then flexural reinforcement can be increased to improve the shear strength but at a diminished rate due to the one-third power. Another alternative is to add shear reinforcement, which is not a typical practice for footings.

6. Check two-way (slab) shear

(a) Determine the dimensions of loaded area for

i) Rectangular concrete columns, the loaded area coincides with the column area (Fig. 11.3b(a))

ii) Steel columns, the perimeter of the effective loaded area ($d_{eff} \times b_{eff}$) is assumed to be halfway between the faces of the steel column and the edges of the steel base plate (Fig. 11.3b(b)):

$$b_{eff} = b_f + \frac{b_{bp} - b_f}{2}$$

where b_f is the width of column flange, and b_{bp} is the width of the base plate.

$$d_{eff} = d_f + \frac{d_{bp} - d_f}{2}$$

where d_f is the depth of column flange, and d_{bp} is the depth of the base plate

(b) Calculate the shear critical section, located at a distance of $d/2$ outside the loaded area (Code 13.2.7.2)

(c) Calculate the factored shear force for two-way shear stress, v_u

(d) Compare v_u to two-way design stress, ϕv_n, calculated using the equations in Code 22.6.5.2. As with one-way shear, the size effect factor may be neglected for two-way shear

NOTE: If the design shear stress is less than factored shear stress, then increase footing thickness and repeat steps starting at (b)

7. Design and detail the footing reinforcement (Fig. 11.3c). Square footings are designed and detailed for moment in one direction and the same reinforcement is placed in the other direction. For rectangular footings, the reinforcement must be designed and detailed in each direction. The critical section for moment extends across the width of the footing at the face of the column (Code 13.2.6.4 and 13.2.7.1)

(a) Calculate projection, L_m, from the column face (Fig. 11.3d): $L_m = \ell/2 - c/2$, where c is the smaller dimension of the column for a square footing. For a rectangular footing,

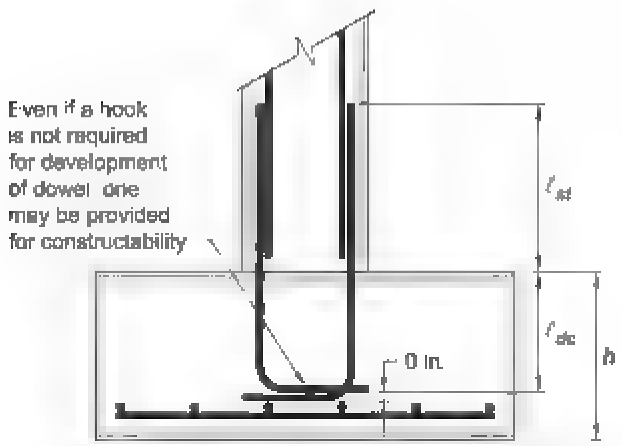


Fig. 11.3e—Column/wall dowels into footing.

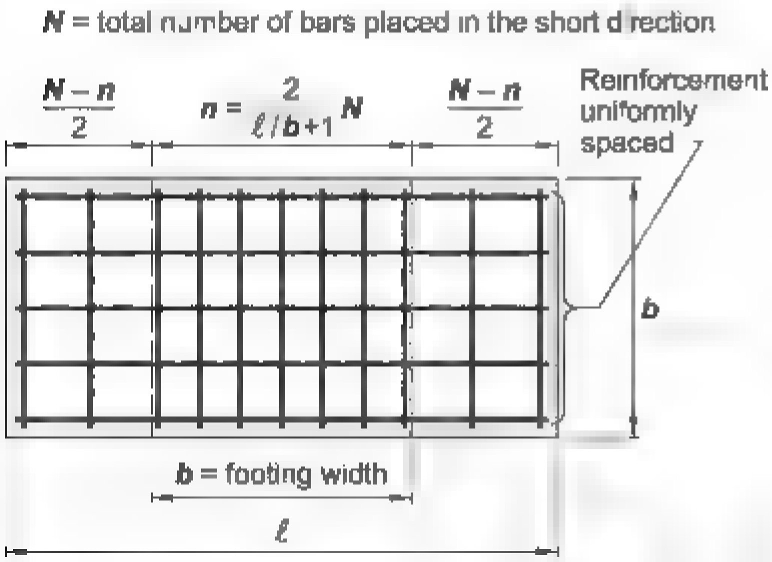


Fig. 11.3f—Bar distribution in short direction.

c is the dimension perpendicular to the critical section in each direction

(b) Calculate total factored moment, M_u, at the critical section

(c) Calculate required A_s

Code 13.3.2.1 and 7.6.1.1 specify minimum flexural reinforcement of $A_{s,min} = 0.0018A_g$, and 7.7.2.3 specifies a maximum bar spacing of $3h$ and 18 in.

8. Check the load transfer from the column to the footing per Code 16.3 (Fig. 11.3e)

(a) Check the bearing strength of the footing concrete: Code 22.8.3.2

(b) Calculate the load to be transferred by reinforcement (usually dowels):

If $\phi B_n > P_u$, only a minimum area of reinforcement is required (Code 16.3.4.1)

(c) Calculate the required reinforcement area and choose bar size and number

(d) Check dowel embedment into footing for compression: Code 25.4.9

NOTE: The footing must be deep enough to develop the dowels in compression, ℓ_{dc}. Hooks are not considered effective in compression and are used to stabilize the dowels during construction

(e) Dowels must be long enough to lap with the column bars in compression, ℓ_{sc}: Code 25.5.5

(f) Choose bar size and spacing

For square footings, A_s must be furnished in each direction. The same size and number of bars should be uniformly spaced in each direction (Code 13.3.2.2 and 13.3.3.2).

For rectangular footings, A_s must be furnished in each direction. Bars in long direction should be uniformly spaced. Bars in the short direction should be distributed as follows (Code 13.3.3.3):

i) In a band of width B_s centered on column

$$\text{No. of bars in } B_s = \frac{2}{\beta+1}$$

(total No. of bars) (round up to an integer)

ii) Remaining bars should be uniformly spaced in outer portions of footing (outside the center band width of footing). The remaining bars should satisfy the minimum reinforcement requirements of Code 9.6.1 (refer to Fig. 11.3f)

(g) Check development length

Calculate the bar's development length, ℓ_d, in tension per Code 25.4. The development length, ℓ_d, must be less than (L_m – end cover). If the (L_m – end cover) is shorter than ℓ_d, use bars of a smaller diameter

11.4—Footings subject to eccentric loading

In addition to vertical loads, footings often resist lateral loads or overturning moments. These loads are typically from seismic or wind forces.

Overturning moments result in a nonuniform soil-bearing pressure under the footing, where soil-bearing pressure is larger on one side of the footing than the other. Nonuniform soil bearing can also be caused by a column located away from the footing's center of gravity.

If overturning moments are small in proportion to vertical loads—that is, the total applied load is located within the kern ($e < \ell/6$)—then the entire footing bottom is in compression and a $P/A \pm M/S$ analysis is appropriate to calculate the soil pressures, where the parameters are defined as follows:

P = the total vertical service load, including any applied loads along with the weight of all the foundation components, and also including the weight of the soil located directly above the footing

A = the area of the footing bottom

M = the total overturning service moment at the footing bottom

S = the section modulus of the footing bottom

If overturning moments are larger—that is, the total applied load falls outside the kern, $e > \ell/6$—then $P/A - M/S$ analysis requires the soil to resist tension (upward movement of the footing), which is not possible.

This soil is only able to transmit compression.

The following are typical steps to calculate footing bearing pressures if nonuniform bearing pressures are present. These steps are based on a footing that is rectangular in plan and assumes that overturning moments are parallel to one of the footing's principal axes. These steps should be completed for as many load combinations as required by the applicable design criteria. For instance, the load combination with the maximum P usually causes the maximum bearing pressure

while the load combination with the minimum P usually is critical for overturning

1. Determine the total service vertical load P
2. Calculate the total service overturning moment M, measured at the footing bottom
3. Determine whether P/A exceeds M/S
4. If P/A exceeds M/S, then the maximum bearing pressure equals $P/A + M/S$ and the minimum bearing pressure equals $P/A - M/S$
5. If P/A is less than M/S, then the soil bearing pressure is as shown in Fig. 11.4. Such a soil-bearing pressure distribution is structurally inefficient. The maximum bearing pressure, shown in the figure, is calculated as follows: maximum bearing pressure $= 2P/[(B)(X)]$, where $X = 3(\ell/2 - e)$ and $e = M/P$

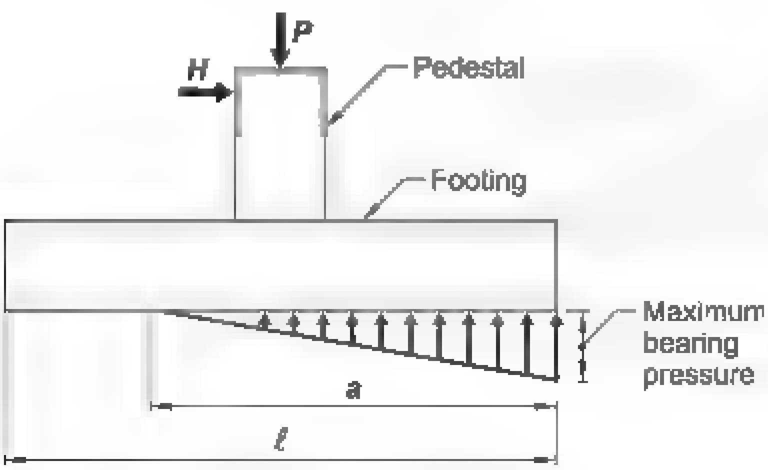


Fig. 11.4—Footing under eccentric loading.

11.5—Combined footing

If a column is near a property line or near a pit or a mechanical equipment in an industrial building, a footing may not be able to support a column concentrically and the eccentricity is very large. In such a case, the column footing is extended to include an adjacent column and support both on the same footing, called combined footing (Fig. 11.5). The combined footing is sized to have the resultant force of the two columns within the kern, or preferably to coincide with the center of the footing area. The combined footing can be rectangular, trapezoidal, or having a strap, connecting the two main column footings together (Fig. 11.5(c)).

Design steps

1. Calculate the total service column loads, P_1 and P_2 (Code R13.2.6)
2. Calculate service column load resultant location Center for rectangular footing

$$x_c = \frac{P_1 x_1 + P_2 x_2}{\Sigma P}$$

If P_1 is much larger than P_2, then trapezoidal combined footing may be used

Determine combined footing length from construction constraints

Calculate the widths B_1 and B_2 such that the center of the footing coincides with the force resultant or is at least within $\ell/6$ of the force resultant

3. Determine combined footing dimensions assuming uniform bearing

Combined rectangular footing length, $\ell = 2x_c$

Combined rectangular footing width

$$x = \frac{\ell}{3}\frac{B_1 + 2B_2}{B_1 + B_2} - \frac{c}{2}$$

4. Steps to design a combined footing to resist one-way and two-way shear and moment is similar to the isolated footing design steps presented previously

11.6—Retaining wall design

Prior to ACI 318-19, cantilever retaining wall design provisions were contained in the chapter covering structural walls. In ACI 318-19, however, retaining wall design provisions were moved to Code Chapter 13 - Foundations. The stem wall of a cantilever retaining wall is designed as a one-way slab in accordance with Code Chapter 7 and the footing is designed as a one-way shallow footing. Further

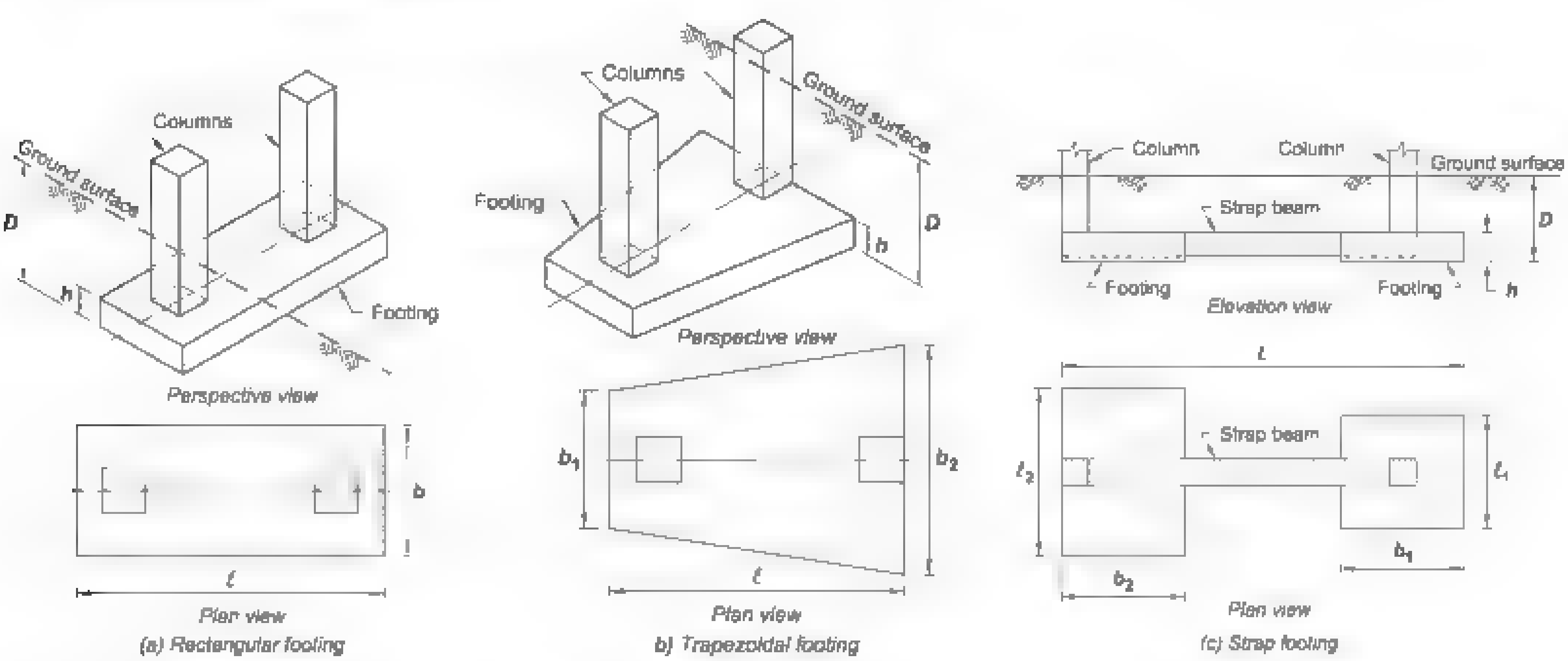


Fig. 11.5—Common types of combined footing geometries.

Table 11.7a—Allowable compressive strength for deep foundation members (Code Table 13.4.2.1)

Deep foundation member type	Maximum allowable compressive strength[1]	
Uncased cast-in-place concrete drilled or augered pile	$P_a = 0.3f_c'A_g + 0.4f_yA_s$	(a)
Cast-in-place concrete pile in rock or within a pipe, tube, or other permanent metal casing that does not satisfy 13.4.2.3	$P_a = 0.33f_c'A_g + 0.4f_yA_s$[2]	(b)
Metal cased concrete pile confined in accordance with 13.4.2.3	$P_a = 0.4f_c'A_g$	(c)
Precast nonprestressed concrete pile	$P_a = 0.33f_c'A_g + 0.4f_yA_s$	(d)
Precast prestressed concrete pile	$P_a = (0.33f_c' - 0.27f_{pc})A_g$	(e)

[1]A_g applies to the gross cross-sectional area. If a temporary or permanent casing is used, the inside face of the casing shall be considered the concrete surface.

[2]A_s does not include the steel casing, pipe, or tube.

details of retaining wall design are covered in Chapter 12 of this Manual.

11.7—Deep foundation member design

In the 2019 Code edition, provisions for the structural design of deep foundations were added. Prior to this, deep foundation elements were designed using a combination of relevant Code provisions along with provisions from ASCE/SEI 7 and the IBC. These design provisions have been gathered into Chapter 13 and Chapter 18 of the Code. As indicated in Section 1.4.7 of the Code, precast piles can be designed for use in all seismic design categories, but cast-in-place pile design is limited to members used in SDC C, D, E, and F.

The primary mode of loading of deep foundation elements is axial compression. This load is delivered to the surrounding soil through end bearing and possibly side friction. In addition, deep foundation members may be loaded in axial tension to resist overturning of the pile cap or uplift due to global overturning. Deep foundation members may also be loaded in shear by wind or earthquake loads, as with any other structural concrete element, these members must be checked for all possible load combinations.

Piles, piers, or caissons must be proportioned for soil bearing strength, possible skin-friction resistance, and resistance to overturning of the pile cap. In addition, excessive settlement due to compression and consolidation of the underlying soil must be addressed. Deep foundation capacity is to be determined using principles of soil or rock mechanics in accordance with the general building code, or other requirements as determined by the authority having jurisdiction. Practically, these design considerations are usually addressed by the geotechnical engineer with input from the structural engineer. The structural strength of the deep foundation element and its connection to the pile cap must be also be considered, which is the purview of the structural engineer and the focus of this section of the Manual.

The successful design of a concrete deep foundation member involves intimate knowledge of the relevant geotechnical and structural design requirements, manufacturing process, transportation details, and installation procedures. Concrete deep foundation members are classified according to the condition under which the concrete is cast. Precast piles are cast in a plant before driving, which allows controlled inspection of all phases of manufacture. Cast-in-place (CIP) drilled piers or caissons are fabricated by placing concrete into a previously driven, enclosed container such as corrugated shells or pipes. CIP deep foundation members can also be fabricated by casting concrete directly against the earth.

Table 11.7b—Strength reduction factors for use in strength design of deep foundation members with axial load only (Code Table 13.4.3.2)

Deep foundation member type	Compressive strength reduction factors ϕ	
Uncased cast-in-place concrete drilled or augered pile[1]	0.55	(a)
Cast-in-place concrete pile in rock or within a pipe, tube,[2] or other permanent casing that does not satisfy 13.4.2.3	0.60	(b)
Cast-in-place concrete-filled steel pipe pile[3]	0.70	(c)
Metal cased concrete pile confined in accordance with 13.4.2.3	0.65	(d)
Precast-nonprestressed concrete pile	0.65	(e)
Precast-prestressed concrete pile	0.65	(f)

[1]The factor of 0.55 represents an upper bound for well understood soil conditions with quality workmanship. A lower value for the strength reduction factor may be appropriate, depending on soil conditions and the construction and quality control procedures used.

[2]For wall thickness of the steel pipe or tube less than 0.25 in.

[3]Wall thickness of the steel pipe shall be at least 0.25 in.

Historically, geotechnical and structural design of deep foundations have been on a service load basis, with structural requirements for the various types of piling specified by building codes or other regulatory agencies on the material allowable stress (ACI 543R-12). Code provisions for deep foundation member design, however, allow the use of either allowable load (Code Section 13.4.2) or strength (Code Section 13.4.3) design method. Some restrictions are in place for allowable axial strength. The deep foundation member must be laterally supported for its entire height and, if bending moment is present, then it must be less than the moment due to an accidental eccentricity of 5% of the pile diameter or width. Furthermore, the load combinations for allowable stress design in ASCE/SEI 7, Section 2.4, must be used. If these restrictions and requirements are met, then the deep foundation member can be designed for the allowable loads calculated from the equations in Table 11.7a. Further restrictions are imposed by the member type limitations shown in the table.

The designer is referred to Code Section 10.4 for the strength design of deep foundation members. For members with axial load only, the strength reduction factors given in Table 11.7b must be used. If the element has tension, shear, or combined axial force and moment, then the typical strength reduction factors from Table 21.2.2 in the Code (Table 11.7b herein) should be used. The Code also warns the designer regarding the strength reduction factors in the Code for cast-in-place concrete drilled or augered piles. Both the Code and ACI 543R indicate that the strength reduction factors

provided in the Code are predicated on quality workmanship, well-understood soil conditions, and sound quality control procedures. Problems with uncased concrete deep foundation members are possible as a result of soil penetrating the column section while concrete or grout is still fresh. Furthermore, reinforcement placement can vary significantly from its specified position during insertion. Verification of placement by visual inspection is not possible with this type of construction. Because the strength reduction factor is a function of both the dimensional reliability of the cross section and the dependence of the member strength on the strength of the concrete actually attained in the member, local experience with the soils and construction techniques may indicate that a lower value be selected for design.

In general, deep foundation members have structural behavior similar to that of columns, but there can be major differences between the two regarding lateral support conditions and construction and installation methods. In the case when the surrounding soil is adequate, the member may be assumed to be fully supported laterally, whereas columns may be laterally unsupported or sometimes supported only at intervals. The failure mode of a column is due entirely to structural inadequacy, while deep foundation members can have two distinctly different failure modes caused by either inadequate capacity of the pile-soil system (excessive settlement) or insufficient structural capacity of the pile. One additional structural design consideration is the temporary loads and stresses imposed during transportation and installation of the pile. Driven precast prestressed piles, for instance, experience driving forces that must be addressed. Stresses generated during handling and driving is one reason for the minimum f_c' for precast prestressed piles specified in Code Section 19.2.1.1 and the minimum effective prestress levels specified in Code Section 13.4.5.4.

From a reliability perspective, single columns are usually more critical than a single pile within a group. Columns do not typically have the redundant load path to transfer load in case of failure, which can lead to partial or full collapse of a structure if lost. A single structural column is often supported by a group of four or more piles, which improves the redundancy of the deep foundation system. Where drilled piers or caissons are used, however, it is also common to have a column supported on a single pier.

Table 11.8—Minimum compressive stress in precast prestressed piles (Code Table 13.4.5.4)

Pile length, ft	Minimum compressive stress, psi
Pile length < 30	400
30 < Pile length < 50	550
Pile length > 50	700

11.8—Deep foundation member detailing

Detailing of deep foundation members is important to ensure proper behavior of the member under axial load, flexure, and shear. Code Section 13.4.4.1 requires that cast-in-place deep foundations be reinforced or enclosed by a structural steel pipe or tube where they are subject to uplift or where M_u is greater than $0.4M_{cr}$.

For precast concrete piles, Code Section 13.4.5 requires that precast non-prestressed piles be reinforced with at least four bars arranged in a symmetrical pattern with a minimum area of $0.008A_g$.

Precast prestressed piles must have a minimum effective prestress as indicated in Table 11.8 assuming a total prestress loss of 30,000 psi in the prestressed reinforcement. This minimum prestressing provides sufficient precompression to reduce the potential for cracking caused by handling and driving under typical conditions. Unusual handling or driving conditions may dictate that a higher precompression is needed, which would require additional prestressed reinforcement. Transverse reinforcement is required to enclose the longitudinal reinforcement and can be smooth or deformed wires or reinforcing bars. In precast prestressed concrete piles, the transverse reinforcement is typically a spirally wrapped wire. Minimum size and spacing is given in Code Tables 13.4.5.6a and b.

For the following deep foundation members in SDC C or higher, additional detailing and design requirements are included in Code Section 18.13.5:

- Uncased cast-in-place concrete drilled or augered piles
- Metal-cased concrete piles
- Concrete-filled pipe piles
- Precast concrete piles

11.9—Examples

Foundation Example 1: *Design of a square spread footing for a seven-story building*

Design and detail a typical square footing of a six bay by five bay seven-story building, founded on stiff soil, supporting a 24 in. square column. The building has a 10 ft high basement. The bottom of the footing is 3 ft below finished grade (refer to Fig. E1.1). The building is assigned to Seismic Design Category (SDC) B.

<u>Given:</u>

Column load

Service dead load $D = 541$ kip
Service live load $L = 194$ kip
Earthquake $E = \pm 18$ kip
(Earthquake induced axial force effect on footing)

Material properties

Concrete compressive strength $f_c' = 4$ ksi
Steel yield strength $f_y = 60$ ksi
Normalweight concrete $\lambda = 1$
Density of concrete = 150 lb/ft^3

Allowable soil-bearing pressures—

D only: $q_{all,D} = 4000$ psf
$D + L$: $q_{all,D+L} = 5600$ psf
$D + L + E$: $q_{all,+E} = 6000$ psf

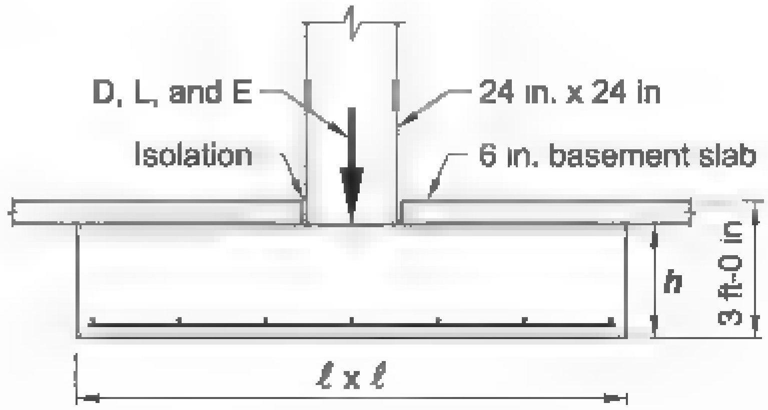


Fig. E1.1—Rectangular foundation plan.

ACI 318	Discussion	Calculation
Step 1: Foundation type		
13.1.1	The bottom of the footing is 3 ft below the basement slab. Therefore, it is considered a shallow foundation.	
13.3.3.1	The footing will be designed and detailed with the applicable provisions of Chapter 7, One-way slabs, and Chapter 8, Two-way slabs, of ACI 318.	

Step 2: Material requirements		
	Concrete compressive strength	
19.2.1.1 19.2.1.3	The value of concrete compressive strength at a given age must be specified in the contract documents. Table 19.2.1.1 provides a lower concrete compressive strength limit of 2500 psi.	Provided: f_c' = 4000 psi > $f'_{c,min}$ = 2500 psi **OK**
	Exposure categories and classes	
19.3.1 19.3.2	The engineer must either assign exposure classes to the footing with respect to Table 19.3.1.1 (ACI 318) so the ready-mix supplier can proportion the concrete mixture, or use the classes to directly specify mixture proportions in the contract documents. Based on the exposure classes, the concrete mixtures must satisfy to the most restrictive requirements of Table 19.3.2.1.	
	Concrete exposure categories There are four categories: F, S, W, and C.	
	Category F	
19.3.1.1	The foundation is placed below the frost line, therefore, it is not exposed to external elements—freezing and thawing cycles. Therefore, class F0 applies.	Class F0 Maximum $(w/cm)_{max}$ = N/A Minimum f_c' = 2500 psi
19.3.2.1	Mixture requirements that must be satisfied for F0 are listed in Table 19.3.2.1.	Air content is not required and there are no limits on cementitious materials
19.3.3.1	Requirements of Table 19.3.3.1 do not apply.	
	Category S	
19.3.1.1 19.3.2.1	Injurious sulfate attack is not a concern. Mixture requirements for S0 are listed in Table 19.3.2.1.	S0 → $(w/cm)_{max}$ = N/A and f_c' = 2500 psi
	Category W	
19.3.1.1 19.3.2.1	The footing may be in contact with water and low permeability is not required.	W0 → $(w/cm)_{max}$ = none and f_c' = 2500 psi
	Category C	
19.3.1.1 19.3.2.1	The concrete is exposed to moisture and there is no external source of chlorides, therefore the class is C1. Mixture requirements for C1 are listed in Table 19.3.2.1.	C1 → $(w/cm)_{max}$ = none and f_c' = 2500 psi Therefore, there is no restriction on w/cm and f_c' = 4000 psi

Conclusion:

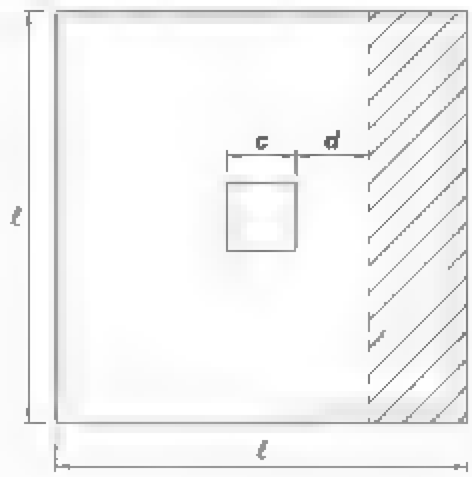
(a) The most restrictive minimum concrete compressive strength is 2500 psi, and no limits on the w/cm. Therefore, in the judgment of the licensed design professional, use 4000 psi concrete compressive strength.

(b) Other parameters, such as maximum chloride ion content and air content, are exposure specific, and thus not compared with other exposure limits.

(c) The f_c' utilized in the strength design must be at least what is required for durability.

Step 3: Determine footing dimensions		
13.3.1.1	To calculate the footing base area, divide the service load by the allowable soil pressure	The unit weights of concrete and soil are 150 pcf and 120 pcf; close. Therefore, footing self-weight will be ignored for initial sizing of footing.
	$\text{area of footing} = \dfrac{\text{total service load } (\sum P)}{\text{allowable soil pressure } q_a}$	$\dfrac{D}{q_{all,D}} = \dfrac{541\text{ kip}}{4\text{ ksf}} = 135\text{ ft}^2$ **Controls**
		$\dfrac{(D+L)}{q_{all,D+L}} = \dfrac{541\text{ kip} + 194\text{ kip}}{5.6\text{ ksf}} = 131\text{ ft}^2$
		$\dfrac{D+L+E}{q_{all,Lat}} = \dfrac{541\text{ kip} + 194\text{ kip} + (0.7)18\text{ kip}}{6\text{ ksf}} = 125\text{ ft}^2$
	Assuming a square footing	$\ell = \sqrt{135\text{ ft}^2} = 11.6\text{ ft}$
	The footing thickness is calculated in Step 5	Therefore, try a 12 x 12 ft square footing

Step 4. Soil pressure		
	Footing stability Because the column doesn't impart a moment to the footing, the soil pressure under the footing is assumed to be uniform and overall footing stability is assumed. Calculate factored soil pressure. This value is needed to calculate the footing's required strength. $q_u = \frac{\sum P_u}{\text{area}}$ Calculate the soil pressures resulting from the applied factored loads.	
5.3.1(a)	Load Case I: $U = 1.4D$	$U = 1.4D = 1.4(541\text{ kip}) = 757\text{ kip}$ $q_u = \frac{757\text{ kip}}{144\text{ ft}^2} = 5.3\text{ ksf}$
5.3.1(b)	Load Case II: $U = 1.2D + 1.6L$	$U = 1.2D + 1.6L = 1.2(541\text{ kip}) + 1.6(194\text{ kip}) = 960\text{ kip}$ $q_u = \frac{960\text{ kip}}{144\text{ ft}^2} = 6.7\text{ ksf}$ **Controls**
5.3.1(d)	Load Case IV: $U = 1.2D + E + L$	$U = 1.2D + 1.0E + 1.0L = 1.2(541\text{ kip}) + 18\text{ kip} + 1.0(194\text{ kip}) = 861\text{ kip}$ $q_u = \frac{861\text{ kip}}{144\text{ ft}^2} = 6.0\text{ ksf}$
5.3.1(e)	Load Case IV: $U = 0.9D + E$	$U = 0.9D + 1.0E = 0.9(541\text{ kip}) + 18\text{ kip} = 505\text{ kip}$ $q_u = \frac{505\text{ kip}}{144\text{ ft}^2} = 3.5\text{ ksf}$
	The load combinations include the seismic uplift force. In this example, uplift does not occur.	Note: The full definition of E includes not only earthquake loads due to overturning but also earthquake loads due to vertical acceleration of ground as per ASCE/SEI 7, Section 12.4.2.
13.3.2.1	Design square isolated footing assuming two-way action. Distribute reinforcement across the entire width in both directions.	

Step 5. One-way shear design		
		 *Fig. E1.2—One-way shear in longitudinal direction.*
7.5.1.1	$\phi V_n > V_u$	
13.2.6.2	To set the depth of the footing, consider one-way and two-way shear. Size effect factor in calculating both one-way and two-way shear strength contribution of concrete may be neglected.	
7.5.3.1 22.5	Shear reinforcement is not typically used in one-way slabs and footings, so all of the shear strength is provided by the concrete contribution $\phi V_n = \phi V_c$	
21.2.1	Strength reduction factor for shear from Table 21.2.1b	$\phi = 0.75$
7.6.3.1	Minimum shear reinforcement is required where $V_u > \phi V_c$. Footings, however, are not typically constructed with shear reinforcement. Provide sufficient depth to avoid the need for minimum shear reinforcement.	
22.5.5.1c	Ignoring size effects, axial load, and using normalweight concrete, the applicable equation from Table 22.5.5.1c becomes $\phi V_c = \phi 8(\rho_w)^{1/3}\sqrt{f'_c}b_w d$ If ρ_w is set to the minimum required flexural reinforcement ratio of 0.0018, then the equation becomes $\phi V_c = \phi 0.97\sqrt{f'_c}b_w d$	

13.2.7.2 Factored shear is calculated for the critical section located at d from the face of the column (Fig. E1.2)

$$\phi V_c \geq V_u = \left(\frac{\ell}{2} - \frac{c}{2} - d\right) bq_u$$

$$\left(\frac{144\text{ in.}}{2} - \frac{24\text{ in.}}{2} - d\right)\frac{12\text{ ft}(6.7\text{ ksf})}{12\text{ in./ft}}$$

$$= 0.75(0.97)\sqrt{4000\text{ psi}}(144\text{ in.})d$$

Required depth of footing is
$d = 30.17$ in.

Use centroid of upper layer of reinforcement to calculate footing thickness
$h = 30.17\text{ in.} + 3\text{ in.} + 1.5(1\text{ in.}) = 34.67\text{ in.}$

Try footing thickness of 36 in.
$d = 36\text{ in.} - 3\text{ in.} - 1.5(1\text{ in.}) = 31.5\text{ in.}$

$$d = 30\text{ in.} - 3\text{ in.} - 1\text{ in.} - 1\text{ in.}/2 = 25.5\text{ in.}$$

$$V_u = \left(\frac{12\text{ ft}}{2} - \frac{24\text{ in.}}{2(12\text{ in./ft})} - \frac{31.5\text{ in.}}{12\text{ in./ft}}\right)(12\text{ ft})(6.7\text{ ksf})$$

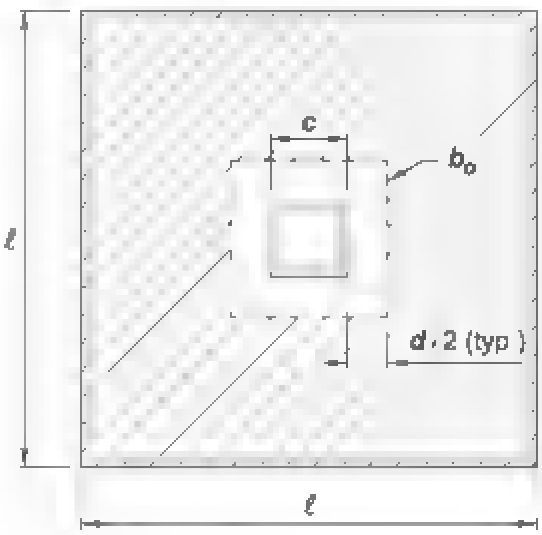
$$= 191\text{ kip}$$

$$\phi V_n > V_u = \left(\frac{\ell}{2} - \frac{c}{2} - d\right) bq_u$$

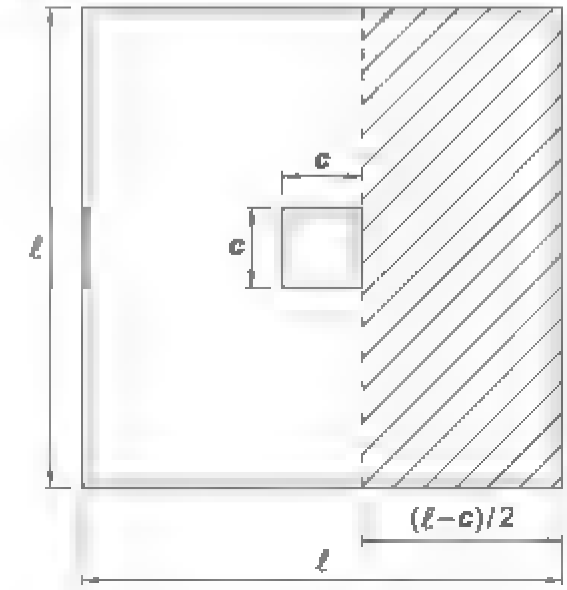
$$\phi V_c = 0.75(0.97)\sqrt{4000\text{ psi}}(12\text{ ft})(31.5\text{ in.})(12\text{ in./ft})$$

$$= 208\text{ kip}$$

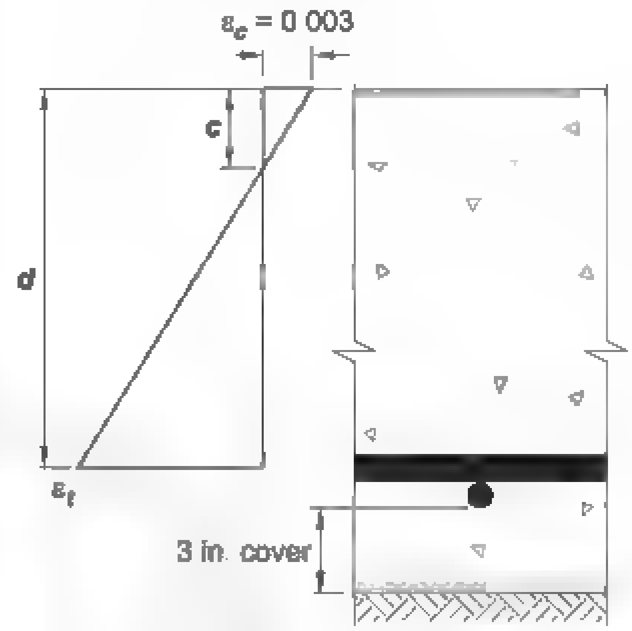
$\phi V_c = 208\text{ kip} > V_u = 191\text{ kip}$ **OK**

Step 6: Two-way shear design		
13.3.3.1 13.2.7.2	The footing will not have shear reinforcement. Therefore, the nominal shear strength for this two-way footing is the concrete contribution to shear strength $v_n = v_c$	 *Fig. E1.3—Two-way shear.*
22.6.1.2 22.6.1.4 22.6.4.1	Under punching shear theory, inclined cracks are assumed to originate and propagate at 45 degrees away and down from the column corners. The area of concrete that resists shear is calculated at an average distance of $d/2$ from column face on all sides (refer to Fig. E1.3).	
	$b_o = 4(c + d)$	$b_o = 4(24 + 31.5) = 222$ in.
	where b_o is the perimeter of the area of shear resistance	
22.6.2.1	ACI 318 permits the engineer to take the average of the effective depth in the two orthogonal directions when calculating the shear strength of the footing, but in this example the smaller effective depth is used	
8.5.3.1.2 22.6.1	Check two-way shear with the selected footing thickness	
22.6.5.2	Calculate the shear strength contribution of concrete using the following formulas $4\lambda_s\lambda\sqrt{f'_c}$ $\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c}$ $\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c}$ Ignoring size effects, the equations become	
	(a) $v_c = 4\lambda\sqrt{f'_c}$	$v_c = 4(1.0)(\sqrt{4000\text{ psi}}) = 253$ psi **Controls**
	(b) $v_c = \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f'_c}$	$v_c = \left(2+\frac{4}{1}\right)(1.0)(\sqrt{4000\text{ psi}}) = 379.5$ psi
	where β is ratio of the long side to short side of column, β = 1	
	(c) $v_c = \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f'_c}$	$v_c = \left(\frac{(40)(25.5\text{ in.})}{198\text{ in.}}+2\right)(1.0)(\sqrt{4000\text{ psi}}) = 452$ psi

22.6.5.3	$\alpha_s = 40$, considered interior column $V_c = 4\lambda\sqrt{f'_c}b_o d$	Equation (a) controls, $v_c = 253$ psi $V_c = \dfrac{4(1.0)(\sqrt{4000\text{ psi}})(198\text{ in.})(25.5\text{ in.})}{1000\text{ lb/kip}} = 1277\text{ kip}$
21.2.1(b)	Use a shear strength reduction factor of 0.75 $\phi V_c = (0.75)4\lambda\sqrt{f'_c}b_o d$ $V_u = q_u[(a)^2 - (c + d)^2]$	$\phi = 0.75$ $\phi V_c = 0.75(1769\text{ kip}) = 1327\text{ kip}$ **OK** $V_u = (6.7\text{ ksf})\left((12\text{ ft})(12\text{ ft}) - \left(\dfrac{24\text{ in.} + 25.5\text{ in.}}{12\text{ in./ft}}\right)^2\right) = 851\text{ kip}$
8.5.1.1	Check if design strength exceeds required strength $\phi V_c > V_u$?	$\phi V_c = 1327\text{ kip} > V_u = 821\text{ kip}$ **OK** Two-way shear strength is adequate.

Step 7: Flexure design		
13.2.7.1	The code permits the critical section to be at the face of the column (refer to Fig. E1.4)	 *Fig. E1.4—Flexure in the longitudinal direction*
	$M_u = q_u\left(\frac{\ell - c}{2}\right)^2 (b)/2$	$M_u = (6.7\text{ ksf})\left(\frac{12\text{ ft} - \frac{24\text{ in.}}{12\text{ in./ft}}}{2}\right)^2 (12\text{ ft})/2 = 1005\text{ ft-kip}$
22.2.1.1	Set concrete compression force equal to the steel tension force at the column face: $C = T$	
22.2.2.4.1	$C = 0.85f_c'ba$ and $T = A_sf_y$ $a = \frac{A_sf_y}{0.85f_c'b}$ and	$a = \frac{A_s(60\text{ ksi})}{0.85(4\text{ ksi})(12\text{ ft})} = 0.15A_s$
7.5.2.1	$\phi M_n = \phi A_sf_y\left(d - \frac{a}{2}\right)$	
21.2.1a	Assume section is tension controlled so that $\phi = 0.9$ Substitute for a in the equation above.	
8.5.1.1(a)	Setting $\phi M_n > M_u = 1005$ ft-kip and solving for A_s.	$\phi M_n \geq (0.9)A_s(60\text{ ksi})\left(25.5\text{ in.} - \frac{(0.15)A_s}{2}\right)$ $A_s \geq 7.2\text{ in.}^2$
13.3.3.2	Distribute bars uniformly across the entire 12 ft width of footing	Use 13 No. 8 bars ($13 \times 0.79 = 10.27\text{ in.}^2$) distributed uniformly across the entire 12 ft width of footing.
8.6.1.1	Check the minimum reinforcement ratio: $\rho_t = 0.0018$	$A_{s,min} = 0.0018(12\text{ ft})(12\text{ in./ft})(36\text{ in.}) = 9.4\text{ in.}^2$ $A_{s,prov} = 10.27\text{ in.}^2 > A_{s,min} = 9.4\text{ in.}^2$
21.2.1(a)	Check the assumption of tension controlled behavior	

Foundations

7.3.3.1	Confirm that section is tension-controlled. The strain in reinforcement is calculated from similar triangles (refer to Fig. E1.5)	$a = 0.15(13)(0.79\text{ in.}^2) = 1.26\text{ in.}$ $c = \dfrac{1.03\text{ in.}}{0.85} = 1.48\text{ in.}$
	$\varepsilon_t = \dfrac{\varepsilon_c}{c}(d - c)$	$\varepsilon_t = \dfrac{0.003}{1.48\text{ in.}}(31.5\text{ in.} - 1.48\text{ in.}) = 0.074$ $\varepsilon_t = 0.075 > 0.005$
22.2.2.4.1 22.2.2.4.3	where: $c = a/\beta_1$ and $a = 0.15A_s$	Section is tension controlled  *Fig. E1.5—Strain distribution across footing cross section*
	Verify that allowable soil pressure is not exceeded when including footing self weight and slab self weight and live load above footing:	
	Footing self weight less soil self weight	$W_f = (12\text{ ft})(12\text{ ft})\left(\dfrac{30\text{ in.}}{12}\right)(0.15\text{ kcf} - 0.12\text{ kcf}) = 10.8\text{ kip}$
	Slab self weight and assume 40 psf live load	$W_s = (12\text{ ft})(12\text{ ft})(0.5\text{ ft}(0.15\text{ kcf}) + 0.04\text{ ksf}) = 16.6\text{ kip}$
	Total weight on supporting soil:	$W_T = 541\text{ kip} + 194\text{ kip} + 13.0\text{ kip} + 16.6\text{ kip} = 764.6\text{ kip}$
	Calculate actual soil pressure:	$q_u = \dfrac{762.4\text{ kip}}{(12\text{ ft})(12\text{ ft})} = 5.3\text{ ksf} < q_{all} = 5.6\text{ ksf}$ **OK**

Step 8. Transfer of column forces to the base		
13.2.2.1 16.3.1.1	Factored column forces are transferred to the footing by bearing on concrete and through reinforcement	
22.8.3.2	The foundation is wider on all sides than the loaded area. Therefore, the nominal bearing strength, B_n, is the smaller of the two equations.	
22.8.3.2(a)	$B_n = \sqrt{\frac{A_1}{A_2}}(0.85f_c'A_1)$ and	
22.8.3.2(b)	$B_n = 2(0.85f_c'A_1)$	
	Check if $\sqrt{\frac{A_2}{A_1}} \leq 2.0$ where A_1 is the bearing area of the column and A_2 is the area of the part of the supporting footing that is geometrically similar to and concentric with the loaded area.	$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{[(12\text{ ft})(12\text{ in./ft})]^2}{(24\text{ in.})^2}} = 6 > 2$ Therefore, Eq. (22.8.3.2(b)) controls.
21.2.1(d)	The bearing strength reduction factor is 0.65	$\phi_{bearing} = 0.65$ $\phi B_n = (0.65)(2)(0.85)(4000\text{ psi})(24\text{ in.})^2$ $\phi B_n = 2546\text{ kip} > 960\text{ kip}$ (Step 4) **OK**
16.3.4.1	Column factored forces are transferred to the foundation by bearing and through reinforcement, usually dowels. Provide dowel area of at least $0.005A_g$ and at least four bars.	$A_{s,dowel} = 0.005(24\text{ in.})^2 = 2.88\text{ in.}^2$ Use eight No. 6 bars
16.3.5.1	Bars are in compression for all load combinations. Therefore, the dowels must extend into the footing a compression development length, ℓ_{dc}, the larger of the two expressions and at least 8 in. (refer to Fig. E1.6):	
25.4.9.2	$\ell_{dc} = \begin{cases} \dfrac{f_y\psi_r}{50\lambda\sqrt{f_c'}}d_b \\ (0.0003f_y\psi_r d_b) \end{cases}$ where ψ_r = confining reinforcement factor; ψ_r = 1.0, because reinforcement is not confined	$\ell_{dc} = \frac{(60{,}000\text{ psi})(1.0)}{50\sqrt{4000\text{ psi}}}(0.75\text{ in.}) = 14.3\text{ in.}$ **Controls** $\ell_{dc} = 0.0003(60{,}000\text{ psi})(1.0)(0.75\text{ in.}) = 13.5\text{ in.}$
	The footing depth must satisfy the following inequality so that the No. 6 dowels can be developed within the provided depth	
25.3.1	$h \geq \ell_{dc} + r + d_{b,dwl} + 2d_{b,bars} + 3\text{ in.}$ where r = radius of No. 6 bent = $6d_b$	$h_{req'd} = 14.3\text{ in.} + 6(0.75\text{ in.}) + 0.75\text{ in.} + 2(0.75\text{ in.}) + 3\text{ in.} = 24.1\text{ in.}$ $h_{req'd} = 24.1\text{ in.} < h_{prov.} = 36\text{ in.}$ **OK**

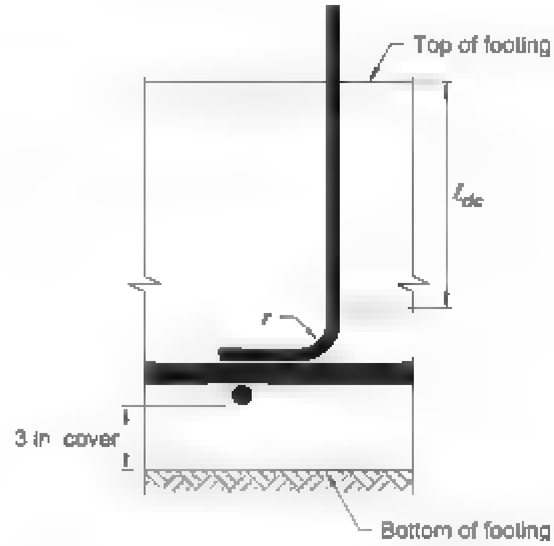


Fig. E1.6—Reinforcement development length.

Step 9: Footing details

Development length

13.2.8.3
13.2.7.1

Flexural reinforcement bar development is required at the critical section. This is the point of maximum factored moment, which occurs at the column face. Bars must extend at least a tension development length beyond the critical section.

No. 6: $\dfrac{c_b + K_{tr}}{d_b} = \dfrac{3.5\text{ in.} + 0}{1.0\text{ in.}} = 3.5$

Use maximum $\dfrac{c_b + K_{tr}}{d_b} = 2.5$

25.4.2.4

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s\psi_g}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$

$$\ell_d = \left(\frac{3}{40}\frac{60{,}000\text{ psi}}{(1.0)\sqrt{4000\text{ psi}}}\frac{(1.0)(1.0)(1.0)}{2.5}\right)d_b = 28.5d_b$$

where ψ_t = casting position,

ψ_t = 1.0 because not more than 12 in. of fresh concrete below horizontal reinforcement

ψ_e = coating factor; ψ_e = 1.0, because bars are uncoated

ψ_s = bar size factor; ψ_s = 1.0 for No. 7 and larger

ψ_g = reinforcement grade factor; ψ_g = 1.0

c_b = spacing or cover dimension to center of bar, whichever is smaller

K_{tr} = transverse reinforcement index

It is permitted to use K_{tr} = 0

However, the expression: $\dfrac{c_b + K_{tr}}{d_b}$ must not exceed 2.5

See footing details in Fig. E1.7.

No. 8 bars: ℓ_d = 28.5(1 in.) = 28.5 in. > 12 in.
Therefore, **OK**

ℓ_d in the longitudinal direction
$\ell_{d,prov}$ = ((12 ft)(12 in./ft) – 24 in.)/2 – 3 in.
$\ell_{d,prov}$ = 57 in. > $\ell_{d,req'd}$ = 28.5 in. **OK**

Use straight No. 8 bars in both directions.

Step 10: Detailing

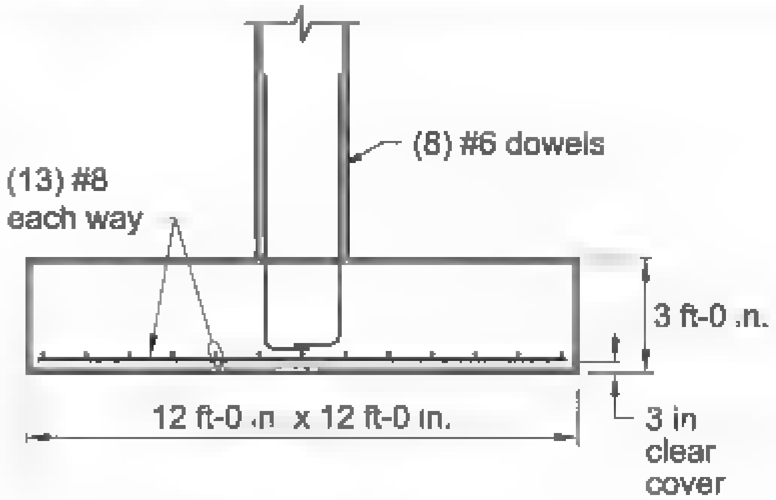


Fig. E1.7—Footing reinforcement detailing.

Foundation Example 2: *Design of a continuous footing*

Design and detail of a continuous footing, founded on stiff soil, supporting a 12 in. concrete wall. The footing is located in Seismic Category D and is 3 ft 0 in. below finished grade. Exposure to freezing and thawing is not an issue (refer to Fig. E2.1).

Given:

Wall load—

Service dead load D = 25 kip/ft

Service live load L = 12.5 kip/ft

Wind W = ±6.4 kip/ft

Axial force effect on footing from wind load

Earthquake E = ±6 kip/ft

Earthquake-induced axial force effect on footing

Note: the wall has no out-of-plane moments or shears.

Material properties—

Concrete compressive strength f_c' = 4000 psi

Steel yield strength f_y = 60,000 psi

Normalweight concrete λ = 1

Density of concrete = 150 lb/ft^3

Allowable soil-bearing pressures—

D only $q_{all,D}$ = 3000 psf

$D + L$ $q_{all,D+L}$ = 4000 psf

$D+L+W$ $q_{all,W}$ = 5000 psf

$D+L+E$ $q_{all,E}$ = 5000 psf

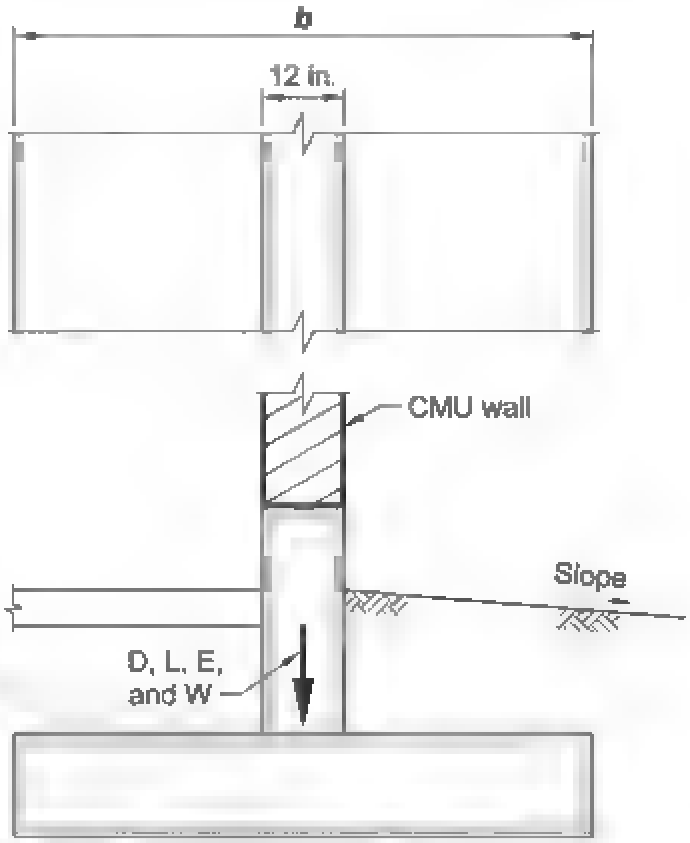
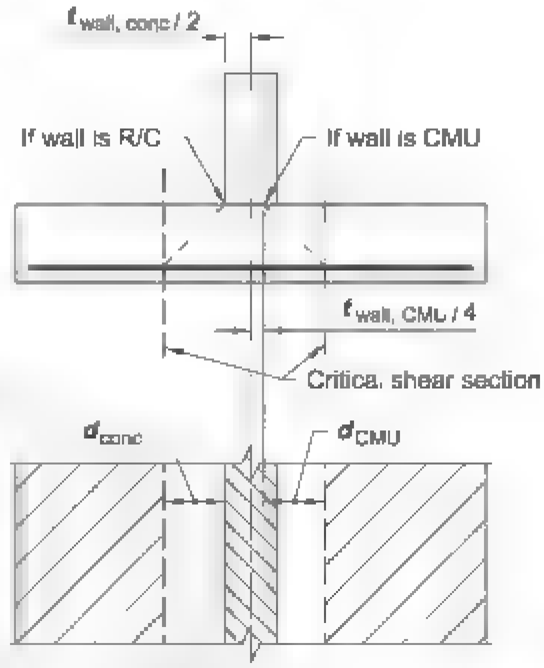


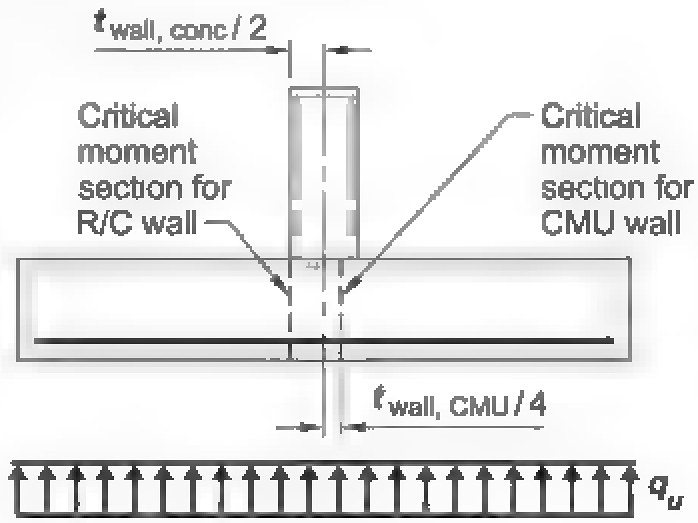
Fig. E2.1—Plan and elevation of continuous footing.

ACI 318	Discussion	Calculation
Step 1: Foundation type		
13.1.1	This strip footing is 3 ft below finished grade. Therefore, it is considered a shallow foundation.	
13.3.2.1	The footing will be designed and detailed with the applicable provisions of Chapter 7, One-way slabs, and Chapter 9, Beams, of ACI 318.	
13.2.3.1	Foundation resisting earthquake forces must comply with Section 18.2.2.3 of ACI 318.	

Step 2. Material requirements		
13.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318). The designer determines the durability classes. Please see Chapter 4 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor. Example 1 of this chapter provides a more detailed breakdown on determining the concrete compressive strength and exposure categories and classes.	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 4000 psi.
Step 3: Determine footing dimensions		
13.3.1.1	To calculate the footing width, divide the service load per foot by the allowable soil pressure. Load combinations are obtained from ASCE/SEI 7, Section 2.4. $\text{area of footing} = \dfrac{\text{total service load } (\sum P \text{ kip/ft})}{\text{allowable soil pressure } q_a}$ The footing thickness is calculated in Step 4, footing design.	Ignoring the footing self weight $\dfrac{D}{q_{all,D}} = \dfrac{25 \text{ kip/ft}}{3 \text{ ksf}} = 8.3 \text{ ft}$ $\dfrac{D+L}{q_{all,D+L}} = \dfrac{25 \text{ kip/ft} + 12.5 \text{ kip/ft}}{4 \text{ ksf}} = 9.4 \text{ ft}$ **Controls** $\dfrac{D + 0.75L + (0.75)(0.6)W}{q_{all,lat}}$ $= \dfrac{25 \text{ kip/ft} + (0.75)(12.5 \text{ kip/ft}) + (0.75)(0.6)(6.4 \text{ kip/ft})}{5 \text{ ksf}} = 7.5 \text{ ft}$ $\dfrac{D + 0.75L + (0.75)(0.7)E}{q_{all,lat}}$ $= \dfrac{25 \text{ kip/ft} + (0.75)(12.5 \text{ kip/ft}) + (0.75)(0.7)(6.0 \text{ kip/ft})}{5 \text{ ksf}} = 7.5 \text{ ft}$ Use $B = 10$ ft

Step 4. Footing design		
	<u>Wall stability</u> Because there is no out-of-plane moment, the soil pressure under the footing is assumed to be uniform and overall wall stability is assumed.	
7.4.2.1	The footing cantilevers on both sides of the wall are designed as one-way slabs.	
5.3.1	<u>Calculate soil pressure</u> *Factored loads*— Calculate the soil pressures resulting from the applied factored loads.	
	Load Case I: $U = 1.4D$	$U = 1.4D = 1.4(25\text{ kip/ft}) = 35\text{ kip/ft}$ or 3.89 ksf
	Load Case II: $U = 1.2D + 1.6L$	$U = 1.2D + 1.6L = 1.2(25\text{ kip/ft}) + 1.6(12.5\text{ kip/ft})$ $= 50\text{ kip/ft}$ or 5.26 ksf **Controls**
	Load Case IV: $U = 1.2D + W + L$	$U = 1.2D + 1.0W + 1.0L$ $= 1.2(25\text{ kip/ft}) + 6.4\text{ kip/ft} + 12.5\text{ kip/ft}$ $= 48.9\text{ kip/ft}$ or 5.15 ksf
	Load Case IV: $U = 0.9D + W$	$U = 0.9D + 1.0W = 0.9(25\text{ kip/ft}) + 6.4\text{ kip/ft}$ $= 28.9\text{ kip/ft}$ or 3.04 ksf
	Load Case V: $U = 1.2D + E + L$	$U = 1.2D + 1.0E + 1.0L$ $= 1.2(25\text{ kip/ft}) + 6\text{ kip/ft} + 12.5\text{ kip/ft}$ $= 48.5\text{ kip/ft}$ or 5.11 ksf
	Load Case VI: $U = 0.9D + E$	$U = 0.9D + 1.0E = 0.9(25\text{ kip/ft}) + (6\text{ kip/ft})$ $= 28.5\text{ kip/ft}$ or 3.0 ksf

	One-way shear design—	
13.2.6.2	To set the depth of the footing, consider one-way shear. Size effect factor may be neglected.	
7.5.3.1 22.5	Shear reinforcement is not typically used in one-way slabs and footings so all of the shear strength is provided by the concrete contribution. $\phi V_n = \phi V_c$	
21.2.1 7.6.3.1	Strength reduction factor for shear from Table 21.2.1b. Minimum shear reinforcement is required where $V_u > \phi V_c$. Footings, however, are not typically constructed with shear reinforcement. Provide sufficient depth to avoid the need for minimum shear reinforcement.	$\phi = 0.75$ 
22.5.5.1c	Ignoring size effects, axial load, and using normal-weight concrete, the applicable equation from Table 22.5.5.1c becomes: $\phi V_c = \phi 8(\rho_w)^{1/3}\sqrt{f_c'}b_w d$ If ρ_w is set to the minimum required flexural reinforcement ratio of 0.0018, then the equation becomes $\phi V_c = \phi 0.97\sqrt{f_c'}b_w d$	
13.2.7.2	Factored shear is calculated for the critical section located at d from the face of the wall (Fig. E2.2) $\phi V_c \geq V_u = \left(\frac{l}{2} - \frac{c}{2} - d\right)bq_u$	*Fig. E2.2—Shear critical section.* $\left(\frac{120\text{ in.}}{2} - \frac{12\text{ in.}}{2} - d\right)\frac{1\text{ ft}(5.26\text{ ksf})}{12\text{ in./ft}} = 0.75(0.97)\sqrt{4000}\text{ psi}(120\text{ in.})(d)$ Required depth of footing is $d = 23.2$ in.
	Consider flexural reinforcement in shear calculations to help reduce required footing thickness. Use No. 8 bars at 12 in. spacing. Solve for the required effective depth d.	$\left(\frac{120\text{ in.}}{2} - \frac{12\text{ in.}}{2} - d\right)\frac{(5.26\text{ ksf})}{12\text{ in./ft}} = 0.75(8)\left(\frac{0.79\text{ in.}^2}{12\text{ in.}(d)}\right)^{1/3}\sqrt{4000}\text{ psi}(12\text{ in.})(d)$ Required depth of footing is $d = 18.1$ in. Calculate required thickness $h = 18.1\text{ in.} + 3\text{ in.} + 0.5(1\text{ in.}) = 21.6$ in. Use footing thickness of 24 in. with at least No. 8 bars at 12 in. spacing.

	Flexure design Note. Masonry wall is shown in Fig. E2.2 and E2.3 to indicate that for masonry walls, the critical section for shear or moment are not the face of the masonry wall, but is $t_w/4$ from the face	 *Fig. E2.3—Moment critical sections*
13.2.7.1	For concrete walls, the factored moment and moment strength are calculated at the face of the wall From maximum factored load M_u is	$M_u = (5.26\text{ ksf})(10\text{ ft}/2-1.0\text{ ft}/2)^2/2 = 53.3$ ft-kip/ft
22.2.2.4.1	Set the concrete compression strength equal to the steel tension strength	
	$C = T$; $0.85f_c'ba = A_sf_y$	$C = 0.85(4000\text{ psi})(12\text{ in.})a = A_s(60{,}000\text{ psi})$ $d = 24\text{ in.} - 3\text{ in.} - 0.5(1.0\text{ in.}) = 20.5\text{ in.}$ $a = 1.47A_s$ in.
7.5.1.1 21.2.1a	$\phi M_n \geq M_u$ Assume section is tension controlled so that $\phi = 0.9$	$(0.9)(60\text{ ksi})A_s\left(16.5\text{ in.}-\dfrac{1.47A_s}{2}\right) = 53.3\text{ ft-kip/ft}$ $A_{s,reqd} = 0.60\text{ in.}^2$ Use bottom bars No. 8 at 12 in. on center. If these bars are not hooked, provide calculations to justify the use of straight bars
7.6.1.1	Check if A_s exceeds the minimum $A_{s,min} \geq 0.0018A_g$	$A_{s,min} = 0.0018(12\text{ in.})(20\text{ in.})$ $= 0.43\text{ in.}^2/\text{ft} < A_{s,reqd} = 0.60\text{ in.}^2/\text{ft}$ **OK**
7.3.3.1	Confirm that section is tension-controlled. The strain in reinforcement is assumed to be proportional to the distance from neutral axis calculated from similar triangles (refer to Fig. E2.4)	$a = 1.47A_s = (1.47)(0.79\text{ in.}^2) = 1.16\text{ in.}$ $c = \dfrac{1.16\text{ in.}}{0.85} = 1.37\text{ in.}$
22.2.1.2	$\varepsilon_s = \dfrac{\varepsilon_c}{c}(d - c)$ where: $c = a/\beta_1$ and $a = 1.47A_s$	$\varepsilon_s = \dfrac{0.003}{1.37\text{ in.}}(16.5\text{ in.} - 1.37\text{ in.}) = 0.033$ $\varepsilon_t = 0.042 > 0.005$ Section is tension-controlled

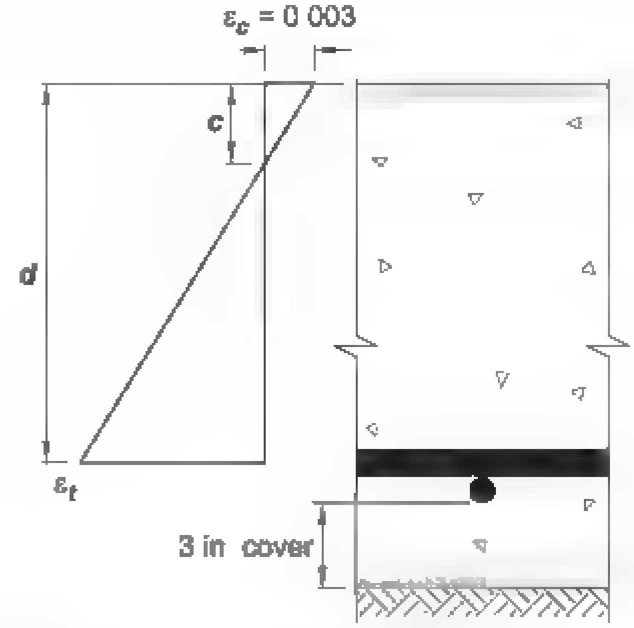


Fig. E2.4—Strain distribution across footing.

	Verify that the allowable soil pressure is not exceeded when including footing self-weight and soil self-weight above footing.	
	Footing self-weight:	$W_F = (10\text{ ft})\left(\frac{20\text{ in.}}{12}\right)(0.15\text{ kip/ft}^3 - 0.12\text{ kip/ft}^3) = 0.5\text{ kip}$
	Soil weight above footing	$W_s = (10\text{ ft})\left(\frac{36\text{ in.} - 20\text{ in.}}{12}\right)(0.12\text{ kip/ft}^3) = 1.6\text{ kip}$
	Total weight on supporting soil	$W_T = 25\text{ kip/ft} + 12.5\text{ kip/ft} + 0.60\text{ kip/ft} + 1.2\text{ kip/ft} = 39.3\text{ ft}$
	Calculate actual soil pressure	$q_u = \frac{39.6\text{ kip/ft}}{10\text{ ft}} = 3.96\text{ ksf} < q_{all} = 4\text{ ksf}$ **OK**
	Note: This is a conservative approach. The footing concrete displaces soil. Therefore, the actual load on soil is the difference between the concrete and soil unit weights multiplied by the footing volume.	

Foundations

Step 5. Footing details		
	Shrinkage and temperature reinforcement along length of footing	
7.6.4.1 7.6.1.1	The area of shrinkage and temperature reinforcement	
	$A_{S+T} \geq 0.0018A_g$	A_{S+T} = (0.0018)(24 in.)(10.0 ft)(12 in./ft) = 5.2 in.^{2} Ten No. 6 bottom longitudinal bars will satisfy the requirement for shrinkage and temperature reinforcement in the long direction.
	Development length Check if the width of the footing provides adequate length for the bottom tension reinforcement beyond the critical tension section.	
25.4.2.4	$\ell_d = \left(\dfrac{3}{40}\dfrac{f_y}{\lambda\sqrt{f'_c}}\dfrac{\psi_t\psi_e\psi_s\psi_g}{\left(\dfrac{c_b+K_{tr}}{d_b}\right)}\right)d_b$	$\ell_d = \left(\dfrac{3}{40}\dfrac{60{,}000\text{ psi}}{(1.0)\sqrt{4000\text{ psi}}}\dfrac{(1.0)(1.0)(1.0)(1.0)}{2.5}\right)d_b$ $= 28.5d_b = 28.5$ in. > 12 in.
	where ψ_t = casting position; ψ_t = 1.0 because not more than 12 in. of fresh concrete is placed below horizontal reinforcement ψ_e = coating factor; ψ_e = 1.0, because bars are uncoated ψ_s = bar size factor; ψ_s =1.0 because bars are larger than No. 7 ψ_g = reinforcement grade factor; ψ_g = 1.0 c_b = spacing or cover dimension to center of bar, whichever is smaller K_{tr} = transverse reinforcement index	
	It is permitted to use $K_{tr} = 0$ However, the expression $\dfrac{c_b+K_{tr}}{d_b}$ must not exceed 2.5	$\dfrac{c_b+K_{tr}}{d_b} = \dfrac{3.5\text{ in.}+0}{1.0\text{ in.}} = 3.5$ Use maximum value of 2.5
	Provided length: $B/2 - t_{wall}/2 - 3$ in.	$\ell_{d,avail}$ = (10 ft)(12 in./ft)/2 – 12 in./2 – 3 in. = 51 in. $\ell_{d,avail}$ = 51 in. > ℓ_d = 28.5 in. **OK** Therefore, the footing is wide enough to use straight bars for development and does not require hooks at both ends

Step 6. Earthquake requirements		
	Earthquake load effects	
13.2.3.2	The foundation is in SDC D therefore, ACI 318, Section 18.13, must be satisfied	
18.13.2	The requirements listed in 18.13.2 for structural walls must be satisfied if calculations show that uplift occurs.	
18.13.2.2	(a) Vertical reinforcement of structural walls resisting forces induced by earthquake effects must extend into the footing and must be fully developed for tension at the interface.	
18.13.2.4	(b) Boundary elements of special structural walls that have an edge within one-half the footing depth from an edge of the footing shall have transverse reinforcement in accordance with 18.7.5.2 through 18.7.5.4 provided below the top of the footing. This reinforcement must extend into the footing and be developed a length equal to the development length, calculated for f_y in tension, of the boundary element longitudinal reinforcement. This condition does not apply for this problem	
18.13.2.5	(c) Where earthquake effects create uplift forces in boundary elements of special structural walls, flexural reinforcement must be provided in the top of the footing to resist actions resulting from the design factored load combinations, and must be less than required by 7.6.1 or 9.6.1. This condition does not apply for this problem	
	See footing details in Fig. E2.5	

Step 7. Detailing

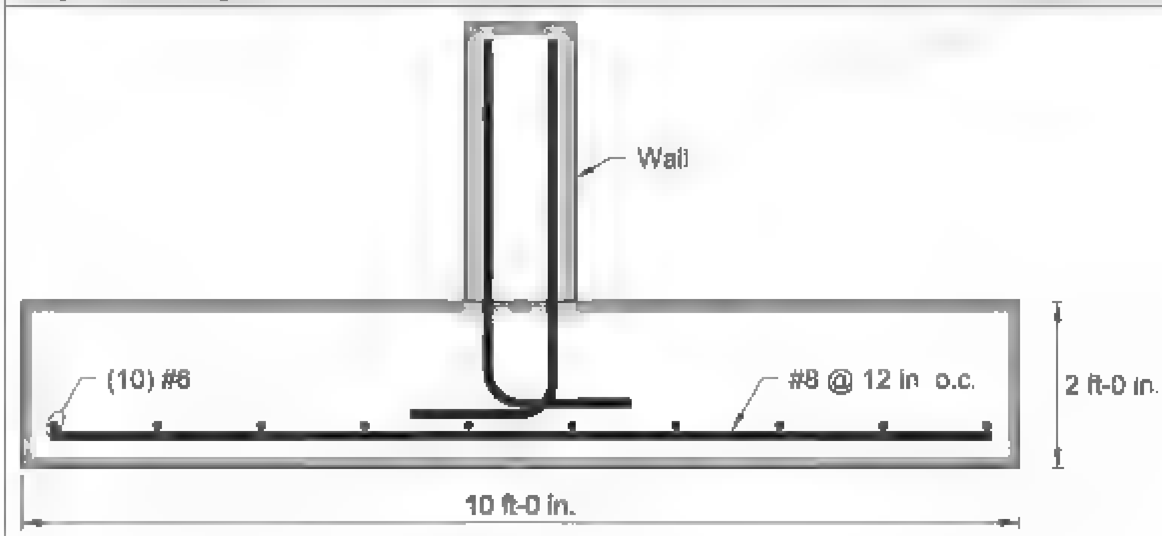


Fig. E2.5—Continuous footing reinforcement

Foundation Example 3: *Design of a continuous footing with an out-of-plane moment*

Design and detail a continuous footing, founded on stiff soil, supporting a 12 in. thick bearing wall, founded on stiff soil, and subject to loading that includes an overturning moment. The bottom of the footing is 3 ft below finished grade (refer to Fig. E3.1).

Given:

Wall load—
Service dead load D = 15 kip/ft (including CMU wall weight)
Wind W = ±3.0 kip/ft
Lateral force on footing stem from wind load effect

Material properties—
Concrete compressive strength f_c' = 4000 psi
Steel yield strength f_y = 60,000 psi
Normalweight concrete λ = 1
Density of concrete = 150 lb/ft^3

Soil data—
q_{all} = 4000 psf
$q_{u,permitted}$ = 6000 psf
Soil unit weight = 100 lb/ft^3

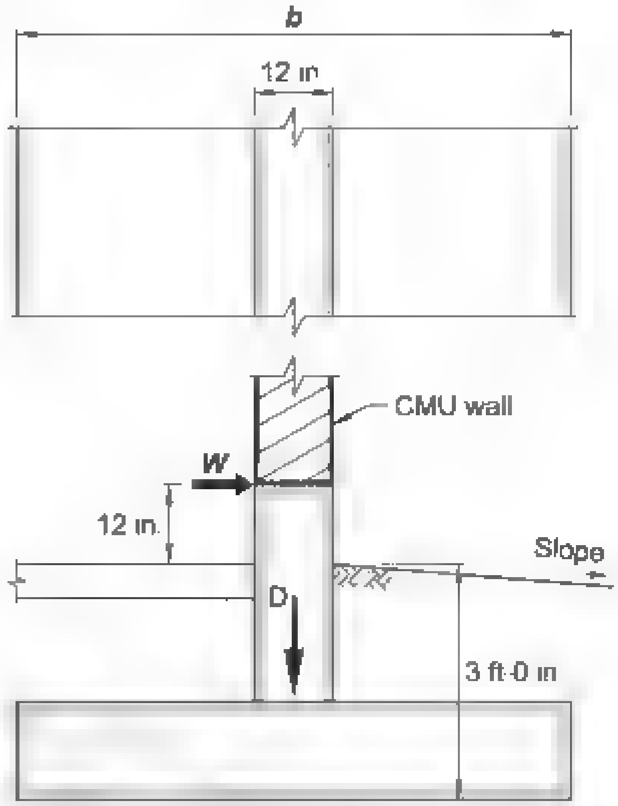


Fig. E3.1—Plan and elevation of continuous footing.

ACI 318	Discussion	Calculation
Step 1: Foundation type		
13.1.1	This strip footing is 3 ft below finished grade. Therefore, it is considered a shallow foundation.	
13.3.2.1	The footing will be designed and detailed with the applicable provisions of Chapter 7, One-way slabs, and Chapter 9, Beams, of ACI 318.	
Step 2: Material requirements		
13.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318).	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, Chapter 19 requirements are satisfied.
	The designer determines the durability classes. Please see Chapter 4 of this Manual for an in-depth discussion of the categories and classes.	Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 4000 psi.
	ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications.	
	There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	
	Example 1 of this chapter provides a more detailed breakdown on determining the concrete compressive strength and exposure categories and classes.	

Step 3: Determine footing dimensions		
13.3.1.1	Footing width is assumed and then verified through calculations. Iterations may be needed.	Try B = 7 ft footing width Area, A = 1(7) = 7 ft^2/ft Section modulus S = 1(7 ft)(7 ft)/6 = 8.167 ft^3/ft
	The footing thickness is also assumed and then verified through calculations in Step 4, Footing design	
13.3.1.2	The footing thickness must be such that the bottom reinforcement has an effective depth of at least 6 in.	Try 15 in. footing thickness

Step 4. Footing design

Wall stability

Because there is an out-of-plane (overturning) lateral force on the stem wall, the overall wall stability must be checked.

To calculate the stability of a footing, the total vertical load is calculated and the resisting moment (M_R) is compared to the resulting overturning moment (M_{OTM}).

Refer to the general building code requirements for foundation stability. For this example, require $M_R > 1.5M_{OTM}$.

Weights on bearing soil below footing

Weight of footing

$$W_{ftg} = \left(\frac{15\text{ in.}}{12\text{ in./ft}}\right)(0.15\text{ kip/ft}^3) = 0.19\text{ ksf}$$

Weight of soil above footing

$$W_{soil} = \left(\frac{36\text{ in.} - 15\text{ in.}}{12\text{ in./ft}}\right)(0.10\text{ kip/ft}^3) = 0.18\text{ ksf}$$

Weight of concrete wall pier

$$W_{conc.pier} = \left(\frac{36\text{ in.} - 15\text{ in.}}{12\text{ in./ft}}\right)(0.15\text{ kip/ft}^3) = 0.26\text{ ksf}$$

Total vertical dead load

$$\sum P = (0.19\text{ ksf})(7\text{ ft}) + (0.18\text{ ksf})(7\text{ ft} - 1\text{ ft}) + (0.26\text{ ksf})(1\text{ ft}) + 15\text{ kip/ft}$$
$$= 2.67\text{ kip/ft} + 15\text{ kip/ft} = 17.7\text{ kip/ft}$$

Vertical distance from bottom of footing to location of applied lateral wind shear.

$$H = 3\text{ ft} + 1\text{ ft} = 4\text{ ft}$$

The overturning moment, M_{OTM}, is measured at base of footing. The lateral wind force must be multiplied by 0.6 (ASCE/SEI 7 Section 2.4.1) to convert to service load level.

$$M_{OTM} = (0.6)(W)(H) = (0.6)(3.0\text{ kip/ft})(4\text{ ft})$$
$$= 7.2\text{ ft-kip/ft (wind load)}$$

The resisting moment, M_R, is calculated as the product of vertical load by distance from the centerline to edge of footing.

$M_R = P(B/2)$

$$M_R = (17.7\text{ kip/ft})(7\text{ ft}/2) = 61.8\text{ ft-kip/ft}$$

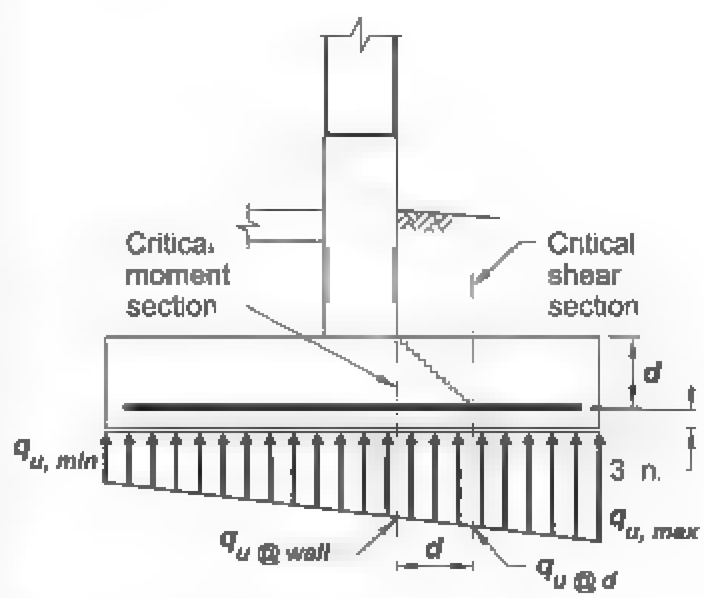
$61.8\text{ ft-kip/ft} > (1.5)(7.2\text{ ft-kip/ft}) = 10.8\text{ ft-kip/ft}$ **OK**

To ensure footing stability, the following inequality must be satisfied.

$M_R > 1.5M_{OTM}$

Step 5. Calculate soil pressure		
13.3.1.1	<u>Service loads</u> The maximum soil pressure is calculated from service forces and moments transmitted by foundation to the soil To calculate soil pressure, the location of the vertical service resultant force is determined The distance to the resultant from the front face of stem	
	$e = \dfrac{M_{ant}}{\sum P}$	$e = \dfrac{7.2\text{ ft kip}}{17.7\text{ kip}} = 0.41\text{ ft}$
	Check if resultant falls within the middle third (kern) of the footing.	$B/6 = 7\text{ ft}/6 = 1.17\text{ ft} > e = 0.41\text{ ft}$ **OK**
	Because $e < B/6$, the footing imposes compression to the soil across the entire width	
	The resulting soil pressure must be less than the allowable bearing pressure provided by the geotechnical report.	
13.3.1.1	Check allowable soil bearing pressures against service load combinations from ASCE/SEI 7	Service actions $M_{Wind} = 3\text{ kip/ft}(3\text{ ft} + 1\text{ ft}) = 12\text{ kip ft/ft}$ $P_{DL} = 17.7\text{ kip/ft}$
	ASCE/SEI 7 Section 2.4.1 Allowable stress combination 5 $D + 0.6W$	$\dfrac{17.7\text{ kip}}{7\text{ ft}(1\text{ ft})} + 0.6\dfrac{12\text{ kip ft}}{(7\text{ ft})^2\ 1\text{ ft}} = 2676\text{ psf}$ $\dfrac{17.7\text{ kip}}{7\text{ ft}(1\text{ ft})} - 0.6\dfrac{12\text{ kip ft}}{(7\text{ ft})^2\ 1\text{ ft}} = 2382\text{ psf}$
	ASCE/SEI 7 Section 2.4.1 Allowable stress combination 7 $0.6D + 0.6W$	$0.6\dfrac{17.7\text{ kip}}{7\text{ ft}(1\text{ ft})} + 0.6\dfrac{12\text{ kip ft}}{(7\text{ ft})^2\ 1\text{ ft}} = 1664\text{ psf}$ $0.6\dfrac{17.7\text{ kip}}{7\text{ ft}(1\text{ ft})} - 0.6\dfrac{12\text{ kip ft}}{(7\text{ ft})^2\ 1\text{ ft}} = 1370\text{ psf}$ $< q_{all} = 4\text{ ksf}$ **OK**

Foundations

Step 6. Factored loads		
13.3.2.1	The footing is designed as one-way slab. Calculate the soil pressures resulting from the applied factored loads.	
5.3.1a	Load Case I $U = 1.4D$ Use $D = P = 17.7$ kip and $M_{OTM} = W = 12$ ft-kip	$U = 1.4(17.7 \text{ kip/ft}) = 24.7$ kip/ft $q_u = (24.7 \text{ kip/ft}) / (7 \text{ ft}) = 3.53 \text{ ksf} < q_u = 6$ ksf **OK**
5.3.1d	Load Case II $U = 1.2D/A + 1.0W/S + 0.5L/A$ where S is the section modulus (Step 2)	$1.2D/A = 1.2(17.7 \text{ kip/ft})/(7 \text{ ft}) = 3.03$ ksf $1.0W/S = 1.0(12 \text{ ft-kip})/(8.167 \text{ ft}^3) = 1.47$ ksf $0.5L = 0$
	$e = 1.0(W)/(1.2(P))$	$e = 1.0(12 \text{ ft-kip})/(1.2(17.7 \text{ kip})) = 0.56 \text{ ft} < (7 \text{ ft})/6 = 1.67$ ft
	Because $e < B/6$, the footing bearing pressure varies as follows (refer to Fig. E3.2)	
	$q_u = 1.2(D/A) \pm 1.0(W/S)$	$q_{u,max} = 3.04 \text{ ksf} + 1.47 \text{ ksf} = 4.51$ ksf (maximum) $q_{u,min} = 3.04 \text{ ksf} - 1.47 \text{ ksf} = 1.57$ ksf (minimum) $q_{u,max} = 4.51 \text{ ksf} < q_{u,permitted} = 6$ ksf **OK**
5.3.1f	Load Case III $U = 0.9D/A + 1.0W/S$	$0.9D/A = 0.9(17.7 \text{ kip/ft})/(7 \text{ ft}) = 2.28$ ksf $1.0W/S = 1.0(12 \text{ ft-kip})/(8.167 \text{ ft}^3) = 1.47$ ksf
	$e = 1.0(W)/(1.2(P))$	$e = 1.0(12 \text{ ft-kip})/(0.9(17.7 \text{ kip})) = 0.75$ ft
	Because $e < B/6$, bearing pressure $q_u = 0.9(D/A) \pm 1.0(W/S)$	$q_{1,2} = 2.27 \text{ ksf} \pm 1.5$ ksf $q_{u,max} = 3.75$ ksf (maximum) $< q_{u,permitted} = 6$ ksf **OK** $q_{u,min} = 0.81$ ksf (minimum)
		 *Fig. E3.2—Soil pressure distribution under factored loads, critical shear section, and critical moment section.*

Step 7: Shear strength		
13.2.6.2	To set the depth of the footing, consider one-way shear. Size effect factor may be neglected.	
7.5.3.1 22.5	Shear reinforcement is not typically used in one-way slabs or footings so all of the shear strength is provided by the concrete contribution $\phi V_n = \phi V_c$	
21.2.1	Strength reduction factor for shear from Table 21.2.1b	$\phi = 0.75$
7.6.3.1	Minimum shear reinforcement is required where $V_u > \phi V_c$ Footings, however, are not typically constructed with shear reinforcement. Provide sufficient depth to avoid the need for minimum shear reinforcement	
22.5.5.1c	Ignoring size effects, axial load, and using normal weight concrete, the applicable equation from Table 22.5.5.1c becomes $\phi V_c = \phi 8(\rho_w)^{1/3}\sqrt{f'_c}b_w d$ If ρ_w is set to the minimum required flexural reinforcement ratio of 0.0018, then the equation becomes: $\phi V_c = \phi 0.97\sqrt{f'_c}b_w d$	
13.2.7.2	Factored shear is calculated for the critical section located at d from the face of the wall (Fig. E3.2) where the factored soil bearing pressure is $q_{u,d} = q_{u,min} + \left(\frac{q_{u,max} - q_{u,min}}{B}\right)\left(\frac{B}{2} + \frac{t_{wall}}{2} + d\right)$ Factored shear is then $V_u = 0.5\left[q_{u,max} + q_{u,min} + \frac{q_{u,max} - q_{u,min}}{B}\left(\frac{B}{2} + \frac{t_{wall}}{2} + d\right)\right]\left(\frac{B}{2} - \frac{t_{wall}}{2} - d\right)$ Equate the factored shear and nominal shear strength contribution of concrete and solve for effective depth d:	$0.5\left[1.57\text{ ksf} + 4.5[illegible]\text{ ksf} + \left(\frac{4.5[illegible]\text{ ksf} - [illegible]}{84\text{ in.}}\right)\left(\frac{84\text{ in.}}{2} + \frac{12\text{ in.}}{2} + d\right)\right]\left(\frac{84\text{ in.}}{2} - \frac{12\text{ in.}}{2} - d\right)$ $= 0.75(0.97\sqrt{4000}\text{ psi})(12\text{ in.})d$ Required effective depth of footing is $d = 13.8$ in.
	Consider flexural reinforcement in shear calculations to help reduce required footing thickness. Use No. 7 at 12 in. spacing. Solve for the required effective depth d	$\rho_w = \frac{0.6\text{ in.}^2}{12\text{ in.}(11.5\text{ in.})} = 0.00435$ $0.75(8)(0.00435)^{1/3}\sqrt{4000}\text{ psi}(12\text{ in.})(d)$ $d = 11.3$ in. Calculate required thickness $h = 11.3\text{ in.} + 3\text{ in.} + 0.5(0.875\text{ in.}) = 14.7$ in. Use footing thickness of 15 in. with at least No. 7 bars at 12 in. spacing.

Step 8. Flexural strength

Flexure design

13.2.7.1 The footing factored moment is calculated at the face of the wall (refer to Fig. E3.3).

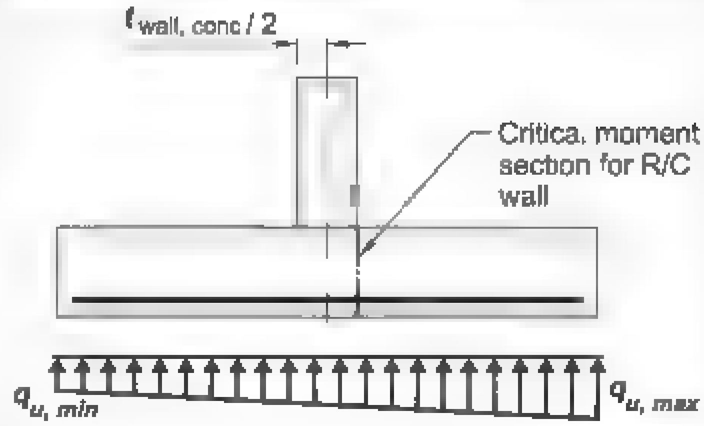


Fig. E3.3—Moment critical section

5.3.1a Calculate M_u at face of wall from
$U = 1.4D/A$
$q_u = 3.53$ ksf (Step 6)

$$M_u = \frac{3.53\text{ ksf}}{2}(3.5\text{ ft} - 0.5\text{ ft})^2 = 15.9\text{ ft-kip/ft}$$

5.3.1d Calculate M_u at face of wall
$U = 1.2D/A + 1.0W/S + 0.5L/A$

$$q_{u,wall} = q_{u,min} + \frac{q_{u,max} - q_{u,min}}{B}\left(\frac{B}{2} + \frac{t_{wall}}{2}\right)$$

$$q_u = 1.57\text{ ksf} + \frac{4.51\text{ ksf} - 1.57\text{ ksf}}{7\text{ ft}}(3.5\text{ ft} + 0.5\text{ ft}) = 3.25\text{ ksf}$$

Factored moment

$$M_u = \frac{1}{2}q_{u,wall}\left(\frac{B}{2} - \frac{t_{wall}}{2}\right)^2 + \frac{1}{3}(q_{u,max} - q_{u,wall})\left(\frac{B}{2} - \frac{t_{wall}}{2}\right)^2$$

$$M_u = \frac{3.25\text{ ksf}}{2}(3.5\text{ ft} - 0.5\text{ ft})^2 + \frac{4.51\text{ ksf} - 3.25\text{ ksf}}{3}(3.5\text{ ft} - 0.5\text{ ft})^2 = 18.4\text{ ft-kip/ft}$$

Controls

5.3.1f By inspection load condition
$U = 0.9D/A + 1.0W/S$ does not control

Note: Counteracting moments due to footing weight and soil weight are conservatively neglected.

22.2 Calculate the required area of flexural reinforcement.

Set concrete compression strength equal to steel tension strength

$C = T$

22.2.2.4.1
22.2.2.3
22.2.2.4.3

$0.85f_c'ba = A_sf_y$

$0.85(4000\text{ psi})(12\text{ in.})a = A_s(60{,}000\text{ psi})$
$a = 1.47A_s$

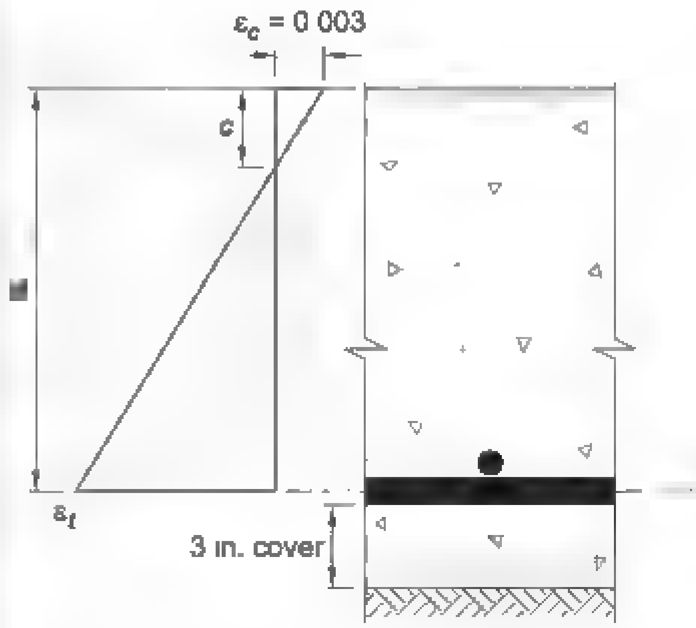
7.5.1.1
22.3.1.1
21.2.1a

$M_u \leq \phi M_n = 0.9A_sf_y(d - a/2)$

Assume section is tension controlled so that $\phi = 0.9$

$\phi M_n = 0.9A_s(60{,}000\text{ psi})(12\text{ in.} - 1.47A_s/2)$
≥ 19.38 ft-kip/ft

$A_s \geq 0.37$ in.^{2}/ft

7.6.1.1	Check if A_s exceeds the minimum. $A_{s,min} \geq 0.0018A_g$	$A_{s,min} = 0.0018(12\text{ in.})(15\text{ in.})$ $= 0.32\text{ in.}^2/\text{ft}$ Use No. 7 at 15 in. on center bottom bars $A_{s,prov} = 0.48\text{ in.}^2/\text{ft} > A_{s,req'd} = 0.32\text{ in.}^2/\text{ft}$ **OK**
21.2.1(a)	Check if the section is tension-controlled	
7.3.3.1	Confirm that section is tension-controlled. The strain in reinforcement is calculated from similar triangles (refer to Fig. E3.4).	
22.2.1.2	$\varepsilon_t = \frac{\varepsilon_c}{c}(d - c)$ where $c = a/\beta_1$ and $a = 1.47A_s$	$c = \frac{1.47(0.48\text{ in.}^2)}{0.85} = 0.83\text{ in.}$ $\varepsilon_t = \frac{0.003}{0.83\text{ in.}}(12\text{ in.} - 0.83\text{ in.}) = 0.040$ $\varepsilon_t = 0.040 > 0.005$ Section is tension-controlled  *Fig. E3.4—Strain distribution through depth of footing*

Step 9: Footing details		
7.6.4.1 24.4.3.2	Shrinkage and temperature reinforcement: The area of shrinkage and temperature reinforcement: $A_{S+T} \geq 0.0018A_g$	A_{S+T} = (0.0018)(15 in.)(7 ft)(12 in.) = 2.27 in.^{2} Eight No. 5 bottom longitudinal bars (area = 2.48 in.^{2}) satisfies the requirement for shrinkage and temperature reinforcement placed perpendicular to the flexural reinforcement.
13.2.8.3 13.2.7.1	Development length Reinforcement development is calculated at the maximum factored moment and the code permits the critical section to be located at the wall face. Bars must extend at least a tension development length beyond the critical section.	
25.4.2.4	$\ell_d = \left(\dfrac{3}{40}\dfrac{f_y}{\lambda\sqrt{f'_c}}\dfrac{\psi_t\psi_e\psi_s\psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right)d_b$ where ψ_t = casting location; ψ_t = 1.0, because not more than 12 in. of fresh concrete is placed below horizontal reinforcement ψ_e = coating factor; ψ_e = 1.0, because bars are uncoated ψ_s = bar size factor; ψ_s = 1.0, because bars are larger than No. 7 ψ_g = reinforcement grade factor; ψ_g = 1.0 c_b = spacing or cover dimension to center of bar, whichever is smaller K_{tr} = transverse reinforcement index	$\ell_d = \left(\dfrac{3}{40}\dfrac{60{,}000\text{ psi}}{(1.0)\sqrt{4000\text{ psi}}}\dfrac{(1.0)(1.0)(1.0)(1.0)}{2.5}\right)d_b$ $= 28.5d_b = 28.5(0.875\text{ in.}) = 25\text{ in.}$
	It is permitted to use K_{tr} = 0 But the expression $\dfrac{c_b + K_{tr}}{d_b}$ must not be taken greater than 2.5 The development length is the greater of the calculated value of Code Eq. (25.4.2.4) and 12 in. Check if No. 7 can be developed using straight bars, without hooks. See footing details in Fig. E3.5	For a No. 7 bar $\dfrac{c_b + K_{tr}}{d_b} = \dfrac{3.44\text{ in.} + 0}{0.875\text{ in.}} = 3.93$ Use maximum value of 2.5 ℓ_d = 25 in. > 12 in. **OK** ℓ_d provided perpendicular to the wall $\ell_{d,prov}$ = ((7 ft)(12 in./ft) – 12 in.)/2 – 3 in. $\ell_{d,prov}$ = 33 in. > $\ell_{d,req'd}$ = 25 in. **OK** use straight No. 7 bars

Step 10: Final design

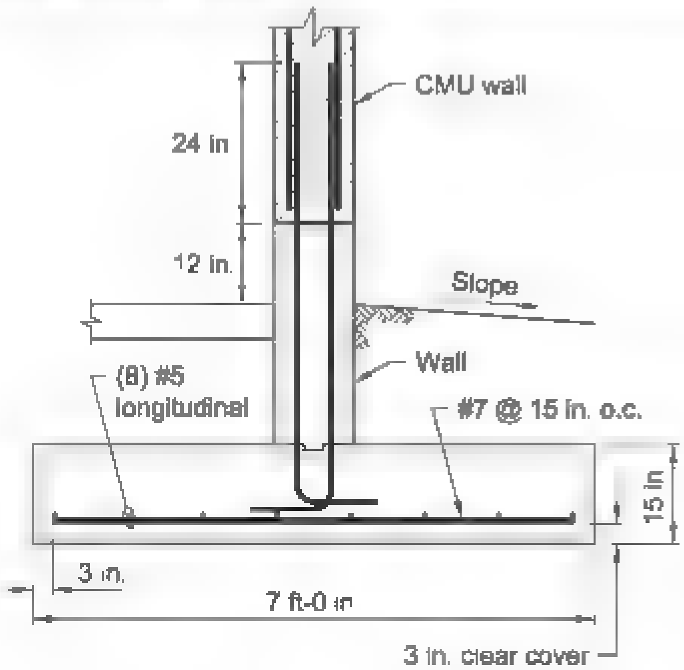


Fig. E3.5—Footing reinforcement detailing

Foundation Example 4—*Design of a rectangular spread footing*

Design and detail a rectangular spread footing founded on stiff soil, supporting an 18 in. square column. The bottom of the footing is 4 ft below finished grade (refer to Fig. E4.1).

<u>Given:</u>

Column load—
Service dead load D = 200 kip
Service live load L = 100 kip
Wind W = ±175 kip
(Axial force effect on footing from wind load)

Material properties—
Concrete compressive strength f_c' = 4000 psi
Steel yield strength f_y = 60,000 psi
Normalweight concrete λ = 1
Density of concrete = 150 lb/ft^3
Soil unit weight = 120 lb/ft^3

Allowable service level soil bearing pressures—
D only $q_{all,D}$ = 4000 psf
$D + L$ $q_{all,D+L}$ = 5800 psf
$D + L + W$ $q_{all,Lat}$ = 8000 psf

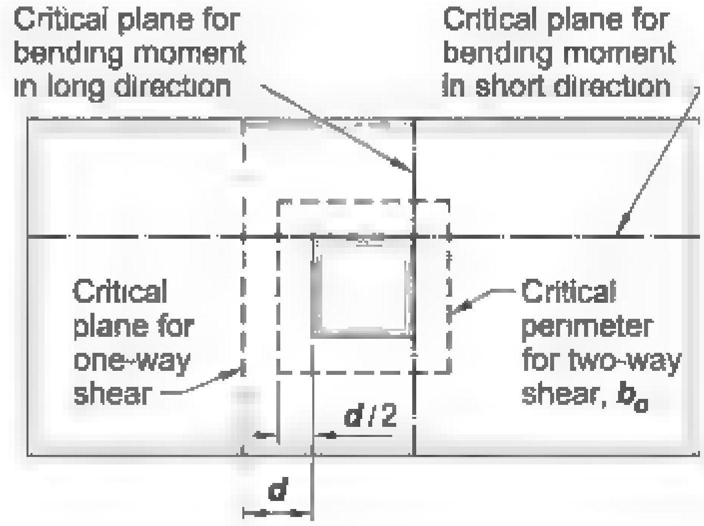
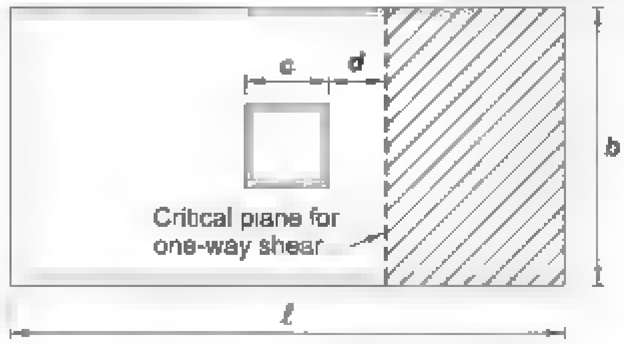


Fig. E4.1—Rectangular footing plan.

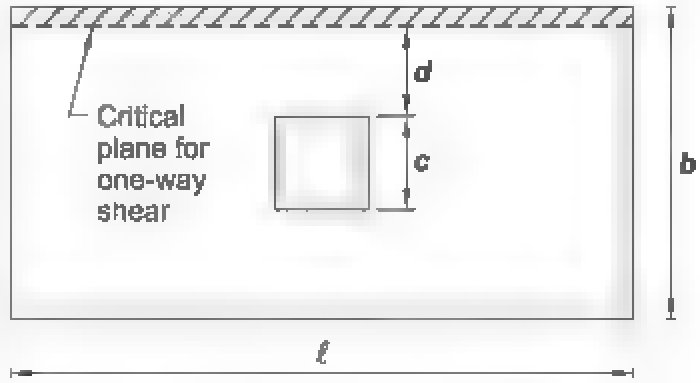
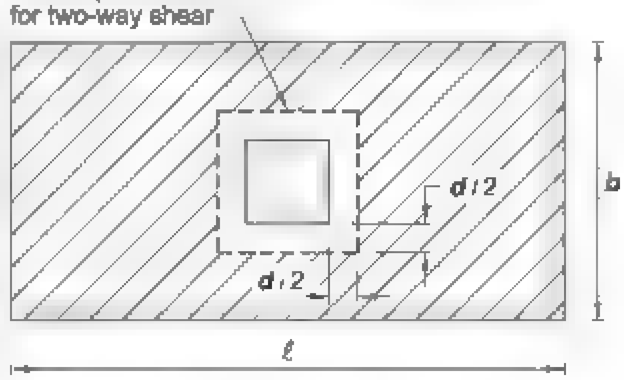
ACI 318	Procedure	Computation
Step 1: Foundation type		
13.1.1	This footing is 4 ft below finished grade. Therefore, it is considered a shallow footing.	
13.3.3.1	The footing will be designed and detailed with the applicable provisions of Chapter 7, One-way slabs, and Chapter 8, Two-way slabs, of ACI 318.	
Step 2: Material requirements		
13.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318). The designer determines the durability classes. Please see Chapter 4 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor. Foundation Example 1 provides a more detailed breakdown on determining the concrete compressive strength and exposure categories and classes.	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 4000 psi.

Step 3: Determine footing dimensions		
13.3.1.1	To calculate the footing area, divide the service load by the allowable soil pressure	The unit weights of concrete and soil are 150 pcf and 120 pcf; close. Therefore, footing self-weight will be ignored for initial sizing. Actual soil pressure is checked end of Step 6
	area of footing $\geq \dfrac{\text{total service load } (\sum P)}{\text{allowable soil pressure } q_a}$	$\dfrac{D}{q_{all,D}} = \dfrac{200 \text{ kip}}{4 \text{ ksf}} = 50 \text{ ft}^2$
5.3.5	$W_{service} = (0.6)W = (0.6)(175 \text{ kip}) = 105 \text{ kip}$	$\dfrac{(D+L)}{q_{all,D+L}} = \dfrac{200 \text{ kip} + 100 \text{ kip}}{5.8 \text{ ksf}} = 51.7 \text{ ft}^2$ **Controls**
		$\dfrac{D+L+W}{q_{all,Lat}} = \dfrac{200 \text{ kip} + 100 \text{ kip} + 105 \text{ kip}}{8 \text{ ksf}} = 50.6 \text{ ft}^2$
	The lateral wind force must be multiplied by 0.6 (ASCE/SEI 7, Section 2.4.1) to convert to service load level	
	Assume that there is a constraint on the width of the footing (B) of 5.5 ft.	$(51.7 \text{ ft}^2)/(5.5 \text{ ft}) = 9.4$ ft say 10 ft Use ($\ell \times B$) 10 ft x 5.5 ft
	The footing thickness is calculated in Step 4, footing design.	$A_{prov.} = 55 \text{ ft}^2 > A_{req'd} = 50.6 \text{ ft}^2$ $B/\ell = (5.5 \text{ ft})/(10 \text{ ft}) = 0.55$

Step 4. Factored soil pressure		
13.3.3.2	Footing stability Because there is no out-of-plane moment, the soil pressure under the footing is assumed to be uniform and overall footing stability is assumed. The footing is designed for flexure as one-way slab (Step 5) and checked for two-way punching shear (Step 6). Calculate soil pressure $q_u = \dfrac{\sum P_u}{\text{area}}$ Factored loads Calculate the soil pressures resulting from the column factored loads.	
5.3.1(a)	Load Case I: $U = 1.4D$	$U = 1.4(200\text{ kip}) = 280\text{ kip}$ $q_u = \dfrac{280\text{ kip}}{55\text{ ft}^2} = 5.1\text{ ksf}$
5.3.1(b)	Load Case II: $U = 1.2D + 1.6L$	$U = 1.2(200\text{ kip}) + 1.6(100\text{ kip}) = 400\text{ kip}$ $q_u = \dfrac{400\text{ kip}}{55\text{ ft}^2} = 7.3\text{ ksf}$
5.3.1(d)	Load Case III: $U = 1.2D + W + L$	$U = 1.2(200\text{ kip}) + (1.0)(175\text{ kip}) + 1.0(100\text{ kip}) = 515\text{ kip}$ $q_u = \dfrac{515\text{ kip}}{55\text{ ft}^2} = 9.4\text{ ksf}$ **Controls**
5.3.1(f)	Load Case IV: $U = 0.9D + W$	$U = 0.9(200\text{ kip}) + 1.0(175\text{ kip}) = 355\text{ kip}$ $q_u = \dfrac{355\text{ kip}}{55\text{ ft}^2} = 6.6\text{ ksf}$
	The load combinations include the possibility of wind uplift force. In this example, uplift does not occur.	Assume that the calculated $q_u = 9.4$ ksf is acceptable per the geotechnical report.
The footing is rectangular in plan. Therefore, it needs to be designed in both directions. Of course, the longer direction will have larger moments and thus is the more critical condition.		

Step 5. One-way shear design		
		 *Fig. E4.2—One-way shear in longer direction.*
13.2.6.2	<u>Long direction</u> To set the depth of the footing, consider one-way and two-way shear. Size effect factor in calculating both one-way and two-way shear strength contribution of concrete may be neglected.	
7.5.3.1 22.5	Shear reinforcement is not typically used in one-way slabs and footings so all of the shear strength is provided by the concrete contribution. $\phi V_n = \phi V_c$	
21.2.1 7.6.3.1	Strength reduction factor for shear from Code Table 21.2.1b	$\phi = 0.75$
	Minimum shear reinforcement is required where $V_u > \phi V_c$. Footings, however, are not typically constructed with shear reinforcement. Provide sufficient depth to avoid the need for minimum shear reinforcement.	
22.5.5.1c	Ignoring size effects, axial load, and using normal-weight concrete, the applicable equation from Code Table 22.5.5.1c becomes $\phi V_c = \phi 8(\rho_w)^{1/3}\sqrt{f_c'}b_w d$ If ρ_w is set to the minimum required flexural reinforcement ratio of 0.0018, then the equation becomes $\phi V_c = \phi 0.97\sqrt{f_c'}b_w d$	
13.2.7.2	Factored shear is calculated for the critical section located at d from the face of the column (Fig. E4.2) $\phi V_c \geq V_u = \left(\frac{l}{2} - \frac{c}{2} - d\right)bq_u$	$\left(\frac{120\text{ in.}}{2} - \frac{18\text{ in.}}{2} - d\right)\frac{(5.5\text{ ft})(9.4\text{ ksf})}{12\text{ in./ft}} = 0.75(0.97)\sqrt{4000}\text{ psi}(66\text{ in.})d$ Required depth of footing is $d = 30$ in. Use centroid of reinforcement layer to calculate footing thickness $h = 30\text{ in.} + 3\text{ in.} + 0.5(0.75\text{ in.}) = 33.375$ in. Try footing thickness of 34 in. $d = 34\text{ in.} - 3\text{ in.} - 0.5(0.44\text{ in.}) = 30.78$ in.

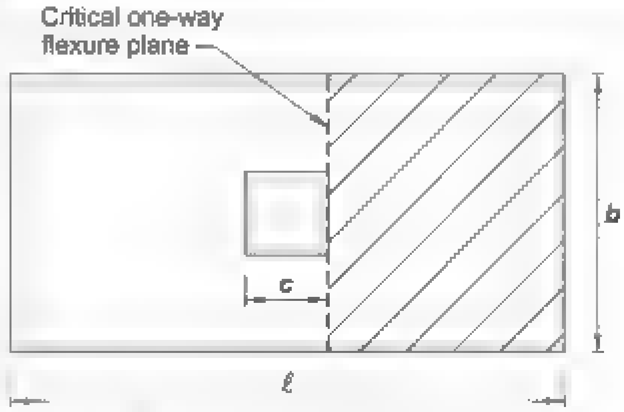
Foundations

	<u>Short direction</u> Check one-way shear The effective depth for the short direction is $d = h - \text{cover} - d_b - d_b/2$ d = 34 in. − 3 in. − 0.75 in. − 0.375 in. d = 29.875 in., say, d = 29.5 in.	
13.2.7.2 7.5.1.1	$\phi V_n \geq V_u = \left(\frac{b}{2} - \frac{c}{2} - d\right)\ell q_u$	*Fig. E4.3—One-way shear in short direction.*
7.4.3.2	Distance of critical shear plane from center of footing (refer to Fig. E4.3):	$\frac{c}{2} + d = \frac{18\text{ in.}}{2(12\text{ in./ft})} + \frac{23.5\text{ in.}}{12\text{ in./ft}} = 2.71\text{ ft}$
	Half of footing width:	$\frac{b}{2} = \frac{5.5\text{ ft}}{2} = 2.75\text{ ft}$ Therefore, one-way shear in the short direction is OK by inspection because the critical shear plane is outside the edge of the footing.
Step 6: Two-way shear design		
13.3.3.1 13.2.7.2	The footing will not have shear reinforcement. Therefore, the nominal shear strength for this two-way footing is the concrete contribution to shear strength. $v_n = v_c$	
22.6.1.2 22.6.1.4 22.6.4.1	Under punching shear theory, inclined cracks are assumed to originate and propagate at 45 degrees away and down from the column corners. The area of concrete that resists shear is calculated at an average distance of $d/2$ from column face on all sides (refer to Fig. E4.4). $b_o = 4(c + d)$	*Fig. E4.4—Two-way shear.*
	where b_o is the perimeter of the area of shear resistance.	b_o = 4(18 + 30.2) = 192 in.
22.6.2.1	ACI 318-14 permits the engineer to take the average of the effective depth in the two orthogonal directions when calculating the shear strength of the footing.	d = 34 in. − 3 in. − 0.75 in. = 30.2 in.
8.5.3.1.2 22.6.1	Check two-way shear with the selected footing thickness.	

22.6.5.2	Calculate the shear strength contribution of concrete using the following formulas $4\lambda_s\lambda\sqrt{f_c'}$ $\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'}$ $\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'}$ Ignoring size effects, the equations become: (a) $v_c = 4\lambda\sqrt{f_c'}$	$v_c = 4(1.0)\left(\sqrt{4000\text{ psi}}\right) = 253\text{ psi}$ **Controls**
	(b) $v_c = \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}$	$v_c = \left(2+\frac{4}{1}\right)(1.0)\left(\sqrt{4000\text{ psi}}\right) = 379.5\text{ psi}$
	where β is ratio of the long side to short side of column, β = 1 (c) $v_c = \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f_c'}$	$v_c = \left(\frac{(40)(30.2\text{ in.})}{192\text{ in.}}+2\right)(1.0)\left(\sqrt{4000\text{ psi}}\right) = 524\text{ psi}$
22.6.5.3	$\alpha_s = 40$, considered interior column	Equation (a) controls; (a) < (b) < (c); $v_c = 253$ psi
	$V_c = 4\lambda\sqrt{f_c'}b_o d$	$V_c = \frac{(253\text{ psi})(192\text{ in.})(30.2\text{ in.})}{1000\text{ lb/kip}} = 1466\text{ kip}$
21.2.1(b)	Use a shear strength reduction factor of 0.75	$\phi = 0.75$
	$\phi V_c = (0.75)4\lambda\sqrt{f_c'}b_o d$	$\phi V_c = 0.75(1466\text{ kip}) = 1100\text{ kip}$
	$V_u = q_u((\ell)(B) - (c+d)^2)$	$V_u = (9.4\text{ ksf})\left[(5.5\text{ ft})(10\text{ ft}) - \left(\frac{18\text{ in.}+30.2\text{ in.}}{12\text{ in./ft}}\right)^2\right]$ $= 365\text{ kip}$
8.5.1.1	Check if design strength exceeds required strength $\phi V_c \geq V_u$?	$\phi V_c = 1100\text{ kip} > V_u = 365\text{ kip}$ **OK** Two-way shear strength is adequate

Foundations

	Calculate the service-level soil pressure:	
	$B = 5.5$ ft, $L = 10$ ft, $h = 34$ in. Weight of displaced soil by the footing.	
	Weight of soil above footing.	$W_{ftg} = (5.5\text{ ft})(10\text{ ft})(34\text{ in.}/12)(0.15\text{ kcf} - 0.12\text{ kcf})$ $= 4.7$ kip, say, 5 kip
	The footing weight is added to the dead load	$W_{soil} = \left(\dfrac{48\text{ in.} - 15\text{ in.}}{12\text{ in./ft}}\right)(5.5\text{ ft})(10\text{ ft})(0.120\text{ kip/ft}^3)$ $= 7.7$ kip $\dfrac{D + W_{ftg} + W_{soil} + L + W}{A_{total}}$ $\dfrac{213\text{ kip} + 4.7\text{ kip} + 7.7\text{ kip} + 100\text{ kip} + 105\text{ kip}}{55\text{ ft}^2} = 7.6\text{ ksf}$ Allowable soil pressure: $q_{all} = 8$ ksf **OK**

Step 7: Flexure design		
	<u>Long direction</u>	 *Fig. E4.5—Flexure in the long direction.*
13.2.7.1	The long direction in the rectangular footing will generate larger moments because of the longer moment arm. The critical section is permitted to be at the face of the column (refer to Fig. E4.5).	
	$M_u = q_u\left(\frac{\ell - c}{2}\right)^2 (B)/2$	$M_u = (9.4\text{ ksf})\left(\frac{10\text{ ft} - \frac{18\text{ in.}}{12\text{ in./ft}}}{2}\right)^2 (5.5\text{ ft})/2 = 467\text{ ft-kip}$
22.2.2.4.1	Set the concrete compression strength equal to the steel tension strength: $C = T$	$a = \frac{A_s f_y}{0.85 f_c' b} = 0.28A_s$
22.2.2.4 22.2.2.4.3	$C = 0.85f_c'ba$ and $T = A_sf_y$ and $f_c' = 4000$ psi	$\beta_1 = 0.85$
7.5.2.1	$\phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$	
21.2.1a	Assume section is tension-controlled so that $\phi = 0.9$	
	Substitute $0.28A_s$ for a in the equation above.	
8.5.1.1(a)	Setting $\phi M_n \geq M_u$ and solving for A_s, where M_u = 467 ft-kip	$(467\text{ ft-kip}) = (0.9)A_s(60\text{ ksi})\left(30.7\text{ in.} - \frac{(0.28)A_s}{2}\right)$
8.6.1.1	Check the minimum area $A_{s,min} = 0.0018A_g$	$A_s \geq 3.5\text{ in.}^2$ $A_{s,min} = 0.0018(5.5\text{ ft})(12\text{ in./ft})(34\text{ in.}) = 4.1\text{ in.}^2$ Use ten No. 6 bars distributed uniformly across the width of footing $A_{s,prov} = 4.4\text{ in.}^2$
13.3.3.3(a) 21.2.1(a) 21.2.2	Reinforcement in the longitudinal direction is uniformly distributed.	
7.3.3.1	Confirm that section is tension-controlled. The strain in reinforcement is calculated from similar triangles (refer to Fig. E4.6).	$c = \frac{0.28(10)(0.44\text{ in.}^2)}{0.85} = 1.45\text{ in.}$
	$\varepsilon_t = \frac{\varepsilon_c}{c}(d - c)$	$\varepsilon_t = \frac{0.003}{1.45\text{ in.}}(30.7\text{ in.} - 1.45\text{ in.}) = 0.061$
22.2.2.4.1 22.2.2.4.3	where $c = a/\beta_1$ and $a = 0.28A_s$	$\varepsilon_t = 0.061 > 0.005$ Section is tension controlled.

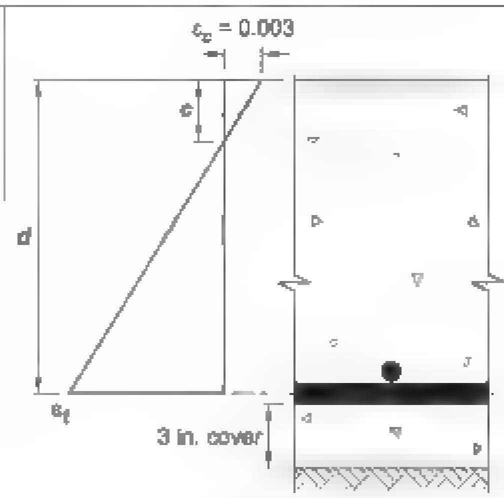


Fig. E4.6—Strain distribution across footing section.

Short direction

Calculate moment in the short direction, at the column face (Fig. E4.7).

Note: the effective depth is less than that calculated for the long direction: $d = 30.7$ in.

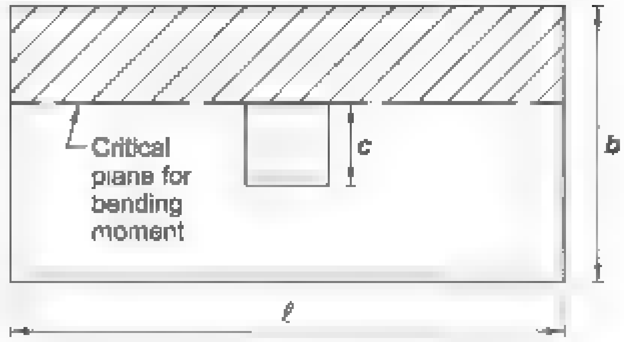


Fig. E4.7—Flexure in the short direction

13.2.7.1 $M_u = q_u\left(\frac{b-c}{2}\right)^2(\ell)/2$

$$M_u = (9.4\text{ ksf})\left(\frac{5.5\text{ ft} - \frac{18\text{ in.}}{12\text{ in./ft}}}{2}\right)^2(10\text{ ft})/2 = 188\text{ ft-kip}$$

22.2.2.4 Set compression force equal to tension force at the column face

$C = T$

22.2.2.4.1 $C = 0.85f_c'ba$ and $T = A_sf_y$

$$a = \frac{A_sf_y}{0.85f_c'b} = 0.147A_s$$

22.2.2.4.3 $f_c' = 4000$ psi $\beta_1 = 0.85$

7.5.2.1 $\phi M_n = \phi A_sf_y\left(d-\frac{a}{2}\right)$

21.2.1a Assume section is tension-controlled so that $\phi = 0.9$

Substitute $0.147A_s$ for a

8.5.1.1(a) Setting $\phi M_n = M_u$ and solving for A_s

$$188\text{ ft-kip} = (0.9)A_s(60\text{ ksi})\left(30.7\text{ in.} - \frac{0.147A_s}{2}\right)$$

$A_s = 1.4$ in.^{2}

8.6.1.1 Check minimum reinforcement area

$A_{s,min} = 0.0018A_g$

$A_{s,min} = 0.0018(10\text{ ft})(12\text{ in./ft})(34\text{ in.})$
$= 7.4\text{ in.}^2 > A_{s,req'd} = 1.4\text{ in.}^2$

Use minimum required reinforcement

$A_s = 7.4$ in.^{2}

13.3.3.3(b) In the short direction, a portion of the reinforcement (γ_sA_s) is distributed within a band width centered on the column

$$\gamma_s = \frac{2}{\beta+1}$$

$$\gamma_s = \frac{2}{\frac{10\text{ ft}}{5.5\text{ ft}}+1} = 0.71$$

The band width is equal to the length of the short side (5.5 ft)

Reinforcement area in 5.5 ft band width = $(7.4\text{ in.}^2)(0.71) = 5.3\text{ in.}^2$

Use twelve No. 6 bars distributed uniformly across the 5.5 ft band width

The remaining reinforcement $(1-\gamma_s)A_s$ is distributed equally on both sides outside the band width. The remaining area of reinforcement must be at least the minimum reinforcement with the bars spacing not exceeding the smaller of $3h$ or 18 in.

Reinforcement area outside the central band
$= (7.4\text{ in.}^2) - 12(0.44\text{ in.}^2) = 2.12\text{ in.}^2$

7.6.1.1	The area of reinforcement outside the band width must, however, satisfy at least the minimum flexural reinforcement $A_{s,min} = 0.0018A_g$	$A_{s,min} = 0.0018(10\text{ ft} - 5.5\text{ ft})(12\text{ in./ft})(34\text{ in.})$ $= 3.3\text{ in.}^2 > A_{s,req'd} = 2.12\text{ in.}^2$ Use four No. 6 bars each side distributed uniformly outside the band width $A_{s,prov} = 4.4\text{ in.}^2 > A_{s,min} = 3.3\text{ in.}^2$ **OK**
Step 8. Column-to-footing connection		
16.3.1.1	Vertical factored column forces are transferred to the footing by bearing on concrete and the reinforcement, usually dowels	
22.8.3.2	The footing is wider on all sides than the loaded area. Therefore, the nominal bearing strength, B_n, is the lesser of the two equations	
22.8.3.2(a)	$B_n = (0.85f_c'A_1)\sqrt{\frac{A_2}{A_1}}$ and	
22.8.3.2(b)	$B_n = 2(0.85f_c'A_1)$	
	Check if $\sqrt{\frac{A_2}{A_1}} \leq 2.0$ where	$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{[(5.5\text{ ft})(12\text{ in./ft})]^2}{(18\text{ in.})^2}} = 3.6 > 2$ Therefore, Eq. (22.8.3.2(b)) controls
	A_1 is the area of the column and A_2 is the area of the footing that is geometrically similar to and concentric with the column	
21.2.1(d)	The reduction factor for bearing is 0.65	$\phi_{bearing} = 0.65$ $\phi B_n = (0.65)(2)(0.85)(4000\text{ psi})(18\text{ in.})^2$ $\phi B_n = 432\text{ kip} > 515\text{ kip}$ (Step 4) **OK**
16.3.4.1	Provide minimum dowel area of $0.005A_g$ and at least four bars. This requirement is to ensure ductile behavior between the column and footing.	$A_{s,dowel} = 0.005(18\text{ in.})^2 = 1.62\text{ in.}^2$ Use four No. 6 bars in each corner of column.
16.3.5.1	Bars are in compression for all load combinations. Therefore, the bars must extend into the footing a compression development length, ℓ_{dc}, the larger of the two and at least 8 in.	
25.4.9.2	$\ell_{dc} = \begin{cases} \frac{f_y\psi_r}{50\lambda\sqrt{f_c'}}d_b \\ (0.0003f_y\psi_r d_b) \end{cases}$	$\ell_{dc} = \frac{(60{,}000\text{ psi})}{50\sqrt{4000\text{ psi}}}(0.75\text{ in.}) = 14.2\text{ in.}$ **Controls** $\ell_{dc} = (0.0003\text{ in.}^2/\text{lb})(60{,}000\text{ psi})(0.75\text{ in.}) = 13.5\text{ in.}$ $\ell_{dc} = 14.2\text{ in.}$ (controls) $> 8\text{ in.}$ **OK**
25.3.1	The footing depth h must satisfy the following inequality so that the vertical reinforcement can be developed $h > \ell_{dc} + r + d_{b,dwl} + 2d_{b,bars} + 3\text{ in.}$ where r = radius of No. 6 bent = $6d_b$	$h_{req'd} = 14.2\text{ in.} + 6(0.75\text{ in.}) + 0.75\text{ in.} + 2(0.75\text{ in.})$ $+ 3\text{ in.} = 23.95\text{ in.}$ $h_{req'd} = 23.95\text{ in.} < h_{prov} = 34\text{ in.}$ **OK**

	Assume that the column is reinforced with four No. 8 bars.	Compression development length for No. 8 bars is
25.5.5.4	As mentioned above, bars are in compression for all load cases. Therefore, the compression lap splices is the larger of the two conditions.	
25.4.9.1 25.4.9.2	1. The development length, ℓ_{dc}, of the larger bar and	$\ell_{dc} = \frac{60{,}000\text{ psi}}{50\sqrt{4000\text{ psi}}}(1.0\text{ in.}) = 19.0\text{ in.}$ **Controls** $\ell_{dc} = (0.0003\text{ in.}^2/\text{lb})(60{,}000\text{ psi})(1.0\text{ in.}) = 18\text{ in.}$
25.5.5.1	2. The compression lap splice of the smaller bar	$\ell_{sc} = 0.0005(60{,}000\text{ psi})(0.75\text{ in.}) = 22.5\text{ in.}$ Use $\ell_{sc} = 24\text{ in.} > 12\text{ in.}$ **OK** Therefore extend No. 6 bars 24 in. into the column
Step 9: Footing details		
13.2.8.3 13.2.7.1 13.2.8.1	<u>Development length</u> Reinforcement development is calculated at the maximum factored moment and the code permits the critical section to be located at the column face. Bars must extend a tension development length beyond the critical section.	
25.4.2.4	$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s\psi_g}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\right)d_b$	$\ell_d = \left(\frac{3}{40}\frac{60{,}000\text{ psi}}{(1.0)\sqrt{4000\text{ psi}}}\frac{(1.0)(1.0)(1.0)(1.0)}{2.5}\right)d_b = 28.5d_b$
25.4.2.1	where ψ_t = casting position, ψ_t = 1.0, because not more than 12 in. of fresh concrete below horizontal reinforcement ψ_e = coating factor; ψ_e = 1.0, because bars are uncoated ψ_s = bar size factor; ψ_s = for No. 6 and larger ψ_g = reinforcement grade factor; ψ_g = 1.0 c_b = spacing or cover dimension to center of bar, whichever is smaller K_{tr} = transverse reinforcement index It is permitted to use K_{tr} = 0	
	But the expression $\frac{c_b+K_{tr}}{d_b}$ must not exceed 2.5	No. 6: $\frac{c_b+K_{tr}}{d_b} = \frac{3.44\text{ in.}+0}{0.75\text{ in.}} = 4.59$ use maximum value of 2.5
	The development length must be the greater of the calculated value of Eq. (25.4.2.4) and 12 in.	No. 6 bars: 28.5(0.75 in.) = 22 in. > 12 in. Therefore, OK
		ℓ_d in the long direction $\ell_{d,prov} = ((10\text{ ft})(12\text{ in./ft}) - 18\text{ in.})/2 - 3\text{ in.}$ $\ell_{d,prov} = 48\text{ in.} > \ell_{d,req'd} = 22\text{ in.}$ **OK** use straight No. 6 bars in long direction
	See footing details in Fig. E4.8	ℓ_d in the short direction No. 6 $\ell_{d,prov} = ((5.5\text{ ft})(12\text{ in./ft}) - 18\text{ in.})/2 - 2\text{ in.}$ $\ell_{d,prov} = 22\text{ in.} > \ell_{d,req'd} = 22\text{ in.}$ **OK**

Step 10: Detailing

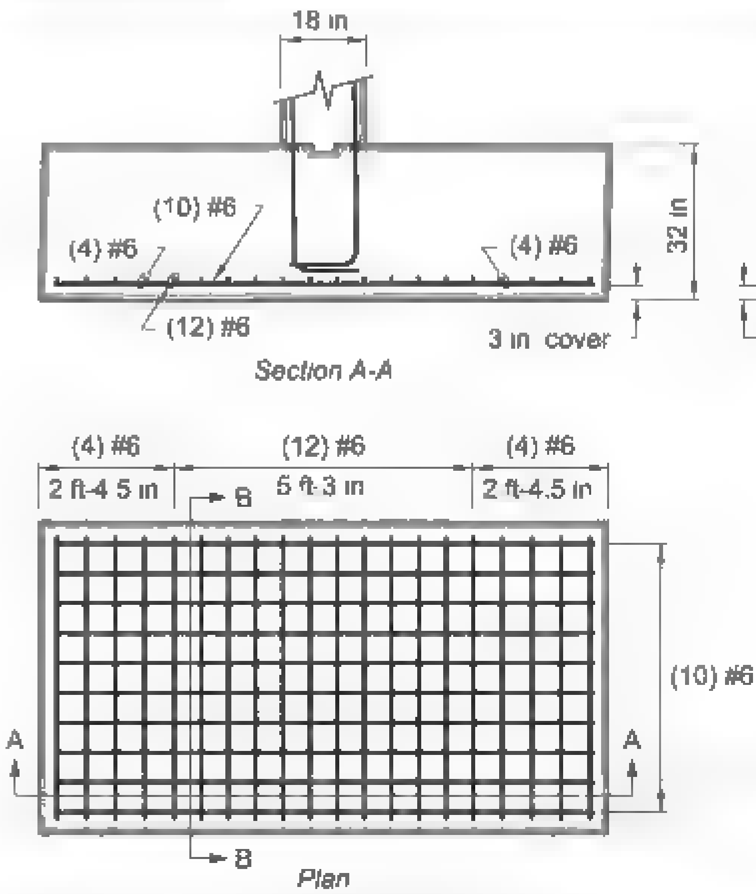


Fig. E4.8—Footing reinforcement detailing.

Square footing

If the problem was solved as square footing, then in Step 3, the following footing dimensions would have been selected: 7 ft 3 in. x 7 ft 3 in.

Following the same calculation procedure, shear strength is satisfied and minimum reinforcement ratio controls the flexure design (thirteen No. 6 each direction). Distribution of reinforcement within a central band does not apply to square footings.

Development lengths and dowel calculations are not affected.

Foundation Example 5: *Design of a combined footing*

Design and detail a rectangular combined footing, founded on stiff soil, supporting two building columns, oriented as shown in Fig. E5.1. The bottom of the footing is 5 ft below finished grade.

The columns only transmit axial force and neither shear nor moment is transmitted from the frame above into the footing. The soil reaction to column loads is assumed to be uniform across the footing bearing area.

Given:

Exterior column load—
Service dead load D_1 = 150 kip
Service live load L_1 = 100 kip
$c_1 \times c_1$ = 18 in. x 18 in.

Interior column load—
Service dead load D_2 = 260 kip
Service live load L_2 = 160 kip
$c_2 \times c_2$ = 20 in. x 20 in.

Material properties—
Concrete compressive strength f_c' = 4000 psi
Steel yield strength f_y = 60,000 psi
Normalweight concrete λ = 1
Density of concrete = 150 lb/ft^3
Allowable soil bearing pressure
q_a = 5000 psf under all loads

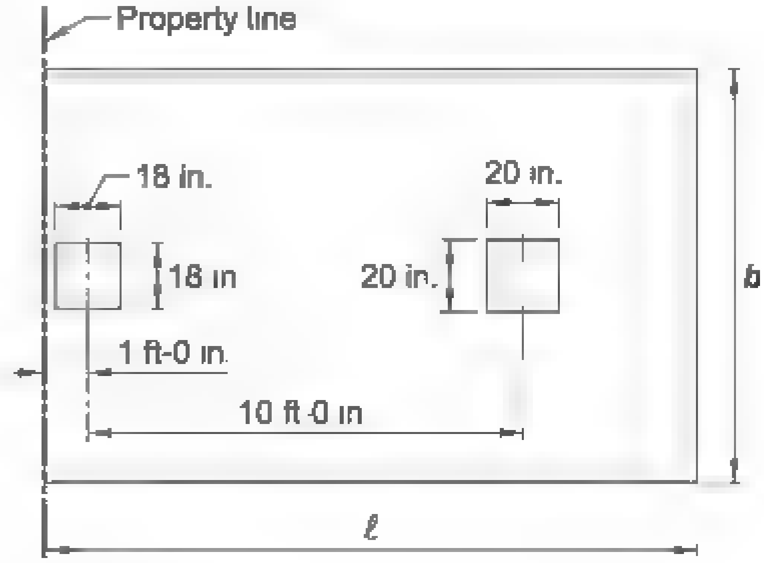


Fig. E5.1—Foundation plan.

ACI 318	Procedure	Calculation
Step 1: Foundation type		
13.1.1	This footing is 5 ft below finished grade. Therefore, it is considered a shallow footing.	
Step 2: Material requirements		
13.2.1.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements (ACI 318). The designer determines the durability classes. Please refer to Chapter 4 of this Manual for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor. Example 1 of this chapter provides a more detailed breakdown on determining the concrete compressive strength and exposure categories and classes.	By specifying that the concrete mixture shall be in accordance with ACI 301 and providing the exposure classes, ACI 318 Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 4000 psi.

Step 3. Determine footing dimensions

13.1.1 13.3.1.1	Service loads To calculate the footing area, assume the columns are supported on isolated square footings. Divide the column service loads by the allowable soil pressure	The unit weights of concrete and soil are 150 pcf and 120 pcf; close. Therefore, footing self-weight will be checked later.
	Exterior column $A_{req'd} = (D_1 + L_1)/q_a$	$A_{req'd} = (100\text{ kip} + 150\text{ kip})/5\text{ ksf} = 50\text{ ft}^2$ Use 7 ft 3 in. x 7 ft 3 in.
	Interior column $A_{req'd} = (D_2 + L_2)/q_a$	$A_{req'd} = (260\text{ kip} + 160\text{ kip})/5\text{ ksf} = 84\text{ ft}^2$ Use 9 ft 3 in. x 9 ft 3 in.
	Because the column is in close proximity to the property line, the exterior column footing cannot be concentric with the column, and the footing needs external bracing to remain stable. This can be supplied by a moment connection between the exterior and the interior footing, but in this case, the two footings are simply combined.	
	The footing thickness is calculated in Step 5, Two-way shear design.	
13.2.6.3	Determine the location of the resultant of the two service column loads by taking the moments about the center of the exterior column	
	$x_c = \dfrac{P_1 x_1 + P_2 x_2}{\sum P}$	$x_c = \dfrac{(260\text{ kip} + 160\text{ kip})(10\text{ ft})}{(150\text{ kip} + 100\text{ kip}) + (260\text{ kip} + 160\text{ kip})} = 6.3\text{ ft}$
	The distance of the resultant from the property line is:	$x = 6.3\text{ ft} + 1\text{ ft} = 7.3\text{ ft}$
	The footing length, L, is taken equal to $2x$ so the soil pressure can be assumed as uniform under the two column loads.	$2(7.3\text{ ft}) = 14.6\text{ ft}$
13.3.4.3	Distribution of bearing pressure under combined footing must be consistent with the soils properties and structure. The footing width is calculated (refer to Fig. E5.2)	
	$B = \dfrac{P}{q_a L}$	$B = \dfrac{(150\text{ kip} + 100\text{ kip}) + (260\text{ kip} + 160\text{ kip})}{(5\text{ ksf})(14.6\text{ ft})} = 9.2\text{ ft}$ Use 9 ft 6 in. x 15 ft 0 in.

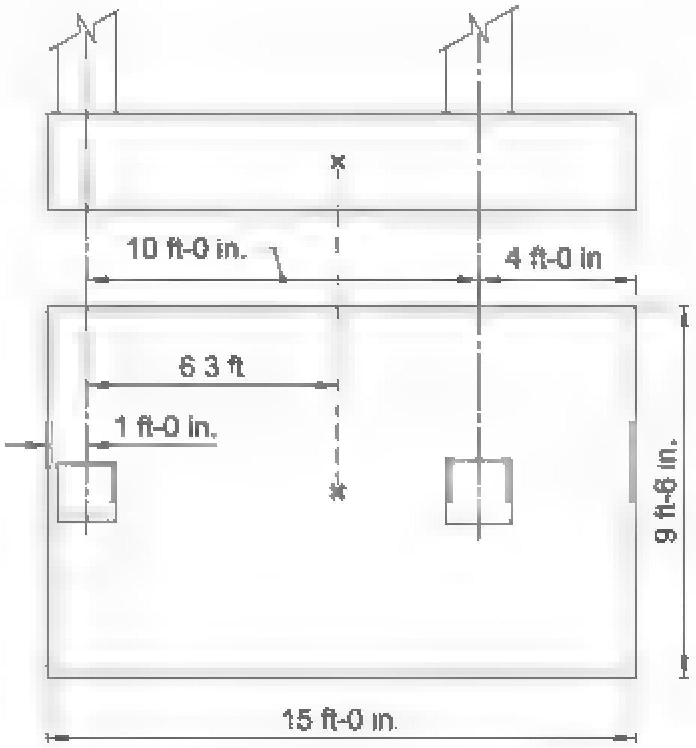
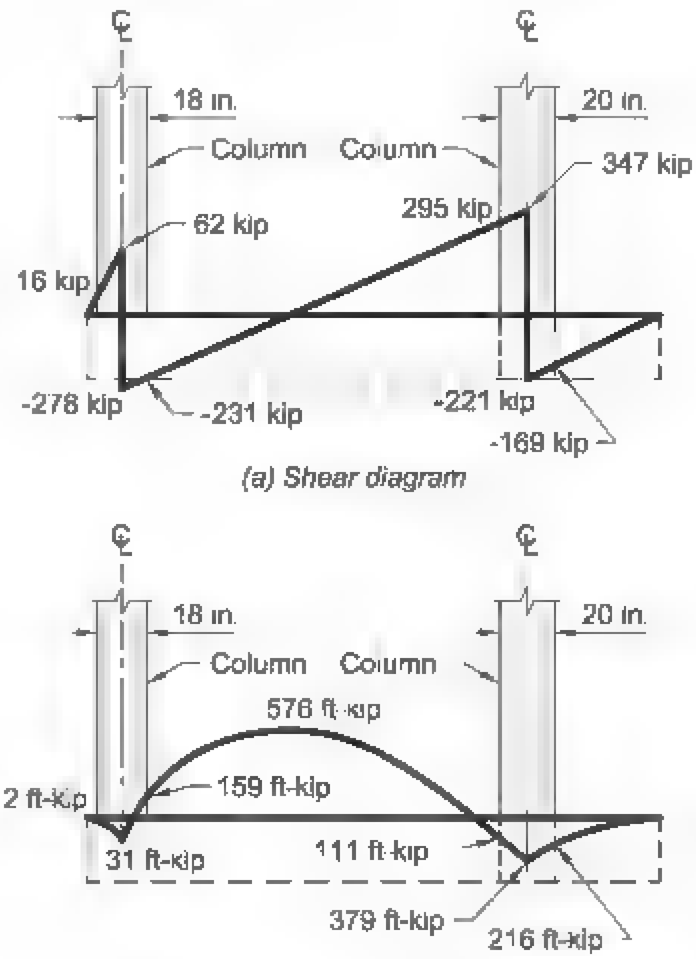


Fig. E5.2—Combined footing dimensions

Step 4: Design forces		
	Calculate soil pressure Factored loads	
13.3.4.3 13.2.6.1	Calculate the soil pressures resulting from the applied factored loads including footing self-weight. Assume 2 ft 6 in. thick footing	
	Footing self-weight:	$W = (0.15\ \text{kip/ft}^3)(15\ \text{ft})(9.5\ \text{ft})(2.5\ \text{ft}) = 53.5\ \text{kip}$ See Note that follows
	Soil self-weight above combined footing:	$W = (0.12\ \text{kip/ft}^3)(15\ \text{ft})(9.5\ \text{ft})(2.5\ \text{ft}) = 42.8\ \text{kip}$
	Total dead load.	dead load = 150 kip + 260 kip + 53.5 kip + 42.8 kip = 506.3 kip
	Total live load.	live load = 100 kip + 160 kip = 260 kip
5.3.1a	Load Case: $U = 1.4D$	$w_u = \dfrac{1.4(506.3\ \text{kip})}{(15\ \text{ft})} = 47.3\ \text{kip/ft}$
5.3.1b	Load Case: $U = 1.2D + 1.6L$	$w_u = \dfrac{1.2(506.3\ \text{kip}) + 1.6(260\ \text{kip})}{(15\ \text{ft})} = 68.2\ \text{kip/ft}$ **Controls**
	Distributed soil pressure per square area below combined footing (refer to Fig. E5.3)	$q_u = \dfrac{1024\ \text{kip}}{(15\ \text{ft})(9.5\ \text{ft})} = 7.2\ \text{kip/ft}^2$
	Note: This is a conservative approach. The footing concrete displaces soil. Therefore, the actual load on soil is the difference between the concrete and soil unit weights multiplied by the footing volume	



(b) Moment diagram

Fig. E5.3—Shear and moment diagrams.

Note: 231 kip and 295 kip are shear forces taken at the exterior and interior column faces, respectively; 159 ft-kip and 111 ft-kip and 216 ft-kip are flexure moment taken at the exterior column face and both interior column faces.

Step 5: Two-way shear design

13.3.4.1 Design of combined footing must satisfy the requirements of Code Chapter 8 for two-way slabs

13.2.6.2 Check footing two-way shear strength at both columns neglecting size effect factor

21.2.1b Shear strength reduction factor

$\phi_{shear} = 0.75$

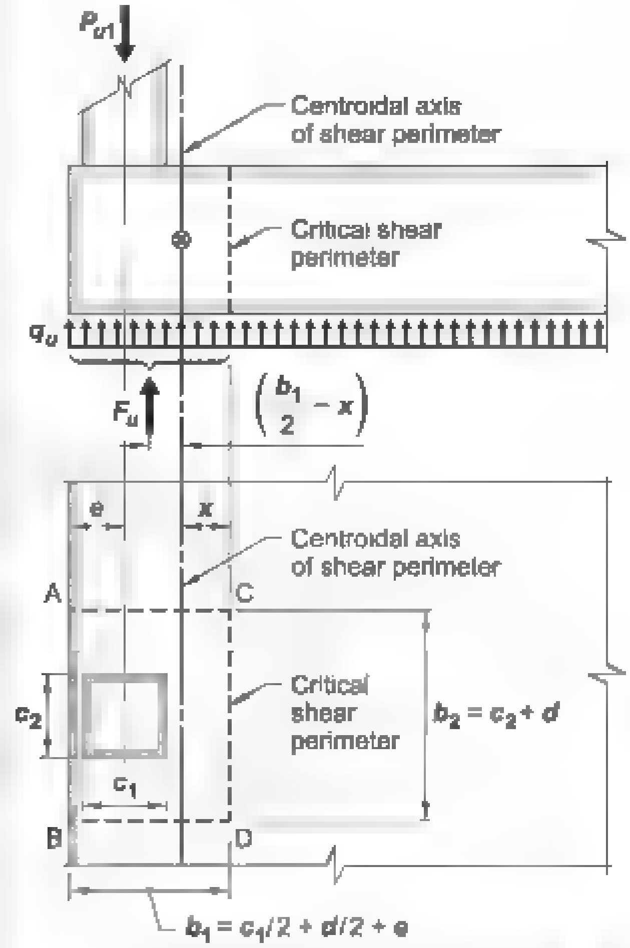


Fig. E5.4: Determining centroidal axis of shear perimeter

Exterior column

22.6.1.4 The footing critical shear perimeter, b_o, at the exterior column is three sided. From the free body diagram, the direct shear force, V_{uv}, is the result of the factored column force less the factored soil pressure force within the critical shear perimeter (refer to Fig. E5.4 and Fig. E5.5). Therefore,
$V_{uv} = P_{u1} - q_u A_1$

where $A_1 = (c_2 + d)((c_1 + d)/2 + e)$
$d = 30 \text{ in.} - 3 \text{ in.} - 1.128 \text{ in.}/2 = 26.4 \text{ in.}$

$$A_1 = (18 \text{ in.} + 26.4 \text{ in.})\left(\frac{18 \text{ in.} + 26.4 \text{ in.}}{2} + 12 \text{ in.}\right) = 1518.5 \text{ in.}^2$$

Assume No. 9 bars and $e = 1$ ft edge distance from the column centerline (refer to Fig. E5.4)

5.3.1 Solving for V_{uv}, where

$P_{u1} = 1.2D_1 + 1.6L$

$$P_{u1} = (1.2)(150 \text{ kip}) + (1.6)(100 \text{ kip}) = 340 \text{ kip}$$

$F_u = q_u A_1$

$$F_u = \frac{(7.2 \text{ ksf})}{144 \text{ in.}^2/\text{ft}^2}(1518.5 \text{ in.}^2) = 76 \text{ kip}$$

Substituting into
$V_{uv} = P_{u1} - q_u A_1$

$$V_{uv} = 340 \text{ kip} - 76 \text{ kip} = 264 \text{ kip}$$

Foundations

	The Code requires the footing moment at the critical shear centroid is transferred into the column by direct flexure and by eccentricity of shear	
	Calculate the centroid axis of shear perimeter (refer to Fig. E5.4 and Fig. E5.5), where	
	$\bar{x} = \frac{2(b_1)^2/2}{2b_1 + b_2}$ where	$\bar{x} = \frac{2(34.2\text{ in.})^2/2}{2(34.2\text{ in.}) + 44.4\text{ in.}} = 10.4\text{ in.}$
	$b_1 = e + \frac{c}{2} + \frac{d}{2}$ and	$b_1 = 12\text{ in.} + \frac{18\text{ in.}}{2} + \frac{26.4\text{ in.}}{2} = 34.2\text{ in.}$
	$b_2 = c + d$	$b_2 = 18\text{ in.} + 26.4\text{ in.} = 44.4\text{ in.}$
	From the free body diagram (refer to Fig. E5.4 and Fig. E5.5), summing the factored column load and soil pressure force about the critical section centroid	
8.4.4.2.1	$M_u^* = P_u(b_1 - e - \bar{x}) - F_u\left(\frac{b_1}{2} - \bar{x}\right)$	$M_u^* = (340\text{ kip})(34.2\text{ in.} - 12\text{ in.} - 10.4\text{ in.}) - 76\text{ kip}\left(\frac{34.2\text{ in.}}{2} - 10.4\text{ in.}\right)$
8.4.4.2.3	The maximum shear stress due to direct shear and shear due to moment transfer is	$M_u^* = 3503\text{ in.-kip}$
	$v_u = \frac{V_{ug}}{A_c} + \frac{\gamma_v M_u^* c}{J_c}$ (Eq. (R8.4.4.2.3))	
	$A_c = b_o d$, is the area of concrete within the critical section b_o, (refer to Fig. E5.4 and Fig. E5.5)	$A_c = (44.4\text{ in.} + 2(34.2\text{ in.}))(26.4\text{ in.}) = 2978\text{ in.}^2$
	The shear perimeter moment of inertia J_c is	
	$J_c = 2\left[b_1\frac{d^3}{12} + d\frac{b_1^3}{12} + (b_1 d)\left(\frac{b_1}{2} - \bar{x}\right)^2\right] + b_2 d\bar{x}^2$	$J_c = 2\left[(34.2\text{ in.})\frac{(26.4\text{ in.})^3}{12} + 26.4\text{ in.}\frac{(34.2\text{ in.})^3}{12} + (34.2\text{ in.})(26.4\text{ in.})\left(\frac{34.2\text{ in.}}{2} - 10.4\text{ in.}\right)^2\right] + (44.4\text{ in.})(26.4\text{ in.})(10.4\text{ in.})^2 = 488{,}727\text{ in.}^4$
	The portion of the moment is transferred by flexure is $\gamma_f M_u^*$, where γ_f is	
	$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}}$	$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{34.2\text{ in.}}{44.4\text{ in.}}}} = 0.63$
8.4.4.2.2	The moment fraction transferred by shear, $\gamma_v M_u^*$, where γ_v is $\gamma_v = 1 - \gamma_f$	$\gamma_v = 1 - 0.63 = 0.37$

	Solving for v_u from Eq. (R8.4.4.2.3) above: where $c = b_1 - \bar{x}$ $c = 34.2\text{ in.} - 10.4\text{ in.} = 23.8\text{ in.}$	$v_u = \dfrac{264\text{ kip}}{2978\text{ in.}^2} + \dfrac{(0.37)(3503\text{ in.-kip})(23.8\text{ in.})}{488{,}727\text{ in.}^4}$ $= 0.089\text{ ksi} + 0.063\text{ ksi} = 0.152\text{ ksi}$
13.2.6.2 22.6.5.2	Ignoring size effect factor, determine two-way shear strength provided by concrete using the following equations: $v_u \le \phi v_c = \phi\begin{Bmatrix} \left(\dfrac{\alpha_s d}{b_o} + 2\right) \\ \left(2 + \dfrac{4}{\beta}\right) \\ 4 \end{Bmatrix}\lambda\lambda_s\sqrt{f_c'}$	$\lambda_s = 1.0$ $\dfrac{\alpha_s d}{b_o} + 2 = \dfrac{(30)(26.4\text{ in.})}{(2)(34.2\text{ in.}) + 44.4\text{ in.}} + 2 = 9 > 4$ **NG** $2 + \dfrac{4}{\beta} = 2 + \dfrac{4}{1} = 6 > 4$ **NG** Therefore, 4 **Controls**
22.6.5.3	$\alpha_s = 30$, edge column $\phi v_c = \phi 4\lambda\sqrt{f_c'}$	$\phi v_c = (0.75)(4)(1)\sqrt{4000}\text{ psi} = 190\text{ psi}$ $\phi v_c = 190\text{ psi} > v_u = 156\text{ psi}$ **OK**
21.2.1	Shear strength reduction factor: 0.75	The factored stress exceeds the footing design shear stress.

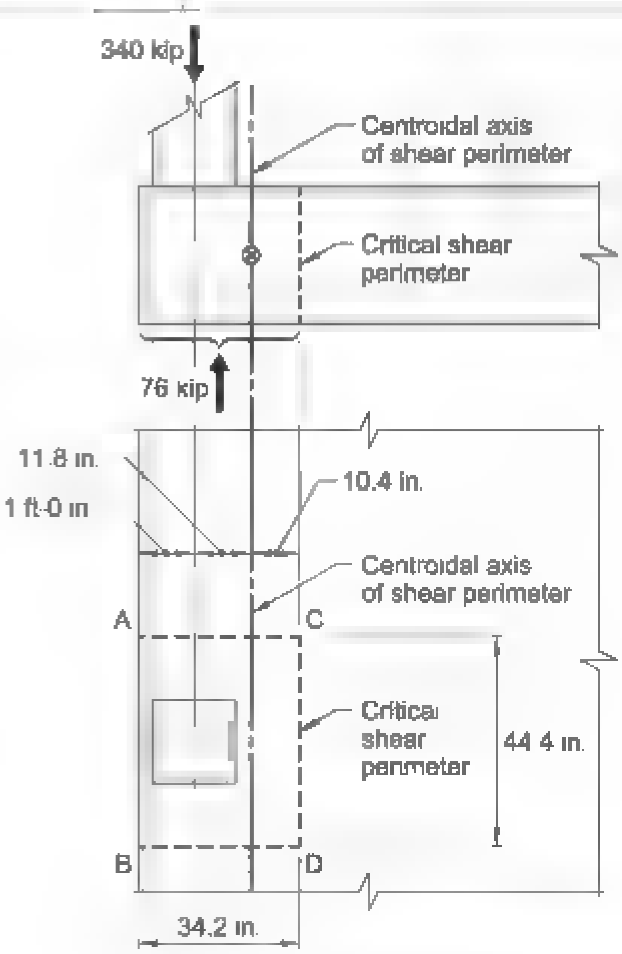


Fig. E5.5—Two-way shear at the exterior column.

8.4.4.1.1	<u>Interior column.</u> The maximum factored shear force at the critical section is equal to the factored column load less the factored soil pressure within the critical section (refer to Fig. E5.6).	
22.6.1.4	$V_u = P_{u2} - q_u(c_2 + d)(c_2 + d)$	$V_u = (1.2)(260\text{ kip}) + (1.6)(160\text{ kip}) - \dfrac{(7.2\text{ ksf})(20\text{ in.} + 26.4\text{ in.})(20\text{ in.} + 26.4\text{ in.})}{(12\text{ in./ft})^2}$
5.3.1	where $P_{u2} = 1.2D_2 + 1.6L_2$	$V_u = 568\text{ kip} - 107.6\text{ kip} = 459.4\text{ kip}$, say, 460 kip
13.2.6.2	Ignore size effect factor	$\lambda_s = 1.0$
22.6.4.1	Critical section, b_o Two-way shear is the least of:	$b_o = (4)(20\text{ in.} + 26.4\text{ in.}) = 185.6\text{ in.}$
22.6.5.2	$v_u \le \phi v_c = \phi\begin{pmatrix}4\\ \left(2+\dfrac{4}{\beta}\right)\\ \left(\dfrac{\alpha_s d}{b_o}+2\right)\end{pmatrix}\lambda\sqrt{f_c'}$	Check if the $\sqrt{f_c'}$ factors are less than 4 4 is used if the other factors are larger than 4 $(2 + 4/\beta) = 6 > 4$, with $\beta = 1$ Eq. (22.6.5.2(b)) does not control $(\alpha_s d/b_o + 2) = (40)(26.4\text{ in.})/185.6\text{ in.} = 5.7 > 4$ Eq. (22.6.5.2(c)) does not control
22.6.5.3	Use $\alpha_s = 40$ (interior column)	Therefore, use the factor 4
	Shear strength Eq. (22.6.5.2(a)) controls	$\phi v_c = (0.75)(4)(1.0)\left(\sqrt{4000\text{ psi}}\right) = 189.7\text{ psi}$ $\phi V_c = (189.7\text{ psi})(4)(20\text{ in.} + 26.4\text{ in.})(26.4\text{ in.})$ 929.5 kip, say, 930 kip
	Check if $\phi V_c > V_u$	$\phi V_c = 930\text{ kip} >> V_u = 460\text{ kip}$ **OK**

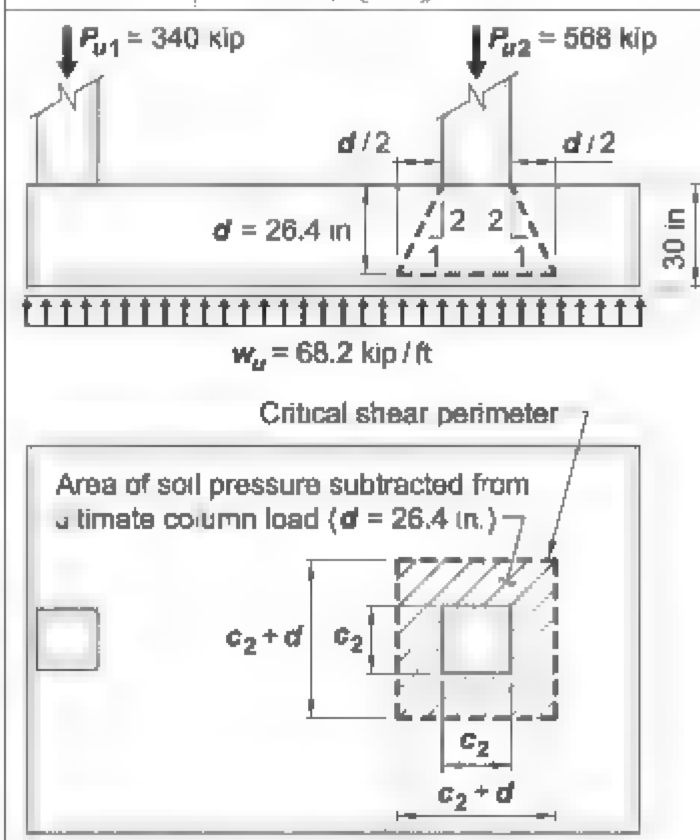


Fig. E5.6—Two-way shear at interior column.

Step 6: One-way shear design		
	<u>One-way shear design.</u>	
13.3.2.1 13.2.7.2 7.4.3.2	One-way shear strength is calculated at a distance d from the interior column face (refer to Fig. E5.7) where the maximum shear force is permitted to be calculated.	
	Check if ϕV_c with d = 26.4 in. exceeds V_u = 295 kip (refer to Fig. E5.3(a)).	
7.4.1.1	Calculate required strength from column factored loads less soil pressure	
	$V_u < V_{u(a)face} - w_u(c_2/2 + d)$	$V_u = 295\text{ kip} - 68.2\text{ kip/ft}\left(\frac{26.4\text{ in.}}{12\text{ in./ft}}\right) =$ [illegible]45 kip
	Calculate shear strength and verify that it exceeds the calculated required strength	
7.5.1.1	$\phi V_n > V_u$	
13.2.6.2	Check the footing depth considering one-way shear. Size effect factor may be neglected	
7.5.3.1 22.5	Shear reinforcement is not typically used in combined footings so all of the shear strength is provided by the concrete contribution $\phi V_n = \phi V_c$	
21.2.1 7.6.3.1	Strength reduction factor for shear from Code Table 21.2.1b	$\phi = 0.75$
	Minimum shear reinforcement is required where $V_u > \phi V_c$.	
	Footings, however, are not typically constructed with shear reinforcement. Provide sufficient depth to avoid the need for minimum shear reinforcement	
22.5.5.1c	Ignoring size effects, axial load, and using normal-weight concrete, the applicable equation from Code Table 22.5.5.1c becomes	
	$\phi V_c = \phi 8(\rho_w)^{1/3}\sqrt{f_c'}b_w d$	
	If ρ_w is set to the minimum required flexural reinforcement ratio of 0.0018, then the equation becomes	
	$\phi V_c = \phi 0.97\sqrt{f_c'}b_w d$	$0.75(0.97\sqrt{4000}\text{ psi})(114\text{ in.})(26.4\text{ in.}) = 138.5\text{ kip}$ **NG**
	Consider contribution of flexural reinforcement to shear strength. Use ten No. 9 bars in the top	$\rho_w = \frac{9(1\text{ in.})}{114\text{ in.}(26.4\text{ in.})} = 0.00299$
		$0.75(8)(0.00299)^{1/3}\sqrt{4000}\text{ psi}(114\text{ in.})(26.4\text{ in.}) = 164.5\text{ kip}$ **OK**
		Therefore, shear reinforcement is not required

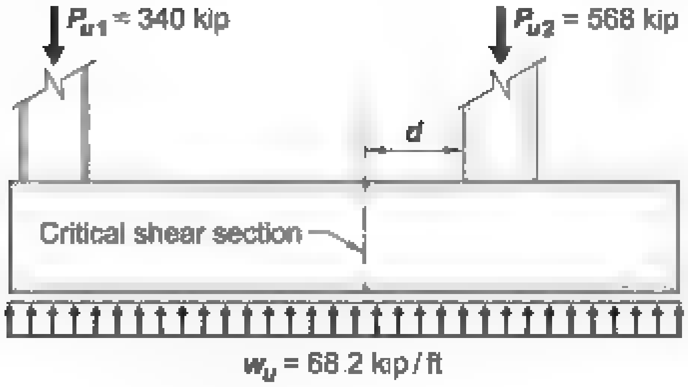


Fig. E5.7—One-way shear.

Summary: The combined footing thickness of 30 in. satisfies both one-way and two-way shear strength without shear reinforcement.

Step 7: Flexure design		
	Calculate the flexural reinforcement in the combined footing	
	<u>Longitudinal direction</u> Note that flexural tension occurs at the top of the footing between the two columns and at the bottom of the footing at both interior and exterior columns (Fig. E5.3(b))	
	<u>Top reinforcement between columns</u> At the section of maximum moment, set the internal compression force equal to internal tension force to calculate the reinforcement area	
22.2.2.4	$C = T$	
22.2.2.4.1 22.2.3.1	$0.85f_c'ba = A_sf_y$	$0.85(4000\text{ psi})(9.5\text{ ft})(12\text{ in./ft})a = A_s60{,}000\text{ psi}$ $a = 0.155A_s$
22.3.1.1	From moment diagram (Fig. E5.3(b)) $M_u < \phi M_n = \phi A_sf_y(d - a/2)$	$M_u = 576$ ft-kip
21.2.1a	Assume section is tension-controlled so that $\phi = 0.9$	
9.5.1.1a	Setting $\phi M_n > M_u$ and substitute for a in the equation above.	$(576\text{ ft-kip})\left(12\dfrac{\text{in.-kip}}{\text{ft-kip}}\right) \geq 0.9A_s(60\text{ ksi})\left(26.4\text{ in.} - \dfrac{0.155A_s}{2}\right)$
	Solve for A_s	$A_s > 4.9\text{ in.}^2$
9.6.1.2	Check if the minimum reinforcement area controls	
	$A_{s,min} = \dfrac{3\sqrt{f_c'}}{f_y}bd$ (9.6.1.2a)	$A_{s,min} = \dfrac{3\sqrt{4000\text{ psi}}}{60{,}000\text{ psi}}(9.5\text{ ft})(12\text{ in./ft})(26.4\text{ in.}) = 9.5\text{ in.}^2$
	$A_{s,min} = \dfrac{200}{f_y}bd$ (9.6.1.2b)	$A_{s,min} = \dfrac{200}{60{,}000\text{ psi}}(9.5\text{ ft})(12\text{ in./ft})(26.4\text{ in.}) = 10.0\text{ in.}^2$ $A_{s,min} = 10.0\text{ in.}^2 > A_{s,req'd} = 4.9\text{ in.}^2$
	Eq. (9.6.1.2b) controls because concrete compressive strength f_c' is less than 4444 psi	Therefore, minimum reinforcement controls
		Use ten No. 9 continuous top bars evenly distributed over the width of the footing.
		$A_{s,prov} = (10)(1.0\text{ in.}^2) = 10\text{ in.}^2$
9.3.3.1	Confirm if section is tension-controlled	$a = \left(0.155\dfrac{1}{\text{in.}}\right)(10\text{ in.}^2) = 1.55\text{ in.}^2$
21.2.2	$c = \dfrac{a}{\beta_1}$	$c = \dfrac{1.55\text{ in.}}{0.85} = 1.82\text{ in.}$

	Calculate the strain in the tension reinforcement and compare to the minimum strain required for tension-controlled section	
22.2.1.2	$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right)$	$\varepsilon_t = 0.003\left(\frac{26.4\text{ in.} - 1.82\text{ in.}}{1.82\text{ in.}}\right) = 0.0405 > 0.005$
	Place bars such that the spacing between them does not exceed $3h$ or 18 in.	Therefore, section is tension-controlled Use No. 9 at 12 in. on center < $3h$ = 90 in. and 18 in. Place first bar placed at 3 in. from the edge.

	<u>Reinforcement at interior column</u>	
13.2.7.1	The moment is taken at the interior face of the interior column	M_u = 216 ft-kip (Fig. E5.3(b))
22.2.2.4	At the section of maximum moment, set the internal compression force equal to internal tension force to calculate the reinforcement area $C = T$	
22.2.3.1 22.2.2.4.1	$0.85f_c'ba = A_sf_y$	$0.85(4000\text{ psi})(9.5\text{ ft})(12\text{ in./ft})a = A_s60{,}000\text{ psi}$ $a = 0.155A_s$
22.3.1.1	$\phi M_n = \phi f_yA_s(d - a/2)$ Substitute for a in the equation above	
21.2.1a	Assume section is tension-controlled so that $\phi = 0.9$	
9.5.1.1a	Setting $\phi M_n > M_u$ and solving	$(216\text{ ft-kip})(12\text{ in./ft}) \geq 0.9(60\text{ ksi})A_s\left(26.4\text{ in.} - \dfrac{0.155A_s}{2}\right)$
	Solve for A_s	$A_s > 1.83\text{ in.}^2$
9.6.1.2	This is less than the minimum reinforcement area calculated above	Therefore, use $A_{s,min} = 10.0\text{ in.}^2 > A_{s,req'd} = 1.83\text{ in.}^2$ Use ten No. 9 continuous bottom bars evenly distributed over the width of the footing. $A_s = (10)(1.0\text{ in.}^2) = 10\text{ in.}^2$ $a = \left(0.155\dfrac{1}{\text{in.}}\right)(10\text{ in.}^2) = 1.55\text{ in.}^2$
9.3.3.1	Check if section is tension-controlled	
	$c = \dfrac{a}{\beta_1}$	$c = \dfrac{1.55\text{ in.}}{0.85} = 1.82\text{ in.}$
	Calculate the strain in the tension reinforcement and compare to the minimum strain required for tension-controlled section.	
22.2.1.2	$\varepsilon_t = 0.003\left(\dfrac{d-c}{c}\right)$	$\varepsilon_t = 0.003\left(\dfrac{26.4\text{ in.} - 1.82\text{ in.}}{1.82\text{ in.}}\right) = 0.0405 > 0.005$ Therefore, section is tension-controlled
	Note: The calculated factored moment at the exterior column face (159 ft-kip) and the exterior moment at the interior column (111 ft-kip) is smaller than the calculated factored interior moment of the interior column face (216 ft-kip). Therefore, minimum reinforcement area controls. Provide 10 No. 9 bottom bars over full length of combined footing and spaced at 12 in. on center < $3h$ = 90 in. and 18 in.	

	<u>Transverse reinforcement</u> In a combined footing, transverse moment distribution may be addressed similar to an isolated spread footing. A strip over the width of the footing is considered to resist the column load. This strip is, however, not independent of the footing itself.	
	Darwin et al. (2015) and Fanella (2011) recommend the width of the strip to be half the effective depth (d/2) on either side of the footing from the face of columns.	Calculate d to the center of the second layer d = 30 in. – 3 in. – 1.128 in. – 1.0 in. = 25.37 in., say, d = 25.3 in. w = 20 in. + 2(25.3 in./2) = 45.3 in.
	<u>Interior column</u> The factored column load distributed over the width of the footing is used to determine the transverse bending moment.	
5.3.1	Factored distributed soil reaction is $q^*_{u,tr} = \frac{P_{u,int}}{B}$	$q^*_{u,tr} = \frac{1.2(260\text{ kip})+1.6(160\text{ kip})}{9.5\text{ ft}} = 59.8\text{ kip/ft}$ say, 60 kip/ft
13.2.7.1	The factored moment at the column face is $M_u = \frac{1}{2}q^*_u\left(\frac{b}{2}-\frac{c}{2}\right)^2$ where $\frac{b}{2}-\frac{c}{2} = \frac{9.5\text{ ft}}{2} - \frac{20\text{ in.}}{2(12\text{ in./ft})} = 3.92\text{ ft}$ Refer to Fig. E5.8	$M_u = \frac{1}{2}(60\text{ kip/ft})(3.92\text{ ft})^2 = 460\text{ ft-kip}$
8.5.1.1 21.2.1a	Calculate required reinforcement $\phi M_n = \phi f_y A_s jd \geq M_u$ Assume section is tension-controlled so that ϕ = 0.9 Coefficient on d: j = 0.9	460 ft-kip = 0.9A_s(60,000 psi)(0.9)(25.3 in.) A_s = 4.5 in.^{2}
13.3.2 17.6.1.1	Check if the minimum reinforcement area controls. Minimum steel $A_{s,min} = 0.0018A_g$	$A_{s,min}$ = 0.0018(45.3 in.)(30 in.) = 2.5 in.^{2}/ft
	Eq. (9.6.1.2b) controls because concrete compressive strength f_c' is less than 4444 psi. Check if section is tension-controlled.	Required reinforcement is greater than the minimum required. Therefore use eight No. 7 spaced at 6 in. on center and placed within the calculated width 46.4 in. $A_{s,prov}$ = (8)(0.6 in.^{2}) = 4.8 in.^{2} > $A_{s,req'd}$ = 4.5 in.^{2} > $A_{s,min}$ = 2.5 in.^{2}
	$c = \frac{a}{\beta_1}$	$a = \frac{(0.9)(60,000\text{ psi})(4.8\text{ in.}^2)}{0.85(4000\text{ psi})(45.3\text{ in.})} = 1.68\text{ in.}$ $c = \frac{1.68\text{ in.}}{0.85} = 1.98\text{ in.}$

	Calculate the strain in the tension reinforcement and compare to the minimum strain required for tension-controlled section	
22.2.1.2	$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right)$	$\varepsilon_t = 0.003\left(\frac{26.4\text{ in.} - 1.98\text{ in.}}{1.98\text{ in.}}\right) = 0.035 > 0.005$
		Therefore, section is tension-controlled
	Exterior column The factored column load distributed over the width of the footing is used to determine the transverse bending moment.	$d = 25.3$ in. $w = 12$ in. + 18 in./2 + 25.3 in./2 = 33.7 in.
5.3.1	Factored distributed soil reaction is:	
	$q^*_{u,TT} = \frac{P_{u,int}}{B}$	$q^*_{u,TT} = \frac{1.2(150\text{ kip}) + 1.6(100\text{ kip})}{9.5\text{ ft}} = 35.8$ kip/ft
13.2.7.1	The factored moment at the column face is	
	$M_u = \frac{1}{2}q^*_u\left(\frac{b}{2} - \frac{c}{2}\right)^2$	$M_u = \frac{1}{2}(35.8\text{ kip/ft})(4\text{ ft})^2 = 286$ ft-kip
	$\frac{b}{2} - \frac{c}{2} = \frac{9.5\text{ ft}}{2} - \frac{18\text{ in.}}{2(12\text{ in./ft})} = 4$ ft	
	Refer to Fig. E5.8	
	Calculate required reinforcement:	
8.5.1.1	$\phi M_n \geq M_u = \phi f_y A_s jd$	
	Coefficient on d: $j = 0.9$	286 ft-kip = $0.9A_s$(60,000 psi)(0.9)(25.3 in.)
21.2.1a	Assume section is tension-controlled so that $\phi = 0.9$	$A_s = 2.8$ in.^{2}
13.3.2	Check if the minimum reinforcement area controls	
17.6.1.1	Minimum steel	
	$A_{s,min} = 0.0018A_g$	$A_{s,min} = 0.0018(33.7\text{ in.})(30\text{ in.}) = 1.9$ in.^{2}/ft
		Use five No. 7 spaced at 10 in. on center: $A_{s,prov} = (5)(0.6\text{ in.}^2) = 3.0\text{ in.}^2 > A_{s,req'd} = 2.8\text{ in.}^2$ $> A_{s,min} = 1.9\text{ in.}^2$
	Check if section is tension-controlled	$a = \frac{(0.9)(60,000\text{ psi})(3.0\text{ in.}^2)}{0.85(4000\text{ psi})(33.7\text{ in.})} = 1.41$ in.
	$c = \frac{a}{\beta_1}$	$c = \frac{1.41\text{ in.}}{0.85} = 1.66$ in.
	Calculate the strain in the tension reinforcement and compare to the minimum strain required for tension-controlled section	
22.2.1.2	$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right)$	$\varepsilon_t = 0.003\left(\frac{25.3\text{ in.} - 1.66\text{ in.}}{1.66\text{ in.}}\right) = 0.043 > 0.005$
		Therefore, section is tension-controlled

Foundations

	For sections outside the effective width at the exterior and interior columns, provide minimum reinforcement area.	$A_{s,min} = 0.0018(12\text{ in.})(30\text{ in.}) = 0.65\text{ in.}^2/\text{ft}$ Use No. 7 at 11 in. on center < $3h$ = 90 in. or 18 in. $A_{s,prov} = \dfrac{0.6\text{ in.}^2}{11\text{ in.}}(12\text{ in./ft}) = 0.654\text{ in.}^2 > A_{s,min} = 0.6\text{ in.}^2$ **OK**
	See footing details in Fig. E5.8	

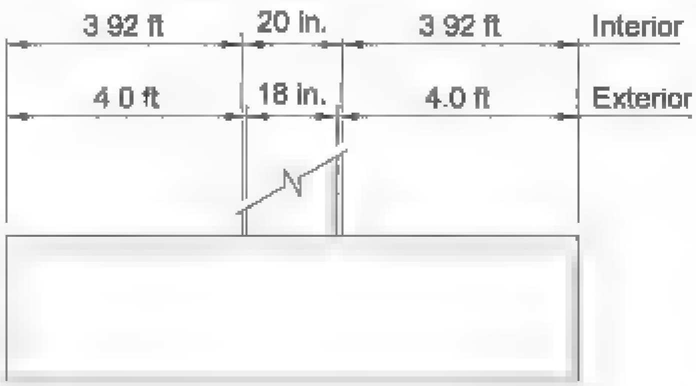


Fig. E5.8—Footing width at columns for transverse reinforcement calculations

Step 8. Footing details

Development length of No. 9 top bars

From the moment diagram in Fig. E5.3(b), the positive moment inflection points at the exterior and interior columns occur at 0.12 ft and 0.45 ft from the respective column centerlines (Fig. E5.9). Therefore, extend top bars to the edge of the footing.

Check if the available distance is sufficient to develop the top bars at midspan in tension.

The development length for No. 9 bar is calculated using a simplified equation as allowed by ACI 318 code rather than the more detailed Eq. (25.4.2.3a).

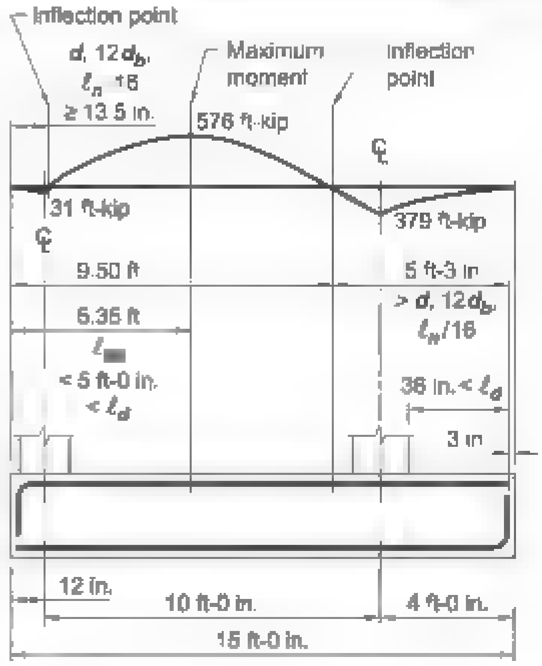


Fig. E5.9—Longitudinal reinforcement of combined footing

Code section	Discussion	Calculation
25.4.2.3	$\ell_d = \left(\dfrac{f_y\psi_t\psi_e\psi_g}{20\lambda\sqrt{f'_c}}\right)d_b$	$\ell_d = \left(\dfrac{(60{,}000\text{ psi})(1.3)(1.0)(1.0)}{(20)(1.0)\sqrt{4000\text{ psi}}}\right)d_b = 61.7d_b = 69.6\text{ in.}$
	where	
25.4.2.5	ψ_t (casting position) = 1.3 for top bars because more than 12 in. of fresh concrete is placed below horizontal bars and ψ_e (coating factor) = 1.0 because bars are uncoated ψ_g = (reinforcement grade) = 1.0 for Grade 60 reinforcement	Use 70 in. = 5 ft 10 in.
25.4.1.4	Check if $\sqrt{f'_c}$ is less than 100 psi	$\sqrt{4000\text{ psi}} = 63.2\text{ psi} < 100\text{ psi}$ **OK**
25.4.2.1	Check if development is less than 12 in.	$\ell_d = 69.6$ in. = 5.8 ft > 12 in. **OK**
		15 ft – 5.35 ft = 9.65 ft > 5.8 ft **OK**
	The available length from maximum moment at midspan to the interior column is greater than the calculated development length (Refer to Fig. E5.9)	Enough distance is available to develop No. 9 bars.
	The available length from maximum moment at midspan to the exterior column is less than the calculated development length, ℓ_d = 5.8 ft.	

25.4.3	Determine required hook development length using the following equations $\ell_{dh} \geq \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5}$ $\ell_{dh} > 8d_b$ $\ell_{dh} > 6$ in.	$\lambda = 1.0$ Bars are uncoated $\psi_e = 1.0$
25.4.3.2	ψ_e – Coating factor ψ_r – Confining reinforcement factor ψ_o – Location factor ψ_c – Concrete compressive strength factor	Spacing of No. 9 bars ~11.5 in., which meets requirement in table $\psi_r = 1.0$ Bars do not meet side cover requirements $\psi_o = 1.25$ Concrete strength is less than 6000 psi $\psi_c = \dfrac{4000}{15{,}000} + 0.6 = 0.867$ Required hook development length: $\dfrac{60{,}000\text{ psi}(1.0)(1.0)(1.25)(0.867)}{55(1.0)\sqrt{4000}\text{ psi}}(1.128)^{1.5} = 22.4\text{ in.}$ Therefore, No. 9 top straight bars can be placed full length and will be developed at the point of maximum moment at the interior column and 90-degree hook at the exterior column.

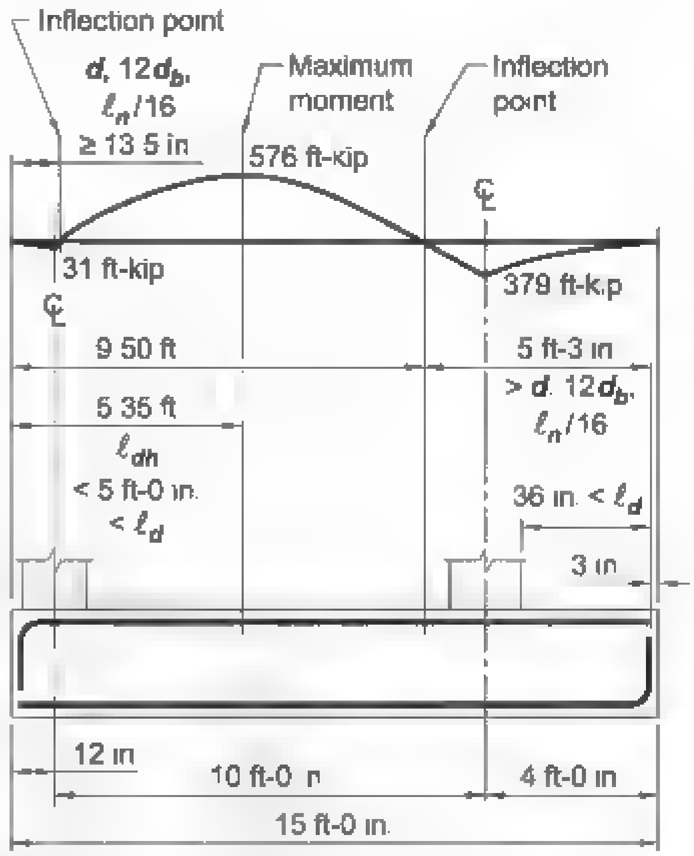
<table>
<tr><td></td><td><u>Development of bottom bars</u>
<u>Longitudinal bars No. 9</u>
The factored moment at the exterior column is negligible, 2 ft-kip at face of column, 32 ft-kip at column centerline (refer to Fig. E5.3(b))

Calculate the development length at the interior column M_u = 279 ft-kip at exterior face (Fig. E5.3(b)).</td><td></td></tr>
<tr><td>25.4.2.3</td><td>$\ell_d = \left(\frac{f_y\psi_t\psi_e\psi_g}{20\lambda\sqrt{f_c'}}\right)d_b$</td><td>$\ell_d = \left(\frac{(60,000\text{ psi})(1.0)(1.0)(1.0)}{(20)(1.0)\sqrt{4000\text{ psi}}}\right)d_b = 47.4d_b$</td></tr>
<tr><td>25.4.2.5</td><td>where
ψ_t = casting location, ψ_t = 1.0, because not more than 12 in. of fresh concrete is placed below horizontal reinforcement
ψ_e = coating factor; ψ_e = 1.0, because bars are uncoated
ψ_g = reinforcement grade factor; ψ_g = 1.0 for Grade 60 reinforcement</td><td><table><tr><th>Bar size</th><th>ℓ_d, in.</th><th>Use, in.</th></tr><tr><td>No. 9</td><td>53.5</td><td>54</td></tr><tr><td>No. 7</td><td>41.5</td><td>42</td></tr></table></td></tr>
<tr><td>25.4.2.1</td><td>Check if development is less than 12 in.</td><td>Both required development length exceeds 12 in. Therefore, OK</td></tr>
<tr><td></td><td>Interior column
Column is located 4 ft from footing edge, which is less than the required calculated development length of 54 in. = 4 ft 6 in. Therefore, provide a hook for the bottom No. 9 bars at the interior column. Refer to prior calculations for top bars.</td><td>ℓ_{dh} = 23 in. < 15 ft − 11.83 ft = 3.17 ft OK</td></tr>
<tr><td></td><td colspan="2">Note: That if the more detailed development length Eq. (25.4.2.3a) is used, then adequate distance is available to place the No. 9 bars without having to bend them</td></tr>
<tr><td></td><td>Transverse reinforcement
From Fig. E5.8, the overhang at the interior column is 3.92 ft = 47 in., which is greater than the required calculated development length = 42 in. Therefore, No.7 bars are placed straight.</td><td></td></tr>
</table>

Foundations

Step 9: Column-to-footing connection

	Interior column	
16.3.1.1	Factored column forces are transferred to the footing by bearing and through reinforcement, usually dowels	
22.8.3.2	The footing is wider on all sides than the loaded area. Therefore, the nominal bearing strength, B_n, is the smaller of the two equations. (a) $B_n = (0.85 f_c' A_1)\sqrt{\frac{A_2}{A_1}}$ and (b) $B_n = 2(0.85 f_c' A_1)$	
	A_1 is the bearing area of the column and A_2 is the area of the part of the supporting footing that is geometrically similar to and concentric with the loaded area.	Center of column is located 3 ft 2 in. from the end of footing and 3 ft 11 in. of the combined footing long sides.
	The sides of the pyramid tapered wedges are sloped 1 vertical to 2 horizontal	3.16 ft/2 = 1.58 ft = 19 in. < 30 in. footing thickness $A_2 = [2(10\text{ in.} + (3.16\text{ ft})(12\text{ in./ft}))]^2 = 9063\text{ in.}^2$
	Check if $\sqrt{\frac{A_2}{A_1}} \le 2.0$ where	$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{9063\text{ in.}^2}{(20\text{ in.})^2}} = 4.76 > 2$ Therefore, Eq. (22.8.3.2(b)) controls. $B_n = 2(0.85 f_c' A_1)$
21.2.1	The bearing strength reduction factor is 0.65	$\phi_{bearing} = 0.65$ $\phi B_n = (0.65)(2)(0.85)(4000\text{ psi})(20\text{ in.})^2$ $\phi B_n = 1768\text{ kip} > 1.2D + 1.6L = 600\text{ kip}$
16.3.4.1	Column factored forces are transferred to the footing by bearing and through dowels. The minimum dowel area is $0.005A_g$ and at least four bars across the interface between interior column and combined footing.	$A_{s,dowel} = 0.005(20\text{ in.})^2 = 2.0\text{ in.}^2$ Use one No. 7 bar in each corner of column
16.3.5.1	The four No. 7 dowels must be developed in the footing depth	$A_s = (4)(0.6\text{ in.}^2) = 2.4\text{ in.}^2 > A_{s,dowel} = 2.0\text{ in.}^2$
	Bars are in compression for all load combinations. Therefore, the bars must extend into the footing at least a compression development length ℓ_{dc}, which is the larger of the following two expressions	
25.4.9.2	$\ell_{dc} = \begin{cases} \dfrac{f_y \psi_r}{50\lambda\sqrt{f_c'}} d_b \\ (0.0003 f_y \psi_r d_b) \end{cases}$	$\ell_{dc} = \dfrac{(60{,}000\text{ psi})(1.0)}{(50)\sqrt{4000\text{ psi}}}(0.875\text{ in.}) = 16.6\text{ in.}$ **Controls** $\ell_{dc} = (0.0003)(60{,}000\text{ psi})(0.875\text{ in.}) = 15.75\text{ in.}$

25.4.9.3	ψ_r = confining reinforcement factor; ψ_r = 1.0, because reinforcement is not confined The footing depth must satisfy the following inequality: $h > \ell_{dc} + r + d_{b,dwl} + d_{b,\#7} + d_{b,\#9} + 3$ in.	$h_{req'd}$ = 16.6 in. + 6(0.875 in.) + 0.875 in. + 0.875 in. + 1.128 in. + 3 in. = 27.728 in., say, 28 in. $h_{req'd}$ = 28.0 in. < $h_{prov.}$ = 30 in. **OK**
20.6.1.3.2	3 in. cover (refer to Fig. E5.10) Check development length of dowel reinforcement into the column The length of dowels in the column is the greater of the development length and lap splice length. Assume that the column is reinforced with six No. 8 bars $d_{b,dowel} < d_{b,column}$	 *Fig. E5.10—Reinforcement development length*
25.4.9.2	Therefore, the lap splice length must be the greater of a) and b) a. No. 8 bars is the larger of: ℓ_{sc} = larger of $\begin{cases} \dfrac{f_y \psi_r}{50\lambda\sqrt{f_c'}} d_b \\ 0.0003 f_y d_b \end{cases}$ where ψ_r = confining reinforcement factor; ψ_r = 1.0, because stirrup spacing is greater than 4 in. (condition 3)	$\dfrac{(60,000\text{ psi})(1.0)}{(50)(1.0)\sqrt{4000\text{ psi}}}(1.0\text{ in.}) = 19.0$ in. 0.0003(60,000 psi)(1.0 in.) = 18.0 in.
25.5.5.1	b. The compression lap splice length of No. 7 is the larger of: ℓ_{sc} = larger of $\begin{cases} 0.0005 f_y d_b \\ 12\text{ in.} \end{cases}$	0.0005(60,000 psi)(0.875 in.) = 26.25 in. 12 in. Use 27 in. = 2 ft 3 in. long lap splice.

	Exterior column	
16.3.1.1	The column factored forces are transferred to the footing by bearing and through dowels.	
22.8.3.2	The footing is wider on all sides than the loaded area. Therefore, the nominal bearing strength, B_n, is the smaller of the following two equations. (a) $B_n = (0.85 f_c' A_1)\sqrt{\frac{A_2}{A_1}}$ and (b) $B_n = 2(0.85 f_c' A_1)$	
	A_1 is the bearing area of the column and A_2 is the area of the part of the supporting footing that is geometrically similar to and concentric with the loaded area.	Center of column is located 1 ft-0 in. from the end of footing and 3 ft-11 in. of the combined footing long sides
	The sides of the pyramid tapered wedges are sloped 1 vertical to 2 horizontal	1 ft/2 = 0.5 ft = 6 in. < 30 in. footing thickness $A_2 = [2(9\text{ in.} + 3\text{ in.})]^2 = 576\text{ in.}^2$
	Check if $\sqrt{\frac{A_2}{A_1}} \leq 2.0$, where	$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{576\text{ in.}^2}{(18\text{ in.})^2}} = 1.33 < 2$ Therefore, Eq. (22.8.3.2(a)) controls
21.2.1	The bearing strength reduction factor for bearing is 0.65	$\phi_{bearing} = 0.65$
16.3.4.1	Column factored forces are transferred to the footing by bearing and dowels. The minimum dowel area is $0.005A_g$ and at least four bars are needed across the interface between column and footing.	$\phi B_n = (0.65)(0.85)(4000\text{ psi})(18\text{ in.})^2\sqrt{\frac{576\text{ in.}^2}{(18\text{ in.})^2}}$ $\phi B_n = 955\text{ kip} > 340\text{ kip}$ **OK**
	The four No. 6 dowels must be developed in the footing	$A_{s,dowel} = 0.005(18\text{ in.})^2 = 1.62\text{ in.}^2$
16.3.5.1	The bars are in compression for all load combinations. Therefore, the bars must extend into the footing a compression development length ℓ_{dc}, which is the larger of the two following expressions	Use one No. 6 bars in each corner of the column. $A_s = (4)(0.44\text{ in.}^2) = 1.76\text{ in.}^2 > A_{s,dowel} = 1.62\text{ in.}^2$ **OK**
25.4.9.2	$\ell_{dc} = \begin{cases} \frac{f_y \psi_r}{50\lambda\sqrt{f_c'}} d_b \\ (0.0003 f_y \psi_r d_b) \end{cases}$ where ψ_r = confining reinforcement factor; ψ_r = 1.0, because reinforcement is not confined	$\ell_{dc} = \frac{(60{,}000\text{ psi})(1.0)}{(50)\sqrt{4000\text{ psi}}}(0.75\text{ in.}) = 14.2\text{ in.}$ **Controls** $\ell_{dc} = 0.0003(60{,}000\text{ psi})(1.0)(0.75\text{ in.}) = 13.5\text{ in.}$
	The footing depth h must satisfy the following inequality $h > \ell_{dc} + r + d_{b,dwl} + d_{b\,No.7} + d_{b,No.9} + 3\text{ in.}$	$h_{req'd} = 14.2\text{ in.} + 6(0.75\text{ in.}) + 0.75\text{ in.} + 0.875\text{ in.} + 1.128\text{ in.} + 3\text{ in.} = 24.5\text{ in.}$, say, 25 in. $h_{req'd} = 25\text{ in.} < h_{ftg,prov.} = 30\text{ in.}$ **OK**
20.6.1.3	3 in. cover (refer to Fig. E5.10)	

	Check development length of dowel reinforcement into the column	
	The length of dowels in the column is the greater of the development length and lap splice length Assume that the column is reinforced with six No 8 bars $d_{b,dowel} < d_{b,column}$	
25 4.9.2	Therefore, the lap splice length must be at least equal to the larger of (a) and (b)	
	(a) For column bars, at least the larger of:	
	$\ell_{dc} = \begin{cases} \dfrac{f_y \psi_r}{50\lambda\sqrt{f_c'}} d_b \\ (0.0003 f_y \psi_r d_b) \end{cases}$	$\dfrac{(60{,}000\text{ psi})(1.0)}{(50)(1.0)\sqrt{4000\text{ psi}}}(0.75\text{ in.}) = 14.3\text{ in.}$ **Controls** $0.0003(60{,}000\text{ psi})(0.75\text{ in.}) = 13.5\text{ in.}$
25 5.5 1	where ψ_r = confining reinforcement factor; ψ_r = 1 0, because reinforcement is not confined	
	(b) The compression lap splice length of dowel must be at least the larger of:	
	$\ell_{sc} = \text{larger of} \begin{cases} 0.0005 f_y d_b \\ 12\text{ in.} \end{cases}$	$0.0005(60{,}000\text{ psi})(0.75\text{ in.}) = 22.5\text{ in.}$ **Controls** 12 in
	See footing details in Fig. E5 11	Use 24 in. long lap splice

Step 10: Details

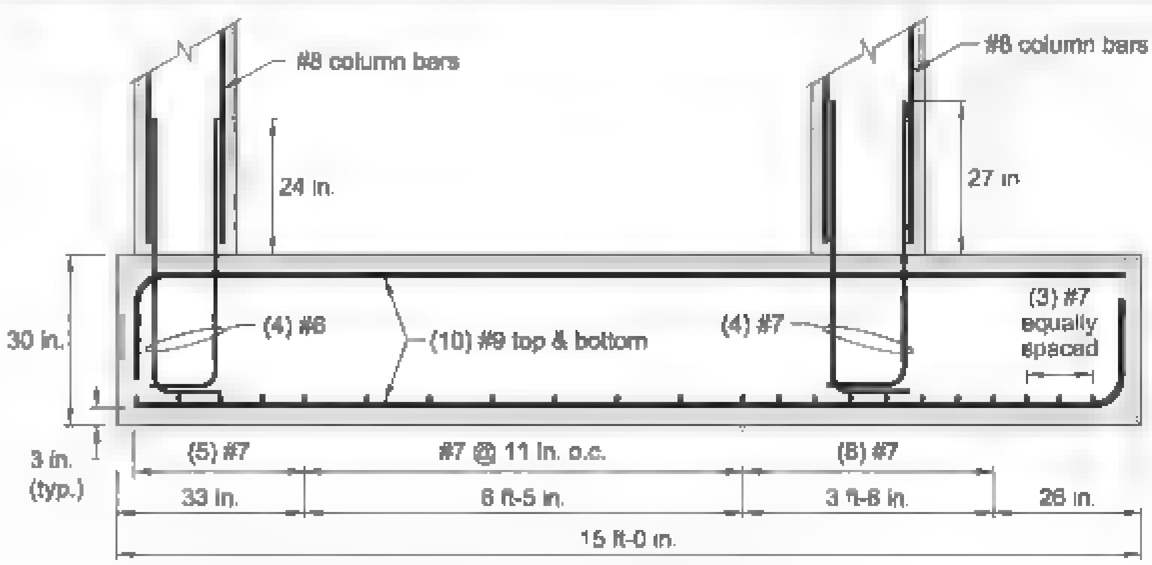


Fig E5.11—Combined footing dimensions and reinforcement

References

Darwin, D , Dolan, C., Nilson, A , eds , 2015, *Design of Concrete Structures*, McGraw-Hill Professional Publishing, 15th edition, New York, 786 pp

Fanella, D , ed 2011 *Reinforced Concrete Structures Analysis and Design*, McGraw Hill Professional Publishing, first edition, New York, 615 pp

Foundations

Foundation Example 6—*Design of an 18 in. square precast prestressed pile under combined axial and lateral loads*

The pile is 60 ft long, driven into loose sand, and subjected to combined axial and lateral loads at the pile head. The precast pile is not fixed to the pile cap and will be considered to exhibit free head conditions for the purpose of the analysis. The building is assigned to Seismic Design Category (SDC) A. Find the maximum deflections and internal demands (versus capacity) first using classical pile analysis techniques and compare to analysis conducted with commercial pile analysis software.

Given:

Material properties—

f_c' = 6000 psi Specified concrete compressive strength

f_{ci}' = 3500 psi Specified concrete compressive strength at prestress transfer

f_y = 60 ksi Specified yield strength of reinforcement

f_{pu} = 270 ksi Specified ultimate strength of ASTM A416 Gr 270 prestressing strand.

Loads—

Load combinations for strength design (Code Table 5.3.1)	P_u, kip	V_u, kip
$U = 1.2D + 1.6L + 0.5S$	394	0
$U = 1.2D + 1.0W + 1.0L + 0.5S$	340	40
$U = 0.9D + 1.0W$	180	40

Load combinations for allowable stress design (ASCE/SEI 7)	P_s, kip	V_s, kip
$S = D + L$	290	0
$S = D + 0.75L + 0.75S$	283	0
$S = D + 0.6W$	200	24
$S = D + 0.75L + 0.75(0.6W) + 0.75S$	283	18
$S = 0.6D + 0.6W$	120	24

Soil properties—

n_h = 30 lb/in.^{3} Modulus of subgrade reaction

ACI 318	Procedure	Computation
Step 1: Check Code applicability		
1.4.7b	The Code applies to precast concrete piles supporting structures assigned to SDC A and B.	

Step 2: Check strength using strength design approach (13.4.3)		
13.4.3.1	Strength design approach is used for this design because the pile resists axial forces and flexure.	
13.4.3.2	Strength design of deep foundation members should be in accordance with Code Section 10.5 using the compressive strength reduction factors given in Code Table 13.4.3.2. Section 10.5 requires the consideration of interaction between load effects such as moment and axial force. Before checking axial-flexure interaction, calculate the maximum design axial strength for this pile.	
	Use strength reduction factor from Code Table 13.4.3.2f for axial force without moment. For combined axial force and moment use Code Tables 21.2.1a and 21.2.2. Further discussion and information on strength design of deep foundations is available in ACI 336.3 and ACI 543R to assist the designer.	For axial $\phi = 0.65$ For axial and moment varies between $\phi = 0.65$ for compression controlled and $\phi = 0.9$ for tension controlled
13.4.5.2	Arrange twelve 0.5 in. diameter prestressing strands around the perimeter of the pile in a symmetrical pattern.	
13.4.5.4	Minimum effective compressive stress from Code Table 13.4.5.4 for this 60 ft long pile is 700 psi.	$A_g = (18\text{ in.})^2 = 324\text{ in.}^2$
13.4.5.5	Effective prestress after total losses of 30,000 psi. Apply these losses to a typical jacking stress of 203 ksi. Depending on the pile driving conditions and methods, significantly higher effective prestress levels may be required to reduce the chance of pile damage during driving. Recommendations for good driving practice can be found in ACI 543R.	$f_{se} = 203\text{ ksi} - 30\text{ ksi} = 173\text{ ksi}$ Effective compressive stress in concrete is $\dfrac{173\text{ ksi}(12)(0.153\text{ in.}^2)}{324\text{ in.}^2} = 980\text{ psi}$ > 700 psi **OK**
10.5.2.1 22.4.2	Determine upper limit on the axial strength of the pile using Code Section 22.4.2. $P_o = 0.85f_c'(A_g - A_{st} - A_{pd}) + f_yA_{st} - (f_{se} - 0.003E_p)A_{pt}$	$A_{st} = 12(0.153\text{ in.}^2) = 1.836\text{ in.}^2$ $P_o = 0.85(6000\text{ psi})(324\text{ in.}^2 - 1.836\text{ in.}^2)$ $-(173\text{ ksi} - 0.003 \times 28{,}500\text{ ksi})(1.836\text{ in.}^2) = 1482\text{ ksi}$ $P_{n\_max} = 0.80(1482\text{ kip}) = 1186\text{ kip}$ $\phi P_{n\_max} = 0.65(1186\text{ kip}) = 771\text{ kip}$ $> P_u = 370\text{ kip}$ **OK** Some strength remains to resist combined axial force and flexure.

Foundations

Step 3. Pile analysis for lateral load using Davisson (1970)		
	Determine the displacement at the pile head using Davisson (1970) $y = CQT^3/E_cI_{cr}$ where y is the lateral displacement, C is the deflection coefficient (selected from Fig. E6.1), Q is the applied lateral load at the pile head, T is a measure of relative stiffness, which is a function of the pile stiffness (E,I) and lateral stiffness of soil (n_h), E is the section elastic modulus, and I is the moment of inertia. Gross moment of inertia is used here to simplify comparison of results with computer software analysis. For design, the nonlinear capabilities of the software would be used to accurately account for concrete cracking. Design the pile to remain uncracked (Class U) under lateral service loads. Here, T has units of in., and is expressed as: $T = (E_cI_g/n_h)^{1/5}$	$I_g = \frac{(18\text{ in.})^4}{12} = 8748\text{ in.}^4$ $E_c = 57{,}000\sqrt{6000}\text{ psi} = 4415\text{ ksi}$ $T = \left[\frac{4415\text{ ksi}(8748\text{ in.}^4)}{30\text{ lbf/in.}^3}\right]^{1/5} = 66.4\text{ in.}$

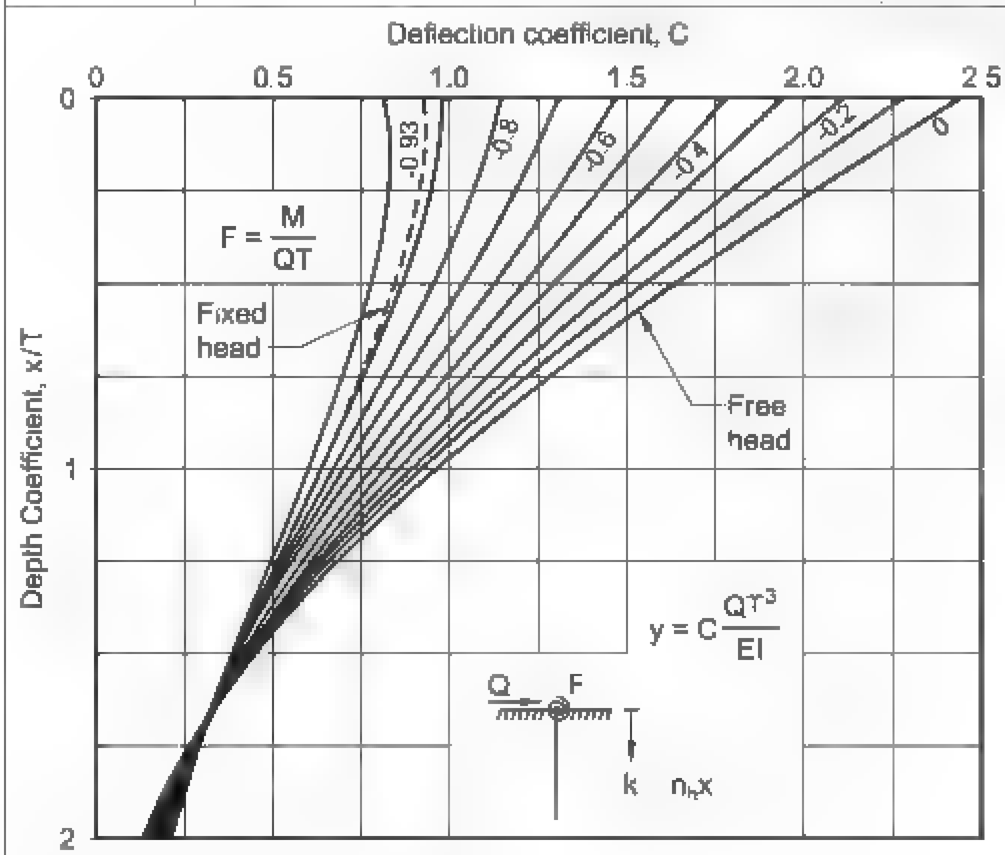


Fig. E6.1—Pile deflection coefficients.

Load combination 1 is axial load with no lateral load other than accidental eccentricity. Consider the factored shear from load combination 2 because it will create the largest moment in the pile.

Selecting deflection coefficient from Fig. E6.1 for free head condition and maximum deflection at the pile head

$C = 2.43$

Determine lateral displacement at pile head using the factored wind load

$$\Delta = 2.43\frac{40\text{ kip}(66.4\text{ in.})^3}{4415\text{ ksi}(8748\text{ in.}^4)} = 0.74\text{ in.}$$

Determine maximum internal load effect in pile using coefficients from Fig. E6.2 and the following equation

$M = CQT$

where M is the pile internal moment, C is the moment coefficient, and Q is the applied lateral load at the pile head. For a free head condition, the maximum moment occurs at x/T of ~1.3

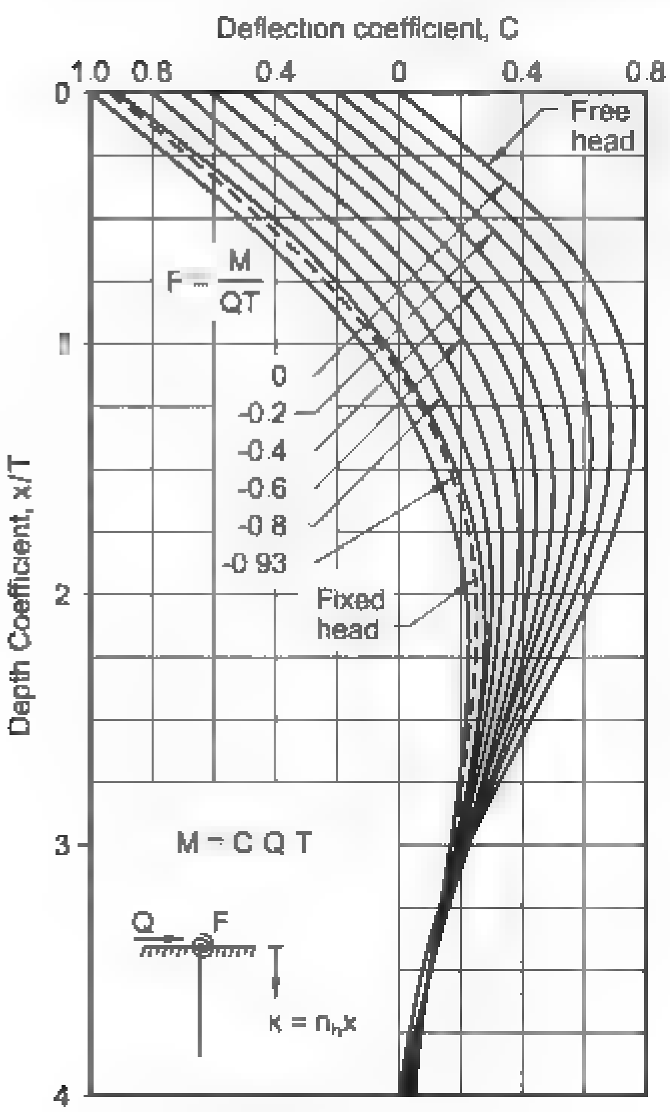


Fig. E6.2—Pile moment coefficients

The maximum moment coefficient from the figure is approximately
$C = 0.77$

Determine coefficient for the location of the maximum moment from the figure

$x = 1.3(66.4\text{ in.}) = 7.2\text{ ft}$

Therefore, maximum moment for load combination 2 is

$M_{U2\_max} = 0.77\ 40\text{ kip}(66.4\text{ in.}) = 170\text{ kip-ft}$

Step 4. Pile design strength		
	Use interaction diagram developed using software for engineering calculations from recommendations in PCI *Recommended Practice for the Design, Manufacture, and Installation of Prestressed Concrete Piling* (1993) and *PCI Calculation of Interaction Diagrams for Precast, Prestressed Concrete Piles* (2015). Figure E6.3 shows the strength interaction diagram for an 18 in. square pile with twelve 0.5 in. diam. prestressing strands. Note that the limiting axial design strength in the curve was calculated in Step 2 of this problem.	
20.5.1.3.4	Protection of reinforcement. Use clear cover requirements for precast prestressed piles assuming permanent contact with ground = 1.5 in.	
20.5.1.3.4	Pile confinement. Assume W4.4 spiral wire for confinement.	

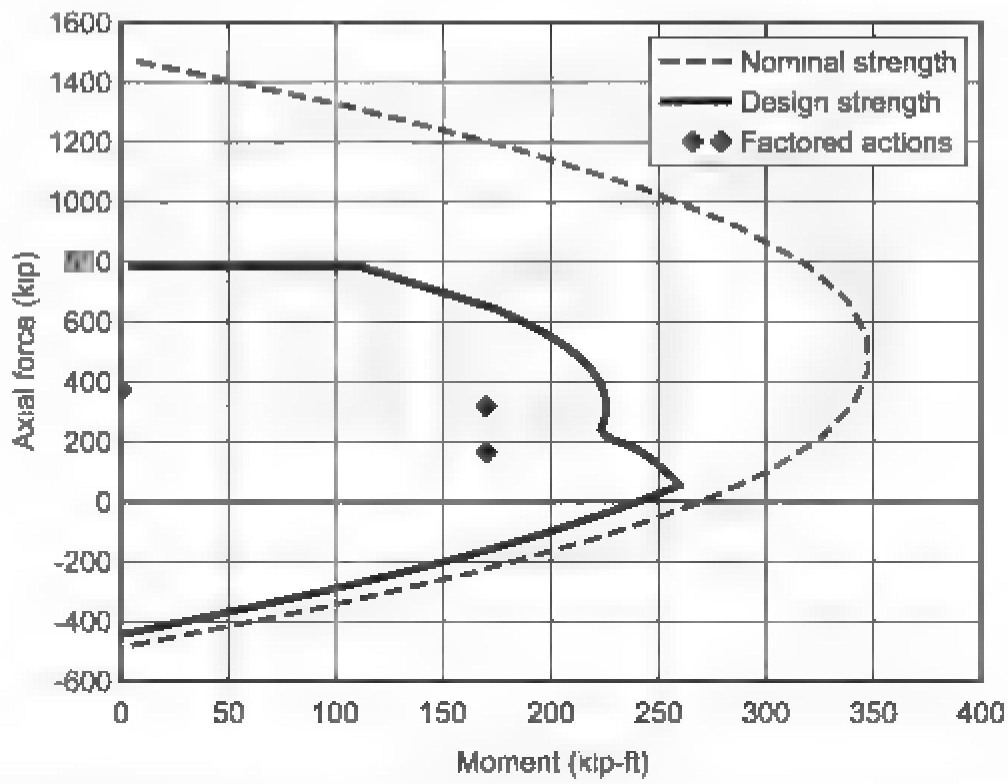


Fig. E6.3—Design strength interaction diagram for 18 in. square prestressed precast concrete pile.

Step 5. Pile service stresses

24.5 Service stress conditions must also be checked because this pile is prestressed.

Use pile analysis coefficients to calculate the service moments in the piles under the considered load combinations.

$M_{S3,max} = 0.77(18\text{ kip})(66.4\text{ in.}) = 77\text{ kip-ft}$

$M_{S4,max} = 0.77(24\text{ kip})(66.4\text{ in.}) = 102\text{ kip-ft}$

24.5.2.1 Prestressed concrete members are classified according to the maximum net tensile stress under load. This classification guides the designer on the section properties to use when calculating stresses or deflections but does not not limit tensile stresses beyond those at prestress transfer. To develop the service interaction diagram, use the recommended allowable stresses from ACI 543R shown in Table 4.3.2.8 below. These allowable stresses also satisfy

24.5.4.1 Code limits.

Table 4.3.2.8—Allowable service-load stresses in prestressed piles*

Loading condition	Permanent, psi	Temporary, psi
Tension		
Concrete tension†	0	$3\sqrt{f_c'}$
Flexure plus compression		
Concrete tension	0	$6\sqrt{f_c'}$
Concrete tension for marine work	0	$3\sqrt{f_c'}$
Concrete compression	$0.45f_c'$	$0.6f_c'$
Flexure plus tension		
Concrete tension	0	$3\sqrt{f_c'}$
Concrete compression	$0.45f_c'$	$0.6f_c'$

*Units for allowable stresses and f_c' in the equations in this table are psi (1 psi = 0.0069 MPa). Because the tension stresses are a function of the square root of f_c', if other units are used for f_c', it is also necessary to change the coefficients in front of the radical. Conversions for the equations are:

$3\sqrt{f_c'} = (\sqrt{f_c'})/4$ Equation in terms of psi

$6\sqrt{f_c'} = (\sqrt{f_c'})/2$ Equation in terms of MPa

†In piles that are expected to be subjected to tension, the ultimate capacity of the prestressing steel should be equal to or greater than the 1.2 times the direct tension cracking force, unless the available strength is greater than twice the required factored ultimate tension load; that is, $f_{pu}A_{ps} \geq 1.2(f_{pc} + 7.5\sqrt{f_c'})A_g$, where f_{pc} and f_{pu} are in psi units.

Allowable stresses for sustained load only. Assume that 25% of the live load and 100% of the dead load are sustained. The resulting combined load effects do not include flexure.

0 Tension

$0.45(6000\text{ psi}) = 2700\text{ psi}$ Compression

Allowable stresses for load combinations containing transient load effects (such as wind or live load)

$-6\sqrt{6000}\text{ psi} = -465\text{ psi}$ Tension

$0.6(6000\text{ psi}) = 3600\text{ psi}$ Compression

Using these allowable stresses develop the interaction diagram shown in Fig. E6.4.

Foundations

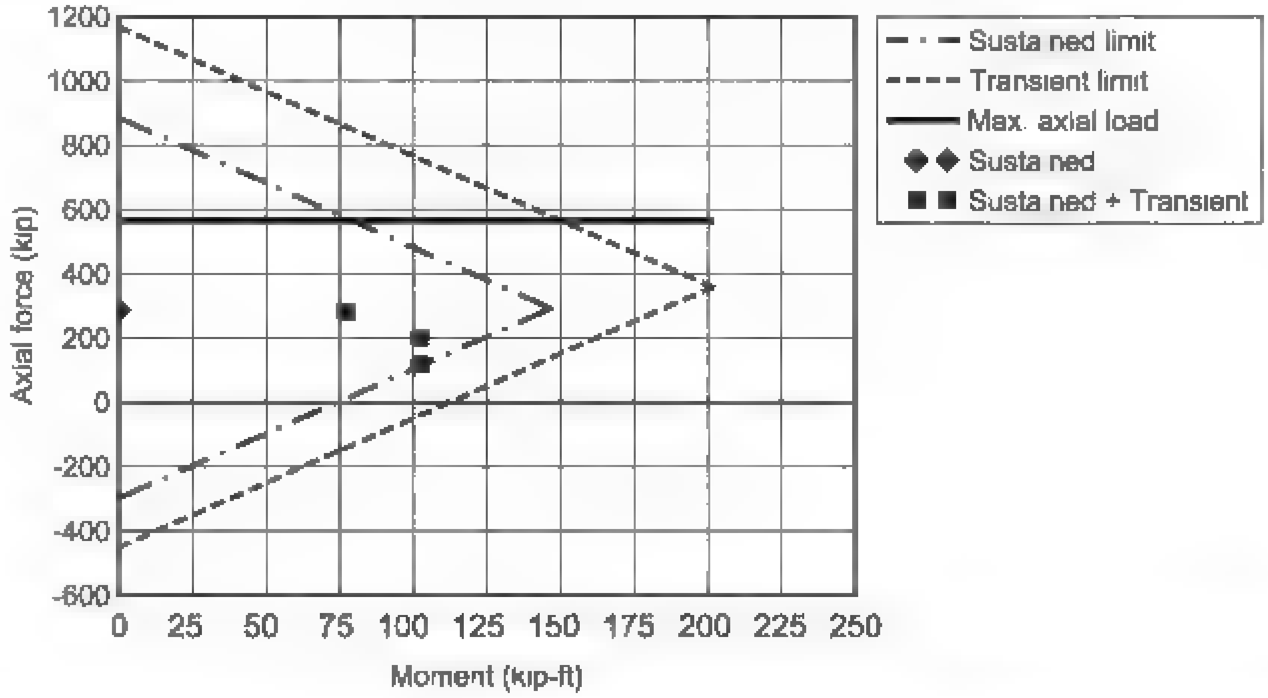


Fig. E6.4—Service interaction diagram of 18 in. precast, prestressed pile.

Step 6: Pile analysis for lateral load using software

The hand methods presented above allow for the engineer to estimate pile response to loading. However, methods such as "equivalent fixity" have shortcomings. As examples, material and geometric properties (modulus of elasticity, moments of inertia) of piles and soil must be assumed as linear; group effects are not readily taken into account. Software packages are available to model nonlinearity associated with soil-structure interaction and overcome the limitations of available hand methods. The results of such an analysis are presented here for comparison.

Results obtained from modeling and analysis of the controlling strength load combination, using the commercial software, are presented below. Profiles of shear, moment, and displacement are shown along with the program-generated (design) interaction diagram.

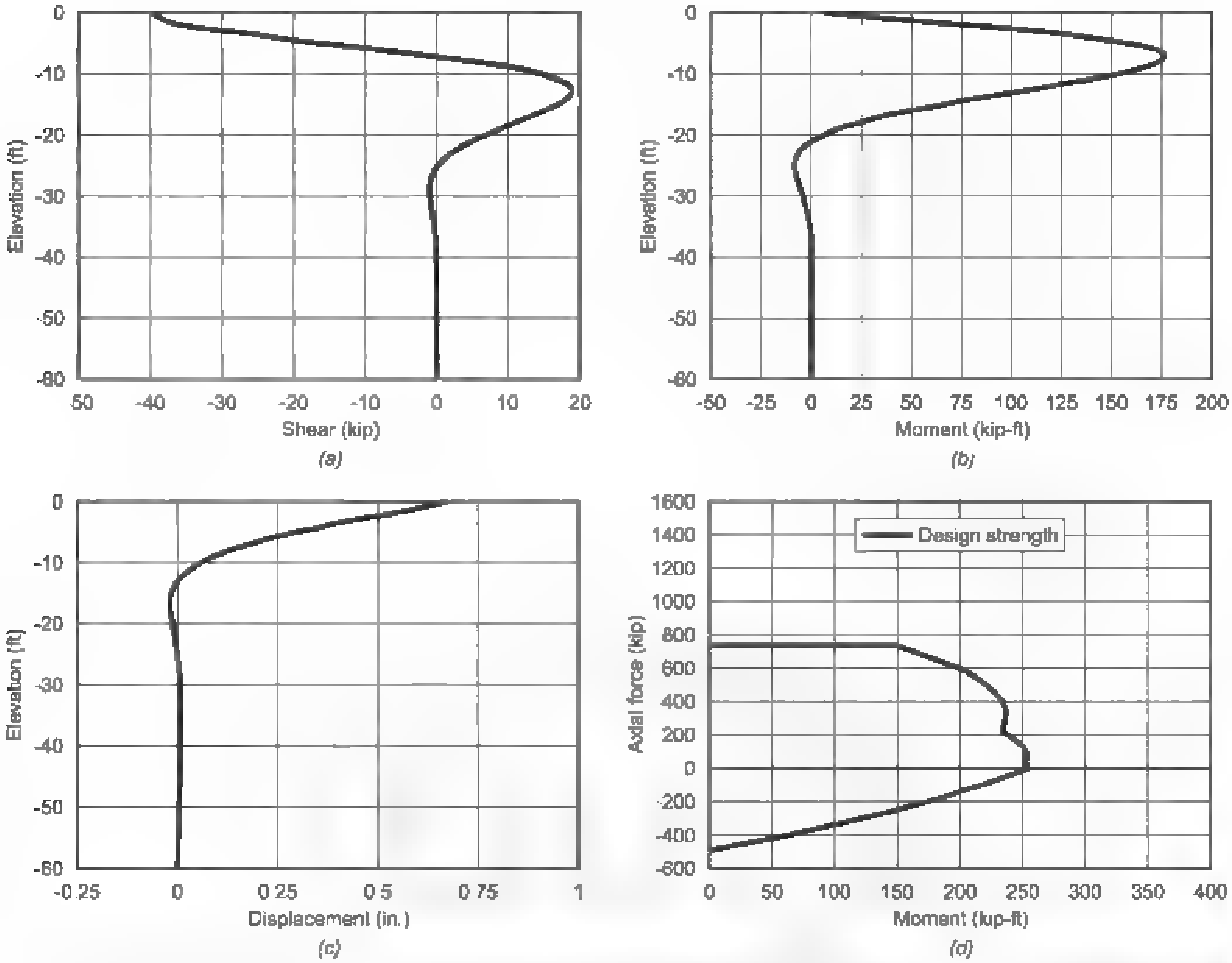


Fig. E6.5—Deep foundation analysis results from commercial software including (a) shear, (b) moment, (c) displacement, and (d) interaction diagram.

		Figure E6.5a shows the shear demand calculated using software. Figure E6.5b shows the maximum moment for the controlling strength load combination for comparison with the results using hand methods. The moment obtained from the hand method (170 kip-ft) and the moment computed using the software (176 kip-ft) exhibit agreement to within 5%. Figure E6.5c shows the maximum displacement for the controlling strength load combination for comparison with the values estimated above using hand methods. The displacement obtained from the hand method (0.74 in.) and the displacement computed using the software (0.70 in.) exhibit agreement to within approximately 5%. Figure E6.5d shows the strength interaction diagram for design (with application of strength reduction factors) for comparison with the diagram formed manually above. The design strength interaction diagram generated by the software shows good agreement with the manually generated interaction diagram. Minor differences are present near the "noses" of the interaction diagrams. Such differences are due to differences in the stress-strain curve assigned to the concrete portions of the pile in the software, as compared to the stress-block approach used for manual curve generation. Because similar demands were computed by the software (relative to available results from hand methods), and also, because the interaction diagrams (manual versus program-generated) are generally in good agreement, the software also produces demand-capacity ratios that agree with the available manual (or hand) methods.
Step 7: Concrete stresses at transfer		
24.5.3.2	Piles are concentrically prestressed. Consequently, no flexural tensile concrete stresses are developed during the process of releasing the prestressing strands. Piles must be lifted out of the bed, however, soon after release and will be subjected to handling stresses at this time (Fig. E6.6). Assume a two-point lifting scheme and check bending stresses against the allowable concrete strength at the time of prestress transfer using Code Table 24.5.3.2.	

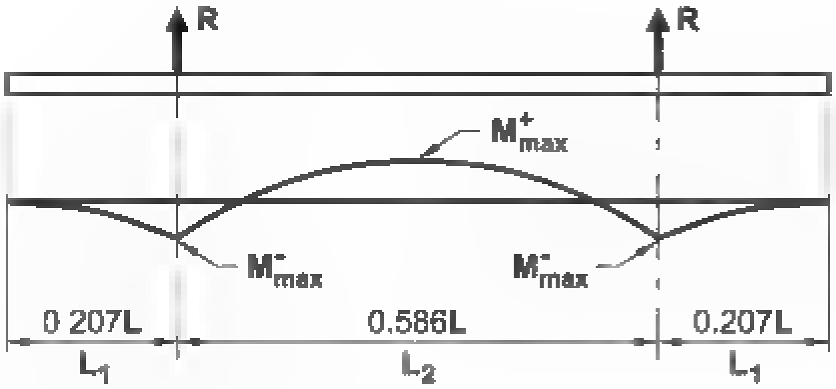


Fig. E6.6—Pile lifting locations to minimize bending moment.

$3\sqrt{3500}$ psi = 177 psi

$M_{max} = 1.5[(1.5\ \text{ft})^2(150\ \text{pcf})]0.5(0.207\ 60\ \text{ft})^2 = 39$ kip ft

Conservatively assume full losses for this check.

$P_e = 172$ ksi(12)(0.153 in.^{2}) = 316 kip

Effective precompression at time of prestress transfer

$f_{pc} = 316$ kip/(18 in.)^2 = 975 psi

Tensile stress due to lifting of pile

$$f = \frac{39\ \text{kip ft}}{\frac{(18\ \text{in.})^3}{6}} = 481\ \text{psi}$$

OK. Pile remains in compression during lifting

Step 8: Transverse reinforcement detailing

13.4.5.6 Confinement reinforcement must be included as shown in Code Table 13.4.5.6(b)

Table 13.4.5.6(b)—Maximum transverse reinforcement spacing

Reinforcement location in the pile	Maximum center-to-center spacing, in.
First five ties or spirals at each end of pile	1
24 in. from each end of pile	4
Remainder of pile	6

Step 9: Design sketch and discussion

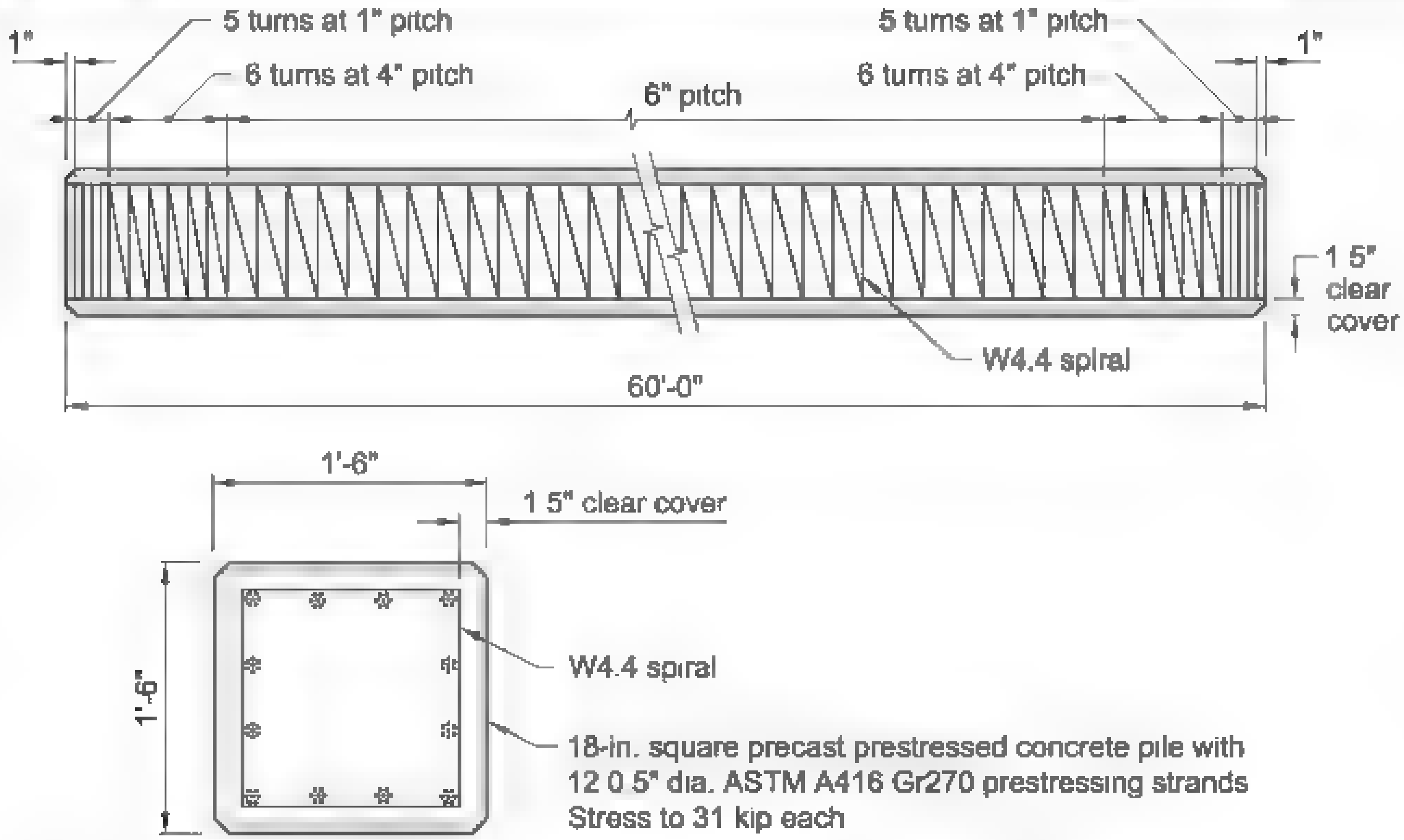


Fig. E6.7—Precast, prestressed pile details.

Figure E6.7 presents the design of an 18 in. square precast, prestressed concrete pile. The structure is categorized as SDC A with lateral load controlled by wind. Because the pile resists lateral load, strength design provisions were used to develop the interaction diagram. In addition, concrete service stress conditions were investigated.

Foundation Example 7 *Design of a circular uncased cast in place concrete augered pile under axial load only*

The pile is founded in loose sand and according to the parameters given in the geotechnical report, the pile is required to be 14 in. diameter and 60 ft long to resist the design axial loads. The building is assigned to Seismic Design Category (SDC) C, but wind and earthquake load effects on this pile are negligible

Given:

Material properties—

f_c' = 4000 psi Specified concrete compressive strength

f_y = 60 ksi Specified yield strength of reinforcement

Service loads—

Dead load 75 kip

Live load 45 kip

Pile geometry—

h_p 14 in. Diameter of pile

ACI 318	Procedure	Computation
Step 1: Check Code applicability		
1.4.7c	The Code applies to cast-in-place deep foundation members that are in SDC C, D, E, or F. Use Code Chapter 13 and Section 18.13 for design	
Step 2: Check strength using allowable axial strength approach (Code Section 13.4.2)		
13.4.2.1	The Code allows the use of allowable compressive strength approach where the deep foundation member is laterally supported for its entire height and the bending moments resulting from applied forces are small. Assume that these conditions are satisfied. Calculate the allowable strength and compare to the service load combinations from ASCE/SEI 7. Allowable pile compressive strengths are given in Code Table 13.4.2.1. These values represent an upper bound for well understood soil conditions and quality workmanship. These values should be reduced if soil conditions or workmanship, or both are anticipated to be less than ideal. This is particularly true for augered piles without casings where cross sectional area of the pile can vary depending on soil conditions and effectiveness of construction procedures. For this example use the allowable compressive strength equation given in the table for uncased augered pile.	Total service axial force. P_s = 75 kip + 45 kip = 120 kip Allowable compressive strength ignoring reinforcement $A_g = 0.25\pi(14\text{ in.})^2 = 154\text{ in.}^2$ $P_a = 0.3(4000\text{ psi})(154\text{ in.}^2) = 184.8\text{ kip}$ $> P_s$ **OK**

Foundations

Step 3. Check strength using strength design approach (13.4.3)		
13.4.3.1	The Code does not place any restrictions on the use of the strength design approach in Code Section 13.4.3 for deep foundation members.	
13.4.3.2	Strength design of deep foundation members should be in accordance with Code Section 10.5 using the compressive strength reduction factors given in Code Table 13.4.3.2. Section 10.5 requires the consideration of interaction between load effects such as moment and axial force. In this case, the pile is loaded axially with negligible flexure. Similar language is provided that cautions the designer to investigate the reliability of the construction workmanship and soil conditions of uncased piles. Further discussion and information on strength design of deep foundations is available in ACI 336.3 and 543R to assist the designer. For this example, use the value given in the table.	$\phi = 0.55$ $P_u = 1.2(75\text{ kip}) + 1.6(45\text{ kip}) = 162\text{ kip}$
10.5.2.1 22.4.2	Determine axial strength of deep foundation member using Code Section 10.5. Since no moment is expected to be applied, use the maximum axial compressive strength from Section 22.4.2. $P_o = 0.85f_c'(A_g - A_{st}) + f_yA_{st}$ (22.4.2.2)	$P_o = 0.85(4000\text{ psi})(154\text{ in.}^2) = 523.6\text{ kip}$ $P_{n,max} = 0.80(523.6\text{ kip}) = 418.9\text{ kip}$ $\phi P_{n,max} = 0.55(418.9\text{ kip}) = 230.4\text{ kip}$ $> P_u$ **OK**

Step 4: Determine seismic detailing requirements		
18.13.5.2	For SDC C and higher, reinforcement is required over the full pile length when tension loads are resisted.	No tension loads are resisted.
18.13.5.7.1	Based on Code Table 18.13.5.7.1, uncased CIP piles must have minimum reinforcement over the length specified in the table.	$0.0025(154\ \text{in.}^2) = 0.385\ \text{in.}^2$ Use four bars within circular ties. Use No. 6 bars to ensure stability of cage in the hole. $A_s = 4(0.44\ \text{in.}^2) = 1.76\ \text{in.}^2$ Longitudinal bars must extend over the longest of: 1/3 pile length = 60 ft/3 = 20 ft 10 ft 3 times the pile diameter = 14 in. × 3 = 3.5 ft flexural length of pile Since moment in the pile is negligible, extend reinforcement 20 ft into the pile from head. Transverse reinforcement must be provided for confinement over the length of three times the pile diameter = 14 in. × 3 = 3.5 ft from the bottom of the pile cap. Use No. 3 closed circular ties at a spacing of 6 in. or 8 × 0.75 in. = 6 in. Over the remaining length of reinforcement, provide No. 3 closed circular ties at a spacing of 16 × 0.75 in. = 12 in.

Foundations

18.13.5.4 25.3.4 25.7.2.4.1		Provide seismic hooks on circular ties. Terminate hooks of circular tie at adjacent longitudinal bars. Stagger overlaps around the perimeter of the pile.
25.4.2.1	Determine required development length of No. 6 bars in pile cap using simplified formulas from Table 25.4.2.3 for No. 6 bars and smaller and Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b.	d_b = 0.625 in. << clear cover $2d_b$ = 1.25 in. << clear bar spacing
25.4.2.3	$\ell_d \ge \left(\dfrac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f'_c}}\right) d_b$	
25.4.2.1(b)	$\ell_d \ge 12$ in.	$\lambda = 1.0$
25.4.2.5	ψ_t – Casting position factor ψ_e – Epoxy coating factor ψ_g – Reinforcement grade factor	Bars are oriented vertically $\psi_t = 1.0$ Bars are uncoated $\psi_e = 1.0$ Bars are Grade 60 $\psi_g = 1.0$ Required development length $\left(\dfrac{60{,}000\text{ psi}(1.0)(1.0)(1.0)}{25(1.0)\sqrt{4000\text{ psi}}}\right)(0.75\text{ in.}) = 28.5\text{ in.}$ Use 30 in.

Step 5. Design sketch and summary

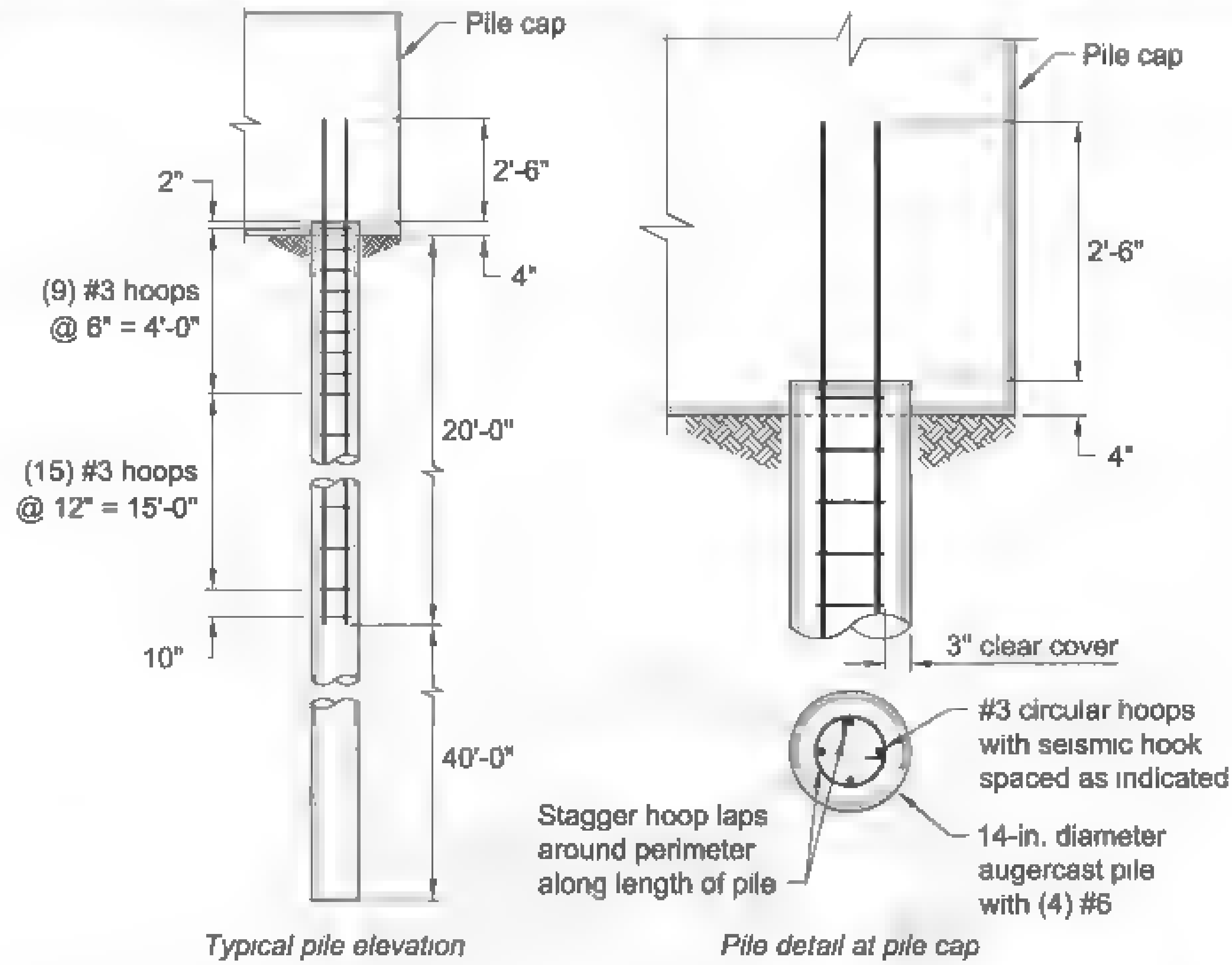


Fig. E7.1 Uncased, augered pile details

Figure E7.1 presents the design of an uncased augered concrete pile in a building assigned to SDC C. Although this pile carries only axial load, the detailing requirements of Code Chapter 18 require that both longitudinal and transverse reinforcement be included in the design for approximately 1/3 of the pile length.

Foundations

Foundation Example 8—*Design of a 24 in. diameter cast-in-place concrete drilled pile under combined axial and lateral loads*

The pile is 80 ft long, founded in loose sand, and subjected to combined axial and lateral loads at the pile head. The drilled pile is considered to exhibit fixed head conditions and must be detailed at the pile to cap connection to ensure such behavior. Pile is fitted with a permanent metal casing to provide hole stability; pile is reinforced with eight No. 10 mild steel bars. The building is assigned to Seismic Design Category (SDC) C. Find the maximum deflections and internal demands (versus capacity) first using classical pile analysis techniques and compare to analysis conducted with commercial pile analysis software.

Given:

Material properties—

f_c' = 4000 psi Specified concrete compressive strength

f_y = 60 ksi Specified yield strength of reinforcement

Loads—

Load combinations (Code Table 5.3.1)	P_u, kip	V_u, kip
$U = 1.2D + 1.6L + 0.5Lr$	400	0
$U = 1.2D + 1.0W + 1.0L + 0.5S$	320	60
$U = 1.2D + 1.0E + 1.0L + 0.2S$	320	30
$U = 0.9D + 1.0W$	205	60
$U = 0.9D + 1.0E$	205	30

Soil properties—

n_h = 30 lb/in.^{3} Modulus of subgrade reaction

Pile geometry—

h_p = 24 in. Diameter of pile

ACI 318	Procedure	Computation
Step 1: Check Code applicability		
1.4.7c	The Code applies to cast-in-place deep foundation members that are in SDC C, D, E, or F. Use Code Chapter 13 and Section 18.13 for design.	

Step 2: Check strength using strength design approach (13.4.3)		
13.4.3.1	Strength design approach is used for this design because the pile resists axial forces and flexure	
13.4.3.2	Strength design of deep foundation members should be in accordance with Code Section 10.5 using the compressive strength reduction factors given in Code Table 13.4.3.2. Section 10.5 requires the consideration of interaction between load effects such as moment and axial force. Before checking axial-flexure interaction, calculate the maximum design axial strength for this pile Use strength reduction factor considering this confinement from Code Table 13.4.3.2b Further discussion and information on strength design of deep foundations is available in ACI 336.3 and 543R to assist the designer	 $\phi = 0.60$
18.13.5.8.2	Pile casing is to be selected by contractor as necessary for hole stability and will not be considered in the design calculations other than the selection of clear cover requirements for reinforcement.	
10.5.2.1 22.4.2	Determine upper limit on the axial strength of the pile using Code Section 22.4.2 $P_o = 0.85f_c'(A_g - A_{st}) + f_yA_{st}$ (22.4.2.2)	 $A_g = 0.25\pi h_p^2 = 452\text{ in.}^2$ $A_{st} = 8(1.27\text{ in.}^2) = 10.16\text{ in.}^2$ $P_o = 0.85(4000\text{ psi})(452\text{ in.}^2 - 10.16\text{ in.}^2) + 60\text{ ksi}(10.16\text{ in.}^2) = 2111.91\text{ kip}$ $P_{n,max} = 0.80(2112\text{ kip}) = 1689.6\text{ kip}$ $\phi P_{n,max} = 0.60(1690\text{ kip}) = 1014\text{ kip}$ $> P_u = 400\text{ kip}$ **OK** Some strength remains to resist combined axial and flexure
Step 3: Minimum reinforcement		
18.13.5.7.1 10.7.3.1	Ignore pile casing to determine minimum reinforcement as given in Table 18.13.5.7.1 Determine minimum longitudinal reinforcement for pile If spiral transverse reinforcement is used, then 6 longitudinal bars are required. If circular ties are used, then four bars are required.	 $A_g = 0.25\pi h_p^2 = 452\text{ in.}^2$ $A_{min} = 0.0025(452\text{ in.}^2) = 1.13\text{ in.}^2$ $< A_{st} = 10.16\text{ in.}^2$ **OK** Eight No. 10 bars meets minimum

Step 4. Pile analysis for lateral load using Davisson (1970)

Determine the displacement at the pile head using Davisson (1970)

$$y = CQT^3/E_cI_{cr}$$

where y is the lateral displacement, C is the deflection coefficient (selected from Fig. E8.1), Q is the applied lateral load at the pile head, T is a measure of relative stiffness, which is a function of the pile stiffness (E_cI) and lateral stiffness of soil (n_h), E is the section elastic modulus, and I is the moment of inertia. Use a pile stiffness of one-half of the gross moment of inertia to account for cracking. One-half of the gross moment of inertia is assumed here for simplicity. In actual design, the appropriate cracked moment of inertia should be used. Also, here, T has units of in., and is expressed as

$$I_g = \frac{\pi(24\text{ in.})^4}{64} = 16,286\text{ in.}^4$$

$$I_{cr} = 0.5(16,286\text{ in.}^4) = 8143\text{ in.}^4$$

$$E_c = 57,000\sqrt{4000}\text{ psi} = 3605\text{ ksi}$$

$$T = (E_cI_{cr}/n_h)^{1/5}$$

$$T = \left[\frac{3605\text{ ksi}(8143\text{ in.}^4)}{30\text{ lb/in.}^3}\right]^{1/5} = 62.8\text{ in.}$$

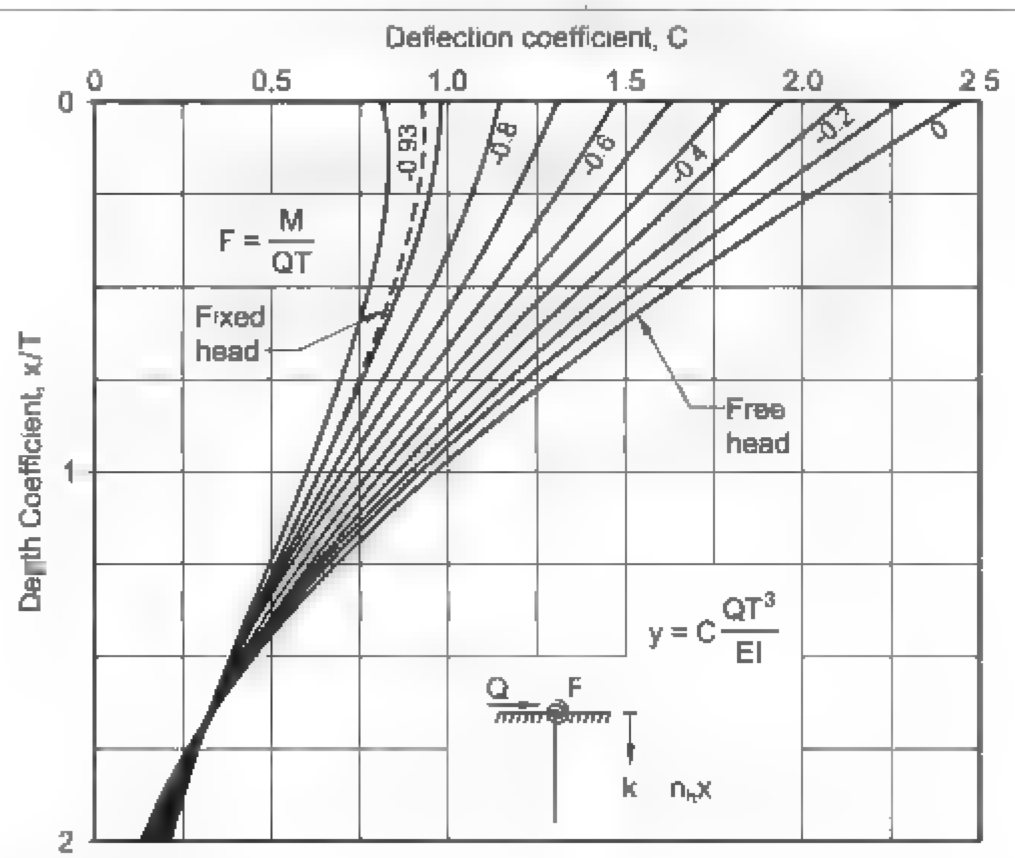


Fig. E8.1—Pile deflection coefficients.

Load combination 2 includes both axial load and lateral load. Consider the factored shear from load combination 2 because it will create the largest moment in the pile.

Selecting deflection coefficient from Fig. E8.1 for fixed-head restraint and maximum deflection at the pile head.

$C = 0.8$

Determine lateral displacement at pile head.

$$y = 0.8\frac{60\text{ kip}(62.8\text{ in.})^3}{3605\text{ ksi}(8143\text{ in.}^4)} = 0.4\text{ in.}$$

Determine maximum internal actions in pile using coefficients from Fig. E8.2 and the following equation.

$M = CQT$

where M is the pile internal moment, C is the moment coefficient, and Q is the applied lateral load at the pile head. For a fixed-head condition, the maximum moment occurs at the pile head.

The maximum moment coefficient from the figure is approximately

$C = 0.94$

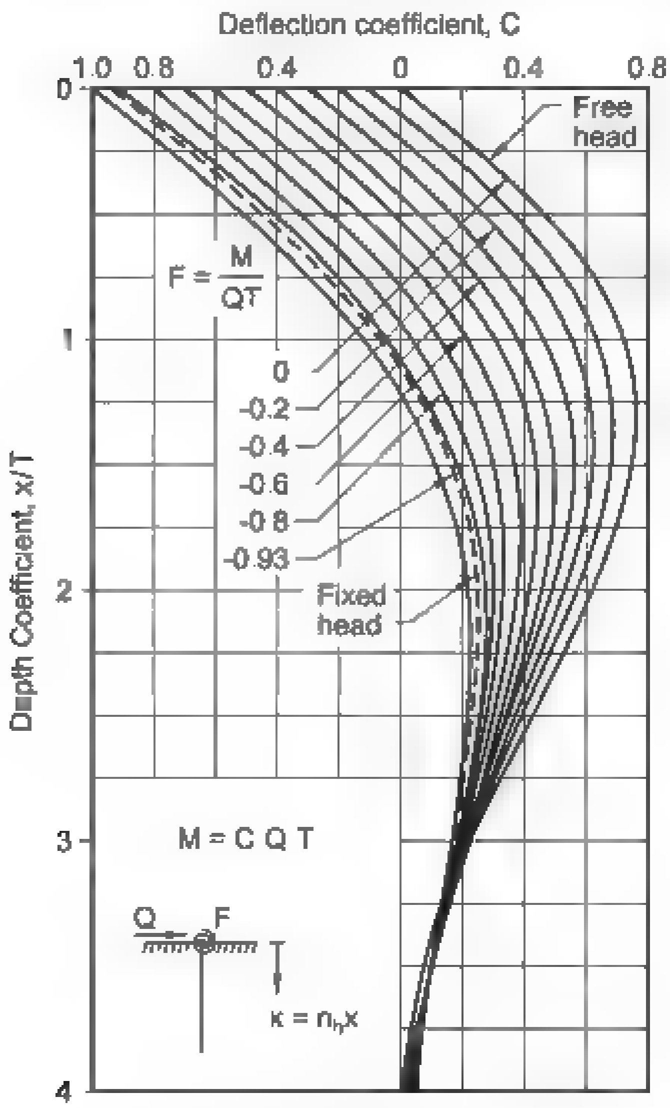


Fig. E8.2—Pile moment coefficients.

Therefore, maximum moment for load combination 2 is

$M_{2,max} = 0.94(60\text{ kip})(62.8\text{ in.}) = 295\text{ kip-ft}$

Step 5. Pile design strength		
20.5.1.3.4	Clear cover requirements for cased pile is given in Code Table 20.5.1.3.4	clear cover = 1.5 in.
	See Volume 3: Design Aids, Page 81 (IAD C4-60.9) for interaction diagram to determine reinforcement requirement. Pile dimensions and reinforcement position provides a slightly higher value of γ, which will yield conservative results.	$\gamma = \dfrac{24\text{ in.} - [2(1.5\text{ in.}) - 1.27\text{ in.}]}{24\text{ in.}} = 0.928$
	Calculate nondimensional parameters for use in the interaction diagram. Use strength reduction factor for ties (rather than spirals) as minimum (0.65).	
	$R_n = \dfrac{M_u/\phi}{f'_c A_g h_p}$ $K_n = \dfrac{P_u/\phi}{f'_c A_g}$	$R_n = \dfrac{\dfrac{295\text{ kip-ft}}{0.65}}{(4000\text{ psi})(452\text{ in.}^2)(24\text{ in.})} = 0.126$ $K_n = \dfrac{\dfrac{320\text{ kip}}{0.65}}{(4000\text{ psi})(452\text{ in.}^2)} = 0.272$ Use eight No. 10 bars evenly spaced around perimeter to give a reinforcement ratio of $\rho_g = \dfrac{8(1.27\text{ in.}^2)}{452\text{ in.}^2} = 0.0225$ Plot all load cases on interaction diagram (Fig. E8.3).

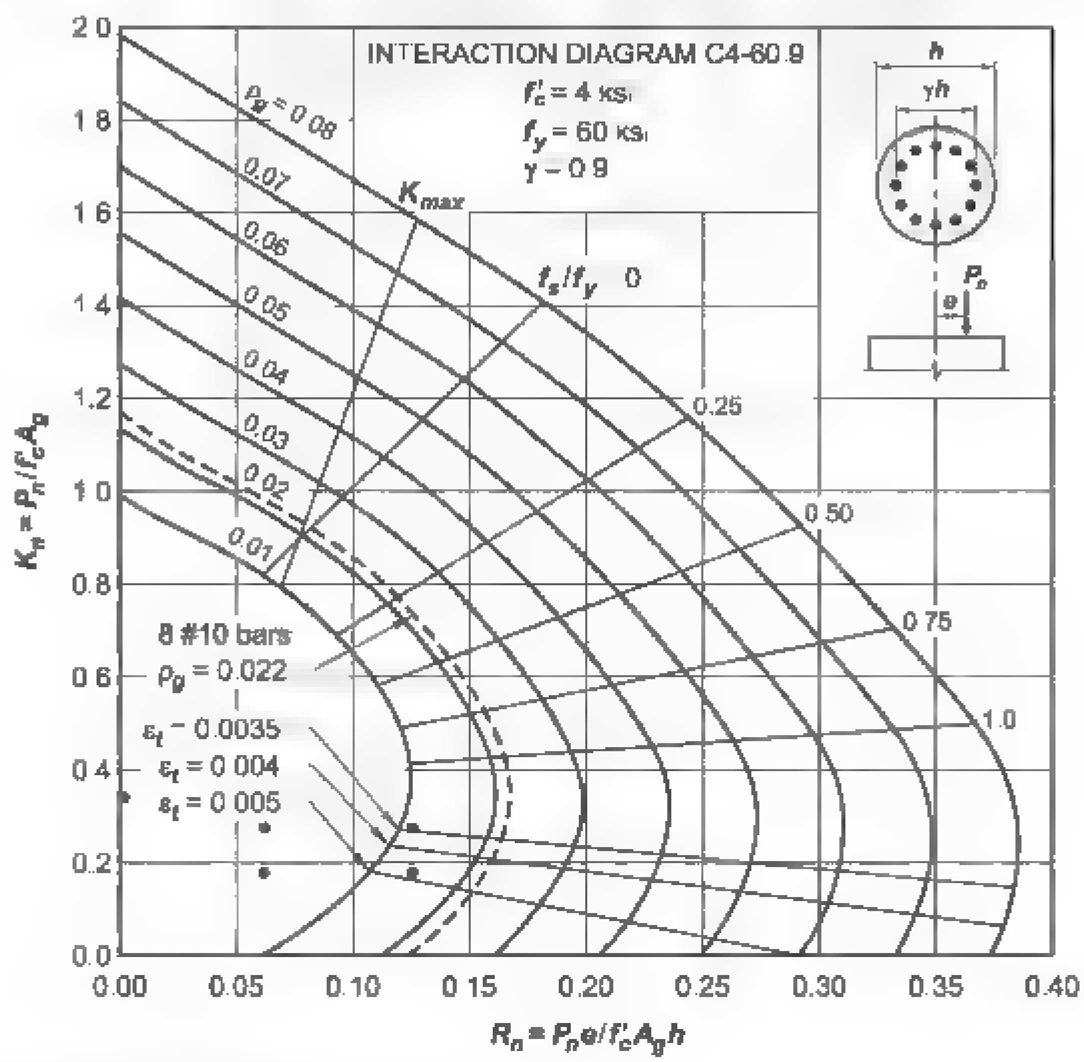


Fig. E8.3—Check interaction of axial force and moment on pile.

Step 6. Pile analysis for lateral load using software		
	The hand methods presented above allow for the engineer to estimate pile response to loading. However, methods such as "equivalent fixity" have shortcomings. As examples, material and geometric properties (modulus of elasticity, moments of inertia) of piles and soil must be assumed as linear; group effects are not readily taken into account. Software packages are available to model nonlinearity associated with soil-structure interaction and overcome the limitations of available hand methods. Results obtained from modeling and analysis of the controlling strength load combination, using the commercial software, are presented in Fig. E8.4. Profiles of shear, moment, and displacement are shown along with the program-generated (design) interaction diagram.	

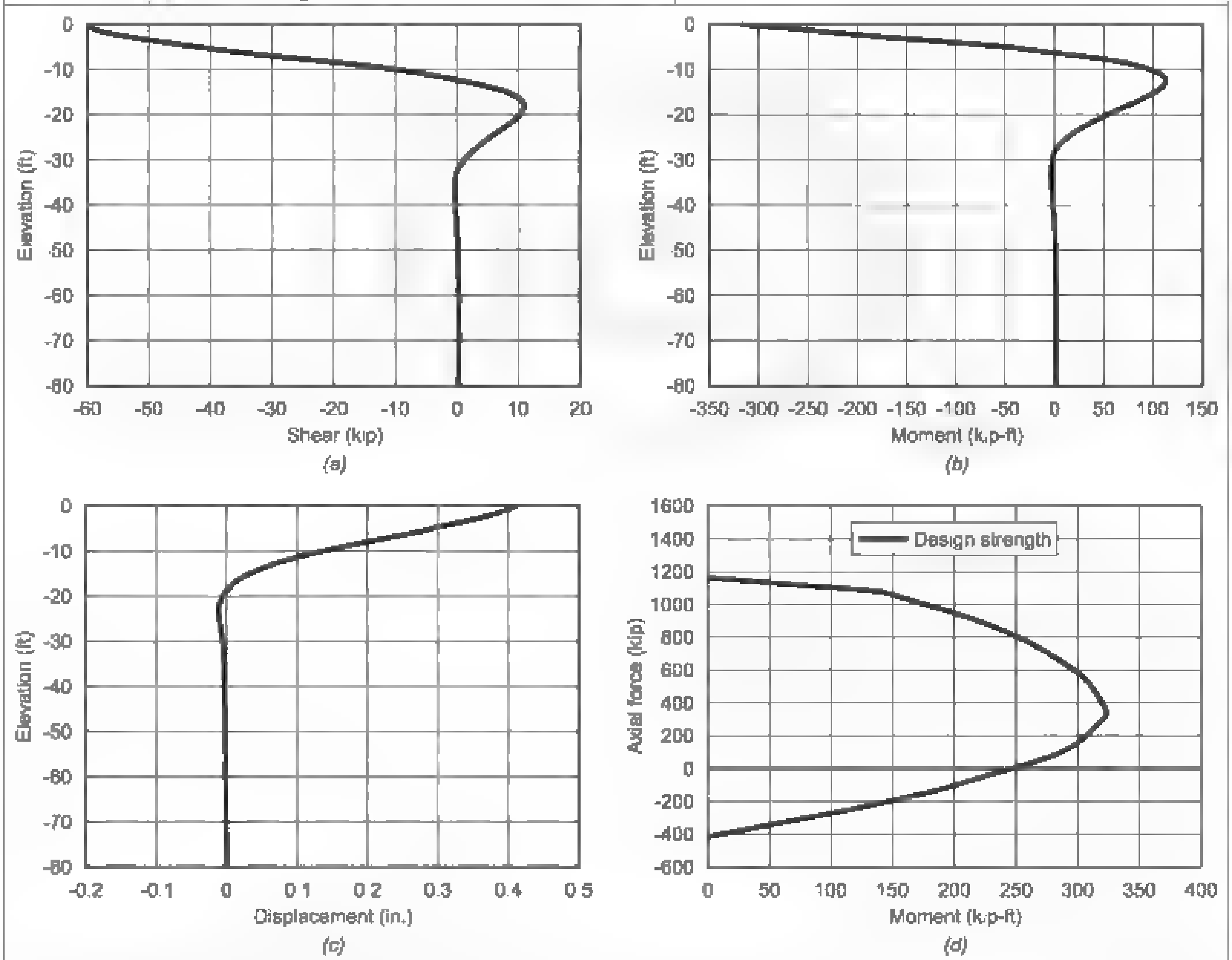


Fig. E8.4—Deep foundation analysis results from commercial software including: (a) shear; (b) moment; (c) displacement; and (d) interaction diagram.

Figure E8.4(a) shows the shear demand calculated using the software. Figure E8.4(b) shows the maximum moment for the controlling strength load combination for comparison with the estimated above using hand methods. The moment obtained from the hand method (295 kip-ft) and the maximum magnitude moment computed using the software (320 kip-ft) exhibit agreement to within 10%.

Figure E8.4(c) shows the maximum displacement for the controlling strength load combination for comparison with the values estimated above using hand methods. The displacement obtained from the hand mmethod (0.4 in.) and the displacement computed using the software (0.4 in.) exhibit agreement to within 1%.

Figure E8.4(d) shows the design strength interaction diagram (with application of resistance factors) for comparison with the design resources. The design strength interaction diagram generated by the software shows good agreement with the manually generated interaction diagram. Note that the software used a resistance factor of 0.65 for all points on the interaction diagram.

Because similar demands were computed by the software (relative to available results from hand methods), and also, because the interaction diagrams (design resource versus program generated) are in good agreement, the software also produces demand-capacity ratios that agree with the available manual (or hand) methods.

Step 7: Seismic detailing		
18.13.5.1	Detailing and other design requirements for uncased cast-in-place drilled or augered concrete piles according to Code Section 18.13.5.1a	
18.13.5.2	For structures in SDC C through F, pile reinforcement must be continuous over the length of the pile to resist tension forces. In this example, none of the load combinations result in axial tension	
18.13.5.3	Minimum longitudinal and transverse reinforcement must be extended over the entire unsupported length for portions of the member in air or water, or in soil that does not provide adequate support. Soil provides lateral support over entire pile length.	
18.13.5.4	Hoops, spirals, or ties that are used in this pile must terminate with seismic hooks	
18.13.5.5	In SDC D, E, or F or where located in Site Class E or F, piles must be detailed to accommodate potentially high flexural and shear demands at points of discontinuity. This includes elevations where soil properties change significantly. ASCE/SEI 7 defines the limits for this provision. This example is SDC C and Site Class C, so these provisions are not applicable	
18.13.5.7.1	Minimum longitudinal and transverse reinforcement requirements must meet those provisions for SDC C in Code Table 18.13.5.7.1. Minimum reinforcement was checked previously. Minimum length of longitudinal reinforcement should be the longest of (a) through (d)	
	(a) 1/3 pile length	$80\text{ ft}/3 = 26.7\text{ ft}$ **Controls**
	(b) 10 ft	10 ft
	(c) 3 times the pile diameter	$3(2\text{ ft}) = 6\text{ ft}$

Foundations

	(d) Flexural length of pile - distance from bottom of pile cap to where $0.4M_{cr}$ exceeds M_u	$M_{cr} = \dfrac{16{,}286\text{ in.}^4(7.5\sqrt{4000}\text{ psi})}{0.5(24\text{ in.})} = 53.6\text{ kip}\cdot\text{ft}$ $0.4(53.6\text{ kip}\cdot\text{ft}) = 21.4\text{ kip}\cdot\text{ft}$ From lateral load analysis on pile, the depth at which the factored moment is less than $0.4M_{cr}$ is approximately 19 ft. Determine coefficient for the cracking moment to be used in Fig. E8.2: $\dfrac{21.4\text{ kip}\cdot\text{ft}}{60\text{ kip}(62.8\text{ in.})} = 0.068$ Distance to $0.4M_{cr}$: $x = 3.5(62.8\text{ in.}) = 18.3\text{ ft}$ Provide reinforcement to a depth of 26.7 ft below bottom of pile cap.
18.13.5.7.1	Code Table 18.13.5.7.1 provides the required spacing and location of transverse reinforcement for SDC C:	
	For the transverse confinement reinforcement zone: Transverse reinforcement must be provided over the length of the reinforcement zone, which is equal to three times the pile diameter from the bottom of the pile cap.	Space closed ties over: 3(2 ft.) = 6 ft length from bottom of pile cap
	The transverse reinforcement in this zone must be closed ties or spirals with a diameter of no less than 3/8 in.	Use No. 4 ties to ensure a stable cage during placement
	Transverse reinforcement spacing must not exceed $8d_b$ or 6 in.	Tie spacing: 8(1.27 in.) = 10.1 in. 6 in. **Controls** Use No. 4 ties at 6 in. for a distance of 6 ft from bottom of pile cap
	For the remainder of the reinforced pile length: The transverse reinforcement in this zone must be closed ties or spirals with a diameter of no less than 3/8 in.	Continue use of No. 4 ties.
	Transverse reinforcement spacing must not exceed $16d_b$.	Tie spacing: 16(1.27 in.) = 20.3 in.
18.13.5.4	Provide seismic hooks on ties for structures in SDC C or above. See Code Section 2.3 for definition.	Use No. 4 ties at 20 in. for the reminder of the longitudinal reinforcement length

25.7.2.4.1	To avoid the possible loss of tie restraint, the ties should not be anchored at a single longitudinal bar. Code Section 25.7.2.4.1 requires the following conditions to be satisfied to ensure adequate tie anchorage: (a) Ends must overlap by at least 6 in. (b) Ends must terminate in a hook that engages a longitudinal bar. (c) Tie end laps must be staggered around the perimeter of the column.	Determine tie overlap: Tie length = π(24 in. −2(1.5 in. + 0.25 in.)) = 64.3 in. Longitudinal bar spacing around circumference of pile: 64.3 in./8 = 8.0 in. > 6 in. minimum overlap Overlap ties by engaging adjacent longitudinal bars.
25.4.3 25.4.3.1 25.4.3.2	Because a fixed-head condition is assumed at the pile cap, the longitudinal bars must be developed in the pile cap. Provide standard hooks embedded in pile cap. Determine required hook development length using the following equations: $\ell_{dh} \geq \left(\frac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}}\right) d_b^{1.5}$ $\ell_{dh} \geq 8d_b$ $\ell_{dh} \geq 6$ in. ψ_e – Coating factor ψ_r – Confining reinforcement factor ψ_o – Location factor ψ_c – Concrete compressive strength factor	Bars are uncoated. $\psi_e = 1.0$ No. 10 bar center-to-center spacing = ~9.5 in > $6d_b$ = ~7.6 in. $\psi_r = 1.0$ Bars meet side (center-to-center) cover requirements > $6d_b$ $\psi_o = 1.0$ Concrete strength less than 6000 psi. $\psi_c = \frac{4000}{15{,}000} + 0.6 = 0.867$ Required hook development length: $\frac{60{,}000\,\text{psi}(1.0)(1.0)(1.0)(0.867)}{55(1.0)\sqrt{4000}\,\text{psi}}(1.27)^{1.5} = 21.4$ in. Use 2 ft 0 in. hook length.

Step 8: Design sketch and summary

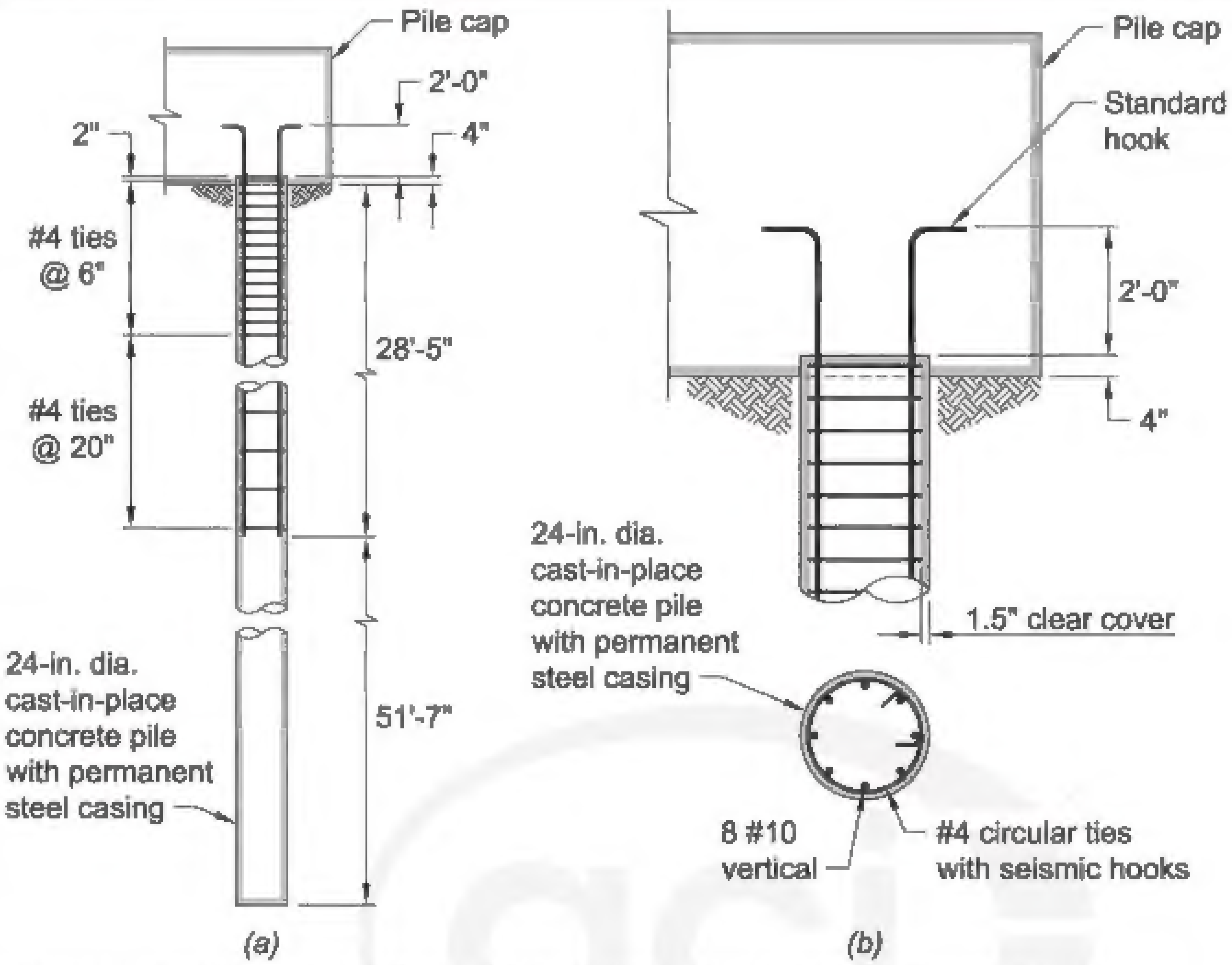


Fig. E8.5—Pile design details including (a) pile elevation and (b) pile reinforcing details.

Figure E8.5 presents the design of a concrete augered pile that is installed with a steel casing. The casing provides sufficient confinement to satisfy the seismic requirements without use of seismic detailing of the transverse reinforcement cage. Minimum longitudinal reinforcement requirements of Chapter 18 must be checked to ensure that sufficient reinforcement is included.

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